The antiscientifical revolution and mathematics *

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I shall start describing an example of a mathematical theory that is easily explained to non-mathematicians. Then, I shall discuss the aversion of the society to mathematics, ending with some remarks on the specific problems of Russian mathematicians.

I consider the first digit of the number representing the area of a country. This digit may be 1,2,...,9. The distribution of the countries of the world according to the first digit of the area figure is extremely nonuniform. The countries for which the first digit is 1 form about 30% of all the countries, and the number of those for which the first digit is 9 is approximately 6 times smaller, with a gradual decline in between.

This distribution does not depend on the area units: you might measure the areas in square kilometers, or square miles, or square inches and so on.

This nonuniform distribution of the first digits had been observed in many other cases and is known as the empirical Benford law.

For instance, the first digits of the populations of countries of the world behave similarly.

The contribution of mathematics to the explanation of these rather mysterious empirical phenomena depends on ideas from the ergodic theory of dynamical systems.

Consider the sequence of the first digits of the powers of 2:

$$1, 2, 4, 8, 1, 3, 6, 1, 2, 5, 1, 2, ...$$

The 1-s form about 30% of this sequence. The density of 9-s is approximately 6 times smaller.

These mathematical facts can be proved rigorously by the theory of dynamical systems.

Namely, consider a rotation of a circle at an angle which is incommensurable with 2π. Iterating the rotation, one gets from a point a sequence of points, its orbit under the dynamical system defined by the rotation. This sequence of points is equidistributed along the circle: the moving point spends in each domain the amount of time that is proportional to the measure of this domain (this theorem of H. Weyl was a predecessor of the general ergodic theorem of Birkhoff).

The application of this equidistribution theorem to the rotation at the angle 2π provides the strange distribution of the first digits of the numbers 2^n. Indeed, the first

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digit of a number depends only on the position of the fractional part of its logarithm (base 10) on the circle of the fractional parts.

Note that the fractional parts of the logarithms of numbers forming a geometrical progression (like 2^n) form an orbit of the corresponding circle rotation dynamical system. This orbit is equidistributed along the circle except the case when the rotation angle is commensurable with 2π (which corresponds to the progressions whose denominator is a rational power of ten). Hence one gets the same mysterious nonuniform distribution of the first digits for any generic geometrical progression.

This mathematical result provides some explanation for the distribution of the first digits of the world countries populations. According to the Malthus law, the population figures of a country in different years form a geometrical progression. Hence the first digits of these numbers verify the mysterious distribution with about 30% of 1-s.

According to the ergodic principle, one may replace the statistics of the time evolution of a single country by the space statistics of different countries populations evaluated simultaneously. Thus the distribution of the first digits of the populations of countries of the world should be the same as that of the first digits of the powers of 2.

To get the distribution of areas one should fix some model of the world partition evolution. In the simplest model, every country is subdivided into two equal areas parts (in unity of time) with the probability 50% and is united with another country of the same area with probability 50%.

For the simplistic model, one can rigorously prove that the mysterious nonuniform distribution of the first digits would occur after some time units. Conjecturally this theorem is also true for a large class of modified models. For instance, one might replace 50% by another subdivision probability, one could also make the parts unequal and one could in addition take into account the geographical distribution.

The computer experiments with these modified models were recently (1997) performed by M. Khesina (Toronto) and by F. Aicardi (Trieste). After few iterations one observes the mysterious distribution of the first digits of the areas. However, the corresponding limit theorem has still to be proved.

The flourishing of mathematics in the last century is at present threatened by the general trend: the suppression of the science and of scientific Al education, by both society and by governments. The situation is similar to that of the history of the Hellenistic culture, destroyed by the Roman Empire, which was only interested in final results, those useful for military applications, navigation and architecture.

The Americanization of the society in most countries, which one observes now, could lead to a similar annihilation of the science and the culture of the present day Humanity.

Liz studied the history of art at Harvard. At a lesson of French language she was asked whether she had visited France ("Yes"), Paris ("Yes"), whether she had seen the Notre Dame de Paris cathedral ("Yes") and whether she liked it ("No"). "Why?" — the question came. "It is so old" — Liz answered.

Mathematics is now, as it was 2000 years ago, the first candidate to be eliminated. The computer revolution allows one to replace the educated slaves by the ignorant ones. The governments of all countries have started to eliminate the mathematical sciences from the high school education curriculum.

The biology department of the University of Göttingen has asked that the mathematicians provide a course in number theory. The mathematicians, initially puzzled by this proposal, discovered that what the biologists wanted was to teach students the addition of fractions. Many of the Göttingen University's students prefer to add the nominators and the denominators of fractions separately, like the American students do: 1/3+1/2=2/5.

The Russian government is now attempting to reduce the high school mathematical education in Russia to the American standard.

The project is to reduce the time devoted to mathematics by a factor of two, using the hours saved to study the horse-breeding (for boys) and macrame (for girls).

The French Education, Science and Technology Ministry is proposing to reduce the mathematics high-school textbooks by a factor of three. Taking into account the explosive growth of all kinds of pseudosciences (like astrology) in many countries, one can expect the advent of a new medieval obscurantism in the coming century.

The present flourishing of the sciences might well be followed by an irreversible decline (similar to what had happened to the painting after the Italian High Renaissance).

Unfortunately, I cannot deny the culpability of the mathematical community in the present aversion of the society and of the governments toward mathematics and the mathematical education.

The human brain contains two lobes - the left and right ones. The left one is responsible for the languages, sequences of reasoning, intrigues and so on. The right lobe is governing the space orientation, the emotions and everything needed in real life.

A typical example of the left lobe hyperdevelopment is represented by the chess player character Luzhin from Nabokov's "Luzhin's defence". This illness, as indeed it is, is the strong quality of the left-brain-dominated persons. It is usually accompanied by an underdevelopment of the right brain and by the accompanying inferiority complex.

In the middle of the twentieth century a strong mafia of left-brained mathematicians succeeded in eliminating all geometry from the mathematical education (first in France and later in most other countries), replacing the study of all content in mathematics by the training in formal proofs and the manipulation of abstract notions. Of course, all the geometry, and, consequently, all relations with the real world and other sciences have been eliminated from the mathematics teaching.

Define the multiplication of natural numbers by the long multiplication rule. The commutativity of the multiplication ($ab = ba$) becomes then a difficult theorem, which one can however deduce logically from the definition. Forcing poor students to learn such proofs, the left-brained criminals had inevitably created the present negative opinion, of society and governments, of mathematics.

One can only understand the commutativity of the multiplication counting the soldiers by the rows and by the columns, or evaluating by two ways the area of the rectangle. All the attempts to avoid this intervention of the real world into mathematics is a sectarian approach that will be rejected by any reasonable person and will produce an aversion to mathematics, to multiplication and to all kinds of proofs. This "abstract" description of mathematics can be used neither for the teaching, nor for any practical purpose.

But the left-brained ill people have succeeded in breeding generations of mathematicians, who understand no other approach to mathematics and are able only to continue to teach it the same way. The aversion to mathematics of the ministers who have suffered through the humiliating teaching of this type in high school is a normal and healthy reaction.

Unfortunately, their aversion to mathematics is acting undiscriminantly on all of it and can kill it completely. One of the dangerous trends is to eliminate the proofs from the high-school mathematics.

The role of the proof for mathematics is similar to that of orthography or even calligraphy for poetry. A person, who had not mastered the art of the proofs in high school, is as a rule unable to distinguish correct reasoning from that which is misleading. Such people can be easily manipulated by the irresponsible politicians.

Mass hypnosis and the disastrous social events may result.

L. Tolstoy observed that the strength of a government depends on the people's ignorance. Moreover, he said, the government is aware of this and would therefore always fight against the people's education.

I think however that the complete destruction of mathematics and of mathematical education would be a mistake similar to the Galileo persecution.

One French editor (who had once organized the edition of the superabstract nonsense and had thus contributed to the present disaster) invited me recently to discuss the situation and presented me his younger assistant lady as having philosophical education. Trying to be gallant, I have immediately mentioned that according to my experience the philosophers are the most ignorant people. I have quoted the statement by a French philosopher of the XIX-th century that I had read in the "Dictionnaire de la bêtise" ("Foolishness dictionary"): the Roman Catholic church had make a mistake, burning Galileo.

"What is foolish here?" - the lady replied. "I also agree that it had been a mistake to burn him."

My reaction provoked her correction: "Of course, I meant Tycho Brahe".

Telling this story at the high tables of three Cambridge colleagues, I have observed that the name of Giordano Bruno (whose monument one can see at the Campo di Fiori in Rome) is only known to the Russians.

By the way, Bruno is not yet rehabilitated (in contrast to Galileo who was finally rehabilitated in 1992).

Up until now, the destruction of culture, science and education (in particular of mathematics and mathematical education) in Russia has been progressing more slowly than in more civilized countries.

The present shameful discrimination against Russian (as well as Indian, Chinese etc) scientists by the western scientific community is evidently harmful for world science. Before the fall of communism we had not be allowed to leave Russia by the communists. At present the door is closed from the other side, by the system of useless "visas" which were not needed in the XIXth century and which are not needed for Americans and other "genuinely white" people.

The English Consulate (at Paris) recently gave me the list of their requirements to get a visa to visit Cambridge and Oxford for a few days. Among several dozen other requirements, they have asked me to provide a copy of the British passport of the inviting person as well as the information on the religious background of the priest signing my marriage certificate.

A hundred years ago mathematicians were able to travel from one country to another with no visas and no humiliations from consulates. At present this is only possible to those born in some selected countries. Russians, Asians and Africans, among others, are undesirable. The Euroamerican idea of human rights is the idea of the human rights of Euroamerican people.

An interesting modern version of slave trade appeared recently. My friends working in biology, chemistry and physics have told me, that the American and European Universities are inviting Russian researchers, paying them a small salary (which is however many times higher then the Russian salaries, which were in July 1998 at the level of one hundred dollars a month and are now probably several times less, the food prices at Moscow being essentially the same as at Paris). These Russian slaves are working hard, but the publications are signed not by them but by the researchers of the inviting laboratory.

The technology of the appropriation of the results of Russian mathematicians is different, but the result is the same: they are mostly attributed to the occidental epigons.

No Russians were plenary speakers at the last International Congress of Mathematicians (Berlin, August 1998). Most papers by the Russian mathematicians were not included into the Abstracts due to the inability of the authors to send money from Russia to the organizers. Such discrimination had never happened even during the worst times of the cold war.

I think, however, that in spite of all these discriminatory measures, Russia will finally reach the level of Europe and even of America, whence the knowledge of mathematics and of the history of Giordano Bruno in Russia would attain the Euroamerican standards.