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ABSTRACTS

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ADDRESS:

Prof. V.M.Babich
Mathematical Institute,
Fontanka quay 27, St.Petersburg 191011
Russia
e-mail: `babich@pdmi.ras.ru`

Prof. V.S.Buldyrev
Institute in Physics, St.Petersburg University
1 Ulianovskaja Ave., Petrodvoretz 198504 Russia
e-mail: `buld@math.nw.ru`
Fax: 7 (812) 428-7240

Dr. V.E.Grikurov	e-mail: `grikurov@math.nw.ru`
Dr. I.V.Andronov	e-mail: `iva@aa2628.spb.edu`
Web site	`http://math.nw.ru/DD`

FOREWORD

The Seminars "Day on Diffraction" are annually held since 1968 in late May or in June by the Faculty of Physics of St.Petersburg State University, St.Petersburg Branch of the Steklov's Mathematical Institute and Euler International Mathematical Institute of the Russian Academy of Sciences.

This booklet contains the abstracts of talks to be presented at oral and poster sessions in 4 days of the Seminar. Author index can be found on the last page.

The full texts of selected talks will be published in the Proceedings of the Seminar. The texts in LaTeX format are due by September 15, 2005 to e-mail `iva@aa2628.spb.edu`. Format file and instructions can be found on the Seminar Web site at `http:/math.nw.ru/DD`. The final judgement on accepting the paper for the Proceedings will be made by the Organizing Committee following the recommendations of the referees.

We are as always pleased to see in St.Petersburg active researchers in the field of Diffraction Theory from all over the world.

Organizing Committee

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75 years to V. M. Babich

On June 13, 2005 the family, friends and colleagues celebrated the 75-th anniversary of Professor Vassilii Mikhailovich Babich. Professor V. M. Babich is an undisputed leader of the St. Petersburg Diffraction School. He is worldwide recognized for his outstanding contributions to areas as diverse as the diffraction and propagation of waves, elasticity, geophysics and partial differential equations. He is an author of about 175 scientific papers and of 6 monographs of which *Boundary Layer Methods in Diffraction Problems* (with N. Ya. Kirpichnikova) Springer 1979, *Short-Wavelength Diffraction Theory* (with V. S. Buldyrev) Springer 1991 are available in English. V. M. Babich is a member of the Editorial Boards of *Wave Motion*, of *Integral Transforms and Special Functions* and of *St. Petersburg Mathematical Journal* and the Editor of the series "Mathematical Problems of Wave Propagation Theory" in *Zapiski Nauch. Semin. POMI (Proceedings of Seminars at St. Petersburg Branch of the Steklov Mathematical Institute)* translated to English in *Journal of Mathematical Sciences.* He is a winner of the Soviet State Prize (1982) and of the V. A. Fock's Prize of Russian Academy of Sciences (1998).

From 1967 Professor Babich has been the Head of the Laboratory of Mathematical Problems in Geophysics at St.Petersburg Branch of the Steklov Mathematical Institute and a Professor of the Department of Mathematical Physics at St. Petersburg State University. During all these years he has been having an amazing research productivity, which he manages successfully combining with raising young researchers, teaching and running a weekly St. Petersburg Diffraction Seminar. He approached his anniversary travelling a lot, being as always productive, energetic, full of ideas and plans. His current research activity includes elastic and electromagnetic waves guided by curved structures, localised wave propagation and diffraction by cones and elastic wedges.

Despite his intensive schedule, being also a head of a large family having four grandchildren, he at the same time remains easily accessible and always finds time and enjoys socializing with his numerous friends and colleagues.

On behalf of the participants of the DD'2005 and of all his colleagues working in the diffraction theory we wish him good health and continued productivity. We are all looking forward to him continuing inspiring us by his new ideas and remaining the leader of St. Petersburg diffraction.

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Contact interaction of the Pauli electron with a plane

Mikhail A. Antonets
Nizhny Novgorod agency for high technology,
Nizhny Novgorod, Russia
e-mail: ama@nant.ru

Some selfadjoint extensions of the Pauli operator for an electron in a uniform magnetic field are considered. This extension correspond to contact interaction of the Pauli electron with a general plane. Asymptotic behavior of negative eigenvalues is studied for large values of the coupling constant.

This work is supported by Russian Foundation for Basic Research, Grant 05-01-00299.

Closed Extensions of the Maxwell Operator for Impedance Boundary Condition

Mikhail A. Antonets, Ludmila V. Ponomareva
Nizhny Novgorod agency for high technology,
Nizhny Novgorod, Russia
e-mail: ama@nant.ru

Boundary value problem for Maxwell's equations with impedance boundary condition on a smooth surface S leads to constructing a nonsimmetric (in general case) extension in the space $L_2^6(R^3)$ for the Maxwell operator $\hat{A}_0^S$ defined on a subspace of pairs of linear differential forms $\{\omega_{\mathcal{E}}, \omega_{\mathcal{H}}\}$ on the space R^3 vanishing near the boundary surface S.

The von Neumann theory being the base for selfadjoint extension construction for simmetric operators has been used here for constructing a nonselfadjoint closed extension, corresponding to impedance boundary value condition. Deficiency spaces of the operator $\hat{A}_0^S$ are shown to be isomorphic to the space $\mathfrak{L}(S)$ consisting of pairs of linear differential forms distributions on the boundary surface S. So the problem of constructing the operator connecting deficiency spaces and defining the above mentionned closed extensions is reduced to the inversion problem for a pseudodifferential operator acting in the space $\mathfrak{L}(S)$ and depending on the impedance operator acting on a space of linear differential forms distributions on the boundary surface S.

A description of domain of this closed extension is done using a linear conditions on restrictions of linear differential form distributions $\{\omega_{\mathcal{E}}, \omega_{\mathcal{H}}\}$ on the surface S as well.

This work is supported by Russian Foundation for Basic Research, Grant 05-01-00299.

Applications of fractional derivative analysis to some electromagnetic problems

Barbara Atamaniuk, Andrzej J. Turski
Institute of Fundamental Technological Research, PAS, Warsaw, Poland

The mathematical theory of the fractional calculus and the theory of fractional ODE and PDE is well developed and there is a vast literature on the subject, e.g. see; [1], [2], [3] and [4]. The theory of PDE equations is a recently investigated problem and the theory mainly concerns fractional diffusion-wave equations, e.g. see [5], [6], [7],[8] and [9].

The main objectives of this paper is a presentation of main rules of fractional calculus, Riess potential and fractional Laplacians, factorization of the Helmholtz equation to obtain four pairs of fractional eigenfunctions allowing to construct a solution to the well known half-plane diffraction problems. Factorizing the Leontovich-Fock equation, we determine semi-differential Green functions, which allow us to find paraxial solutions for given beam boundary conditions.

Factorizing Helmholtz equation and PWE, we obtain fractional eigenfunctions for diffraction problems (edge waves) and beam propagations.

1. An eigenfunction $y(\xi)$ of the linear operator $L[y(\xi)]$ is such a function that the repeated operations preserve the function, e.g. $L[y(\xi)] = Cy$ with the exactness to a multiplicative constant C. In the case of fractional operation, the definition is extended to preservation of the function but an additive constant or a term of power of ξ, e.g. $(\pi\xi)^{-1/2}$, is subtracted at each step.

2. Factorizing the 2-D Helmholtz equation: $\Delta_{x,y}\Phi(x,y) + k^2\Phi(x,y,) = 0$, we obtain the following pairs of eigenfunctions: and the fractional eigenfunctions: $\sin(ax \pm by)$, $\cos(ax \pm by)$ and fractional eigenfunctions:

$$\int_{\sqrt{kr+ax\pm by}}^{\infty} \sin\left(ax + by \pm t^2\right) dt, \qquad \int_{\sqrt{kr+ax\pm by}}^{\infty} \cos\left(ax + by \pm t^2\right) dt,$$

where $k^2 = a^2 + b^2$ and $r^2 = x^2 + y^2$. The fractional eigenfunctions represent "edge waves", which are related to half-plane and can be used to solve diffraction problems.

3. Factorizing PWE: $2ik\partial_z u(x,y,z) + \Delta_{x,y}u(x,y,z) = 0$, we derive the following Green functions:

$$G(x,y,z) = \frac{i}{\pi}\left(\int_{x\sqrt{\frac{k}{z}}}^{\infty} e^{it^2} dt\right)\left(\int_{y\sqrt{\frac{k}{z}}}^{\infty} e^{it^2} dt\right), \quad G(x,y,z) = \frac{k}{\pi}\int_{\frac{x+y}{2}\sqrt{\frac{k}{z}}}^{\infty} e^{it^2} dt.$$

The derivatives $\partial_x^m \partial_y^n$ of $G(x,y,z)$, where m and n are natural numbers, satisfying the PWE and may be related to higher order Gaussian-Hermite optical beams. By use of the fractional Green functions, we may obtain known and new solutions for optical beams.

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New methods in quasiphoton theory

V. M. Babich

PDMI, St.Petersburg, Russia

Formal power series technique gives the possibility to develop some version of ray method in the case of complex eiconal. This "ray expansions with complex eiconal" contain in particular series, which can describe quasiphotons. Thus the constructing of quasiphoton expansion (if the classical ray expansion of the class of waves under consideration is known) becomes almost algorithmic.

Conditions of existence of the wave along the stiffener.

A. Badanin, A. Sakharova

Arkhangelsk State Technical University, Russia

We consider oscillations of the acoustic medium in the half space, bounded by the thin elastic plate, backed by the thin straight-line homogeneous stiffener. Conditions of the existence of the wave along the stiffener are considered. We seek the solution of the problem in the form of the decomposition on plane waves. Substituting this decomposition into the boundary-contact conditions we obtain the solution of the problem. Then we find the parameters of the liquid, the stiffener and the plate, when the surface waves propagate along the stiffener (the stiffener waves). The first terms in the asymptotics of the corresponding conditions, as the density of the acoustic medium is close to zero, have the following form. Let M be the surface density, D be the cylindrical rigidity of the plate, ω be the frequency of the wave field, ρ_p be the density of the stiffener, b_p be the thickness, H_p be the height of the stiffener, E_p be Young's modulus, I_p be the moment of inertia and K_p be the moment of rotate of the stiffener. Then if $\frac{M}{D} < \frac{\rho_p\, b_p\, H_p}{E_p\, I_p}$, then there exist the symmetric stiffener waves , if $\frac{M}{D} \geq \frac{\rho_p\, b_p\, H_p}{E_p\, I_p}$, then such waves do not exist. Moreover, if $\omega > \sqrt{\frac{M}{D}\,\frac{K_p}{\rho_p\, I_p}}$ then there exist the antisymmetric stiffener waves, if $\omega \leq \sqrt{\frac{M}{D}\,\frac{K_p}{\rho_p\, I_p}}$ then such waves do not exist.

Spectral properties of the fourth order operator with the periodic δ-potential

A. Badanin

Arkhangelsk State Technical University, Russia

We consider the spectral properties of the operator $Ty = y'''' + \sum \delta(x-n)y$, acting in $L^2(\mathbf{R})$. It is well known that the spectrum of T consists of the non-degenerated intervals, bands, separated by the gaps. We construct the Lyapunov function for T, which is the analytic function on the two-sheeted surface. The properties of this function are similar to the properties of the Lyapunov function for the Hill operator. We prove that the spectral bands of T have the multiplicity 2 or 4. Moreover, we prove that the Lyapunov function has real and non-real branch points.

This is a joint work with E.Korotyaev from Humboldt Universität zu Berlin.

Excitation of semi-infinite electromagnetic crystal by plane electromagnetic wave

P.A. Belov

Samsung Electronics Co., LTD., Telecomm.Network Business, Mobile Communication Div., #94-1, Imsoo-Dong, Gumi-City, Gyeong-Buk, 730-350, South Korea
e-mail: belov@rain.ifmo.ru

C.R. Simovski

St. Petersburg State University of Information Technologies, Mechanics and Optics, Sablinskaya 14, 197101, St. Petersburg, Russia
e-mail: simovsky@phd.ifmo.ru

An analytical theory describing excitation of a semi-infinite electromagnetic crystal by plane electromagnetic wave is presented in this paper. A three-dimensional crystal with orthorhombic elementary cell, formed by scatterers which can be substituted by point dipoles with known polarizability and fixed orientation is considered. The closed-form analytical formulae for the amplitudes of excited modes and Floquet harmonics of scattered field for the semi-infinite electromagnetic crystal are derived by solv ing a dispersion equation for the infinite electromagnetic crystal. Generalized Ewald-Oseen extinction principle for electromagnetic crystals under consideration has been formulated.

Electromagnetic crystals are usually studied with the help of numerical methods [1]. Analytical models exist only for the very narrow class of electromagnetic crystals. There are only few geometries which can be solved strictly analytically as well as some types of electromagnetic crystals which can be studied analytically only under a certain approximation. In this paper an analytical method is suggested, which allows studying dispersion and reflection properties of three-dimensional crystals formed by point scatterers with known polarizabilites using the local field approximation [2, 3, 4]. The first attempt to achieve this goal has been made by Mahan and Obermair in [2]. Analytical expressions for reflection coefficients and amplitudes of excited modes for a semi-infinite crystal were obtained in terms of wave vectors of the infinite crystal eigenmodes. That is the unique rigorous solution at the present time. Mahan and Obermair have considered only the normal incidence and have taken into account only the fundamental Floquet harmonic interaction between layers. Moreover, the method suggested in [2] is based on introduction of fictitious zero polariz ations at the imaginary crystallographic planes in free space over the semi-infinite crystal. As shown below such approach appears to be not a strict one.

In this paper we follow to the main idea of [2] to express amplitudes of excited modes and scattered Floquet harmonics in terms of wave vectors of the infinite crystal eigenmodes. In order to describe the interaction between layers we consider oblique incidence and take into account all Floquet harmonics. In the current study equations for real (non-fictitious) layers are derived and as an intermediate step a generalized Ewald-Oseen extinction theorem for electromagnetic crystals under consideration is proved. As a result a linear system of equations for amplitudes of excited eigenmodes has been derived. The obtained system is solved analytically with the help of an original method of function recovery based on its zeros, poles and a value at a definite point. Solving that system makes possible to determine amplitudes of all excited eigenmodes and all scattered Floquet harmonics.

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Far field of a point source acting on a half-space covered by an inhomogeneous layer

Anna A. Klimova

Instructional Center, University of Washington, Seattle, USA

Aleksei P. Kiselev

St.Petersburg Branch of the Steklov Mathematical Institute, Russia

We consider two-dimensional, time harmonic acoustic wave field in layered half-space $z > 0$, which is homogeneous for $z > h, h > 0$, described by the equation:

$$(\mu(z)u_z)_z + \mu(z)u_{xx} + \rho(z)\omega^2 u = 0, \quad z > 0.$$

Here the function μ is equal to $\mu(z) = \mu_j$ for $z_{j-1} < z < z_j, \quad k = 1, ..., n$ (with $z_0 = 0$) and $\mu(z) = \mu_\infty$ for $z > z_n$. Also $\rho(z) = \rho_j$ for $z_{j-1} < z < z_j, \quad k = 1, ..., n$ (with $z_0 = 0$) and $\rho(z) = \rho_\infty$ for $z > z_n$. The standard self adjoint matching conditions at points where μ jumps are satisfied:

$$[u]|_{z=z_k} = 0, \quad [\mu u_z]|_{z=z_k} = 0, \quad k = 1, \ldots, n.$$

The wavefield is generated by a point source is acting on the surface $z = 0$, and the radiation condition is assumed at infinity.

We are seeking the solution in the form of Fourier integral:

$$u(x, z) = \int_{-\infty}^{+\infty} \hat{u}(z, \xi) \exp(i\xi x) d\xi.$$

We are interested in the far-field asymptotics of u, which can be found, as we show, by the stationary phase method. The main result is that *for $z > h$, $\hat{u}(z, \xi)$ has no branch cuts, which implies that no head waves propagate in the homogeneous half-space $z > h$.*

Fronts and profiles of the waves in 2-D inhomogeneous dispersionless media created by localized sources

S. Yu. Dobrokhotov

Institute for Problems in Mechanics RAS, Moscow
e-mail: `dobr@ipmnet.ru`

We consider the Cauchy problem $u|_{t=0} = u_0(x/\varepsilon), u_t|_{t=0} = 0$ for the wave equation equation $u_{tt} = \nabla C^2(x) \nabla u, \quad x \in \mathbb{R}^2$, with the smooth positive varying velocity $C(x)$.

The asymptotic solution of this problem for $\varepsilon << 1$ was obtained in the papers [1],[2]. The corresponding formulas were based on the construction of the parametrix, expansion with respect to smoothness and the solution of some additional minimization problem. We show that the last problem can be solved explicitly which allows one to simplify final asymptotic formulas crucially. We discuss the geometrical objects appearing in this construction and the problem of a "visualization" of asymptotics.

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Reflection and transmission of water waves over an uneven ocean bed

Swaroop Nandan Bora

Department of Mathematics, Indian Institute of Technology Guwahati,
Guwahati 781 039, India
e-mail: swaroop@iitg.ernet.in

The problem of free surface flow over an obstacle or a geometrical disturbance at the bottom of an ocean are important for their possible applications in the areas of coastal and marine engineering, and as such these were being studied by the researchers even since the last century. The two dimensional scattering of water waves by a finite number of periodic ripples or sandbars on an otherwise horizontal flat bed has been a subject of investigation for many years. The problem of reflection of surface waves by patches of large bottom undulations has received an increasing amount of attention recently, as its mechanism is important in the development of shore-parallel bars. In the classical work of Lamb, the free surface elevation for the two dimensional problem of steady flow over the bottom irregularities was obtained assuming the irrotational motion.

This paper is concerned with the scattering of a train of progressive waves by a small deformation of the bottom of a laterally unbounded ocean using two dimensional linear water wave theory. Assuming the irrotational motion, a simplified perturbation analysis is employed to obtain the first order corrections to the velocity potential by using the Green's integral theorem in a suitable manner and the reflection and transmission coefficients, in terms of integrals involving the shape of the function $c(x)$ representing the bottom deformation. Three particular forms of the shape function are considered and the integrals for reflection and transmission coefficients are evaluated for these three different shape functions. Among these cases for the particular case of a patch of sinusoidal ripples at the bottom, the reflection coefficient up to the first order is found to be an oscillatory function of β which is twice the ratio of the wave numbers $\mathbf{k}$ and λ. When this ratio becomes 0.5 that means when $\beta = \mathbf{1}$, the reflection coefficient becomes a multiple of the number of ripples and high reflection of the incident wave energy occurs if this number is large.

Waves produced by a source on the moving and expanding circular frame

Victor V. Borisov

Fock Institute of Physics, St.-Petersburg University, Russia

The goal of the present report is to derive the transient solution to the inhomogeneous wave equation that describe wave perturbations formed by a source distributions on the special circular frame expanding and moving with the wave front velocity (for electromagnetic waves, the velocity of light). Starting from Smirnov's method of incomplete separation of variables [1] we represent the wave functions and sources as Fourier's series and applying the Fourier-Bessel transformation yield the problem for the 1D telegraph equation. Its solution is constricted with the help of the Riemann formula. Then making the inverse Fourier-Bessel transformation and performing calculations, we obtain the expansion coefficients that together with Fourier series yield the solution of the wave equation in terms of the modes in cylindrical coordinate system. The result obtained allows to investigate the space-time structure of the formed waves. Application of the scalar solution to a description of electromagnetic waves is also discussed.

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Diffraction characteristics of an impedance loaded parallel plate waveguide

Alinur Büyükaksoy, Gökhan Çinar

Gebze Institute of Technology, PO. Box. 141 Gebze, 41400 Kocaeli, Turkey
e-mail: alinur@gyte.edu.tr; gokhan@gyte.edu.tr

Gökhan Uzgören

İstanbul Kültür University, İstanbul, Turkey
e-mail: gokhanuzgoren@iku.edu.tr

In the present work the diffraction of plane electromagnetic waves by an impedance loaded parallel plate waveguide formed by a two-part impedance plane and a parallel half plane with different face impedances is analyzed.

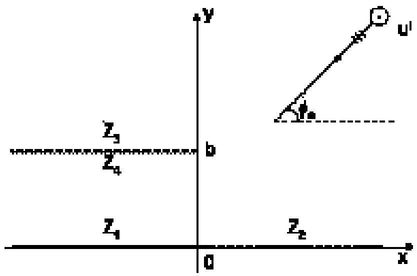

Figure : Geometry of the diffraction problem

This problem is a generalization of a previous work by the authors [1] who considered the same geometry in the case where the half plane is perfectly conducting. In [1] the related boundary value problem is formulated as a matrix Wiener-Hopf equation which is uncoupled by the introduction of infinite sum of poles. The exact solution is then obtained in terms of the coefficients of the poles, where these coefficients are shown to satisfy infinite system of linear algebraic equations.

When the half plane has non vanishing surface impedances, the resulting matrix Wiener-Hopf equation becomes intractable. To overcome this difficulty one recourses of a hybrid formulation consisting of employing the Fourier transform technique in conjunction with the mode matching method (see e.g. [2],). By expanding the total field into a series of normal modes in the waveguide region and using the Fourier transform elsewhere, we get a scalar modified Wiener-Hopf equation of the second kind. The solution involves a set of infinitely many expansion coefficients satisfying infinite system of linear algebraic equations.

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Solution of a matrix Wiener-Hopf equation connected with the plane wave diffraction by an impedance loaded parallel plate waveguide

Alinur Büyükaksoy, Gökhan Çinar

Gebze Institute of Technology, PO. Box. 141 Gebze, 41400 Kocaeli, Turkey
e-mail: alinur@gyte.edu.tr; gokhan@gyte.edu.tr

In the present work the diffraction of a plane electromagnetic wave by an impedance loaded parallel plate waveguide formed by a two-part impedance plane and a parallel perfectly conducting half-plane is analyzed.

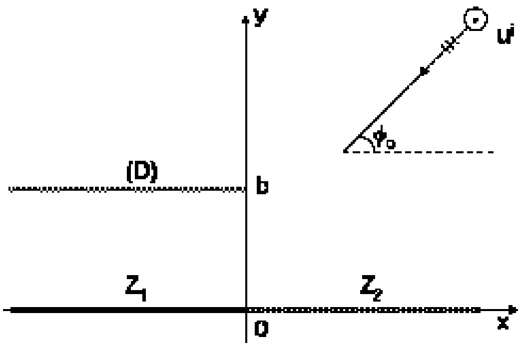

Figure : Geometry of the diffraction problem

The formulation of the boundary-value problem in terms of Fourier integrals leads to a matrix Wiener-Hopf equation which is uncoupled by the introduction of infinite sum of poles. The exact solution is then obtained in terms of the coefficients of the poles, where these coe.cients are shown to satisfy infinite system of linear algebraic equations. This system is solved numerically and the influence of the parameters such as the waveguide spacing and the surface impedances of the two-part plane on the diffraction phenomenon is shown graphically.

Some notes on the commutative matrix factorization

A.V. Shanin, E.M. Doubravsky

Moscow, Russia

We study the classical problem of matrix factorization, i.e. the representation of the matrix G in the form $G = G^+G^-$, where G^+ and G^- are functions analytical in the upper and lower variable, respectively. This problem is connected with a lot of famous diffraction problems, however unfortunately nowadays no general solution for it is known.

The most effective and widely known approach to the matrix factorization is the Khrapkov method. This method is applicable to a relatively narrow class of matrices G. The idea is to construct a commutative set of matrices to which G, G^+ and G^- all belong.

In our talk we do not propose a general solution for the matrix factorization problem, but demonstrate a powerful analytical tool to study the properties of the solution (yet unknown). Among the properties that can be established by the new method we mark out the possibility to perform a commutative factorization (a necessary condition) and the behaviour at infinity.

Mathematical physics of "nondiffracting beams"

S. Chávez-Cerda

Instituto Nacional de Astrofísica, Óptica y Electrónica,
Apdo. Postal 51/216, Puebla, México 72000

Light beams with apparent nondiffracting properties have intrigued the scientific community since they were introduced in 1987. In this work we will discuss their properties based on the mathematical-physics formalism of the Helmholtz differential equation that gives rise to them. What at some point were seen as rather odd features of these beams, when re-interpreted as superposition of traveling wave solutions they become clear and physically consistent. Here, the four different families of nondiffracting beams will be analyzed, namely plane waves, Bessel, Mathieu and Transverse Parabolic beams. The consequences of the analysis that will be presented go towards a better physical understanding of these optical fields and thus to include or discard potential practical applications.

Since what above have been termed nondiffracting or diffraction-less beams [1] may be physically misleading, they will be referred to as Propagation-Invariant Optical Fields (PIOFs). This kind of wave fields have been of interest since numerous applications, ranging from metrology to atom guiding, have been proposed and demonstrated. Beams can be made such that their transverse intensity distribution remains unchanged in amplitude and shape upon propagation for very long distances within a predefined region of space.

The most common of these wave fields are the plane waves. Curiously enough, the concept of plane waves is very well accepted and understood in physics. Very rarely one questions its physical achievability considering that a true plane wave should have infinite extent (otherwise it would be a very large flattened beam), but then this takes us to something physically unrealizable. Plane waves are the solution of the Helmholtz equation in Cartesian coordinates with boundaries at infinity. When they are related to a physical experiment then they have to be approximations to true plane waves as is the case of a collimated coherent beam. These same criteria apply to the other three families of PIOFs. When this is done it is easier to understand the true nature of the PIOFs, of which Bessel beams are the most known and discussed in the literature. Recently, Mathieu and Transverse Parabolic beams have been demonstrated [2–6].

A general solution of the Helmholtz equation $\nabla^2 E + k^2 E = 0$ that represents all of the mentioned propagation invariant optical fields is the reduced Whittaker integral

$$E(\mathbf{r}_\perp, z) = E_0 \exp(ik_z z) \int_0^{2\pi} A(\varphi) \exp\left[ik_\perp(x\cos\varphi + y\sin\varphi)\right]\, d\varphi, \tag{1}$$

where $\mathbf{r}_\perp$ represents the two dimensional transverse vector position. The wave vector is also decomposed in a transverse and a longitudinal component of magnitude $k_\perp$ and k_z, respectively. $A(\varphi)$ is the transverse angular spectrum. From this integral it can be deduced that the wave fronts are conic [7]. From Eq. (1) is observed that the intensity, proportional to $|E|^2$, is independent of the propagation coordinate z. Then it represents a wave field whose transverse structure propagates unchanged, i. e. any solution of the Helmholtz equation that has the property of being propagation invariant, being $A(\varphi)$ its corresponding spectrum function. However, requesting that this solution also agrees with that obtained by separation of variables then that will define a very particular solution for each case that describes propagation invariant optical fields.

The mathematical solutions obtained above have to fulfill certain theorems, like Sommerfeld radiation condition or Huygens principle for two dimensional differential wave equations, in order to make them physically consistent. Once the theorems have been properly established then the propagation characteristics of the PIOFs can be easily understood and it can be seen that they do not violate any physical law as sometimes it is mentioned in the literature [8,9]. Another more relevant issue is that once the propagation characteristics of these beams are understood their applications to real problems may be easier to visualize [10,11]. A thorough analysis based on the mathematical physics of the Helmholtz wave equation and their solutions in cylindrical coordinates (circular, elliptic and parabolic) will be the core of the work to be presented.

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Diffraction under G_0–strictly singular perturbations

Ketill Ingolfsson

Department of Mathematics, CCP 1700 Spring Garden, Philadelphia PA 19130 USA

In the traditional use of positive definite inner-product metric for quantum dynamics the spontaneous decay can be detected as a path in the unit sphere of a separable Hilbert space over the complex field. If G is the skew self-adjoint infinitesimal generator of the decay and G_0 is the generator of an exponentially bounded semi-group, then it is possible to associate the decomposition $G \subset G_0 + iV$ with a partition of $R(G)$, the resolvent of G, into a power expansion in $iVR(G_0)$, which under the inverse Laplace-transform yields an G_0–strictly singular perturbation series. Theoretically the directional field of the dynamical solution may show a diffraction pattern at the junction of the main term with the remainder-term and so indicate the existence of a non-exponentially decaying process. In experiments, however, such deviation has never been found. On the contrary, the quality of the exponential decay is, in a residuum under the inverse Laplace-transform, the most reliable marking of an elementary physical particle. We might therefore suggest that the correct dynamical theory is embedded in a positive semi-definite metric, and a measurement then takes place through a projection onto a positive definite subspace. Such projections may be mathematically formulated along the lines of the old Gupta-Bleuler theory, and have in fact been discussed before by B.L.van der Waerden and E.Cartan. The apparent discrepancy is therefore probably not more a mathematical but rather a physical problem, which could be solved by clearing the ambiguity of the measurement process.

Optical solitons with dual-power law nonlinearity

Anjan Biswas

Dept. of Physics & Mathematics, Center of Excellence in ISEM,
Tennessee State University, Nashville, TN 37209-1561, USA

The multiple scale perturbation analysis is carried out to study optical solitons that are governed by the nonlinear Schrodinger's equation with dual-power law nonlinearity. The WKB type ansatz of the perturbed soliton captures the corrections to the pulse where the soliton perturbation theory fails. The perturbation terms that are considered here are nonlinear damping and saturable amplifiers. Finally, the numerical simulations support the theory.

Transparent body synthesis for the cases of a circular cylinder and a sphere

Eugenii V. Chernokozhin

Dept. of Computational Mathematics and Cybernetics, Moscow State University, Russia
e-mail: celd@cs.msu.su

The suppression of the electromagnetic field scattered by simplest perfectly conducting bodies by means of appropriate internal reradiators is studied. The reradiators are supposed to be connected with the ambient space by small apertures or narrow slots in the boundaries of the bodies. It is required to minimize the scattering in all directions — not only the backscattering. This problem is referred to as the transparent body synthesis. One should emphasize the difference between the transparent (i.e. not scattering) and the black (i.e. not reflecting) bodies.

The problem of transparent body synthesis is formulated as follows: for the given perfectly conducting body, it is necessary to find a reradiator such that, under the incidence of a certain electromagnetic field $(\vec{E}, \vec{H})$, the power P scattered to the far zone by the body with the reradiator be less than the power P_0 scattered by the initial body by a specified factor ε. Usually, the incident field is understood as a plane electromagnetic wave of a certain frequency range. The direction and polarization of the wave can be both arbitrary and fixed. Thus, a body can be transparent with respect to a certain type of incident field. The body's transparency can be characterized by the attenuation factor $K = 10\lg(P/P_0)$ [dB].

The problem of transparent body synthesis for an infinite circular cylinder is solved separately for two different polarizations.

In the case of the TH-polarization, the reradiator is sought as a periodic system of cylindrical or ring cavities connected with the ambient space by narrow transverse ring slots in the cylindrical boundary. For different frequencies of the incident field within $k_0R \leq 10$, parameters of the cavities providing the attenuation factor $K = -30$ dB at these frequencies have been found. This structure is isotropic with respect to the azimuth angle.

In the case of the TE-polarization, two different approaches are used. In the first one, the reradiator is constructed of cavities consisting of cylindrical ring sectors connected with each other by slots in the side walls. In the second one, the reradiator consists of coaxial cylinders with longitudinally slotted boundaries. The first approach gives a nonisotropic structure but has no significant restrictions in the frequency and ensures a theoretically unlimited attenuation. The second approach yields a structure approximately isotropic with respect to the azimuth angle, but the difficulty of determining the cavities' parameters increases with the frequency and the required attenuation.

The problem of synthesizing a transparent anisotropic sphere is also solved. The reradiator is constructed from spherical ring sectors connected with the ambient space by narrow parallel ring slots in the spherical boundary. The transparency effect can be attained only if the sphere is specially oriented with respect to an incident linearly polarized plane wave.

Theoretically and by numerical solution of the diffraction problem, it is demonstrated that, in all cases, the transparency effect is observed in a narrow frequency band.

Scattering and localization in network subjected to magnetic field

V. V. Demidov

Mordovian State University, Saransk, Russia, 430000

The dynamics of a quantum particle in a periodic potential in the presence of a uniform magnetic field were the subject of researches for several decades and revealed many remarkable effects. A surprising effect has been presented in [1] for a two-dimensional lattice with hexagonal symmetry (the so called $\mathcal{T}_3$ lattice) displayed in Fig. 1. If the flux quantum per rhombus is equal to one half, the energy spectrum of a tight-binding model with nearest-neighbor hopping collapses into three highly degenerate levels. The recent results of the conductance measurements of normal metallic networks etched on a two-dimensional GaAs/GaAlAs electron gas were published in [2]. Remarkably, for the particular $\mathcal{T}_3$ network, the magnetoresistance presents large ϕ_0-periodic oscillations (ϕ_0 is the flux quantum). This is the first observation of strong ϕ_0-periodic oscillations in macroscopic systems. The authors of papers [1-4] describe above mentioned effects in the framework of tight-binding approach by formation of so called Aharonov-Bohm cages when the flux ϕ per rhombus equals $\phi_0/2$. The spectral and scattering properties of the Schrödinger operator defined on a finite graph with $\mathcal{T}_3$ geometry are of special interest. To analyze the scattering problem, we construct a Hamiltonian on the finite graph connected to 6 input and 6 output leads using theory of self-adjoint extensions of operators [5,6]. We choose gluing parameters for the nodes of the graph in such a way that the electron wave-functions satisfy wide-using Griffits conditions at the nodes. The main results of our scattering analysis are as follows. A single rhombus connected with two leads in the magnetic field is similar to the two-terminal Aharonov-Bohm ring considered e.g. in [7,8]. The probability of electron transfer between two diametrically opposite nodes of rhombus vanishes under the flux per rhombus equals to one half. The interference effects in the finite graph (containing 74 rhombus) lead to complex flux dependence of the transmission coefficient T which is reminiscent of the structure of energy spectrum (Fig. 2, here $ka = 0.87\pi$, a is the length of graph bound). Note that the transmission coefficient is not a periodic function of the wave vector k but a periodic function of the flux ϕ. In contrast to [3], we show that influence of dispersive edge states may be compensated for suitable selection of electron energy (Fig. 3, here $ka = 1.78\pi$). Moreover we found the energy domains where the transmission coefficient is equal to zero for every flux so that the system is completely reflecting.

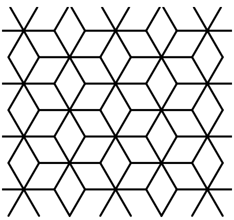
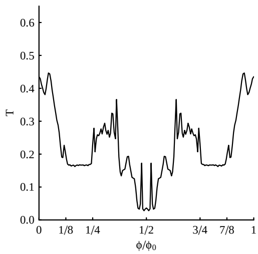


Fig. 1.



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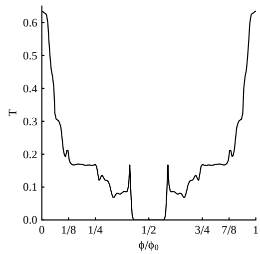

Fig. 3.

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The Wiener-Hopf technique for impenetrable wedge problems

V. G. Daniele, G. Lombardi

Dipartimento di Elettronica, Politecnico di Torino,
C.so Duca degli Abruzzi 24.10129 Torino, Italy
e-mail: guido.lombardi@polito.it

In the last five years the first author of this paper, has developed a general theory based on the Wiener-Hopf technique for studying electromagnetic problems in arbitrary angular regions [1,3].

In general this technique yields a new class of functional equations called Generalized Wiener-Hopf equations (GWHE). The (GWHE) differ from the classical Wiener-Hopf equations (CWHE) since the involved plus and minus functions are defined into two different complex planes. It was remarkable that in some cases a suitable mapping reduces the Generalized Wiener-Hopf equations to the classical ones. For instance it happens in all the impenetrable wedge problems that have been solved by the Sommerfeld-Malyuzhinets (SM) method. Closed form W-H solutions for these problems and also for other problems, which are not solved with the SM method, have been obtained by using an explicit factorization of the matrix kernels [2,3].

For arbitrary impenetrable wedges problems, the W-H formulation involves the factorization of matrix kernels of order four. With the exception of some classes of problems, including the all ones solved by the SM method, closed form factorizations of kernels are not available and we need to resort to approximate factorization techniques. Several techniques to obtain approximate factorizations of arbitrary matrices are available in literature [4]. New different approximate methods are defined with respect to reference [5]. In particular the W-H factorization problem provides its immediate reduction to Fredholm equations without applying regularizations to the kernel operator.

In general the powerfulness of the approximate W-H factorization technique depends on the kernel's spectrum of the related Fredholm equation. When we are dealing with impenetrable wedge problems, we experienced that a particular mapping makes the Fredholm equation suitable to be solved numerically.

The aim of this work is to present an efficient method based on the W-H technique to solve the all impenetrable wedge problems which are still unsolved in literature.

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Comparative microacoustic investigations of elastic properties of nano-, poly- and crystalline-silicon thin films

A. Doghmane, I. Hadjoub, S. Bouhedja, Z. Hadjoub

Laboratoire des Semi-Conducteurs, Département de Physique, Faculté des Sciences, Université Badji-Mokhtar, Annaba, BP12, DZ-23000 Algeria
e-mail: lsc_contact@yahoo.fr

Thin films of polycrystalline and nanostructured silicon are of great importance in micro- and nano-technology applications. Therefore, they received a great deal of interest in the investigation of their electronic, magnetic, and optical properties but only very little work is reported on their elastic properties. In this context, we first deduce their SAW velocities longitudinal, VL, transverse, VT and Rayleigh, VR. It is found that these velocities change according to grain dimensions, atomic structure and defect densities characterizing each Si type; with the highest values corresponding to crystalline Si followed by those of poly-Si, and finally the smallest values were obtained for nano-Si. Moreover, in order to establish dispersion curves, we calculate the acoustic material signatures, via Sheppard and Wilson formula that describes the output response as a function of the defocusing distance in acoustic microscopy configuration. Such signatures are determined for different film thickness of all the above Si types deposited on crystalline Si substrates. Their spectral analysis led to the determination of the propagating wave velocities which were plotted in terms of normalized thickness. It is found that the general trend of the curves is dispersive: with increasing normalized thickness, the surface wave velocity decreases from the value of the substrate velocity to that of the deposited film. However, for very thin film of poly Si, we noticed an anomalous behavior consisting of a small initial increase followed by the usual velocity decreases.

Analytical and empirical evaluation of materials elastic moduli via one-parameter derived-formulae

A. Doghmane, M. Doghmane, I. Hadjoub, F. Hadjoub

Laboratoire des Semi-Conducteurs, Département de Physique, Faculté des Sciences, Université Badji-Mokhtar, BP 12, Annaba, DZ-23000, Algeria
e-mail: a_doghmane@yahoo.fr

Young's modulus, E, and shear modulus, G, are usually expressed in terms of the velocities of different propagating wave velocities. Such velocities can be determined via nondestructive ultrasonic characterization techniques such as scanning acoustic microscopy, SAM. However, it is difficult to determine the velocity values of all the propagating modes from a single and unique measurement. In fact, in SAM technique, the measured $V(z)$ signatures is usually representative of a single dominating mode. Hence, to overcome such limitations we derive in this work, simple expressions of E and G in terms of the velocity of just one propagation mode: longitudinal, V_L, transverse, V_T or Rayleigh, V_R. To do so we used Viktorov formula and apply some physically acceptable approximations to find (i) for Young's modulus: $E = 0.757\rho V_L^2$, $E = 2.586\rho V_T^2$ and $E = 2.99\rho V_R^2$ and (ii) for shear modulus: $G = 0.293\rho V_L^2$ and $G = 1.156\rho V_R^2$. The validity of these derived formulae was successfully tested via their application to a great number of materials whose elastic constants were also determined by conventional expressions. Data comparison obtained via approximated and conventional relations led good precisions of 1% or even less in some cases to reach 0.007%. Empirical expressions were also deduced from a graphical plot of elastic constants of over sixty solid materials. The data fitting gave very close proportionality constants to those mentioned above. Thus, confirming the validity of such one parameter formulae. The use of such expressions would remove many limitations in certain ultrasonic micro-characterization techniques.

Soliton-like excitations and instability in a layered media and on surface of cylindrical shell

V.V. Kiseliev, D.V. Dolgikh

Institute of Metal Physics, Urals Branch of the Academy of Sciences,
Ekaterinburg, 620219, Russia

An undulatory bending of the separate material layers are observed in a different of deformation ways. We assume that the strong loaded layers undergo a transversal corrugation by retarding influences of neighboring weakly loaded (and since stabile) layers. To discover the pecularities of this mechanism, we solved the problem of dynamics of the strong loaded material layer in the form of a plate which is constrained by two half-spaces with lesser elastic modules. The local bends of the middle layer are assumed to be comparable to its thickness. Therefore, the consideration was carried out in the framework of Murnaghans theory of finite deformations. The Murnaghan theory is the most attractive because it uses the expression for nonlinear- elastic energy containing all invariants of deformation tensor compatible with a symmetry of media without recourse to a priori geometrical hypotheses, that is difficult estimate quantitatively. The initial equations of the finite deformation theory are too complicated to be analytically useful. We proposed the special variant of perturbation theory. In this method, we reduced initial equations into the simple nonlinear model with the controlled precision referring to the small parameters reflecting the typical space-time scales of deformation, geometrical and physical nonlinearity of a media. The simplified model takes into account correctly the main interactions and at the same time gives exact solutions which describe a middle layer bending. In this process phenomenological constants of Murnaghans theory are combined into the a small number of parameters which play the role of effective modules of media. The novel approach is in following: we solve the nontrivial boundary problem, where the shape of surface of a strong loaded layer of media is not known beforehand, but is determined during the process of the calculations. Pecularities of the initial corrugation of middle layer result from a balance between boundary and dimensional effects responsible for a space dispersion of media, effects of the nonlinear interaction of the neighboring unstable modes and effects of the non-local interaction between layers. As far as we know, dynamics of such nonlinear-elastic deformations of material is not investigated previously. We established the possibility for formation of solitons of the "transversal corrugation" inside layered media which proceed by the plastic flow of material. An undulatory bending of the middle layer of media and its "subdivision" into solitons of corrugation will occur, starting from some external critical stress. It depends on a thickness of layer and material parameters of media, but must be less than instability threshold of the middle layer. Solitons of corrugation propagate with velocities less than a certain critical value (they can be motionless too). In this case solitons are the concentrators of stress and at the same time precursors of the subsequent plastic deformation of material; therefore they carry along information about geometry of layers and the stress-state of material. The conditions for formation of chains type structures from solitons of corrugation inside of layered media are found. Our approach may be applied to the analysis of the nonlinearly — elastic dynamics of cylindrical shells, when small amplitude "patterns" from space-localized bandings are formed on the shell surface. We constructed the simplified nonlinear model. With this model, the ring-shape solitons (small amplitude transversal folders) on surface of the longitudinal compressed cylindrical shell have been predicted and analytically described. The conditions of soliton formation have been investigated depending on the external stress, geometrical and material parameters of the shell.

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Analytic model for a solitary wave generation in fully nonlinear shallow-water theory

Gennady A. El

Department of Mathematical Sciences, Loughborough University,
Loughborough LE11 3T, UK
Institute of Terrestrial Magnetism, Ionosphere and Radio Wave Propagation, RAS,
Troitsk, Moscow Region, 142190 Russia

We present an analytic approach to deal with the unsteady problem of the solitary wave generation in the Green - Naghdi equations describing propagation of *fully nonlinear* shallow-water waves:

$$\begin{aligned} &\eta_t + (\eta u)_x = 0\,, \\ &u_t + u u_x + \eta_x = \frac{1}{\eta}\left[\frac{1}{3}\eta^3(u_{xt} + u u_{xx} - (u_x)^2)\right]_x . \end{aligned} \tag{1}$$

Here η is total depth and u is layer-mean horisontal velocity, all variables are non-dimensionalised by their typical values. The system (1) is obtained from Euler equations using the asymptotic expansions in small dispersion parameter $\epsilon = h_0/L \ll 1$ where h_0 is the (dimensional) equilibrium depth and L is typical wavelength. No small-amplitude decompositions are made.

The system (1) is not integrable via spectral transform and, apart from the water wave dynamics, appears in various applications where strongly nonlinear processes are important (bubbly fluids, Solar MHD). We consider the "semi-classical" formulation of the problem when the generation of solitary waves is viewed as a part of a more general process of the undular bore development from a large-scale initial perturbation. Based on previous analytic results for completely integrable systems and strong numerical evidence for the systems that are not integrable but structurally similar to integrable ones we suggest that the fully nonlinear shallow-water undular bore can be modelled by the expansion fan solution of the corresponding modulation (Whitham) system. Although in absence of the Riemann invariants, the full modulation solution is not available, we are able to find a set of nontrivial integrals of motion allowing one to obtain analytically the major physical parameters of the undular bore determining its intensity and location. Due to an unsteady character of the problem these *do not coincide* with formal jump conditions following from conservation laws. As a consequence, we find the amplitude of the lead solitary wave given the initial jumps in η and u, the result available previously only for completely integrable dynamics via the spectral transform or full modulation solution. A comparison with direct numerical simulations is made.

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Determination of elastic properties of bio-materials via surface acoustic waves

F. Hadjoub, M. Gherib

Laboratoire des Semi-Conducteurs, Département de Physique, Faculté des Sciences, Université Badji-Mokhtar, BP 12, Annaba, DZ-23000, Algeria
e-mail: lsc_contact@yahoo.fr

Recently, the uses of biomaterials, bioceramics, metals, bioactive glass, biopolymers, have been revolutionizing the biomedical field in deployment as implants for humans. Bioglasses are interesting versatile class of biomaterials. Some of the bioactive glasses with specific composition are embedded in a biomaterial support to form prosthetics for hard tissues. Such prosthetics are biocompatible, show excellent mechanical properties and are useful for orthopedic and dental prosthetics. Therefore, in this work, glasses (x_1%SiO_2 x_2%Na_2O x_3%CaO 6%P_2O_5) with various compositions (x%) are investigated by a non-destructive technique. The acoustic material signatures $V(z)$ of theses bio-glasses, deduced theoretically, showed oscillatory beahvior as a result of surface acoustic mode constructive and destructive interferences. The spectral treatment of theses signatures, via fast Fourier transform, led to the determination of Rayleigh velocities, V_R, then we estimate their elastic constants (Young's modulus, E, and shear modulus, G). The results thus obtained were found to be dependent on various compositions such that:

$V_R = 3220$ m/s, $E = 84.50$ GPa and $G = 33.45$ for: 43%SiO_2 25.5%Na_2O 25.5%CaO6%P_2O_5,
$V_R = 3215$ m/s, $E = 85.60$ GPa and $G = 33.59$ for: 45%SiO_2 24.5%Na_2O 24.5%CaO 6%P_2O_5,
$V_R = 3411$ m/s, $E = 82.85$ GPa and $G = 32.61$ for: 50%SiO_2 22%Na_2O 22%CaO 6%P_2O_5,
$V_R = 3288$ m/s, $E = 77.27$ GPa and $G = 30.22$ for: 55%SiO_2 19.5%Na_2O 19.5%CaO 6%P_2O_5,

Therefore, it can be concluded that one can easily change the mechanical properties of such glasses by just changing their composition; such a procedure leads to the optimisation of their charactezristics for any potential applications.

Rayleigh model of vibrations of N-stepped bar

I. Fedotov, Y. Gai, S. Joubert

Department of Mathematical Technology P.B.X680, Pretoria 0001 Tshwane University of Technology, Pretoria, South Africa
e-mail: fedotovi@tut.ac.za

M. Shatalov

CSIR Manufacturing and Materials P.O. Box 395, Pretoria 0001, South Africa
Department of Mathematical Technology P.B.X680, Pretoria 0001 FIN-40014 Tshwane University of Technology, South Africa
e-mail: mshatlov@csir.co.za

The traditional approach to the analysis of longitudinal vibrations of N-stepped structures is based on the assumption that the longitudinal vibrations of every section are described by the wave equation and that lateral motion could be neglected. This hypothesis is true if the aspect ratios of every section are high and characteristic frequencies are small enough.

In this paper we consider the small aspect ratios of the sections and relatively high excitation frequencies of the lateral vibrations. This leads to a more complex Rayleigh model of longitudinal vibrations of thick bars where the classical wave equation is replace by a fourth order PDE so that the effects of lateral inertia are kept in mind. The analytical tools of vibration analysis are based on finding eigenfunctions with piecewise continuous derivatives, which are orthogonal with respect to a specially chosen weight function. These eigenfunctions satisfy the boundary conditions at the end points and boundary conditions at the junctions. The solution of the problem is formulated in terms of Green function. This algorithm is efficient in the design of low frequency transducers of Tonpilz type produced in South Africa. An example is given to show the practical implementation of the algorithm to a three-stepped structure.

Problems of diffraction type for pseudodifferential operators

I. Fedotov, Y. Gai, S. Joubert

Department of Mathematical Technology P.B.X680, Pretoria 0001 Tshwane University of Technology, Pretoria, South Africa
e-mail: fedotovi@tut.ac.za

In this paper we consider problems of diffraction type for pseudo-differential operators with variable symbols. In more detail, we consider simultaneously two pseudo-differential equations elliptic with parameters in different domains with a common boundary. In the statement of this problem, a bounded domain breaks the homogeneity of a medium provided that the solution satisfies the conditions of a maximal smoothness on the boundary of this domain. This is equivalent to finding a solution belonging to the Sobolev space H_s with the maximal value of s. We also prove that the solution of a Dirichlet problem can be obtained as the limiting case of a diffraction problem as the parameter of the external domain tends to zero.

Uncertainty relations for generalized oscillators

V.V. Borzov
Department of Mathematics, St.Petersburg University of Telecommunications,
191065, Moika 61, St.Petersburg, Russia
e-mail: vadim@VB6384.spb.edu

E.V. Damaskinsky

Department of Mathematics, University of Defense Technical Engineering, 191123, Zacharievskaya 22, St.Petersburg, Russia
e-mail: evd@pdmi.ras.ru

The generalized coherent states for the oscillator-like systems connected with the known classes of the orthogonal polynomials were considered in our talks at the previous seminars "Day on Diffraction" (DD'02-DD'04). We discussed the cases of the classical polynomials in a continuous as well as in a discrete variables and also their deformed analogues. Mainly we studied the Barut-Giradello coherent states (and more general Klauder - Gazeau coherent states) and, as a rule, did not touch, so called, minimum uncertainty coherent states (MUCS) and coherent states of Perelomov type. In our talk on DD'05 we want to discuss the connection of constructed coherent states with uncertainty relation. This investigation was promoted by stimulating question of prof. M.Berry after our talk on DD'02. The main result is that for all generalized oscillators connected with considered families of orthogonal polynomials, the generalized coherent states of the Barut-Giradello type are at the same time the MUCS. Another result to be interest for possible physical applications consists in the assertion that the generalized coherent states can have both sub-poissonian and super-poissonian statistics even in the case of known orthogonal polynomial systems. The type of statistics is determinated by the sign of the well-known in quantum optics Mandel parameter Q_M. If $Q_M > 0$, the statistics is sub-poissonian, and it is super-poissonian if $Q_M < 0$. The case $Q_M = 0$ corresponds to the poissonian statistics. For the investigated cases of classical polynomials in a continuous and in a discrete variable we obtain the following result. For Charlie polynomials (as well as in the case of Hermitean polynomials related to the standard coherent states) $Q_M = 0$. $Q_M > 0$ for Laguerre and Meixner polynomials whereas in the case of Jacobi and Krawtchouk polynomials $Q_M < 0$. Finally, in the case of q-deformed Hermite polynomials the sign of the Mandel parameter is defined by the magnitude of deformation parameter: for $0 < q < 1$ one has $Q_M < 0$, and for $q > 1$ $Q_M > 0$.

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New elements in the theory of the coaxial waveguide with azimuthally magnetized ferrite

Georgi Nikolov Georgiev

Faculty of Mathematics and Informatics, University of Veliko Tirnovo,
BG-5000 Veliko Tirnovo, Bulgaria
e-mail: gngeorgiev@yahoo.com

Mariana Nikolova Georgieva-Grosse

Meterstrasse 4/2, D-70839 Gerlingen, Germany

The theory of the azimuthally magnetized circular ferrite waveguides, propagating normal TE_{0n} modes, studied for development of nonreciprocal digital phase shifters [1–4], may successfully be built by means of the complex confluent hypergeometric functions (CHFs) [3,4]. The characteristic equation of the coaxial structure of finite thickness of the central switching conductor has been derived in terms of complex Kummer and Tricomi CHFs $\Phi(a,c;x)$ and $\Psi(a,c;x)$, respectively [3] with $a = c/2 - jk$, $c = 3$, $x = jz$, (k — real, z — real, positive). For the purposes of boundary-value analysis the dependence of purely imaginary zeros $\xi_{kn}^{(c)}(\rho)$ of the equation in x, ($n = 1, 2, 3, \ldots, \rho$, — switching conductor to guide radius ratio) on the parameter k has been investigated in a finite interval of variation of the latter, symmetrically situated with regard to the point $k = 0$ [3].

Here, it is proved numerically that if $k \to -\infty$, the numbers $\xi_{kn}^{(c)}(\rho)$ tend to zero and the products $|k|\xi_{kn}^{(c)}(\rho)$ and $|a|\xi_{kn}^{(c)}(\rho)$ have a finite real positive limit $L = L(c, \rho, n)$. Physically, this leads to the appearance of an envelope curve in the phase diagram, restricting the phase characteristics for negative magnetization of ferrite from the side of higher frequencies. Similar dependence of the products mentioned is not found if $k \to +\infty$, respectively envelope curve of the characteristics for positive magnetization of the filling does not exist. This new property of coaxial guide allows to formulate the condition for phaser operation of the structure in terms of the $L(c, \rho, n)$ numbers. A comparison with the case of circular guide shows that the introduction even of a thin central conductor increases significantly the value of the latter. For example, provided $\rho = 0$, it holds $L(3, 0, 1) = 6.5936$, whereas if $\rho = 0.1$, $L(3, 0.1, 1) = 7.6500$. As a result the area of propagation for negative magnetization, respectively the one for phaser operation, determined by the coaxial geometry, expands considerably. In addition, an original simple method for calculation of differential phase shift in normalized form $\Delta\overline{\beta}$ is suggested which permits to figure the latter in the whole area of phaser operation of the guide. It uses the envelope curve and enables to derive a formula for finding $\Delta\overline{\beta}$ in terms of the $L(c, \rho, n)$ numbers. Besides, the approach needs also a restricted application of iterative techniques, elaborated so that the necessary computation time is reduced to a minimum and the error admitted is of the order of a few percent.

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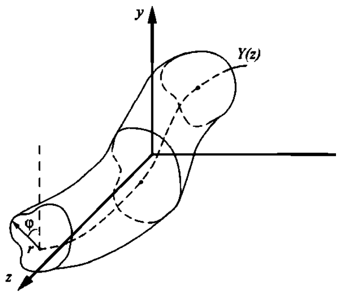
Wave packets in elastic flat bent waveguide

P. A. Gladkov, G. I. Mikhasev

Vitebsk State University 33, Moskovsky Ave., 210038 Vitebsk, Belarus

An asymptotic solution of a boundary-value problem for the wave equation in elastic medium is revealed in this paper. The medium is bounded by the surface of the waveguide, bent in theplain in Cartesian coordinates. The boundary conditions are the standard conditions of the absence of displacements.



In the stationary case solutions of the wave equations governing the wave processes concentrated in a neighborhood of a ray are well-known [1]. Non-stationary solutions localized in the vicinity of the space-time ray and named as "quasiphotons" were described in [2, 3]. Using the complex WKB method, stationary and non-stationary solutions were also constructed by Maslov [4].

The present work is based on the asymptotic method, used in [5] for the examination of the wave processes in a cylindrical shell. This approach allows to reduce the three-dimensional boundary-value problem to the two-dimensional or to the univariate one. This method was applied also to research the behaviour of running wave packets in non-cylindrical shell with slashed ages [6], in infinite cylindrical shell under variable internal pressure [7] and in the straight waveguide with round cross-section [8].

The asymptotic solution of the boundary-value problem for the wave equation is constructed in the form of localized families of short waves running in the longitudinal direction. The condition for existence of modes of free oscillations in the narrow waveguide near some cross-section is determined. The effects of reflection of the wave packet and localization of the wave process near the "weakest" plane are revealed.

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Multiple scattering of ultra-cold neutrons in the framework of random zero-range potential theory

J. Brüning

Humboldt-Universität zu Berlin, Germany

S.Yu. Dobrokhotov, T.Ya. Tudorovskiy

Institute for Problems in Mechanics, RAS, Moscow, Russia

V.A. Geyler

Mordovian State University, Saransk, Russia

The interest to the theoretical investigation of the scattering parameters for ultra-cold neutrons (UCN) has motivated by possibilities to perform a series of important experiments using these particles; see e.g., the recent electric dipole moment experiment using stored particles [1], which is a test for the Standard Model of electroweak interaction, or the experiment on the measurement of quantum states of neutrons in the Earth's gravitational field [2], which tests some consequences of Quantum Gravitation Theory. The main problem related to UCN results from the storage of UCN in closed vessels and is the follows: The losses due to the collisions of neutrons with the walls of the trap may significantly exceed ones predicted by theory [3,4]. One possible way to explain this discrepancy is to take into account properties of Fomblin which coats the walls of the vessels [5]. In this connection we study an explicitly solvable scattering problem considering the coating as a random ensemble of point scatterers. We suppose the position of scatterers to be fixed, but the strength of the corresponding zero-range potentials are identically distributed random variables with the Lorentzian (Caushy) distribution $f(t) = \dfrac{1}{\pi}\dfrac{\kappa}{(t-\alpha)+\kappa^2}$. Here $(4\pi\alpha)^{-1}$ is the scattering length, κ^{-1} is the scale parameter. Using the supersymmetric trick [6] we show that the Fermi pseudopotential u_0 in the considered case is $u_0 = 4\pi nl(1+i\kappa l)^{-1}$ where n is the density of the scatterers. The imaginary term $i\kappa l$ is responsible for the energy loss in the neutron vessel. On the other hand, according to [3], a small imaginary term of order kl, where k is the wave vector of the neutron, is present in u_0 and this contradicts to the expression for u_0 given in [7]. We show that the approach of [3] is valid if one can define the scattering parameters separately at each fixed quasimomentum of the Bloch–Floquet decomposition over the Brillouin zone of the periodic layer.

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Reflection coefficient modification for mobile path loss calculation

E. Torabi, A. Ghorbani

Faculty of Electrical and electronic eng., Amirkabir Univ. of Technology, Tehran, Iran

Ray tracing methods are relatively efficient for calculating links parameters in cellular mobile communication. These methods are known as high frequency solution for the Maxwell equations. According to these methods electromagnetic rays are traced on interaction with obstacles which cause reflections and diffractions to the waves and then sum up the individual contributions. In these methods and similar approach for calculating the received fields in the mobile base station, the Coefficients of reflection and diffraction are important keys in evaluation the path loss and path delay introduced by obstacles. Even though up to now many researchers tried to modify these coefficients for better estimation, but most of these modifications tend toward diffraction coefficient. It can argue that the reflection coefficient is more important parameter than diffraction coefficient. Since the reflected fields not only are the dominant terms in at the receiver but they are stronger than the diffracted waves in the direct, reflect and shadow regions. In the other hands as we know most of the mobile paths consist of irregular buildings in the urban environment and the walls of these buildings are inhomogeneous layers. Therefore using the Fresnel reflection coefficient in such environment is not a good choice and therefore new approach needs to be considered. One of these methods are Riccati equations which relay on solving the nonlinear differential equation as will be explained in this paper.

Also reflection coefficient is the main input to the diffraction coefficient therefore path loss calculation accuracy relay to high degree on it especially in UTD methods. Therefore in this paper we attempt to modify the reflection coefficients by solving the Riccati equations in order to be able to calculate the path loss in mobile communication in the urban environment.

In doing so the reflection coefficient for inhomogeneous medium soled and compared with measurements. The results show good agreement with measurements as compare with Fresnel coefficient especially in the phase. So with this new method path loss and path delay can be estimated to better degree.

Fermat's principle for waves in nonstationary media

Oleg A. Godin

CIRES, University of Colorado at Boulder,
325 Broadway, Boulder, CO 80305-3328, USA

Alexander G. Voronovich

NOAA/Environmental Technology Laboratory,
325 Broadway, Boulder, CO 80305-3328, USA

The principle of least time, also known as Fermat's principle or the shortest optical path principle, was discovered by French mathematician Pierre de Fermat in about 1650. This simple and general principle proved to be of exceptional heuristic and philosophical value, and it materially influenced the subsequent development of mathematics, mechanics, optics, and other branches of physics. In modern formulation, Fermat's principle states that the value of the optical path length between any two points is stationary on an actual ray. For nondispersive waves, travel time is proportional to the optical path length and therefore is also stationary on an actual ray. The principle is of particular importance for acoustic tomography because it allows one to neglect perturbations in ray geometry when performing a linear inversion of acoustic travel times for sound speed variations.

Derivations of Fermat's principle available in the literature apply to neither acoustic waves in medium with time-dependent sound speed nor other waves in the fluid with nonstationary parameters. Moreover, the very formulation of the principle in this case is not obvious as travel time along a trial ray ceases to be a single-valued functional of the trial ray geometry.

In this paper, Fermat's principle is extended to nonstationary media that support waves with frequency-independent velocity. The approach used to prove the stationarity of travel times with respect to deformation of the actual ray trajectory is based on a comparison of the rays that follow from the variational principle and from the eikonal equation. Inhomogeneous, moving, and anisotropic media are considered. The identities that relate phase and group velocities and their derivatives in general anisotropic, inhomogeneous, nonstationary media are established. It is shown that not only the travel time, but the eikonal as well, is stationary on the actual ray in media with time-dependent parameters if all trial rays are required to arrive at the receiver simultaneously.

Another approach is developed to extend the results to certain dispersive waves. The approach allows one to formulate and prove the stationarity of travel time on the actual ray in time-dependent environments for a class of wave dispersion laws that includes a power-law dependence of the phase velocity on wave frequency.

Some corollaries and applications of Fermat's principle in nonstationary media are discussed briefly.

Waves in almost-incompressible solids: applications to ocean remote sensing

Oleg A. Godin

CIRES, University of Colorado at Boulder and NOAA/Environmental Technology Lab., 325 Broadway, Boulder, CO 80305-3328, USA

David M. F. Chapman

Defence Research and Development Canada - Atlantic, P.O. Box 1012, Dartmouth, Nova Scotia B2Y 3Z7, Canada

In the upper tens to few hundred meters of clay and silt marine sediments with high porosity, the shear wave velocity is much smaller than the compressional wave velocity. The shear velocity has very large gradients close to the ocean floor leading to strong P-SV coupling in such "soft" sediments. The combination of small shear rigidity and strong gradients leads to a number of unexpected wave phenomena.

One such phenomenon is the resonance structure of the spectrum of ambient seismo-acoustic noise. The spectrum of the horizontal component of seabed velocity shows several prominent peaks in the frequency domain whereas the spectra of both the acoustic pressure and the vertical component of seabed velocity show very weak or nonexistent features at the same frequencies. Another interesting phenomenon is the existence of slow interface waves at the ocean/sediment boundary and their dispersion curves. Observation of ambient noise resonances and measurement of interface wave dispersion provide valuable insights into the shear properties of surficial marine sediments that are difficult to determine by other means.

To understand the experimentally observed phenomena, a theory of elastic wave propagation in continuously-stratified soft sediments has been developed that fully accounts for the P-SV coupling and singularities associated with vanishing shear velocity. It is shown that elastic waves in soft sediments consist of "fast" waves propagating with velocities close to the compressional velocity and "slow" waves propagating with velocities on the order of the shear velocity. For the slow waves, the theory predicts existence of surface waves at the ocean/sediment boundary. An explicit, exact solution is obtained for the surface waves in the case of linear increase of shear rigidity with depth (i.e. a square-root shear speed profile). Asymptotic and perturbation techniques are used to extend the result to more general environments. In the case of the ambient noise features, a uniform asymptotic expansion is obtained that accurately predicts frequencies of the resonances.

Measurements of ambient noise resonances can be easily inverted for the thickness of the layer of soft sediments and an estimate of the shear speed profile. The speed of vertically-polarized interface waves is found to be sensitive to sediment density and shear rigidity. Theoretical dispersion relations agree well with available experimental data and are shown to lead to a simple and robust inversion of shear-speed profiles in the sediment from interface wave travel times.

Generalized Ince-Gaussian beams

Miguel A. Bandres, Julio C. Gutiérrez-Vega

Photonics and Mathematical Optics Group, Tecnologico de Monterrey,
Monterrey 64849, México

The standard and elegant Hermite-Gaussian, Laguerre-Gaussian, and Ince-Gaussian beams constitute the three orthogonal and biorthogonal, respectively, complete families of paraxial solutions for the scalar Helmholtz equation. The elegant solutions differ from the standard solutions in that the former contain polynomials with a complex argument but coinciding with that of the Gaussian function, whereas in the latter the argument is real [1-6].

As other solutions to satisfy the paraxial wave equation, Pratesi and Ronchi in Ref. [7] and, Wunsche in Ref. [8] have presented independently the generalized Hermite-Gaussian beams (gHBs) and generalized Laguerre-Gaussian beams (gLGBs) in which the argument of the polynomials is generally complex. The standard Gaussian beams and elegant Gaussian beams are particular cases of these generalized solutions.

In this work we introduce the generalized Ince-Gaussian beams (gIGBs) that form the third family of exact generalized solutions of the paraxial wave equation. The transverse distribution of these fields is described by Ince polynomials, with a generally complex argument [9,10]. The gIGBs are not the solution of a Hermitian operator at an arbitrary z plane. We derived the adjoint operator and the adjoint eigenfuncitons. The gIGBs form a complete biorthogonal set with their adjoin eigenfunctions, therefore, any paraxial field can be described as a superposition of gIGBs with the appropriate weighting and phase factors. The gHGBs and gLGBs correspond to limiting cases of the gIGBs when the ellipticity parameter tends to infinity or to zero, respectively. The gIGBs include the conventional and elegant Ince-Gaussian beams as particular cases when the parameters, which appear in the expression for the gIGBs, are chosen adequately. The expansion formulas among the three generalized families are also derived.

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Vector Helmholtz-Gauss optical beams

Miguel A. Bandres, Julio C. Gutiérrez

Photonics and Mathematical Optics Group, Tecnológico de Monterrey,
Monterrey 64849, México

Laser beams are commonly studied within the framework of the scalar and paraxial approximations of the wave equation. Hermite-Gauss, Laguerre-Gauss, and Ince-Gauss beams constitute the three fundamental and orthogonal families of paraxial solutions of the scalar wave equation [1,2]. Localized beam solutions have been also constructed as the product of a complex amplitude depending on the propagation coordinate, a fundamental Gaussian beam, and a complex scaled version of the transverse shape of a nondiffracting beam [3-5]. For most applications which do not involve the polarization properties of beams, the scalar framework is quite adequate. However, for polarization-dependent applications knowledge of the vector beam solutions is essential.

The problem of finding vector paraxial solutions of Maxwell equations has been treated by several authors [6-8]. In this work we reconsider this fundamental issue and show that vector paraxial wave equation admits two new and general TM and TE localized vector-beam solutions that can be expected to be particularly important when the polarization of the field is of major concern. These solutions constitute the vector generalization of the scalar Helmholtz-Gauss beams introduced in Refs. [4,5]. We apply the separation of variables method to obtain the transverse and longitudinal components of the paraxial electric and magnetic propagating fields. Under the appropriate limits, vector Helmholtz-Gauss beams reduce to the special cases of scalar Helmholtz-Gauss beams [3-5], nondiffracting vector Bessel beams [7], vector Bessel-Gauss beams [8], and TE and TM modes supported by dielectric waveguides [9]. The general expressions are used to describe explicitly for the first time to our knowledge the polarization and propagation characteristics of vector Mathieu-Gauss and vector Parabolic-Gauss beams. The conditions for the validity of the paraxial approximation are also discussed.

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New integrable versions of sin-Gordon's types equations

E. Sh. Gutshabash

Inst. Research for Physics, St.Petersburg Univ., St.Petersburg, Russia
e-mail: gutshab@EG2097.spb.edu

Lax pairs for the series of new versions of sin-Gordon's types equations are proposed. Radial-symmetrical equation, radial-symmetrical equation with dissipation and some others are among them. The exact solutions of some of them are constructed with usage of the Darboux Transformation method.

Zero resistance ammetter of metallic alloys in aqueous solutions: novel technique

Khaled Habib
Materials Science Lab., Department of Advanced Systems,
KISR, P.O. Box 24885 SAFAT. 13109 Kuwait
e-mail: khaledhabib@usa.net

It is well known that electronic instrumentations, i.e. Ammeter, Potential-meter, have been used for years to measure electrochemical properties of metallic electrodes in aqueous solutions. One of the disadvantages of using electronic instruments for the measurement of electrochemical properties is the invasive nature of those instruments to the electrochemical systems of the metallic electrodes in aqueous solutions. In recent work published elsewhere, it has been shown that laser optical interferometry can be used as an optical transducer to characterize the electromagnetic field, i.e., phase and amplitude of the reflected light waves of a surface of a metallic electrode moving further away from the light source, which develops as a result of the electron conduction in metallic electrodes in aqueous solutions due to the anodic reaction, corrosion processes, between the electrodes and the aqueous solutions. The characterization of such electromagnetic field (phase and amplitude of the reflected light waves of a surface) and a mathematical correlation of the electromagnetic field to any electrochemical properties, i.e., corrosion current density, double layer capacitance, alternating current impedance, and so on, would lead to the measurement of the electrochemical properties by optical interferometry, by the non-invasive method.

In the present work, the corrosion current density of a low carbon steel, a pure aluminum, a stainless steel, a Copper-Nickel alloy were obtained in 1M NaOH, 1M KCl, 1M NaCl, 1M H_2SO_4 solutions, respectively. The obtained corrosion data from the optical interferometry technique, as a zero resistance Ammeter were compared with corrosion data obtained on the same alloys in the specified solutions from an electronic zero resistance-Ammeter as well as from the linear polarization method. The comparison among the three techniques indicates that there is a contrast in the results among the investigated alloys. In general, the results of the optical interferometry were found to fall in between the corrosion values of the zero resistance ammeter and the linear polarization method, because the technique works based in the electromagnetic principle, with the absent of electronic noise. As a result, the optical interferometry can be considered as a useful zero resistance-Ammeter for measuring the corrosion current density of metallic electrodes in aqueous solutions at the open circuit potential of the electrodes in the aqueous solutions.

Application of velocity dispersion curves to the determination of critical thickness for rayleigh wave excitation in thin films

Z. Hadjoub, I. Touati, A. Doghmane
Laboratoire des Semi-Conducteurs, Département de Physique,
Faculté des Sciences, Université Badji-Mokhtar,
BP 12, Annaba, DZ-23000, Algeria
e-mail: z_hadjoub@yahoo.fr

Velocity dispersion curves, representing the variation of wave mode velocity, V, with film thickness, d, and wavelength, λ_T, are usually plotted as V versus normalized thickness, d/λ_T, where λ_T is the wavelength of transverse waves. Depending on the acoustic properties of the film relative to those of the substrate, the wave velocity increases (stiffness effect) or decreases (loading effect) with the ratio d/λ_T. In this work, we consider the loading effect for rapid materials when wave velocity in the film material is smaller than the corresponding wave velocity in the substrate material and chose the lowest mode known as Rayleigh mode. The calculation procedure for each normalized film thickness (ranging from 0 to 2) consisted of (i) calculating $V(z)$ signatures from Sheppard and Wilson formula, (ii) analyzing such signatures via fast Fourier transform and (iii) deducing propagating Rayleigh velocities. This investigation was carried out on several thin materials: SiC, AlN, Si_3N_4, Al_2O_3, TiN, MgO, Zn, Mg, W, Cr, ...with 2225 m/s $< V_R <$ 6810 m/s on a fast substrate of Be with $V_R = 7844$ m/s. The usual dispersion curves of slow on fast systems were obtained: at $d/\lambda_T = 0$, the deduced curves start from the velocity of leaky Rayleigh waves on the Be substrate, and as $d/\lambda_T = 2$ increases the velocity decreases to asymptotically approach the velocity of leaky Rayleigh waves of each layer material. However, it was found that the critical normalized thickness, $(d/\lambda_T)_{\text{crit}}$, at which the transition from the initial decrease to the final saturation of the dispersion curves occurs differ from one layer to the other. Therefore, to establish a general relation describing such a behavior, we defined an acoustic parameter describing the ratio of the velocities and densities of the films and the substrate such that $\xi = (V_f/V_s)/(\rho_f/\rho_s)$. Hence, a plot of $(d/\lambda_T)_{\text{crit}}$ versus ξ led to a straight line dependence of the form:

$$(d/\lambda_T)_{\text{crit}} = 1.2 - 1.25\xi = 1.2 - 1.25(V_f/V_s)/(\rho_f/\rho_s).$$

The importance of such a formula lies in its universality, its applicability to a great number of materials in order to differentiate between the acoustic properties of the layer and those mixed with the substrate.

Dark field scanning acoustic microscopy investigations of surface acoustic wave propagation in solid materials

Z. Hadjoub, S. Bouhedja, I. Hadjoub, A. Doghmane

Laboratoire des Semi-Conducteurs, Département de Physique,
Faculté des Sciences, Université Badji-Mokhtar,
Annaba, BP12, DZ-23000 Algeria
e-mail: `lsc_contact@yahoo.fr`

Recent studies based on the suppression of certain surface acoustic wave propagating modes were reported in order to show, qualitatively, the variations of fundamental parameters characterising materials elastic properties. In this quantitative work, we suppressed the central beam of an acoustic lens to obtain a dark field, in the scanning acoustic microscopy, SAM, technique. Hence, by placing successive absorbing stops of different diameters, we calculated acoustic materials signatures, $V(z)$ from Sheppard and Wilson formula, using corresponding filtered reflectance functions for annular lenses. Normal operating conditions of a conventional SAM were chosen: a lens half-opening angle of 50°, water as coupling liquid and operating frequency, f, of 142 MHz to 500 MHz. Such a method is applied to several materials (Be, SiC, AlN, Si_3N_4, Si, SiO_2, quartz, duraluminium, Al, Heavy flint) representing heavy and light as well as rapid and medium materials characterised by Rayleigh velocities varying from 2730 m/s to 7850 m/s. To enrich this investigation we used two methods, analytical and spectral, which gave identical results. We also carried out a complete investigation not only on the behaviour of $V(z)$ curves (period and damping), but also on relative errors as a function of occulted angles. It was shown as the dark spot increases the periods in $V(z)$ curves increase to finally completely disappear for large stop diameters. From the $V(z)$ curves, we were able to determine Rayleigh wave attenuation, α, from the exponentially decaying $V(z)$ free interference curves obtained with annular lenses. The dependence of such coefficients on Rayleigh velocities, V_R, as well as on operating frequencies were then investigated to establish a useful empirical relation between these parameters of the form: $\Delta\alpha/\Delta f = 1.35 \cdot 10^{-3} V_R - 3.39$. This formula, which can be applied to several materials (knowing their V_R), establishes the interdependence between three important acoustic parameters: frequency, attenuation and SAW velocities.

Complex rays in Minkowski space

Elman Hasanov
College of Arts and Science, Mathematics Department, Isik University,
Buyukdere Caddesi, 34398, Maslak, Istanbul, Turkey
e-mail: elman.hasanov@isik.edu.tr

A new geometrical theory of complex rays based on three dimensional Minkowski space has been developed. It is shown that the solutions of the classical eikonal equation

$$(\nabla S)^2 = 1 \tag{1}$$

subject to the initial condition $S(x, y, 0) = \varphi(x, y)$ where $\varphi(x, y)$ is twice continuously differentiable function, can be considered as rays in the three dimensional Minkowski space with a signature $(+, -, -)$. If $\varphi(x, y)$ is real valued, then ray is called real if $1 - \varphi_x^2 - \varphi_y^2 > 0$ and complex if $1 - \varphi_x^2 - \varphi_y^2 < 0$. The real rays lay in the light cone and the complex rays lay outside of the light cone, so complex rays, as real ones, may have quite definite direction in the mentioned space. In this interpretation, solving eikonal equation has quit simple geometric meaning: it is drawing normals to the initial surface in the Minkowski space. Therefore, caustic surfaces are the locus of the focal points of $\varphi(x, y)$ in the sense of the indefinite metrics of Minkowski space. This allows as immediately write out not only the equation for caustic surface via the function $\varphi(x, y)$, but also to solve inverse problem, i.e. to determine $\varphi(x, y)$ by the given caustic surface by using simple algebra. If the function $\varphi(x, y)$ is complex valued, $\varphi(x, y) = u(x, y) + iv(x, y)$, then $1 - \varphi_x^2 - \varphi_y^2 = 1 - u_x^2 - u_x^2 + v_x^2 + v_y^2$ still is real, and we say that ray is real if $1 - \varphi_x^2 - \varphi_y^2 > 0$ and complex otherwise. In contrast to the real case, in the case of complex $\varphi(x, y)$, complex rays may have real components and real rays may have complex components. As in the real case, we can formally define Minkowski normals to the complex valued $\varphi(x, y)$ and via this determine complex radii of curvature and complex caustics. The developed approach is applied to the Gaussian shape beams. In this case $\varphi(x, y) = iv(x, y)$ $(v(x, y) > 0)$ and $1 - \varphi_x^2 - \varphi_y^2 = 1 + v_x^2 + v_y^2 > 0$, so the Gaussian shape beam propagating in the direction $\mathbf{e} = (0, 0, 1)$ corresponds to real rays with complex components. In order to get the standard Gaussian beam with "waist" R, one may set $\varphi(x, y) = \sqrt{x^2 + y^2 - R^2}$ which represents a pseudosphere of imaginary radius iR if $x^2 + y^2 > R^2$ and a pseudosphere of radius R otherwise. So the Gaussian beam of "waist" R has simple geometric meaning which illuminates its ray structure: it is normals to the pseudospheres in Minkowski space, moreover the normals to the pseudosphere of radius R represents real rays and and and the normals to the pseudusphere of radius iR represents the complex rays, i.e. the diffracted part of the propagation. The corresponding solution of (1) is

$$S = \sqrt{x^2 + y^2 + (z + iR)^2}$$

which is known in the literature as a complex distance. Then the field

$$u(x, y, z) = \frac{1}{R - iz} e^{ikS}$$

paraxially represents the Gaussian beam propagating in the direction of the $z-$axis. The Gaussian beams can be generalized somehow to solve the diffraction problem for Gaussian shaped beams in the plane $z = 0$, if one sets $\varphi(x, y) = ax + by + iv(x, y)$. It can be shown,that the corresponding solution represents the Gaussian shape beam, propagating in the direction $\mathbf{e} = (a, b, 1)$ if $v(x, y) = f(ay - bx)$, and $f > 0$ is any function satisfying

$$f'^2(ay - bx) > \frac{a^2 + b^2 - 1}{a^2 + b^2}$$

Transverse patterns produced by interference of arrays of Bessel beams

Morales Marcelino Anguiano, Otero M. Maribel Méndez

Postgrado en Optoelectrónica, FCFM, BUAP, 72000 Puebla; Pue. México

Sabino Chávez-Cerda, Castillo M. David Iturbe

Instituto Nacional de Astrofisica, Óptica y Electrónica, 51 y 216 72000 Puebla; Pue. México
e-mail: `sabino@inaoep.mx; diturbe@inaoep.mx`

In this paper we present experimental results on the field distributions obtained when ordered arrays of equal zero-order Bessel beams are superimposed. Depending on the propagation angle, position and relative phase of interfering beams the resultant field can be considered as propagation invariant or not. Numerical simulation supports the experimental observations.

Zero-order Bessel beams can be produced using an annular slit and a positive lens or using an axicon. In this work we used both conditions in order to have different angles between the beams: the first condition produced paralel beams, the second condition produced beams that interfere with some angle.

In order to obtain the set of Bessel beams, whose position and relative phase were controlled independently, we used two coupled Michelson interferometers. The resultant system allowed obtaining four independent beams with the same intensity at the output of the interferometer. This output was used to illuminate the annular mask or the axicon where the number of beams, position, angle and relative phases could be adjusted at will. The interference of up to four Bessel beams could be analyzed with the same system.

Initially, we analyzed the case of parallel co-propagation of the interfering beams. No matter the number of beams, as expected the interference resulting pattern always was propagation invariant. In the case when two beams interfered, the resultant pattern showed a family of ellipses and hyperbolae that intersect perpendicularly. More complicated distributions were obtained when the number of interfering beams was increased, where the positions of the beams were not concentric. In this case when the four beams interfered the resultant intensity pattern exhibited square or circular symmetry, depending on the position and relative phase between the beams.

In the case when the axicon was used to generate the Bessel beams, the resultant interference pattern was not propagation invariant. The reason for this was that the propagation axis of the beams were not parallel. When just two beams were superimposed the output pattern did not exhibit the ellipses and hyperbolas observed in the case of parallel propagation and the distribution changed as the propagation distance increased.

Due to the variety of patterns obtained by the superposition of Bessel, we think that these distributions can be used for example in manipulation of micro-particles or in testing of optical elements. In conclusion, we have presented an experimental and numerical study of the patterns obtained by the superposition of several zero-order Bessel beams and verified that propagation invariant distributions are obtained when parallel beams are used.

Noncommutative nano- and micro-structures in resonance wave channels

Mikhail Karasev

Moscow Institute of Electronics and Mathematics, Russia

We discuss quantum (noncommutative) effects which accompany the classical wave propagation in resonance channels. Nano- and micro- zone structure, the adiabatics, tunneling, trapping are considered. Some details around these topics can be found in the author's paper: arXiv, math. QA/0412542, Part II.

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Resonance magneto-atoms and algebras with non-Lie commutation relations

M. Karasev
Moscow Institute of Electronics and Mathematics, Russia

We consider a model of the so-called artificial 2-dimensional atom, which is just a central dot surrounded by electrons. We insert an additional homogeneous magnetic field to this model and analyse the structure of the spectrum. We observe algebras with non-Lie commutation relations whose irreducible representations control the splitting of the energy levels of such a magneto-atom under perturbations. Using the hypergeometric Kaehlerian structures and coherent states we calculate these representations and derive a model equation which determines the leading corrections to the spectrum. This equation is explicitely resolved for excited energy levels by the global semiclassical technique.

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Radiation of seismic waves from a source in a fluid-filled borehole surrounded by infinite poroelastic medium

B. Kashtan, A. Bakulin, S. Ziatdinov, S. Golovnina
St.Petersburg, Russia

The paper focuses on deriving a far-field seismic radiation of body waves into surrounding poroelastic medium from fluid-filled borehole in-low frequency approximation. Tube waves propagating along a borehole for different types of interface conditions are also described. Far-field radiation pattern of a point pressure source is derived based on steepest descent method.

Theory and results. Seismic sources inside a fluid-filled borehole are used extensively in seismic exploration. These sources are used for vertical seismic profiling and cross-well tomography. Mathematical treatment of such problem for elastic (non-porous) surrounding medium is given by Lee and Balch (1982). To describe the influence of a borehole Ben-Menahem and Kostek (1991) introduced a concept of effective seismic source that radiates the same wavefield (in far zone) in unbounded medium without borehole as the actual seismic source does in a presence of the well. In addition to the pattern diagram the propagating tube waves inside the hole sometimes generate very high amplitude body waves (Parrot, 1980).

We describe influence of permeability and porosity of surrounding medium on body wave radiation patterns using far-field approximation. We also describe tube waves propagating along the borehole.

The analysis of acoustic wave propagation in fluid-filled porous media is based on Biot theory. Constitutive relations, the balance equation and generalized Darcy law of the modified Biot theory yield a coupled system of differential equations which describe the wave propagation in surrounded medium (Parra, 1993). The solution of the problem is constructed in the form of repeated integrals: the outer integral is along the frequency, the inner integral is along vertical wavenumber. The two different types of boundary conditions are considered (open and closed pores). The dispersion equation of the problem is analyzed in low-frequency seismic regime. The tube-wave field is described as residual of dispersion equation in the low-frequency approximation.

Radiation patterns of fast and slow compressional fast and shear body waves are described using steepest descent method. The contours of steepest descent for phase functions are numerically constructed for different types of waves. Radiation patterns are computed in the far-field approximation for point pressure source.

Radiation patterns for fast and slow compressional and shear waves are presented for different values of permeability and porosity of surrounded poroelastic medium.

The asymptotical results are compared with the finite difference computations. The comparison shows the wide validity of the approximate results for seismic applications.

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Additional components of the Rayleigh wave

S. Ziatdinov, B. Kashtan

St.Petersburg, Russia

In our work we present the solution of the Rayleigh wave problem in the homogeneous isotropic half space. Unlike classical solution of the Rayleigh problem, solution, given in [2], has the additional shear components of Rayleigh wave displacement vector that are more low-frequency than main components of Rayleigh wave. That's why, this low-frequency waves could be considered as additional in the terms of ray-method. The purpose of our work is to show the fact that the solution presented in [2] of the isotropic elastic equations for the halfspace is an asymptotic estimation of a well-known solution for wavefield, excited by tangent force applied to the free surface of the elastic halfspace.

We pick out the wavefield of Rayleigh wave from the analytic solution. This solution contains longitudinal and shear components and we compare this asymptotic solution with full solution from [1].

Basing on analytical solution, we can say that radial part Rayleigh wave displacement vector contains main and additional components. Additional component is one order less in frequency and one order more damped with distance than the main. At the time the amplitude of azimuth part of Rayleigh wave displacement vector fully coincide with additional component of radial part.

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Restoration of electrical conductivity and elastic parameters by iterative approach

Yurii V. Kiselev, Vladimir N. Troyan

Institute of Physics, St.Petersburg University, Ul'yanovskaya 1, 198504, Russia
e-mail: `Kiselev@JK6907.spb.edu; Troyan@hq.pu.ru`

The results of numerical simulation on restoration of electrical conductivity and elastic parameters (Lame parameters and mass density) of the local inhomogeneities correspondingly with the help of electromagnetic and elastic sounding signals are considered.

The direct problems for the Maxwell and Lame equations are solved by the finite difference method. Restoration of the desired parameters is implemented by the diffraction tomography method in the time domain [1], [2] using the first-order Born approximation.

We consider restoration of the inhomogeneity parameters with the help of the iterative procedure. Each step of the iterative procedure includes solution of the direct problem and correction of the desired parameters by the diffraction tomography method. For this purpose the algebraic methods with consequent regularisation schemes are used.

Restoration of the electrical conductivity is studied in the low frequency case. Main attention is given to study of convergence and accuracy of the considered algorithms.

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Application of Lagrange structures for analyses of electromagnetic wave fields to reflection of the plane monochromatic waves problem

O. A. Kazakov

Moscow State Technical University "STANKIN", Moscow, Russia

e-mail: `lsoef@mail.ru`

For periodic electromagnetic processes in homogeneous, isotropic and neutral medium with constant or time-dependent conductivity and dielectric and magnetic (μ) permittivity one can obtain the following representation of average wave lagranjian on vector-function $\mathbf{E}(r,t)$ (the solution of wave equation)

$$\bar{L}_w(\mathbf{r}) = \frac{1}{T}\int\limits_0^T dt L_w(\mathbf{r},t), \qquad L_w(\mathbf{r},t) = \frac{1}{\omega_0 c}\left[\frac{1}{4\pi}\left(\frac{\partial \mathbf{E}}{\partial t}\frac{\partial \mathbf{D}}{\partial t} - \frac{\partial \mathbf{H}}{\partial t}\frac{\partial \mathbf{B}}{\partial t}\right) - \frac{1}{2}\left(\mathbf{E}\frac{\partial \mathbf{j}}{\partial t} - \mathbf{j}\frac{\partial \mathbf{E}}{\partial t}\right)\right],$$

where $\omega_0 = 2\pi/T$, T is period. Reactive powers (actions) $\mathrm{Q} = -\int\limits_{\mathrm{V}} dv \bar{L}_w(\mathbf{r})$ of objects of ideal electromagnetic oscillator are positive for inductance and negative for capacitance. We can consider electromagnetic field in the volume of oscillator as simple harmonic standing wave. Surfaces $\bar{L}_w(\mathbf{r}) = 0$ separate areas of strength bulges of electric and magnetic fields, where $\bar{L}_w(\mathbf{r}) > 0$ corresponds capacitance and $\bar{L}_w(\mathbf{r}) < 0$ corresponds inductance. Therefore we can use space functions $L_w(\mathbf{r},t)$ or $\bar{L}_w(\mathbf{r})$ for detection of space structures (Lagrange structures) of electromagnetic field, which are similar to standing waves structures.

Each physical characteristic help detect some physical phenomena, which are invisible while using other characteristics. Thus lagranjian and Lagrange structures help to detect some new special angles of incidence in addition to limiting angle of perfect reflection $\varphi_r = \arcsin(1/n_{12})$ and angle of Bruster $\varphi_{\mathrm{B}} = \arcsin\left(1\big/\sqrt{n_{12}^2+1}\right)$ when dealing with reflection of plane monochromatic wave problem.

Next notations are used in our case: the plane yz is a plane of incidence; the plane xz is a plane separating medium 1 (with incident and reflecting waves, $y < 0$) and medium 2 (with refracted wave, $y > 0$); ε_1, ε_2 are dielectric constants; $n_{12} = \sqrt{\varepsilon_1/\varepsilon_2}$ is relative coefficient of refraction ($\mu_1 = \mu_2 = 1$); φ is the angle of incidence.

If $n_{12} > 1$ and $\varphi > \varphi_{\mathrm{r}}$ there exists the angle of appearance of a new layer in Lagrange structure (Fig. 1)

$$\varphi_n = \arcsin\sqrt{\frac{\left[\left(n_{12}^2+1\right)^2 + 2n_{12}^2\cos 2\beta \pm \sqrt{\left(n_{12}^2+1\right)^2\left(n_{12}^2-1\right)^2 + 4n_{12}^4\cos^2 2\beta}\right]}{2n_{12}^2\left(n_{12}^2+1\right)\left(1+\cos 2\beta\right)}}.$$

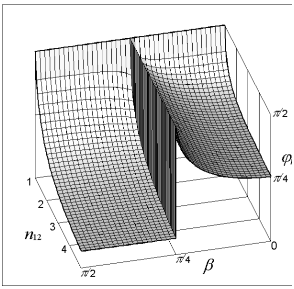

Fig. 1.

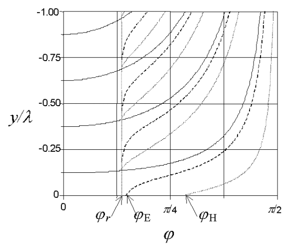

Fig. 2.

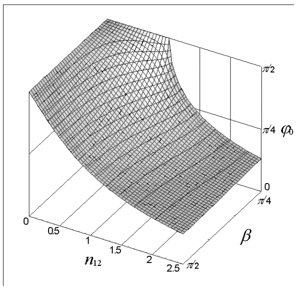

Fig. 3.

Here sign "+" is used when $0 \le \beta < \pi/4$, and sign "−" — when $\pi/4 \le \beta < \pi/2$. For H-wave ($\beta = 0$) $\varphi_n = \varphi_{\rm H}$, and for E-wave ($\beta = \pi/2$) $\varphi_n = \varphi_{\rm E}$ (Fig. 2). Lagrange structure in medium 1 don't exist $\bar{L}_w\,(y<0) \equiv 0$, when $\pi/4 \le \beta < \pi/2$ and angle of incidence is equal to $\varphi_{01} = \arcsin\sqrt{\left(n_{12}^2 + 1 - \sqrt{\left(n_{12}^2+1\right)^2 - 4n_{12}^2\sin^2 2\beta}\right)\Big/ 2n_{12}^2\sin^2 2\beta}$ (Fig. 3). For E-wave: $\varphi_0 = \varphi_{\rm B}$.

When $\beta \in [\mathrm{arctg}(1/n_{12}), \ \mathrm{arctg}(n_{12})]$ there exists the angle of incidence $\varphi_{02} = \arcsin\sqrt{1/\left[\left(n_{12}^2+1\right)\cos^2\beta\right]}$, for which $\bar{L}_w\,(y>0) \equiv 0$. If $\beta = \mathrm{arctg}(1/n_{12})$, then $\varphi_{02} = \varphi_r$ and if $\beta = \mathrm{arctg}(n_{12})$, then $\varphi_{02} = \pi/2$.

The following angle of incidence may be of interest $\varphi_c = \arcsin\sqrt{1/\left(n_{12}^2 + \cos 2\beta\right)}$, for which lagranjian is uninterrupted $\Delta\bar{L}_w\,(\varphi_c, \beta, y=0) = \bar{L}_w\,(\varphi_c, \beta, y=0-) - \bar{L}_w\,(\varphi_c, \beta, y=0+) = 0$ on the medium separating plane. This angle exists if $\pi/4 \le \beta \le \arcsin\left(\min\left(n_{12}/\sqrt{2}, 1\right)\right)$. When $\beta = \arcsin\left(\min\left(n_{12}/\sqrt{2}, 1\right)\right)$: $\varphi_c = \pi/2$ and if $\beta = \pi/4$ then $\varphi_c = \varphi_r$.

The heat source on the matrix space

S. Khekalo

Saint-Petersburg department of Steklov's Mathematical Institute, Russia
e-mail: khekalo@pdmi.ras.ru

1. The heat source. Let V_m be a cone of positive defined real symmetrical $m \times m-$matrices; $M_{n,m}$ be a space of real rectangular $n \times m-$matrices $(n \ge m \ge 1)$; tr, det be signs of the trace and the determinant of a matrix; $*$, $'$ be signs of the convolution and the transpose correspondingly.

DEFINITION 1 ([1]). *The function*

$$h_a(x) = \frac{1}{(4\pi)^{nm/2}\,(\det a)^{n/2}}\ \mathrm{e}^{-\frac{1}{4}\mathrm{tr}(a^{-1}\,'xx)}, \quad a \in V_m, \ \ x \in M_{n,m}$$

is called the heat source on the space $V_m \times M_{n,m}$.

THEOREM 1.*The heat source satisfies conditions:*

$$h_a(x) \ge 0; \qquad h_a * h_b(x) = h_{a+b}(x);$$

$$\int\limits_{M_{n,m}} h_a(x)dx = 1; \qquad \lim_{a\to 0}\int\limits_{'xx \ge b} h_a(x)dx = 0, \ \ b \in V_m.$$

COROLLARY.*In the sense of generalised functions we have the equality*

$$\lim_{a\to 0} h_a(x) = \delta(x)$$

where δ *is the Dirac delta-function on the* $M_{n,m}$.

2. The heat operators on the $V_m \times M_{n,m}$. For $j, k, s = 1, ..., m$ let

$$\mathcal{D}_k = \det\begin{pmatrix} \partial/\partial a_{11} & \cdots & \frac{1}{2}\partial/\partial a_{1k} \\ \vdots & \ddots & \vdots \\ \frac{1}{2}\partial/\partial a_{1k} & \cdots & \partial/\partial a_{kk} \end{pmatrix}$$

and

$$\Delta_k = \det\begin{pmatrix} '\partial_1\partial_1 & \cdots & '\partial_1\partial_k \\ \vdots & \ddots & \vdots \\ '\partial_k\partial_1 & \cdots & '\partial_k\partial_k \end{pmatrix}, \quad '\partial_s\partial_j = \sum_{l=1}^{n}\frac{\partial^2}{\partial x_{sl}\partial x_{lj}},$$

are generate elements of the Gindikin and the Cayley-Laplace semigroups of linear differential operators correspondingly [2],[3].

We denote by

$$\mathcal{D}^p = \mathcal{D}_1^{p_1}...\mathcal{D}_m^{p_m}; \qquad \Delta^p = \Delta_1^{p_1}...\Delta_m^{p_m}, \ \ p_j \in \mathbf{Z}_+.$$

DEFINITION 2. *The linear differential operator*

$$\mathcal{H}^p = \mathcal{D}^p - \Delta^p$$

is called the heat operator on the $V_m \times M_{n,m}$.

EXAMPLE. For $m = 1$ and $p_1 = 1$ the operator $\mathcal{H}^p$ is the classical heat operator on the $\mathbf{R}_+ \times \mathbf{R^n}$:

$$\mathcal{H}^p|_{m=1,p_1=1} = \frac{\partial}{\partial a_{11}} - \frac{\partial^2}{\partial x_{11}^2} - ... - \frac{\partial^2}{\partial x_{1n}^2}$$

THEOREM 2. *The heat source* $h_a(x)$ *is the solution of the homogeneous heat equation* $\mathcal{H}^p h_a(x) = 0$.

3. Homogeneity of the heat operator. Let $O(n)$ be a group of ortogonal $n \times n-$matrices and T_m^+ be a group of lower triangular $m \times m-$matrices.

The group of transformations $g(\rho, \tau)$, $\rho \in O(n)$, $\tau \in T_m^+$, acts on the space $V_m \times M_{n,m}$ by the rule

$$g(\rho,\tau)\begin{pmatrix} a \\ x \end{pmatrix} = \begin{pmatrix} '\tau & 0 \\ 0 & \rho \end{pmatrix}\begin{pmatrix} a \\ x \end{pmatrix}\tau.$$

THEOREM 3. *The heat operator* $\mathcal{H}^p$ *is homogeneous to the action of the group of transformations* $g(\rho, \tau)$ *on the* $V_m \times M_{n,m}$:

$$\mathcal{H}^p[gf](a,x) = \left[\prod_{k=1}^{m}(\tau_{11}...\tau_{kk})^{p_k}\right]\mathcal{H}^p[f]('\tau a\tau, \rho x\tau)$$

where $gf(a,x) = f('\tau a \tau, \rho x \tau)$.

4. The existence theorem. Let $f(x)$ be a bounded smooth function on the $M_{n,m}$. We define the Poisson integral on the $V_m \times M_{n,m}$ by the equality

$$(W_a\, f)\,(x) = \int\limits_{M_{n,m}} h_a(x-y)f(y)dy.$$

It is clear, that the maximum principle

$$\inf_{M_{n,m}} f(x) \le (W_a\, f)\,(x) \le \sup_{M_{n,m}} f(x)$$

and the semigroup property

$$(W_a\,(W_b\, f))\,(x) = (W_{a+b}\, f)\,(x)$$

are executed.

THEOREM 4. *The function* $u(a,x) = (W_a\, f)(x)$ *is the solution of the heat equation* $\mathcal{H}^p u(a,x) = 0$ *on the* $V_m \times M_{n,m}$ *with the initial heat radiation*

$$u(0,x) \equiv \lim_{a\to 0} u(a,x) = f(x)$$

in the vertex of the cone V_m.

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Diffraction by a corrugated interface: the regularities of complete transformation of an incident plane wave to the diffraction lobe with influence of profile corrugation form

E. V. Koposova, S. N. Vlasov

Institute of Applied Physics of the Russian Academy of Sciences, Nizhny Novgorod, 46, Ulyanov str, Nizhny Novgorod, 603950, Russia
e-mail: ar@sandy.ru

We present the results of a study of the regimes of complete transformation of a plane wave to the diffraction lobe when the wave is incident on a corrugated metal surface or a corrugated boundary of two dielectric media. The purpose is to study the influence of the corrugation profile form, and also the present of losses in the material to the regularities of complete transformation regimes. For study of influence of profiles the series of various profiles distinguished from a sinusoid by a breadth of a resonance region and presence of the additional elements in it undertakes. The profiles having conjugate resonance regions with a scale about a wavelength are considered, on which owing to interaction of waves it is possible to guess existence of qualitative new conditions of a diffraction in relation to observed on a sinusoid, and also profile containing small-scale deviations, which it is possible to suppose by a model of a really produced gratings. For study of influence of losses the permittivity in the task of scattering on the corrugated boundary of dielectrics is put complex. The comparative analysis of results obtained in the previous paper of the authors for the sinusoidal form and without losses [1] is carried out. The tactics of purposeful numerically - intuitive searching of the form of the profile with possibilities of complete not-autocollimation transformation in a wide band of parameters under conditions, at which on the sine profile the complete transformation is possible only in a condition of an autocollimation is demonstrated. A numerical-intuitive pattern of appearance and evolution of fully-reflection regimes is revealed. The common regularities and influence of the profile to a condition of an enlightenment of normally incident waves on a corrugated dielectric surface are explored.

The examination is based on a numerical method of solving the integral equation [2,3] by means of a specially created interactive processing system in Visual Fortran 6.6B. The program has graphics friendly interface which is designed for Windows [4]. It allows the man to interfere with evaluations process for modification, changing of parameters during calculations according to intuitive representations.

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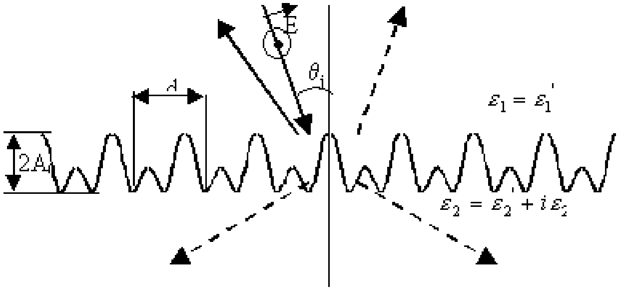
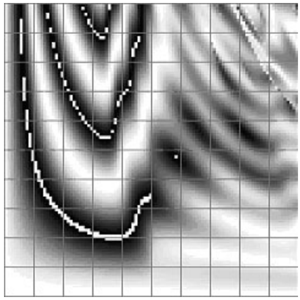


Corrugated boundary, incident and scattered waves.



Total reflection -1st-order harmonic.

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On stability of a charge topological soliton in the system of two interacting scalar fields

N. B. Konyukhova, S. V. Kurochkin

Dorodnicyn Computing Centre of RAS, Vavilov str., 40, Moscow, 119991 Russia
e-mail: `nadja@ccas.ru; kuroch@ccas.ru`

V. A. Gani, V. A. Lenskii

Institute for Theoretical and Experimental Physics,
Bol'shaya Cheremushkinskaya str., 25, Moscow, 117259 Russia
e-mail: `gani@heron.itep.ru`

A brief presentation of some results of [1] on statement and analytical-numerical study of a self-adjoint singular spectral problem for a system of three linear second-order ordinary differential equations (ODEs) defined on the entire real axis is given. The spectral parameter enters ODEs in a non-linear way forming quadratic operator Hermitian pencil. The problem arises in the stability analysis (using linear perturbation theory) of the exact regular time-dependent solution in one space dimension (charged topological soliton) for a system of two nonlinear wave equations; the solution was obtained in [2] for a certain field theory model suggested in [3]. The analysis of the spectral problem shows that the solution is dynamically stable.

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Dynamic behavior of composite piezoelectric actuators

I. D. Kochetkov

Dept. of Applied Mathematics, Institute Economy Communal and Engineering,
Srednaya Kalitnikovskaya 30, 109807 Moscow, Russia

The theory of laminated piezoelectric actuators constructed by the asymptotic method is used to study dynamic electroelastic state and optimize the efficiency of an actuator as energy converter. An important characteristic of a piezoelectric actuator efficiency is the electromechanical coupling coefficient (EMCC).

It was shown in [1-2], that there is the general energy method for determining the EMCC which is valid for any static and dynamic state, for any electroelastic structure.

As an example the EMCC for cylindrical piezoelectric transducer and three layered piezoelectric beam are calculated and optimized as a function of different parameters. The EMCC depending on a size, a shape of electrodes, and thicknesses of elastic and piezoelectric layers is analyzed using the general energy method.

It is widely believed that the EMCC for dynamic state of any structure is less than the EMCC for its static state. In this paper it is shown that the EMCC for dynamic behavior can be extended to a maximum static value due to a special shape of electrodes.

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On the inverse spectral theory for singularly perturbed operators

V. Koshmanenko

Institute of Mathematics, Kyiv, Ukraine

Let A be an unbounded from above self-adjoint operator in a separable Hilbert space $\mathcal{H}$ and $E_A(\cdot)$ its spectral measure. We discuss the inverse spectral problem for singular perturbations $\tilde{A}$ of A ($\tilde{A}$ and A coincide on a dense set in $\mathcal{H}$). We show that for any $a \in \mathbb{R}$ there exists a singular perturbation $\tilde{A}$ of A such that $\tilde{A}$ and A coincide in the subspace $E_A((-\infty, a))\mathcal{H}$ and simultaneously $\tilde{A}$ has an additional spectral branch on $(-\infty, a)$ of an arbitrary type. In particular, $\tilde{A}$ may possess the prescribed spectral properties in the resolvent set of the operator A on the left from a point a. Moreover, for an arbitrary self-adjoint operator T in $\mathcal{H}$ there exists $\mathbf{A}$ such that T is unitary equivalent to a part of $\mathbf{A}$ acting in an appropriate invariant subspace.

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Application of the R-functions method for solving a mixed inverse diffraction problem

M. A. Basarab
Bauman Moscow State Technical University,
107005, Vtoraya Baumanskaya St., 5, Moscow, Russia
e-mail: bmic@mail.ru

V. F. Kravchenko
Institute of Radio Engineering and Electronics RAS,
103907, Center, GSP-3, Mokhovaya St., Moscow, Russia
e-mail: kvf@pochta.ru

Application of the R-functions theory [1-4] for solving the mixed inverse problem of flat electromagnetic wave diffraction by a doubly-connected scatterer is considered. Here, we suppose that both parts of the scatterer are infinitely long cylinders with perfectly conducting thin surfaces generated by smooth or piecewise smooth planar closed curves. The incident field does not depend on the longitudinal coordinate. Properties of one single arbitrary star-shaped scatterer are known while the geometry and boundary conditions for another scatterer must be found [5].

A combined approach is used when on the first stage, a far-zone field scattered by the first body of arbitrary shape is evaluated on the base of the modified method of discrete sources in combination with the R-function method [2-4]. R-operations of conjunction and disjunction are applied for analytical description of the contour of a star-shaped domain generated by theoretical-set intersection or join of two or more simple regions. Then this boundary is deformed so that we deal with a first-kind integral equation with different ranges of kernel arguments definition instead of the original diffraction singular integral equation. The first-kind integral equation is solved by the collocation technique and the far-zone field scattered by the first body is found. On the second stage of the process, the geometry and boundary conditions for the second scatterer are defined using the schemes proposed in [5, 6].

The method presented and justified may be expanded onto the more general case of a multiply connected complex-shaped scatterer with arbitrary boundary conditions on different parts of its boundary.

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Asymptotic solutions of real symmetric systems with multiplicity

Andriy Kryvko, Valeri V. Kucherenko

Instituto Politecnico Nacional, Escuela Superior de Fisica y Matematicas, Mexico D.F.
e-mail: valeri@esfm.ipn.mx

To construct the asymptotic solution of a system of ordinary differential equations with small parameter h near the derivatives one considers the determinant of its main symbol $\|Q_{ij}(\xi, x)\|$ and determines the roots of the equation $\det\|Q_{ij}(\xi, x)\| = 0$ with respect to the variables ξ: $\xi = \lambda_j(x)$, $j = 1, \ldots, \nu$. Suppose that at the point ξ_0, x_0 it holds true that:

$$\det\|Q_{ij}(\xi_0, x_0)\| = 0, \quad \nabla_{\xi,x}\det\|Q_{ij}(\xi_0, x_0)\| = 0, \quad \text{but} \quad Hess\det\|Q_{ij}(\xi, x)\||_{\xi_0,x_0}$$

is a nondegenerate matrix. In this case the problem is reduced to the problem of construction of the asymptotic for the second order system with the main scalar symbol $q^2(\xi, x) - r^2(\xi, x) - s^2(\xi, x)$. Let $\frac{\partial q}{\partial \xi}(\xi_0, x_0) \neq 0, \frac{\partial r}{\partial \xi}(\xi_0, x_0) = 0, \frac{\partial s}{\partial \xi}(\xi_0, x_0) = 0$ then the method described in the paper [1] can be applied to construct the asymptotic solution of the system at a point x.

In the paper [2] the problem of construction of the asymptotic solutions for symmetric hyperbolic systems of partial differential equations with double multiplicity(for example the equation of crystal-optics) is reduced to the problem of construction of the asymptotic solutions for the model problems:

$$\frac{\partial u}{\partial t} + \begin{bmatrix} \frac{\partial}{\partial x_1} & t\frac{\partial}{\partial x_2} \\ t\frac{\partial}{\partial x_2} & -\frac{\partial}{\partial x_1} \end{bmatrix} u = 0, \frac{\partial w}{\partial t} + \begin{bmatrix} \frac{\partial}{\partial x_1} & x_1\frac{\partial}{\partial x_2} \\ x_1\frac{\partial}{\partial x_2} & -\frac{\partial}{\partial x_1} \end{bmatrix} w = 0,$$

where $u|_{t=0} = \exp\left(\frac{i}{h}S_0\right)\psi(x)$, $w|_{t=0} = \exp\left(\frac{i}{h}S_0\right)\psi(x)$, $\psi \in C_0^\infty, S_0 \in C^\infty$.

Using the well-known relations

$$\begin{bmatrix} +r & s \\ s & -r \end{bmatrix}^2 = \left(r^2 + s^2\right) I$$

this problems can be reduced to an asymptotic problem of equations with the main scalar symbols $\xi_t^2 - t^2\xi_2^2 - \xi_1^2$ or $\xi_t^2 - x_1^2\xi_2^2 - \xi_1^2$. The method described in the paper [1] can be applied to solve the first one. The WKB method cannot be applied to construct the asymptotic of the Cauchy problem of the equation with the main scalar symbol $\xi_t^2 - x_1^2\xi_2^2 - \xi_1^2$ at a point $\xi_1 = 0, x_1 = 0$. By the Laplace transform this asymptotic problem can be reduced to the asymptotic problem for the stationary equation of oscillator with oscillating right-hand side. The last problem can be solved using the nonstationary Schrödinger equation for oscillator. In the papers [3, 4] the solution is represented through the Fourier integral operators with nonsmooth phases and amplitudes. Our approach allows to avoid these operators and to obtain the solution through the Maslov's canonic operator (or Fourier integral operator) with smooth phase and amplitude functions.

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Damping of whistler modes guided by a lossy anisotropic plasma cylinder

A. V. Kudrin, V. A. Es'kin, M. Yu. Lyakh
University of Nizhny Novgorod,
23 Gagarin Ave., Nizhny Novgorod 603950, Russia
T. M. Zaboronkova
Technical University of Nizhny Novgorod,
24 Minin St., Nizhny Novgorod 603950, Russia

We consider the propagation of modes guided by an axially magnetized cylindrical plasma channel with allowance for absorption of electromagnetic energy. The main attention is paid to modes in the whistler range. The interest in the subject is motivated by the fact that whistler mode waves play an important role in many applications, ranging from space plasma research to radio-frequency (rf) plasma sources used in general plasma physics experiments, plasma processing, etc. For some of these applications, the understanding of the damping mechanisms of modes in the whistler range is crucial.

It follows from our previous studies of the guided propagation of electromagnetic waves in plasma channels located in a background magnetoplasma that, at whistler frequencies between the lower hybrid frequency and half the electron gyrofrequency, one should necessarily take into account both electromagnetic whistler mode waves, also known as helicon waves, and quasi-electrostatic whistler mode waves (a type of lower hybrid wave), which are comprised simultaneously in the modal fields, in order to correctly determine the properties of guided modes [1, 2]. In this work, we apply the results of the above-mentioned analysis to eigenmodes of a magnetized cold-plasma column located in free space. Losses in the plasma are accounted for by introducing electron collisions. We determine the dispersion properties, damping rates, and field structures of the eigenmodes and discuss how these characteristics depend on the collision rate and the plasma distribution inside the column. It is shown that for a slightly lossy plasma in the column, one can distinguish weakly and strongly damped eigenmodes whose damping rates are determined by the relative contribution of quasi-electrostatic waves to the total modal fields. Moreover, due to the presence of quasi-electrostatic waves, even the collisional damping of weakly damped eigenmodes turns out to be notably greater than that of a pure helicon wave in a homogeneous magnetoplasma. It is established that such behavior of the mode damping rates is typical of bounded helicon discharge plasmas [3], the wave damping in which has been widely disputed in recent years [3–5]. The results obtained are shown to be useful for clarification of the features of the rf power absorption in radially inhomogeneous, collisional magnetized plasma structures capable of guiding electromagnetic waves.

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Curvilinear wave electrodynamics (CWED) — electrodynamics of electromagnetic waves, propagating along curvilinear trajectories

Alexander G. Kyriakos

Saint-Petersburg State Institute of Technology, St. Petersburg, Russia
e-mail: agkyriak@yahoo.com

As it is known according to Maxwell-Lorentz electromagnetic (EM) theory in very strong EM field an EM wave can bend its trajectory. *We suppose that under some conditions a linear EM wave is able to twirl and move along the closed curvilinear trajectory.* In our researches [1, 2] we show that in matrix forms the equations of such curvilinear waves mathematically fully coincides with quantum equations. Due to this fact all the optics effects of the curvilinear waves (diffraction, interference, dissipation, birefringence etc.) have parallels in the elementary particle theory and can be described in operator form of the quantum field theory.

The main concurrent both electromagnetic and quantum forms of CWED is the following.

Consider the equation of linear EM wave, propagating along y-direction, in matrix form:

$$\left[(\hat{\alpha}_o\hat{\varepsilon})^2 - c^2\left(\hat{\vec{\alpha}}\hat{\vec{p}}\right)^2\right]\Phi = 0,$$

where $\hat{\varepsilon} = i\hbar\frac{\partial}{\partial t}$, $\hat{\vec{p}} = -i\hbar\vec{\nabla}$ are the operators of the energy and momentum correspondingly, $\hat{\alpha}_o$, $\hat{\vec{\alpha}}$ are Dirac matrices, $\Phi^+ = \begin{pmatrix} \mathrm{E}_x & \mathrm{E}_z & -i\mathrm{H}_x & -i\mathrm{H}_z \end{pmatrix}$ is the (conjugate) EM field vectors matrix. The wave equation may be disintegrated on two Dirac-like equations without mass: $\Phi^+\left(\hat{\alpha}_o\hat{\varepsilon} - c\hat{\vec{\alpha}}\hat{\vec{p}}\right) = 0$ and $\left(\hat{\alpha}_o\hat{\varepsilon} + c\hat{\vec{\alpha}}\hat{\vec{p}}\right)\Phi = 0$, which are, as it is not difficult to check, the right Maxwell equations of the linear electromagnetic waves: retarded and advanced.

In case if the EM wave propagates through strong EM field (e.g. of a nucleus), the wave is twirled and for the time derivation of fields we have $\frac{\partial\vec{E}}{\partial t} = -\frac{\partial E}{\partial t}\vec{n} - E\frac{\partial\vec{n}}{\partial t} = -\frac{1}{4\pi}\frac{\partial E}{\partial t}\vec{n} + \frac{1}{4\pi}\omega_p E\cdot\vec{\tau}$, where $\omega_p = \frac{m_p c^2}{\hbar}$ is angular velocity, $\varepsilon_p = m_p c^2$ is photon energy, m_p is some mass, corresponding to the energy ε_p. Thus, from wave equation after the twirling we obtain the Klein-Gordon-like equation: $\left(\hat{\varepsilon}^2 - c^2\hat{\vec{p}}^2 - m_p^2c^4\right)\Psi = 0$, where Ψ is EM matrix after twirling. The disintegration of last equation gives the Dirac-like equations for particle and antiparticle with photon mass: $\left[\left(\hat{\alpha}_o\hat{\varepsilon} + c\hat{\vec{\alpha}}\hat{\vec{p}}\right) + \hat{\beta}m_pc^2\right]\psi = 0$ and $\psi^+\left[\left(\hat{\alpha}_o\hat{\varepsilon} - c\hat{\vec{\alpha}}\hat{\vec{p}}\right) - \hat{\beta}m_pc^2\right] = 0$, where ψ is EM matrix after EM wave breaking. As the analysis shows the free term in these equations contain the particle-antiparticle interaction energy. After the particle-antiparticle removing on large distance we obtain $\left[\left(\hat{\alpha}_o\hat{\varepsilon} + c\hat{\vec{\alpha}}\hat{\vec{p}}\right) + \hat{\beta}m_ec^2\right]\psi = 0$ and $\psi^+\left[\left(\hat{\alpha}_o\hat{\varepsilon} - c\hat{\vec{\alpha}}\hat{\vec{p}}\right) - \hat{\beta}m_ec^2\right] = 0$, which are simultaneously the Dirac electron equation and EM wave Maxwell equations with imaginary currents. In framework of CWED $\hat{\beta}m_ec^2 = -\varepsilon_{in} - c\hat{\vec{\alpha}}\vec{p}_{in} = -e\varphi_{in} - e\hat{\vec{\alpha}}\vec{A}_{in}$, where $(\varepsilon_{in}, \vec{p}_{in})$ describes the energy and momentum of inner field of EM electron-like particle. When we consider the electron-like particle from great distance, the field $(\varepsilon_{in}, \vec{p}_{in})$ works as a mass, and we obtain linear Dirac-like equations. Inside the electron-like particle the term $(\varepsilon_{in}, \vec{p}_{in})$ is needed for the detailed description of the inner field of an electron-like particle and carries us to non-linear EM equations [1, 2].

In the framework of CWED many experimental results of physics find explanation: the charge conservation law, the difference between boson and fermion particles, the neutrality of the Universe, the existence of particles and antiparticles, the EM twirled wave - particle duality, etc.

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Interaction of focused high-frequency ultrasound with flat interfaces and plane-parallel objects - theory and experimental data

V. M. Levin, Yu. S. Petronyuk

Laboratory of Acoustic Microscopy, Institute of Biochemical Physics, RAS,
4 Kosygin St , Moscow, 117977 Russia
e-mail: vlevin@online.ru

Interaction of focused ultrasound with obstacles includes non-ray phenomena even in the case of plane objects - plane interfaces or plane-parallel layers and plates. The effects are essential when a plane obstacle is placed nearby the focal region of the ultrasonic probe beam. There two sources of non-ray effects: 1) formation of diverse types of near-surface acoustic modes - leaky Rayleigh waves, lateral (head) waves, leaky guided modes, etc; and interference of them with specularly reflected radiation; 2) phase shifts between diverse wave components of the reflected beam.

A way to study the interaction is receiving the reflected radiation by the same ultrasonic focusing system. Its output signal can be employed to get information on the focused beam interaction and features of local microstructure and properties of the object. The signal is formed as a superposition of contributions of reflected beam plane wave components. It is expressed as an integral of the spatial spectrum of the incident beam, the aperture function of the focused system and fast oscillating phase multiplicand. Asymptotic methods have been applied to calculate the output signal. Results of the calculations are compared with experimental data.

Two possible experimental setups have been under investigation depending on angle aperture of the convergent probe beam.

A *high-aperture beam* gives rise an echo signal reflected from the specimen face only; reflection at internal interfaces and specimen bottom does not give well-shaped impulses because of loosing convergence by the transmitted ultrasonic beam due to high level of refraction aberrations at the specimen face. Performed asymptotic calculations reveal formation the specularly reflected signal as well as signals caused by excitation of leaky Rayleigh waves, longitudinal and transverse lateral (head) waves. In the case of the harmonic probe signal interference of these constituents of the reflected signal results in specific oscillations of the output signal when moving the focusing system towards the specimen face. The oscillations are employed for measuring local values of sonic wave velocities along the specimen surface and characterizing surface elasticity properties.

In the case of *small aperture* the probe beam penetrates into the specimen body keeping its convergent structure. Besides the front surface echo it generates echo signals reflected at internal interfaces and the specimen bottom. To resolve echo pulses ultra short probe signals are employed. Measuring time intervals between particular echoes makes it possible to determine local values of bulk elastic mode velocities and elastic moduli with micron resolution. The critical parameter in the measurement is shape of the echo pulses. It is shown formation of particular pulses is defined by occurrence of stationary points of the fast oscillating phase of the reflected signal. The stationary points correspond to formation of focuses, caustics or convergence points by the beam that penetrates into the specimen body. Number of ray participating in stationary points formation prescribes their power and specific features and, respectively, characteristic shape if particular echo signals. It has been shown these signals $U(t)$ may be expressed through the spectral component B_ω of the probe signal by the formula

$$U(t) = \int d\omega \cdot \frac{B_\omega}{\omega^\alpha} \cdot e^{-i\omega t},$$

where index α is determined by a type of the corresponding stationary point. For isotropic matter the index is able to take values $\alpha = 0$ (reflection from the front surface of the plate), 1 (paraxial focal point inside the specimen body) and 1/2 (the caustics crosses the plate bottom).

Catastrophes in sonic wave reflection at liquid-solid interfaces

V. M. Levin, T. A. Senjushkina

Laboratory of Acoustic Microscopy, Institute of Biochemical Physics, RAS,
4 Kosygin St , Moscow, 117977 Russia
e-mail: vlevin@online.ru

Angle dependence of the reflection and transmission coefficients, R and T, describes plane wave interaction with a flat interface. Experimental data evidence that ultrasound reflection essentially changes in going from one pair of contacting media to another. But no attempts have been made to follow reflection coefficient transformation when varying the parameters of contacting media. The paper contains theoretical analysis of ultrasound reflection at a *liquid – isotropic solid* interface. The angle dependence $R(\theta)$ is controlled by three governing parameters – the density and sonic velocity ratios. Its general structure is determined by occurrence and positions of special angles – the critical angles θ_L and θ_T, the Rayleigh angle θ_R, null-reflection angles; and by presence of local maxima and minima within intervals between these angles.

Generally, successive change of governing parameters induces continuous modification of $R(\theta)$-curves – for an incidence angle θ the reflection coefficient is continuous function of the governing parameters. But at several particular angles: $\theta = \theta_L$, θ_R, $\theta_T/2$, $\pi/2$; the continuity of $R(\theta)$ breaks down for some values of the parameters. The function $R(\theta)$ suffers the catastrophe – the curve $R_{\lambda_0}(\theta) = R(\theta;\lambda_0)$ for a critical value λ_0 of the parameter λ essentially differs from curves corresponding to ordinary values $\lambda \neq \lambda_0$. For small deviations of λ from λ_0 the curve $R(\theta;\lambda)$ coincides with the critical curve $R_{\lambda_0}(x)$ over the whole incidence angle range $(0;\pi/2)$, excluding an immediate vicinity of the catastrophe point. As λ goes to λ_0 the width of the vicinity decreases but the difference between $R(theta,\lambda)$ and $R_{\lambda_0}(x)$ holds finite inside the vicinity.

The catastrophes have not been actually observed; but transitions from one form of the angle dependence to another are realized through them. The paper contains classification of possible forms and ways of their mutual transformation. The classification involves physical interpretation of the reflection coefficient features on the base of such phenomena as leaky and skimmed waves, null reflection and others. The classification can serve for prediction interfacial ultrasonic phenomena for different pairs of matter. It has been illustrated by $R(\theta)$ curves for distinct liquid – solid pairs.

Propagation of a short pulse in a medium with a resonance relaxation

V. A. Larichev, G. A. Maksimov

Moscow Engineering Physics Institute (State university), Russia
e-mail: maximov@dpt39.mephi.ru

The new analytical representation of fundamental solution (Green's function) describing the short pulse propagation in medium with single process of resonant relaxation is presented. This analytical solution is based on the generalized local response function of linear media [Acoustical Physics 1998, V.44, N 6, p.709-716]. It contains well-known Lorentz's and Debye's models of relaxing media, like particular cases. The changing of pulse shape at propagation, described by obtained solution, shows a variety of forms of pulse propagation and general lows of pulses dynamics beginning from pure relaxation behavior and up to resonant one when Zommerfeld's and Brilluin's precursors could be observed separately.

The comparison of theoretical predictions with experimental results is represented for the ulra-short acoustic pulses propagating in fluid with air bubbles.

Effects of elastic anisotropy in reflection of short pulses of focused ultrasound from uniaxial plates

Yu. S. Petronyuk, V. M. Levin

Laboratory of Acoustic Microscopy, Institute of Biochemical Physics, RAS,
4 Kosygin St , Moscow, 117977 Russia
e-mail: julia@sky.chph.ras.ru

Experimental and theoretical study of interaction of short focused ultrasonic pulses with plane-parallel objects shows that the signal detected by focused transducer is a series of time-resolved echoes, which result from reflection of the probe pulse at the surface and bottom of an object (solid plate) and propagation of different types of an elastic waves through the plate. Measuring delay times between the echoes allows finding values of sonic velocity in the object with estimated accuracy.

Formation of the output reflected signal has been studied by asymptotic method. Occurrence of individual echo-pulses is determined by stationary points of the fast oscillating phase of the reflected signals. Presence of stationary points in the phase means convergence of a part of ultrasonic rays penetrated into the specimen body at the back side of the object. Number of the rays, which form the convergence point, prescribes magnitude and characteristic profile of recorded echo-pulses.

In anisotropic plates formation of stationary points of the phase function of reflected radiation and, respectively, echo pulses, is determined by group velocity of elastic mode in material of the plate. The phase velocities of corresponding modes determine the time intervals between the echoes. It has been shown that type of the stationary points of the phase is prescribed by symmetry of elastic properties of the plate material. Correspondingly, the echo-pulse shape critically depends on topology of the elastic slowness surfaces for the plate under investigation.

Analysis of output signal structure and possible profiles of echo impulses has been performed for two orientation of transversally isotropic plate. It has been found that in isotropic orientation (probe focused ultrasonic beam is directed along the C_∞ axis) profiles of the echo-pulses coincide with impulse shapes that are characteristic for isotropic plates, and substantially differ from the pulse profiles characteristic for orthogonal orientation of the specimen (probe beam propagates across the C_∞ axis. The difference is caused by difference in power of stationary points and in power of corresponding sets of ultrasonic rays, which are formed convergence points (or in number of the rays when this set is countable). Convergence points for isotropic orientation are formed by cones of incident rays, while convergence points for anisotropic orientation result from crossing individual rays only. Classification of feasible types of echo pulse shape is given for transversally isotropic materials with convex slowness surfaces.

Surface plasmon polaritons on thin metal cylinder with oxide coating

E. I. Logacheva, V. S. Makin

Research Institute of Complex Testing of OptoElectronic Devices,
Sosnovy Bor city, Leningrad district, 188540, Russia

P. Kohns

"Diospek", Honnefer str. 14, D-53179 Bonn, Germany

The existence and propagation of surface electromagnetic waves in cylindrical geometry on thin metal conductor wire coated by oxide films has been considered. The dispersion relation has been obtained and analyzed for the cases when optically thin cover film has metal properties (the plasma frequency of the film is essentially differ from that of substrate) or dielectric permittivity have real part of the order (-1).

Temperature dependence of metal surface absorptivity

V. S. Makin, Yu. I. Pestov

Research Institute of Complex Testing of OptoElectronic Devices,
Sosnovy Bor city, Leningrad district, 188540, Russia

It is difficult to obtain an experimental data for a temperature dependence of the metal surface absorptivity A from the optical surface absorptivity and reflectivity measurements. In this report the temperature dependence for $A(T)$ for few high reflective metals (Au, Pt, Ni) were obtained via the measurements of temperature dependence of the surface electromagnetic wave (SEW) propagation lengths in middle IR ($\lambda = 10,6$ mcm). The difficulties caused by the existence in experiments in addition to SEW diffracted at grazing angles bulk waves originating from input grating are discussed.

Earlier stages and evolution of laser-induced material damage in universal polariton model

V. S. Makin, Yu. I. Pestov

Research Institute of Complex Testing of OptoElectronic Devices,
Sosnovy Bor city, Leningrad district, 188540, Russia

The earlier stages of laser-induced damage of condensed matter in framework of universal polariton model and its evolution dynamics is analyzed and discussed. The consideration is based on the explanation of published and our own experimental results for binary (InP, InSb) and elemental (Si, Ge) semiconductors. The consideration is nearly restricted by the evolution from excited waveguide modes to surface polaritons interference with incident radiation followed by formation of resonant ordered surface structures and enlarged thin melted layer surface covering.

About polariton model of laser-induced condenced-matter surface damage

V. S. Makin, Yu. I. Pestov

Research Institute of Complex Testing of OptoElectronic Devices,
Sosnovy Bor city, Leningrad district, 188540, Russia

The review of the author's works study of highly nonequilibrium selforganized phenomena which involve, in particular, formation of dynamical and residual surface periodic structures under a laser action onto the condensed matter (semiconductors, metals, dielectrics) has been made. The electrodynamical aspect and the role of material media in universal polariton model of laser-induced damage were considered.

On homoclynic orbits behaviour for evolution equations with distribution parameters

R. S. Makin

Ulyanovsk State University, Dimitrovgrad Branch, Russia

Properties of nonlinear dynamic solutions to evolution equations with distribution of parameters in infinite dimension space are discussed. On the based of hyberbolic sets theory we find the evolution equations group with transversal crossing of stable and unstable manifolds under periodic perturbations. That means there is a nontrivial hyperbolic set homeomorphism to Cantor set. The difficulties of dynamical system analysis for complicated nonlinear phenomenas are discussed.

Limiting amplitude principle in diffraction on a wedge

A. E. Merzon
Institute of Physics and Mathematics, University of Michoacán of San Nicoláas de Hidalgo, ed. C-3, Ciudad Universitaria, 58090, Morelia, Michoacán, México
e-mail: anatoli@ifm.imich.mx

We continue to investigate a nonstationary scattering of plane waves by a wedge (see [2]-[4]). We denote by $W := \{y = (y_1, y_2) : y_1 = \rho\cos\theta, y_2 = \rho\sin\theta, \rho > 0, 0 < \theta < \phi\}$ the plane angle of a magnitude $\phi \in (0, \pi)$. We consider an incident plane wave $u_-(y,t)$ of the form

$$u_-(y,t) = e^{i(k_0\cdot y - \omega_0 t)} f(t - n\cdot y),\ t \in \mathbb{R},\ \ y \in Q := \mathbb{R}^2 \setminus W;\ \ \omega_0 > 0,\ \ k_0 \in \mathbb{R}^2,\ \ |\omega_0| = |k_0|.$$

We assume that $k_0 = (\omega_0\cos\alpha, \omega_0\sin\alpha)$, where $\max(0, \phi - \pi/2) < \alpha < \min(\pi/2, \phi)$. In this case $u_-(y,0) = 0,\ y \in \partial Q$. The profile $f \in C^\infty(\mathbb{R})$, $f(s) = 0, s < 0$, and $f(s) = 1, s > \mu$, for some $\mu > 0$. We consider the following wave problem in Q with (for example) the Dirichlet boundary conditions:

$$\left\{\begin{array}{lllllll} \Box u(y,t) & = & 0, & y \in Q,\ \ t > 0, & u(y,t) & = & 0, \quad y \in \partial Q\ \ t > 0; \\ u(y,0) & = & u_-(y,0), & y \in Q, & \dot u(y,0) & = & \dot u_-(y,0), \quad y \in Q. \end{array}\right. \tag{1}$$

Denote by $\mathcal{C}$ the Sommerfeld contour in the following (turned) form $\mathcal{C} = \mathcal{C}_1 \cup \mathcal{C}_2$, where $\mathcal{C}_1 = \{w_1 - i\pi/2\ |w_1 \geq 1\} \cup \{1 + iw_2\ |\ -5/2\pi \leq w_2 \leq -\pi/2\} \cup \{w_1 - 5/2i\pi\ |\ w_1 \geq 1\}$. The contour $\mathcal{C}_2$ is a reflection of $\mathcal{C}_1$ with respect to the point $-\frac{3\pi}{2}$. We choose the orientation of the contours $\mathcal{C}_{1,2}$ counter clock-wise. We denote by $\dot Q \equiv \overline{Q} \setminus \{0\}$, $\{y\} := |y|/(1 + |y|),\ \ y \in \mathbb{R}^2$.

Let us consider some $\varepsilon \geq 0$ and $N \geq 0$.

Definition. $\mathcal{E}_{\varepsilon,N}$ is the space of functions $u(t,y) \in C(\overline{Q} \times \overline{\mathbb{R}^+})$ with the finite norm

$$\|u\|_{\varepsilon,N} := \sup_{t\geq 0}\sup_{y\in Q}(|u(t,y)| + (1+t)^{-N}\{y\}^\varepsilon|\nabla_y u(t,y)|) < \infty.$$

Let us denote $\Phi := 2\pi - \phi$, $q := \dfrac{\pi}{2\Phi}$ and $H(w,\alpha,\Phi) = \coth(q(w + \pi i/2 - i\alpha)) - \coth(q(w - 3\pi i/2 + i\alpha)),\ \ w \in \mathbb{C}$. We prove the following theorem.

Theorem. i) There exists the unique solution to the problem (1) $u(t,\rho,\theta) \in \mathcal{E}_{1-\frac{\pi}{\Phi},1-\frac{\pi}{\Phi}}$

ii) The Limiting Amplitude Principle holds:

$$u(\rho,\theta,t) - e^{-i\omega_0 t}A(\rho,\theta) \to 0,\ \ t \to 0,$$

uniformly for $\rho \leq \rho_0$, where

$$A(\rho,\theta) := \frac{i}{4\Phi}\int_{\mathcal{C}} e^{i\omega_0\rho\cos(\theta-\alpha)}H(\beta + i\theta)d\beta,\ \ \rho \geq 0,\ \ \phi \leq \theta \leq 2\pi.$$

The limiting amplitude A is a solution of the classic diffraction stationary problem of the plane wave by a wedge of the Sommerfeld-Maljuzhinetz type described in detail in [1], section 1.4.

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Analysis of parametrical dependencies of Stoneley wave attenuation in fluid-filled borehole due to its scattering on rough well surface

E. V. Podyachev, G. A. Maximov, E. Ortega

Moscow Engineering-Physics Institute (State University), Russia
e-mail: maximov@dpt39.mephi.ru

Attenuation of Stoneley waves at their propagation along a borehole is currently considered as an important information source about porosity and permeability of surrounding layers, which are crossed by a borehole. Usually the attenuation mechanism is associated with a fluid flow between borehole and porous permeable surrounding medium. But there are some other mechanisms which lead to attenuation of wave field in a borehole. In particular, the attenuation of Stoneley wave can be occurred due to it's scattering on a rough surface of borehole. Hence, there is the necessity to estimate the contribution of scattering at data interpretation of Stoneley wave's attenuation in a borehole.

As it was reported on ICSV10 the problem can solved in the framework of small perturbation limit with use of the mean field approach. The expressions for attenuation coefficient of Stoneley wave due to St→St processes of scattering as well as for St→P, St→SV, St→R_i scattering processes were obtained.

The main goal of the report is analysis of frequency dependencies of partial attenuation factors on ratios of correlation length and borehole radius to wavelength for different correlation functions of roughness.

Complete regularization of boundary integral equations in the diffraction problems on curved surfaces

D. N. Lesonen, G. A. Maximov

Moscow Engineering Physics Institute (State university), Russia
e-mail: maximov@dpt39.mephi.ru

The new approach for regularization of the exact Fredgolm's integral equations of the second kind for field or its normal derivative on the scattered surface is suggested recently by the authors. This approach allows to obtain the stable solutions of integral equations including the resonance domains, when the direct numerical solution of exact initial equations brings an unstable results. Mathematically the regularization concludes in the splitting of exact Fredgolm's integral equations on the system of two equivalent equations. One of them includes the integral operator, restricted by geometrically illuminated part of surface. The other one is equivalent by form to the initial integral equation but with diffraction source. The solution of the first integral equation with the restricted integral operator is considered as regularized one, which coincides with exact solution of the problem in the short wave limit. From the physical point of view the regularization of the exact initial integral equations concludes in their replacement by restricted analogs with eliminated contributions of geometrically shadowed domains.

In the given work the regularization of the rested after splitting integral equation is represented and it is shown that the process of such regularization is auto-similar one, so that it is possible to carry out the complete regularization of the problem.

Wave field excitation in thin fluid-filled crack of finite size and its interaction with a borehole

A. V. Derov, G. A. Maximov

Moscow Engineering-Physics Institute (State University), Russia
e-mail: maximov@dpt39.mephi.ru

The problem on excitation and propagation of pressure wave field in the fluid-filled crack under action of external seismic wave is considered in the report.. Based on averaging procedure the non-uniform pseudo-differential wave equation for a pressure field in a crack fluid is derived in the long-wave approximation by a crack disclosing, taking into account an external seismic field. It is shown, that at low frequencies the eigen mode, propagating along crack, possesses a strong frequency dispersion. By numerical calculations, it is shown, that the ends of fracture excite a pressure field in a crack fluid under action of external seismic wave. This fact can be used for determination of crack geometrical size by hydrophones records in a well.

On attenuation of waves propagating in fluid mixtures

Lev A. Molotkov

St.Petersburg Branch of V.A.Steklov Mathematical Institute, Russia

In order to investigate the wave propagation in fluid mixtures, it is useful to approximate these mixtures by periodic block media and to construct effective models for these block media. In construction of the effective models in restricted media, the passage to the limit is fulfilled under conditions, the number of blocks increases to infinity but sizes of blocks are reduced to zero. This passage to the limit corresponds to mixtures of fluids. As result of this passage to the limit, the equations describing the effective model are established in [1]. The lack of these equations is that they do not take into account attenuation of the waves in the mixtures.

In order to take account attenuation of waves we assume that on boundaries between particles of different fluids, a force of friction proportional to difference of velocities of these particles arises. This assumption was made by M.Biot for taking account of attenuation in porous media [2]. As result, in the equations of the effective model, additional terms describing the attenuation are introduced. If the flows of energy can be neglected then it is easily to prove that the density of the total energy of the wave field in the effective model decreases steadily with time at every point.

The equations of the effective models of blocks media with taking account of attenuation are constructed in two- and three- dimensional cases. In consequence complexity of these equations, we consider in this paper two often encountered partial cases. In the first case blocks filled by fluid 1 and blocks with fluid 2 alternate along the coordinate axes. In the second case the blocks with fluid 1 are surrounded on all sides by blocks with fluid 2. In both cases the expressions for attenuation coefficients are established. These coefficients are proportional to square of difference of the fluid densities, depend on percent composition of the fluids, and independent of fluid moduli. The formulas of the attenuation coefficients are valid also in the case that one of the fluids is replaced by gas. In this case the attenuation coefficient is very large because the density of gas is very small. The method used in this paper can be applied in investigation of the mixtures of three and more fluids.

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Diffraction through structured planar gratings: numerical approach based on ray tracing

R. González-Moreno, J. Alonso, E. Bernabeu

Universidad Complutense de Madrid, Spain

We present and compare two methods for computing the distortion suffered by a Gaussian beam diffracted by a planar binary grating with arbitrary geometry. These approaches share the same representation of the Gaussian beam. Using a Monte Carlo Method we generate a ray bundle that can be understood as a numerical sampling of the beam's Wigner Distribution Function. The two methods differ in the way the ray bundle is diffracted. In one hand, we follow the opposite way that is made to obtain a DOE. From the grating geometry, we construct the equivalent refractive surface, so we can apply exact or approximate versions of the Snell's law to each ray in the bundle. In the other hand, we use the hypothesis known as local diffraction. For each ray, the standard equation for conical diffraction in linear gratings is applied. The period and orientation of each "effective" linear grating are the local period and orientation of the actual grating's lines at the ray incident point. This approach is named as the Local Geometrical Diffraction. Examples and comparisons of the two methods are carried out in a radial grating, which has been thoroughly studied so far.

Some elliptic traveling wave solutions to the Novikov-Veselov equation

J. Nickel, H. W. Schürmann

Department of Physics, University of Osnabrück,
Barbarastrasse 7, D-49069 Osnabrück, Germany
e-mail: jnickel@uos.de, hwschuer@uos.de

V. S. Serov

Department of Mathematical Sciences, University of Oulu,
PO Box 3000, FIN-90014, Finland
e-mail: vserov@cc.oulu.fi

An approach is proposed to obtain some exact explicit solutions in terms of elliptic functions to the Novikov-Veselov equation (NVE$[\psi(x,y,t)]=0$) [1], [2]. An expansion ansatz $\psi \to g = \sum_{j=0}^{2} a_j f^j$ [3] is used to reduce the NVE to the ordinary differential equation $(f')^2 = R(f)$, where $R(f)$ is a forth degree polynomial in f [4]. The well-known solutions of $(f')^2 = R(f)$ lead to periodic and solitary wave like solutions ψ of the NVE. Subject to certain conditions containing the parameters of the NVE and of the ansatz $\psi \to g$ the periodic solutions ψ can be used as start solutions to apply the (linear) superposition principle proposed by Khare and Sukhatme [5].

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A uniform in time asymptotic for the problem of centered rarefaction appearance

Georgii Omel'yanov

University of Sonora, Mexico

We construct a uniform in time asymptotic which describes the interaction of two isothermal shock waves with the same directions of motion. We show that any smooth regularization of the problem implies the realization of the stable scenario of interaction. In particular, we describe uniformly in time the rarefaction wave appearance.

A radio coverage prediction method in urban microcellular environments using electromagnetic techniques

E. Papkelis, I. Ouranos, H. Moshovitis, K. Karakatselos, P. Frangos

Division of Information Transmission Systems and Materials Technology, Dept. of Electrical and Computer Engineering, National Technical Univ. of Athens,
9, Iroon Polytechniou Str., 157 80 Zografou, Athens, Greece
e-mail: `spapkel@central.ntua.gr`

The accurate prediction of radiocoverage in cellular environments is of great importance nowadays due to the expansion of mobile systems which has led to growing demand for more efficient use of the radio spectrum. Several empirical and deterministic techniques have been developed for the estimation of signal propagation characteristics in micro and pico cellular scenes. The most widely used empirical models are described by equations derived from statistical analysis of a large number of measurements. These models are simple and yield quick results because they do not require detailed information about the environment but lack accuracy when the examined region differs from the one where the results were obtained. On the other hand, the deterministic models are based on the analytical application of electromagnetic techniques to a site-specific description of the radio environment, they are time consuming but they can simulate all well-known area networks. Our model belongs to the deterministic category and it is based on the analytical methods of Physical Optics (PO) and Physical Theory of Diffraction (PTD) for the calculation of first and higher order propagation mechanisms (scattering from surfaces, diffraction from wedges, ground effects) from the objects that the geometrical model consists of. The urban environment modeling adopts a user-defined outdoor geometry with a random street grid and building blocks of a parallel-piped shape erected along the street sides. Shadow regions coming both from the primary source (transmitter) and the secondary sources, which are the scattering sources (centers of illuminated surfaces) and diffraction sources (diffracting wedges), have been defined using a novel shadowing algorithm. The simulation program also allows both the selection of particular categories of field propagation phenomena that should be included for the purpose of evaluating their contribution to the total strength of the electromagnetic field at the receiver, and the accurate definition of magnetic parameters of the emitted field. The 2-dimensional model's results have been compared with other models based on ray-tracing techniques and good accuracy is obtained. Measurements of the electromagnetic fields using a spectrum analysis in an urban environment are in progress and the data analysis will provide a better verification of our simulation results. A 3-dimensional space formulation with the transmitting and receiving antennas below rooftop level is also being developed which will give a more realistic approach of the radio channel environment taking into account the structural and electrical properties of buildings and terrain, as well as the ground contribution. In addition, the near field contribution using either numerical or analytical methods, like Stationary Phase Method (SPM), for accuracy improvement near the building walls, where the PO method cannot give accurate results, is taken into account in our simulation code. Finally, extended research studies are currently undertaken in the area of optimized descriptions of the site topology and electromagnetic channel characteristics for a more flexible implementation in typical urban microcellular environments.

Nonlinear waves in plane granular media

Alessia Casasso, Franco Pastrone

Dipartimento di matematica - Universita di Torino
Via Carlo Alberto 10, 10123 Torino, Italy
e-mail: franco.pastrone@unito.it; casasso@dm.unito.it

A wide class of phenomena can be described by means of microstructural models, where the microstructure can be described by vector fields over the body. These vector fields are the unknown variables of the problem and their number depends on the restrictions due to the possible physical meaning of each of them. The material is supposed to be "hyperelastic": therefore we admit the existence of an energy function and we obtain the field equations through a variational principle. These equations are the Euler-Lagrange equations of a suitable energetic functional (see [2]).

The main task of this research is the study of the behaviour of granular media modelled as Cosserat microstructured solids (see [5]): in particular we study phenomena described with three or four field equations. In the first case the rotation of the single grain depends on one parameter only, the angle of rotation, in the second one we deal with dilatant granular media. In order to obtain constitutive equations deduced from the principles of Continuum Mechanics, we must take into account both the interactions matrix-grains and grain-grain.

In our work the study of strain waves may be useful in developing a suitable method in order to estimate the micro-parameters, since shape, amplitude and velocity of the strain wave can carry information about the microstructure. For this purpose, the most interesting are the waves that keep their shape and velocity on propagation.

The general Euler - Lagrange equations can be reduced to simpler equations, namely such that we can handle them with our tools, assuming some special form of the strain energy function and by means of well known techniques, as the so called "slaving principle". Hence we can obtain some preliminary results, as shown in the one-dimensional case in [3, 5], about the possibility of propagation of waves with constant shape.

To study the PDE wave equations we follow the method of reduction introduced by Samsonov [1]. As in the one-dimensional case, already widely treated (see [1, 4]), we reduce the PDE's to a couple of second order Lie equations, hence to a system of Abel equations. Since in general we do not know the solutions of this system, we turn our attention to find the cases in which it is possible to integrate the Abel equations by means of the Weierstrass equations. Simulation techniques can be usefully employed to exhibit interesting characteristics of the travelling waves.

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On Duffin-Kemmer-Petiau equation in curved space-time

Yu. V. Pavlov

Institute of Mechanical Engineering, RAS, St.Petersburg, Russia
e-mail: pavlov@lpt.ipme.ru

Duffin-Kemmer-Petiau (DKP) equation is a relativistic wave equation of first order in derivatives to describe spin 0 and 1 bosons [1]. Recently there have been an increasing interest to DKP theory in external electromagnetic and gravitational fields (see [2] and refs. there). The DKP theory generalized by [2] to Riemannian space-time is equivalent for spin 0 sector to Klein-Gordon-Fock equation with minimal coupling to curvature. But the non-minimal coupling is considered often in quantum field theory in curved space-time (see e. g. [3] and refs. there). In general case the scalar field in curved space obeys to equation

$$(\nabla_\mu \nabla^\mu + V_g + m^2 + U'(\varphi^* \varphi))\, \varphi(x) = 0\,. \tag{1}$$

Here V_g is a function of invariant combinations of the metric tensor and the curvature tensor of an N-dimensional space-time, $U(\varphi^*\varphi)$ is self-interaction. The case $V_g = 0$ is the minimal coupling. If one has the condition that metrical energy-momentum tensor of the scalar field does not contain derivatives of metric higher than the second order, one can take $V_g = \xi R + \zeta (R_{\lambda\mu\nu\kappa} R^{\lambda\mu\nu\kappa} - 4 R_{\mu\nu} R^{\mu\nu} + R^2)$ (Gauss-Bonnet-type coupling [4]). In this work we find generalized DKP eqs., which are equivalent to (1) for the massive and massless cases:

$$i\beta^\mu \nabla_\mu \psi - m\psi - \left(V_g + U'(\overline{\psi} P \psi)\right) \frac{P\psi}{m} = 0, \qquad i\beta^\mu \nabla_\mu \psi - \gamma\psi - \left(V_g + U'(\overline{\psi} P \psi)\right) P\psi = 0, \tag{2}$$

where $\beta^\mu = e^\mu{}_a \beta^a$, $e^\mu{}_a(x)$ are N-frame fields, $(N+1)\times(N+1)$ matrices β^a and γ obey relations

$$\beta^a\beta^b\beta^c + \beta^c\beta^b\beta^a = \beta^a\eta^{bc} + \beta^c\eta^{ba}, \quad (\eta^{ab}) = \mathrm{diag}(1, -1, \ldots, -1), \quad \beta^a\gamma + \gamma\beta^a = \beta^a, \quad \gamma^2 = \gamma,$$

$\nabla_\mu \psi$ are covariant derivatives of DKP field [2], $P = (-1)^{N-1} (\beta^0)^2 (\beta^1)^2 \cdots (\beta^{N-1})^2$ is Umezawa projector, and we choose γ such that $\gamma P = 0$. The Lagrangians corresponding to eqs. (2) are

$$L = \sqrt{|g|} \left[\frac{i}{2} \left(\overline{\psi} \beta^\mu \nabla_\mu \psi - \nabla_\mu \overline{\psi}\, \beta^\mu \psi \right) - m \overline{\psi} \psi - \frac{1}{m} V_g\, \overline{\psi} P \psi - \frac{1}{m} U(\, \overline{\psi} P \psi) \right], \tag{3}$$

$$L = \sqrt{|g|} \left[i\overline{\psi} \gamma \beta^\mu \nabla_\mu \psi - i \nabla_\mu \overline{\psi}\, \beta^\mu \gamma \psi - \overline{\psi} \gamma \psi - V_g\, \overline{\psi} P \psi - U(\, \overline{\psi} P \psi) \right]. \tag{4}$$

The applications of DKP eqs. are discussed to the Hamiltonian form of the scalar field equation, particle interpretation in curved space-time, etc.

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Deconvolution of instrumental functions in X-ray diffractometry by using the regularization technique

Vladyslav Piskarov, Joachim Wagner, Rolf Hempelmann

Saarland University, Physical Chemistry, Germany
e-mail: v.piskarov@gmail.com

The deconvolution of the instrumental function in X-Ray diffraction profile analysis is a basic step in order to obtain reliable results on the microstructure (crystallite size, lattice microstrain, etc.) and is a typical example of ill-posed inverse problems. The implementation of an eigen function method with different regularization techniques is investigated and a simple regularization algorithm is proposed.

A simulation of an instrumental-broadened profile superimposed with random noise and background signals is used to investigate the reliability and efficiency of the proposed technique. For the simulation an experimentally defined instrument function based on an accurate mathematical model for Cu emission profile, the geometry of the diffractometer and the physical properties of the specimen are used. The parameters for this instrumental function are obtained by least squares fitting of experimental data sets resulting from the reference materials LaB6 and Al2O3.

Compared to established algorithms, the proposed route is faster and more reliable in terms of stability, especially in the case of large experimental noise. The evaluation of experimental diffraction data of nanocrystalline gold with respect to grain size and microstrain and the comparison with standard evaluation technique is done.

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Spectral asymptotics for layered magnetic structures

L. V. Gortinskaya, I. Yu. Popov, E. S. Tesovskaya

Department of Higher Mathematics, St.-Petersburg State University of Information Technology, Mechanics and Optics,
14 Sablinskaya, St.-Petersburg, 197101, Russia

A system of two-dimensional quantum waveguides coupled through system of small windows is considered. The asymptotics of the resonance close to the threshold are obtained for the case of the Neumann boundary condition. The method of matching of asymptotic expansion of boundary value problem solutions is used. The justification is made with using of non stationary method.

The case of the periodic set of coupling windows is considered. The bands close to the threshold are described in the framework of the asymptotic approach. For fixed value of the quasi momentum the problem reduces to the description of the resonance. The independence of the resonance position of the quasi momentum was studied. It is proved that there is a gap between the band and the threshold. The parameters of the gap are determined.

Three dimensional layers with Neumann boundary conditions coupled through finite number of small apertures are investigated. The asymptotics of the resonance close to the threshold is obtained. The scattering problem is considered. The result is a series of the direction diagrams for different parameters of the system.

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True amplitude Gaussian beam imaging

M. I. Protasov, V. A. Tcheverda

Laboratory of Dynamic Problems of Seismology, Institute of Geophysics SB RAS,
prosp. Koptyug,3, 630090, Novosibirsk, Russia
e-mail: pmi702@uiggm.nsc.ru

Introduction. We present an original approach of true amplitude seismic imaging by means of weighted summation of multi-shot multi-offset data. Weights are computed by Gaussian beam (GB) tracing from current points within target area towards acquisition system. The special choice of GB provides possibility to take into account both illumination conditions and structure of covering layers.

Statement of the problem. The wave propagation velocity below is supposed to be decomposed as macro-velocity $c_0(x,z)$ and reflectivity/scatterer component $c_1(x,z)$. In order to describe scattered/reflected wave field $u_1(x,z;x_s,0;\omega)$ Born's approximation is used. Input multi-shots multi-offsets data will be dealt with later are multi-shot multi-offset reflected/scattered waves $\varphi(x_r;x_s;\omega)=u_1(x_r,0;x_s,0;\omega)$. True/preserving amplitudes imaging in the consequent will be treated as a procedure providing images with intensities being proportional to sharpness of the background perturbations $\frac{c_0(x,z)}{c_1(x,z)}$.

Description of the method. For constructing an image in the point $\overline{x}$ trace two rays toward free surface and then construct Gaussian beams [1] along them. Remind that Gaussian beam is a specific solution of the Helmholts equation. Twice Green's formula application gives the following integral identity:

$$\int\limits_{z=0} \frac{\partial u_s^{(gb)}(x_s;\overline{x};x_{0s};\omega)}{\partial z}\bigg|_{z=0} dx_s \int\limits_{z=0} \frac{\partial u_r^{(gb)}(x;\overline{x};x_{0r};\omega)}{\partial z}\bigg|_{z=0} \phi(x_r;x_s;\omega)dx_r =$$
$$= 2\omega^2 F(\omega)\int\limits_X \frac{1}{c_0^2(y)}\frac{c_1(y)}{c_0(y)} u_s^{(gb)}(y;\overline{x};x_{0s};\omega)u_r^{(gb)}(y;\overline{x};x_{0r};\omega)dy. \qquad (1)$$

It should be mentioned that GB are constructed in a way to place their narrowest to the imaging point $\overline{x}$. Now, we can use fact that GB is concentrated in the vicinity of ray and therefore integration in the right hand side of (1) can be done in small vicinity of the point $\overline{x}$ only. As macro-velocity model is supposed to be rather smooth, it is rater reasonable to treat the medium within this vicinity as being homogeneous. That means one can use explicit expression for Gaussian beams and come to the following integral identity that defines true-amplitude imaging:

$$\int\limits_{\omega_1}^{\omega_2} e^{-i\omega[\tau(x_s;\overline{x})+\tau(\overline{x};x_r)]}d\omega \int\limits_{z=0} \frac{\partial u_s^{(gb)}(x_s;\overline{x};x_{0s};\omega)}{\partial z}\bigg|_{z=0} dx_s \int\limits_{z=0} \frac{\partial u_r^{(gb)}(x;\overline{x};x_{0r};\omega)}{\partial z}\bigg|_{z=0} \phi(x_r;x_s;\omega)dx_r =$$
$$= \int\limits_{\omega_1}^{\omega_2}\int\limits_{\varepsilon(\overline{x})} \frac{c_1(y)}{c_0(y)} K(\overline{x};y;\omega)dyd\omega. \qquad (2)$$

It is really true-amplitude imaging cause in right side of the identity (2) is averaging of the contrast of background perturbation with kernel $K(\overline{x};y;\omega)$, which doesn't depend on the overburden and illumination conditions.

Numerical experiments. Numerical experiments we are going to present during the talk are done with widespread synthetic data Sigsbee2a we were provided with by SMAART Joint Venture. This model is very popular in migration community as touchstone for testing new algorithms. Presented numerical results clearly confirm effectiveness of the suggested method and demonsttrate it is really true-amplitude imaging.

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Solving elliptic PDEs with discontinuous coefficients using finite element and multigrid method

B. Pentenrieder, N. Zavyalov

The aim of this research was to develop an algorithm that enables us to solve the equation $\mathrm{div}(\rho \mathrm{grad} u) = f$ on a three-dimensional domain. Hereby, the coefficient ρ depends on the spatial coordinates and is allowed to assume two different discrete positive values so that we can deal with internal transitions from one material property to another. The right hand side of the PDE may be an arbitrary function f. We have implemented Dirichlet and homogeneous Neumann boundary conditions, but - in principle - it is easy to extend the functionality of the program also to mixed boundary conditions. The domain is dicretized by cubes on which we take trilinear finite element ansatz functions. The most difficult task was to calculate the diagonal elements of the stiffness matrices on the coarser grid levels, since the corresponding cells in general consist of different materials. It turned out insufficient to use approximations for them with the multigrid convergence disappearing. The computation of the intermediate stiffness matrices and thereby of the diagonal elements of the coarser levels is complex and expensive, but pays off because it is required only once during an initial run and actually restores the multigrid convergence that one can observe when setting ρ equal to 1 in the whole domain.

In order to evaluate the behavior of the algorithm we studied the heat conduction within ceramic blocks neglecting convection and radiation processes. In this context the two ρ values are the thermal conductivities of the brick material respectively the encapsulated air. Of course, one is not restricted to this application, but may use the program also for other problems described by the same PDE and also exhibiting internal jumps of one or two orders of magnitude in the material property ρ.

As a result of our studies a package with friendly interface to the user was developed. As an output several figures exposing the thermal flux were obtained with the help of OpenDX. The effective thermoconductivity which is important for engineering applications has been computed for ceramic blocks produced in the industry.

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Magnetization and vortices in Kac's model

Michel Rouleux, Hicham Elbouanani, Pierre Picco

CPT (Marseille), France

We consider a classical spin system over a lattice with configuration space S^1, and ferromagnetic, long range interactions. The hamiltonian is of the form

$$H(\sigma_{\Lambda_{\text{int}}}|\sigma_{\Lambda_{\text{ext}}}) = -\frac{1}{2}\gamma^d \sum_{x,y\in\sigma_{\Lambda_{\text{int}}}} J(\gamma|x-y|)\sigma(x)\overline{\sigma(y)} - \gamma^d \sum_{(x,y)\in\Lambda_{\text{int}}\times\Lambda_{\text{ext}}} J(\gamma|x-y|)\sigma(x)\overline{\sigma(y)}$$

Here Λ_{int} is a (large) bounded domain of $\mathbf{Z}^d$, Λ_{ext} its complement, $1/\gamma$ the range of interaction, satisfying $\text{diam}(\Lambda_{\text{int}}) \leq 1/\gamma$, and $J \geq 0$ a smooth potential on $\mathbf{R}^d$ with integral 1 (Kac's potential.) A map $\sigma : \mathbf{Z}^d \to S^1$ is called a configuration ; on Λ_{ext} we impose a vorticity, i.e. deg $(\sigma|_{\Lambda_{\text{ext}}}) = k \in \mathbf{Z}$.

An outstanding problem in Statistical Mechanics consists in studying the Gibbs measure at temperature $\frac{1}{\beta}$, i.e. given $\sigma|_{\Lambda_{\text{ext}}}$, the quantity

$$G_{\beta,\Lambda_{\text{int}}}(\cdot|\sigma_{\Lambda_{\text{ext}}}) = e^{-\beta H(\cdot|\Lambda_{\text{ext}})}/Z_{\beta,\Lambda_{\text{int}}}(\sigma_{\Lambda_{\text{ext}}})$$

Here we content with studying the variational problem for the free energy $\mathcal{F}_\beta(m_{\Lambda_{\text{int}}}|m_{\Lambda_{\text{ext}}})$ of bloc-spin configurations, i.e. after averaging σ over finite boxes of diameter δ, with $\text{diam}(\Lambda_{\text{int}}) \leq (1/\delta) << (1/\gamma)$.

The free energy $\mathcal{F}_\beta(m_{\Lambda_{\text{int}}}|m_{\Lambda_{\text{ext}}})$ is derived from $H(\sigma_{\Lambda_{\text{int}}}|\sigma_{\Lambda_{\text{ext}}})$ and the mean field free energy $f_\beta(m) = -\frac{1}{2}|m|^2 + \frac{1}{\beta}I(m)$ by usual procedures ; here $I(m)$ is the entropy function.

In a first part, we present some numerical simulations. They show that the minimizing configurations for $\mathcal{F}_\beta(m_{\Lambda_{\text{int}}}|m_{\Lambda_{\text{ext}}})$ have vortices in Λ_{int}, with total degree equal to k.

In a second part, we take the continuous limit $\Lambda_{\text{int}} \to \mathbf{Z}^d$, $\delta \to 0$. We proceed as for the Ginzburg-Landau functional, looking first for a radial symmetric configuration minimizing the (renormalized) free energy, i.e. a solution for Euler-Lagrange equation. Next we linearize Euler-Lagrange equation around a radial symmetric configuration and study the bottom of the spectrum of the corresponding operator, using Perron-Froebenius arguments.

A new approach to the essential spectrum of Schredinger, Klein-Gordon, and Dirac operators

V.S. Rabinovich

National Polytechnic Institute of Mexico

We proposed a new approach to the problem of location of the essential spectrum for a wide class of pseudodifferential operators. This approach is based on the theory of limit operators (see, [1]). As applications we consider the essential spectrum of electromagnetic Schredinger, Klein-Gordon, and Dirac operators with potentials belonging to different classes. In particular, we study the operators with slowly oscillating potentials, nonstandard perturbations of the periodic potentials, potentials of multiparticle problems. Note that this method gives a very simple proof of the well-known Theorem of Hunziker, van Winter, Zjislin for multiparticle Hamiltonians.

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Wave propagation in optical waveguides with slowly varying geometry

Vladimir S. Rabinovich
Instituto Politecnico Nacional, ESIME Zacatenco,
Edif 1, Av. IPN, 07738, MEXICO, D.F.
e-mail: vladimir.rabinovich@gmail.com

We consider the problem of electromagnetic waves propagation from narrow-band modulated sources in optical waveguides with stratified core and homogeneous claddings. We suppose that interface between the core and cladding has a slowly oscillating geometry depending on a small parameter $\epsilon \ll 1$. Applying the asymptotic methods for operator-valued Hamiltonians we obtain asymptotic expressions for all components of the electromagnetic field.

Projection to orthogonal function basis method for nonlinear multi-mode fiber

S. B. Leble, B. Reichel
Technical University of Gdańsk
ul. G.Narutowicza, 11 80-952, Gdańsk-Wrzeszcz, Poland

Projecting to orthogonal function basis allow us to derive coupled nonlinear Schrödinger equation (NLSE) for multi-mode fibers. The basis, which we introduce, by electromagnetic field expansion, for waveguide modes, depends on a waveguide geometry (we consider fiber with cylindrical geometry which correspond to Bessel function basis). In this paper we analyse fiber with a weak nonlinearity descent from the Kerr effect. Some aspects of boundary conditions (waveguide modes excitation) are presents. Analytic and numerical results (figure) for single-mode and double-mode fiber are given.

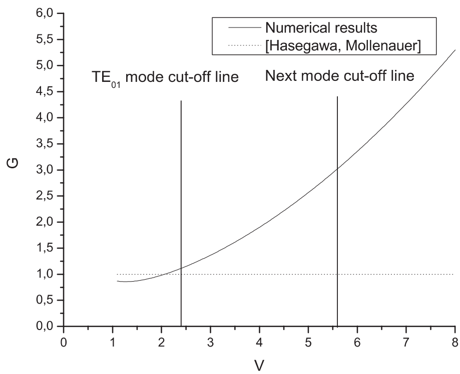

Figure : Comparison between results, for single mode waveguide with following value for physical parameters: $\omega = 12.2 * 10^{14}$Hz, $n_1 = 1.5$(ref. index for core), $n_2 = 1.4$(ref. index for cladding) and waveguide radius: from $1.2 * 10^{-6}$m to $10 * 10^{-6}$m, $\mathbf{G}$ is normalized to one for $\frac{g\omega n_{kerr}}{c}$ and V is normalized frequency and defined as $V = \frac{\omega}{c} r_0 \sqrt{n_1^2 - n_2^2}$.

Nonlinear evolution of components of electromagnetic field of helicoidal wave in plasma

S.B. Leble, D. W. Rohraff

Faculty of Applied Physics and Mathematics, Gdansk University of Technology
ul. G. Narutowicza 11/12, 80-952 Gdansk-Wrzeszcz, Poland
e-mail: lebIe@mif.pg.gda.pl

In this paper we study a nonlinear helicoidal wave in plasma. We consider the hydrodynamical approximation of a single ion plasma. It means that Knudsen number $Kn \ll 1$, so collisions plays main role and plasma may be considered as a conductive fluid. In the context of the soliton theory, linear and nonlinear models are traced, with a special attention paid to the problem of the wave asinchronism. Equations are derived for the directed quasi-one-dimensional waves. By developing the model with asinchronism, we obtain nonsymmetric, "inclined" solitons. We study also a form of a binary Darboux Transformation (bDT), for applying it to the N-wave interactions. We start from the Lax pair, which is a useful to construct N-wave solutions (soliton-like solutions), because it is a compatibility condition for considered equations. The results of a bDT, which has been analyzed numerically, are shown on pictures.

δ'-Shock wave type solutions of hyperbolic systems of conservation laws

E. Yu. Panov

Novgorod State University, Russia
e-mail: pey@novsu.ac.ru

V. M. Shelkovich

St.-Petersburg State Architecture and Civil Engineering University, Russia
e-mail: shelkv@vs1567.spb.edu

For hyperbolic system of conservation laws

$$u_t + (f(u))_x = 0, \quad v_t + (f'(u)v)_x = 0, \quad w_t + (f''(u)v^2 + f'(u)w)_x = 0$$

a *definition of a δ'-shock wave type solution* by integral identities is introduced, where δ' is a derivative of the delta function. It is a new type of singular solutions such that the second component v of the solution may contain Dirac measures, and the third component w may contain the linear combination of Dirac measures and their derivatives, while the first component u of the solution has bounded variation. The Rankine–Hugoniot conditions for δ'-shock are derived and analyzed from geometrical point of view. We prove δ'-shock *balance relations* connected with *area transportation*. To solve these problems, we use the *weak asymptotics method* developed in [1]–[5], and extended to the case of this type of singular solutions. These results show that solutions of hyperbolic systems of conservation laws can develop not only Dirac measures (as in the case of δ-shocks) but also their derivatives.

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Propagation of localized perturbations of the hydrodynamics equations with variable Coriolis parameter

E. S. Semenov, A.I. Shafarevich, S. Yu. Dobrokhotov

Institute for Problems in Mechanics RAS, Moscow

B. Tirozzi

Department of Physics, University "La Sapienza", Rome, Italy
e-mail: `b.tirozzi@libero.it`

We consider the localized perturbations of the hydrodynamics equations on beta plane. We discus the question of stability and preserving of the initial form of the perturbation.

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Lanczos-Arnoldi pseudospectral method for initial value problems in electrodynamics and its applications

Sergei V. Shabanov

Department of Mathematics, University of Florida,
Little Hall 358, Gainesville, FL 32611, USA
e-mail: `shabanov@phys.ufl.edu`

Maxwell's equations for dispersive and absorptive media are reformulated in the form $i\partial\Psi/\partial t = H\Psi$ where Ψ is a multidimensional vector field, $\Psi : \mathbb{R}^3 \to \mathbb{R}^n$, t is time, and $H = H_0 + iV$, H_0 and $V \geq 0$ are self-adjoint (differential) operators in the Hilbert space of square integrable Ψ's. Based on the semi-group property of the fundamental solution, $\Psi(t + \Delta t) = e^{i\Delta t H}\Psi(t)$, a numerical (time-domain) algorithm for the initial value problem is developed in which the exponential $e^{i\Delta t H}$ is approximated by means of the Lanczos-Arnoldi method. The action of powers of H on the wave function Ψ is computed via the fast Fourier algorithm where the sampling efficiency at the medium interfaces is enhanced by a suitable change of variables. The unconditional stability of the algorithm is proved. The proposed algorithm has a dynamical control of accuracy: It allows for variable time steps and/or variable computational costs per time step with error control. Several applications to photonics are considered, in particular, the extraordinary transmission (reflection) properties of gratings and groves are investigated by simulating the scattering of broad-band pulses on periodic structures made of dispersive and absorptive materials. Two types of resonances are observed that are associated with the structure geometry and material.

Analysis of the TE-wave propagation in nonlinear dielectric waveguides using the method of nonlinear integral equations

H. W. Schurmann
Department of Physics, University of Osnabruck,
Barbarastrasse 7, D-49069 Osnabruck, Germany
Yu. Smirnov
Dept. of Mathematics and Mathematical Modeling, Penza State University,
ul. Krasnaya, 40, 440017 Penza, Russia
Yu. Shestopalov
Institute of Engineering Sciences, Physics, and Mathematics, Karlstad University,
SE-65188 Karlstad, Sweden

Propagation of electromagnetic waves in linear media along cylindrical dielectric waveguides is a relevant topic of classical electrodynamics [1, 2]. Cylindrical waveguide structures consisting of nonlinear media is also an object of intense studies [2–8]. However until recently any rigorous proofs of the existence of modes in nonlinear dielectric waveguides have not been obtained, as well as the correct mode classification. Planar structures consisting of several plane-parallel layers filled with nonlinear Kerr-law media is another well-studied family of waveguides (see [2]). Whereas, for example, in the special case of a three-layer planar waveguide with Kerr nonlinearity in the layers the field can be described in terms of elliptic functions [6], it seems that that there is no analytical solution for the corresponding cylindrical waveguide.

In this work, we consider propagation of TE-polarized electromagnetic waves in a cylindrical waveguide of circular cross-section filled with a Kerr-type nonlinear, nonabsorbing, nonmagnetic, and isotropic dielectric. We look for axially (azimuthal) symmetric solutions and reduce the problem to a cubic-nonlinear integral equation (IE) using Green's function of the Bessel equation. IE is solved by iterations and a function sequence is obtained which uniformly converges to the solution of the IE. The dispersion relations (DRs) associated to the exact and iterate solutions are derived and solved, subject to certain constraints. The roots of the exact DR are approximated by the roots of the DRs generated by the iterate solutions. All statements concerning the existence and convergence are based on results of our previous studies [7, 8]. Numerical results (solutions to DRs, field patterns, dependence of the propagation constant and cut-off radius on the nonlinearity parameter, power flow) are also presented.

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Analysis of mixed boundary-value problems for a system of elliptic equations in the layer associated with boundary-contact problems of elasticity

Y. Shestopalov, N. Kotik

Division for Engineering Science, Physics and Mathematics,
Karlstad University, SE-651 88 Karlstad, Sweden

We consider the contact problem of a rigid cylinder pressed against an elastic layer which arises in flexography. This problem is equivalent to the contact of two rigid cylinders, one of which has an elastomeric covering, or to the squeezing of an elastomeric sheet. The results are of great importance for many engineering applications, including textile, paper and printing machines, gears and elastohydrodynamic lubrication. Knowledge of factors such as contact area, penetration and contact pressure is important for design considerations.

The processes which take place in elastics bodies can be described by Lame's equations with respect to longitudinal and transverse displacements using appropriate boundary conditions. The resulting non-classical boundary value contact problems can be reduced to integral equations with a logarithmic singularity of the kernel [1]. Analysis of existence and uniqueness for the logarithmic integral equation is based on the theory of singular integral operators [2], [3].

In this work we obtain a simplified solution to the mixed boundary-value problem (BVP) for a system of elliptic equations in the layer associated with boundary-contact problems of elasticity. The existence and uniqueness of the solution to the BVP are proved. The stress–strain components are expressed in terms of the BVP solution. The BVP is converted to a simplified boundary contact problem and the explicit solution to the latter is obtained. External forces on the boundary are simulated using specially defined 'hat' functions. Fields of displacements, elongations, normal stresses, shearing strain, and shearing stress are calculated at every point of the elastic layer.

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Transient waves produced by a source on circle expanding for finite time

Irina I. Simonenko

Institute of Physics, St.-Petersburg University, Russia

We construct transient solution of the initial-value problem to the inhomogeneous wave equation. The source is distributed on a circle expanding with the velocity of wave perturbation (the velocity of light for the electromagnetic waves). The expanding time is finite. The circle begins its expansion at the initial moment of time and belongs to the plane.

We obtain the solution of wave equation in the cylindrical coordinate system by using the incomplete separation of variables, the Riemann formula and the relation containing three Bessel functions.

The obtained expressions differ from the traditional representations in terms of the spherical harmonics. We describe the peculiarities of the space-time structure of the waves produced by such sources and discuss the specific features of the excited waves. We compare the found result with expressions for the case of the source belonging to the circle on the expanding sphere.

Viscoelastic model of impact excitation of a solid body and thin plate

Alexey A. Loktev, Yuriy A. Rossikhin, Marina V. Shitikova
Voronezh State University of Architecture and Civil Engineering,
st. 20-letija Oktjabrja 84, Voronezh 394006, Russia

The present paper is devoted to mathematical modeling of the impact of a solid body upon a plate, which dynamic behaviour is described by the Uflynd-Mindlin wave equations taking the rotary inertia and shear deformations into account, by means of a viscoelastic buffer involving a viscous damper and an elastic cylindrical spring, which basis is embedded in the plate. The viscoelastic buffer does not lose its stability during deformation, its stiffness expressed in terms of the operator, and thus the relationship for contact force takes on the integral form with a function of relaxation for Makswell's model.

An elastic isotropic plate and an isotropic plate, for which takes place viscoelastic properties of the material are considered only for shear deformations, are investigated. Viscoelastic properties of the material under shear deformations are described by the representation of the shear modulus and, hence, Young's modulus in terms of the operator, and the Hooke's law takes on the integral form with an arbitrary kernel of relaxation, in so doing Poisson's ratio does not depend on viscoelastic properties of the material.

As a method of the decision the ray method and method of splicing asymptotic expansion received for small times in a contact area and outside of it are used.

In the present work, the procedure proposed in [1] for the analysis of transverse impact of a solid body upon a nonlinear elastic buffer positioned on an elastic isotropic plate, is generalized to the case of shock interaction of a solid body with a viscoelastic buffer positioned on an elastic and viscoelastic plate.

During the interaction of the body with the buffer and, hence, the plate, a quasilongitudinal and quasitransverse waves representing the surfaces of strong discontinuity begin to propagate. In a plate of the surface of strong discontinuity represent cylindrical surfaces - strip, whose generators are parallel to the normal to the median surfaces and guides locating in the median surface are circumferences extending with the normal velocities. Behind the wave fronts, the solution is constructed in terms of ray series representing power series, whose coefficients are the different order discontinuities in the time-derivatives of the required functions, and the variable is the time passed from the moment of arrival of a wave to the given points of the plate. To determine the ray series coefficients for the desired functions, it is necessary to differentiate the governing equations with respect to time, to take their difference on the different sides of the wave surface, and to apply the condition of compatibility. As a result of the procedure described, we are led to the system of recurrent differential equations, which solution gives us the discontinuities in time-derivatives of the desired values within arbitrary constants. The arbitrary constants are determined at splicing on border of contact area of the solution for required function inside a contact disk and outside of it. The found jumps allow us to write down the required functions as the ray series with the coefficients expressed algebraically.

The simple and compact analytical expressions for contact force are defined. The carried out numerical researches allow to make the conclusion about influence of parameters of a construction, including buffer's and plate's viscoelastic properties, on dynamic characteristics of interaction. Five-term truncated ray series for the desired functions have allowed one to calculate with the given accuracy the stresses in the contact area of the plate and the dynamic safety margin of the thin plate.

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Feynman formulas for the statistical Hopf equation

O. G. Smolyanov

M.V.Lomonosov Moscow State University, Russia

A Feynman formula is a representation of a solution of the Cauchy problem for an evolution partial differential (or pseudodifferential) equation by a limit of finite dimensional integrals whose integrands contain some Gaussian, or complex Gaussian, exponents, when the multiplicity of integrals tends to infinity. This limit can be interpreted as a Feynman, or Gaussian, type integral over trajectories and such interpretation transforms the Feynman formulas into what one calls the Feynman-Kac formulas.

The statistical Hopf equation describes evolution of the Fourier transform of the (time depending) measure, which is a solution of the Fokker-Plank equation for the Navier- -Stokes equation with a Gaussian white noise (considered as an infinite dimensional stochastic differential equation).

In the talk a Feynman formula for solutions of the Cauchy problem for the statistical Hopf equation will be discussed. In the proof a Chernoff theorem (formula) is used, which is as related, to representations for solutions of Scrödinger type equations by integrals over trajectories in the phase space and also to representations for solutions of Hopf equations, as the famous Trotter formula is related to representation for solutions of some simple Schrödinger equations by integrals over trajectories in the configuration space.

Toward efficient numerical generation of low-reflecting boundary conditions for anisotropic media

Olga V. Podgornova, Ivan L. Sofronov

Keldysh Institute of Applied Mathematics, RAS
Miusskaya sq. 4, Moscow, Russia
e-mail: sofronov@spp.keldysh.ru; olgapv@ok.ru

Recently a spectral approach of constructing low-reflecting boundary conditions for the wave equation in anisotropic media has been proposed [1], [2]. The conditions are generated by two successive stages: firstly an exact operator of boundary conditions is numerically obtained for a discrete counterpart of governing equations; afterwards this boundary operator is approximated by a "cheaper" one in order to achieve reasonable computational costs for its application. In other words the second stage consists of a sharp compression of original huge matrix of the exact non-reflecting discrete operator. We use, in particular, spatial Fourier finite series, as well as sum-of-exponentials representation for occurring temporal kernels while performing such compression.

It was shown in our numerical experiments on an example of wave propagation in media with two different speeds of sound in half-spaces that non-local discrete operators of obtained low-reflecting boundary conditions do provide required accuracy without any enormous computational efforts [2].

However the first stage still remains a very expensive part of our approach since it requires numerical calculation of a correspondent discrete Green's function. In this talk we report our recent progress on faster and more accurate ways of finding the Green's function. Numerical results are demonstrated for a model problem of the wave propagation in two-layer media.

The work is supported by the RFBR grant 04-01-00567.

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A direct method for calculation of harmonic electromagnetic field in cylindrical geometry with multiple exciting source positions

Nikolai A. Zaitsev, Ivan L. Sofronov

Keldysh Institute of Applied Mathematics, RAS
Miusskaya sq. 4, Moscow, Russia
e-mail: sofronov@spp.keldysh.ru; zaitsev@spp.keldysh.ru

We present and discuss a numerical method developed for fast and accurate calculation of harmonic electromagnetic field in a metallic pipe with rotational symmetry [1].

Main features of considered class of tasks are the following:

- problem is formulated in unbounded domain (infinitely long metal pipe) though the region of interest is small enough;
- problem consists of multiple sub-problems with different exciting sources distributed along z-axis (at the same pipe geometry);
- each source can be sufficiently far away from the region of interest;
- setup includes discontinuity of electromagnetic properties (iron-air interfaces);
- second derivatives of solution are required up to the discontinuous interfaces;
- solver must be fast and reliable because it is developed for future inversion tasks (prediction of electromagnetic properties and geometry of the pipe).

We propose a finite-difference method that provides abovementioned requirements. Numerical results of comprehensive verification of the algorithm are given. Let us list some features of the algorithm:

- second-order scalar elliptic equation for the angular component of the electrical field is considered;
- exact discrete boundary conditions are developed on the faces of a cylindrical computational domain; the conditions can be inhomogeneous if the exciting sources are out of the computational domain;
- second-order difference equations on a non-uniform grid are resolved by a direct method with machine accuracy; the operation count for solving L tasks on the grid with M radial cells and N z-axis cells is estimated by $O(NM^2(M+L))$ operations.

The work is supported by the RFBR grant 04-01-00567.

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Analytical and numerical analysis of the wave profiles near the fronts appearing in tsunami problems

B. Tirozzi

Department of Physics, University “La Sapienza”, Rome, Italy
e-mail: b.tirozzi@libero.it

We consider the linear system describing the propagation of the water waves created by localized sources in an unbounded water layer with slow varying depth. Using asymptotic and numerical methods we quantitatively analyze the dependence the wave profile near the front on the form of the sources and the rays which organize the front. This analysis is applied to the tsunami problem. The work is done together with S.Yu. Dobrokhotov and T.Ya. Tudorovskiy.

Piecewise continuous distribution function method and ultrasound at half-plane

M. A. Solovchuk, S. B. Leble
Kaliningrad State University, Russia

The system of hydrodynamic-type equations for a stratified gas in gravity field is derived from BGK equation by method of piecewise continuous partition function[1]. The obtained system of the equations generalizes the Navier-Stokes at arbitrary density (Knudsen numbers).

The kinetic equation for the distribution function with the model integral of collisions in BGK form looks like (here and below we use standard notations):

$$\frac{\partial f}{\partial t} + \vec{V} \cdot \frac{\partial f}{\partial r} - g\frac{\partial f}{\partial V_Z} = \nu \cdot (f_l - f) \tag{1}$$

Let's search for the solution of the equations (1) as combinations of two locally equilibrium distribution functions, each of which gives the contribution in its own area of velocities space:

$$f(t,\vec{r},\vec{V}) = \left(\begin{array}{ll} f^+ = n^+ \left(\frac{m}{2\pi kT^+}\right) \exp\left(-\frac{m(\vec{V}-\vec{U}^+)^2}{2kT^+}\right), & V_Z \geq 0 \\ f^- = n^- \left(\frac{m}{2\pi kT^-}\right) \exp\left(-\frac{m(\vec{V}-\vec{U}^-)^2}{2kT^-}\right), & V_Z < 0 \end{array} \right.$$

Multiplying equation (1) on 6 eigen functions we obtain the system of differential equations. The obtained system of the equations according to the derivation scheme is valid at all frequencies of collisions and within the limits of high frequencies should transform to the hydrodynamic equations. It is a system of hydrodynamical type and generalizes the classical equations of a viscous fluid on any density, down to a free-molecule flow.

The verification of the model is made for a limiting case of a homogeneous medium and the results were compared with the results of sound-propagation measurements in Argon [2,3] and with existing theories.

We linearize the macroscopic equations and find the dispersion relation. In figure comparison of theoretical results on distribution of a sound with experimental data[2,3] is made. Results are in a good agreement with experiment and former theories at arbitrary Knudsen (Kn) numbers.

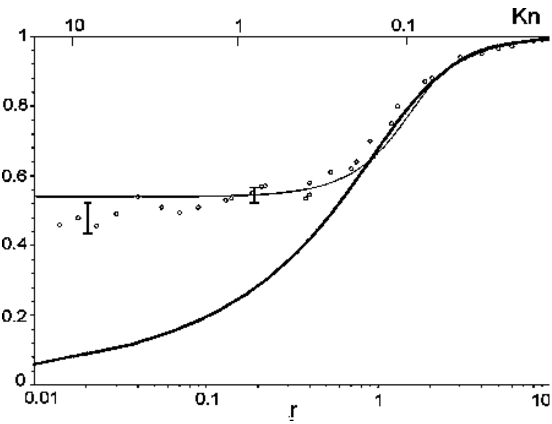

Figure: Sound of speed. thick line – Navier-Stokes, thin line – this paper, circle – measurements in Argon.

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Evolution operator for the multidimensional nonlinear Hartree-type equation with quadratic potential

Alexander L. Lisok, Andrey Yu. Trifonov

Department of Mathematical Physics, Tomsk Polytechnic University, Tomsk, Russia
e-mail: lisok@mph.phtd.tpu.edu.ru; trifonov@mph.phtd.tpu.edu.ru

Alexander V. Shapovalov

Department of Theoretical Physics, Tomsk State University, Tomsk, Russia
e-mail: shpv@phys.tsu.ru

A method of construction of semiclassical asymptotics for a Hartree-type equation (HTE) with smooth coefficients and a cubic nonlocal nonlinearity based on the Maslov's complex germ theory is applied to the multi-dimensional HTE with the quadratic potential

$$\{-i\hbar\partial_t + \hat{\mathcal{H}}_\varkappa(t,\Psi)\}\Psi = 0, \quad \hat{\mathcal{H}}_\varkappa(t,\Psi) = \hat{\mathcal{H}}(t) + \varkappa\hat{V}(t,\Psi), \qquad \Psi \in L_2(\mathbb{R}^n_x).$$

Here, operators

$$\hat{\mathcal{H}}(t) = \frac{1}{2}\mathcal{H}(\hat{z},t) = \langle \hat{z}, \mathcal{H}_{zz}(t)\hat{z}\rangle + \langle \mathcal{H}_z(t), \hat{z}\rangle,$$

$$\hat{V}(t,\Psi) = \int_{\mathbb{R}^n} d\vec{y}\,\Psi^*(\vec{y},t)V(\hat{z},\hat{w},t)\Psi(\vec{y},t) =$$

$$= \int_{\mathbb{R}^n} d\vec{y}\,\Psi^*(\vec{y},t)\frac{1}{2}\Big\{\langle \hat{z}, W_{zz}(t)\hat{z}\rangle + 2\langle \hat{z}, W_{zw}(t)\hat{w}\rangle + \langle \hat{w}, W_{ww}(t)\hat{w}\rangle\Big\}\Psi(\vec{y},t)$$

are functions of non-commuting operators

$$\hat{z} = (-i\hbar\frac{\partial}{\partial\vec{x}}, \vec{x}), \qquad \hat{w} = (-i\hbar\frac{\partial}{\partial\vec{y}}, \vec{y}), \qquad \vec{x},\vec{y} \in \mathbb{R}^n,$$

a function Ψ^* is complex conjugate to Ψ, $\varkappa$ is a real parameter, $\hbar$ is a "small" parameter, $\hbar \in [0,1[$, $\mathcal{H}_{zz}(t)$, $W_{zz}(t)$, $W_{zw}(t)$, $W_{ww}(t)$ are given $2n\times 2n$-matrices, and $\mathcal{H}_z(t)$ is a $2n$-vector. An exact solution of the Cauchy problem for the HTE is found in the class of semiclassically concentrated functions. The nonlinear evolution operator is obtained in explicit form. Parametric families of symmetry operators are found for the Hartree-type equation under consideration. With the help of symmetry operators, families of exact solutions of the equation are constructed. Exact expressions are obtained for the quasi-energies and their respective states.

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The Hartree type equation with quadratic potential in adiabatic approximation and Berry phase

Fedor N. Litvinets, Alexander V. Shapovalov

Department of Theoretical Physics, Tomsk State University,
Tomsk, Russia
e-mail: litvinets@mph.phtd.tpu.edu.ru; shpv@phys.tsu.ru

Andrey Yu. Trifonov

Department of Mathematical Physics, Tomsk Polytechnic University, Tomsk, Russia
e-mail: trifonov@mph.phtd.tpu.edu.ru

Evolution of explicit eigenfunctions $\Psi_{E_n(R)}(x,R)$, $n=\overline{0,\infty}$, of the Hartree type operator

$$\hat{\mathcal{H}}_\varkappa(R)=\hat{\mathcal{H}}(R)+\varkappa\hat{V}(R,\Psi)$$

with quadratic potential

$$\hat{\mathcal{H}}(R)=\frac{\mu\hat{p}^2}{2}+\frac{\sigma x^2}{2}+\frac{\rho(x\hat{p}+\hat{p}x)}{2},$$
$$\hat{V}(R,\Psi)\Psi=\frac{1}{2}\int\limits_{-\infty}^{+\infty}dy\,\Big[ax^2+2bxy+cy^2\Big]|\Psi(y,t)|^2\Psi(x,t)$$

is studied at adiabatic change of parameters $R=(\mu,\ \sigma,\ \rho,\ a,\ b,\ c)$. Here $\varkappa$ is a parameter of nonlinearity. A solution of the Cauchy problem

$$\Big\{-i\hbar\partial_t+\hat{\mathcal{H}}_\varkappa(R(t))\Big\}\Psi=0,\quad \Psi\Big|_{t=t_0}=\Psi_{E_n(R(t_0))}(x,R(t_0)) \tag{1}$$

is obtained as expansion in a small parameter $1/T$, where $T\gg 1$ is a time of adiabatic evolution. The Berry phase is found for the nonlinear system (1) in explicit form

$$\gamma(C)=\Big(n+\frac{1}{2}\Big)\oint\limits_C\Big[\frac{1}{2\Omega(s)}-\frac{\varkappa c(s)}{4}\frac{\mu(s)}{\Omega^3(s)}\Big]\Big(d\rho-\frac{\rho(s)d\mu}{\mu(s)}\Big). \tag{2}$$

Here $\Omega(s)=\sqrt{[\sigma(s)+\varkappa a(s)]\mu(s)-\rho^2(s)}$, and C is a contour in the parameter space $R=(\mu,\ \sigma,\ \rho,\ a,\ b,\ c)$. In linear case ($\varkappa=0$) the obtained expressions grade into well-known results of Ref. [1] (see also [2]).

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Wave packets localized near a surface for the multidimensional nonlinear Schrödinger equation in semiclassical approximation

Alexey V. Borisov, Alexander V. Shapovalov

Department of Theoretical Physics, Tomsk State University, Tomsk, Russia
e-mail: borisov@phys.tsu.ru; shpv@phys.tsu.ru

Andrey Yu. Trifonov

Department of Mathematical Physics, Tomsk Polytechnic University, Tomsk, Russia
e-mail: trifonov@mph.phtd.tpu.edu.ru

An approach is considered to construct semiclassical solutions, asymptotical in a small parameter $\hbar$, $\hbar \to 0$, to the $(1{+}n)$- dimensional nonlinear Schrödinger equation (NLSE) with an external potential $V(\vec{x}, t)$

$$\left(i\hbar\partial_t + \frac{\hbar^2}{2m}\Delta + 2g^2|\Psi(\vec{x}, t, \hbar)|^2 - V(\vec{x}, t)\right)\Psi(\vec{x}, t, \hbar) = 0. \tag{1}$$

The solutions are obtained in a class of functions localized in a neighborhood of an unclosed surface of parabolic type. Functions of the class are of one-soliton form along the direction of a normal to the surface at its vertex and are not normalizable in L_2. The NLSE (1) is linearized on the class of functions accurate to $O(\hbar^{3/2})$, $\hbar \to 0$, and a linear Schrödinger equation associated with the NLSE is derived. A dynamic system for a phase curve describing vertex evolution is obtained.

The solution construction approach is used results of Refs. [1], [2], [3] based on the Maslov's complex germ theory. An example is considered for a special case of the potential $V(\vec{x}, t)$.

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On the destruction of the adiabatic approximation and regular modes for super excited longitudinal motion in quantum waveguides

J. Brüning, S.Yu. Dobrokhotov, T.Ya. Tudorovskiy

Institute for problems in mechanics RAS, Moscow, Russia

We consider the spectral problems for magnetic Schrödinger operators in thin quantum waveguides with nonconstant width. We assume that the width varies slowly in comparison with the width of the waveguide. This assumption allows us to use adiabatic approximation to reduce initial equations to one-dimensional effective ones for the wide range of the eigenstates (longitudinal energies). The lowest states are described by means of Born-Oppenheimer method or by means of "long-wave" approximation for waveguides with almost constant width. The middle and the top of this range is described by means of semiclassical and Born approximation. In last situation the strong magnetic field implies the nonconstant effective mass. We show that the adiabatic approximation doesn't work for high longitudinal energies and analyze this effect using the Maslov complex germ theory. We interpret the destruction of the adiabatic approximation as the display of the Fermi acceleration.

Stability of patterns under random perturbations

S. A. Vakulenko, A. A. Abramian

Institute of Problems of Mechanical Engineering, Russian Academy of Sciences
199178 Saint Petersburg, Russia

This paper concerns with reaction-diffusion systems. Such systems are important for many applications, for example, for biology [1], chemistry, phase transitions and economics. In this applications we often observe appearance of complicated patterns [1].

The aim of this paper is to answer a question posed in [2], where the authors have made a fundamental hypothesis that any patterns generated by any chemical systems are unstable, for large times, under random perturbations.

To formulate mathematically this problem, we consider reaction-diffusion systems

$$u_t = D\Delta u + f(x,u) + \sum_{l=1}^{p} \xi_k(t) g_k(u), \tag{1}$$

where $u = u(x,t)$ is unknown function of space variable $x \in \Omega$ and time $t \geq 0$, $u \in \mathbf{R}^m$, Ω is a bounded domain $\subset \mathbf{R}^n$ with a regular boundary $\partial\Omega$, D is a diagonal diffusion matrix with positive entries, f, g_k are smooth or polynomial functions, $\xi_k \in \mathbf{R}^p$ are random processes with continuous trajectories. We set the zero Neumann boundary conditions: $u_n = 0$ on $\partial\Omega$.

Let us denote by u_{eq} a stationary solution of (1), which describes a pattern. To define a measure of stability of the pattern with respect to random perturbations, we follows the standard approach [3]. We introduce a bounded set Π in the space R^m. We suppose that system state $u(x,t)$ will be destroyed if there exists such a $x \in \Omega$ and $t > 0$ that $u(x,t) \notin \Pi$. This is a mathematical expression of the fact that the system must support "homeostasis". The stability measure $P_T(u_{eq})$ of pattern $u_{eq}(x)$ on the time interval $[0,T]$ is the probability that the solution $u(x,t,u_{eq})$ of (1) with initial data u_{eq} lies in Π for all $x \in \Omega$ and $t \in [0,T]$.

Under some conditions on random processes ξ_k, we prove the following. For $p > 1$ any pattern generated by a "generic" system with smooth f, g_k is unstable for large times, i.e., $P_T \to 0$ as $T \to \infty$. This result holds for any bounded Π. The key tool in proving is the known result on so-called polydynamical systems [4]: trajectories of "generic" polydynamic system connect any given points. So, the assertion of [2] holds, in certain sense, for "almost all" systems. However, for $p > 1$ there exist exceptional systems for which this assertion is invalid. For $p = 1$ we have "many" stable patterns and systems: the set of f, g_k for which our assertion is invalid, is open in an appropriate topology.

For systems with polynomial nonlinearities analogous results hold under some additional restrictions on the domain Π.

These results can be extended to more complicated systems which could be important for economical applications (where we replace Δ to a nonlocal integral operator $K(x,x')u(x')dx'$).

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Analytical solution to the diffraction on a slot

I. L. Verbitskii
Kharkov Regional Institute of Continuous Education,
Pushkinskaya str. 65 apt. 14 Kharkov, 61024, Ukraine
e-mail: verb@reg.kharkov.ua

Report is devoted to the diffraction of the point-source field on a slot in the ideally conducting plane screen. With the aid of the Partial Scattering Operators Method [1] the explicit analytical solution of this classical problem has been obtained.

Let us consider the system consisting of the ideally conducting half plane S_1: $x \geq 0$ and plane S_2: $y = -a$ illuminated by the point source $u_0 = \frac{1}{4i} H_0^{(1)} \left(k\sqrt{(x-x')^2 + (y-y')^2} \right)$.

If we assign the Dirichlet conditions on S_2 than accordingly to the symmetry principle the problem will be equivalent to the excitation of the slot $-2a < x < 0$, $y = 0$ by the difference of sources $u_0(x, y, x', y') - u_0(x, y, -2a - x', y')$, and if we assign the Neumann conditions on the S_2 then the problem will be equivalent to the excitation of the same slot by the sum $u_0 + u_0(x, y, -2a - x', y')$. Thus half-sum of the solutions of these problems gives solution to the problem of excitation of the slot by the point source u_0.

Let us consider the Dirichlet problem on S_1, which corresponds to the E-polarization.

Let T_1 be the scattering operator of S_1, i.e. the operator transforming the incoming signal to the scattered one, and T_2 be the scattering operator of S_2.

Denote u_1 the field scattered by S_1 and u_2 — the field scattered by S_2. Then full field u in considered system is

$$u = u_0 + u_1 + u_2 \tag{1}$$

and as has been shown in [1] the fields u_1 and u_2 obey the equations

$$u_1 = T_1(u_0 + u_2), \qquad u_2 = T_2(u_0 + u_1). \tag{2}$$

From (2) issues the equation for u_1:

$$u_1 = t_1 u_0 + T_1 T_2 u_0 + T_1 T_2 u_1. \tag{3}$$

Using known solutions to the excitation of half-plane by the plane wave and by the point source [2] and expressing u_1 in the form

$$u_1 = \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{e^{i[-\lambda(x+a) + r|y|]}}{r} \varphi(\lambda)\, d\lambda, \tag{4}$$

where $r = \sqrt{k^2 - \lambda^2}$, $\varphi(\lambda)$ is unknown function, we can represent operators T_1, T_2 and $T_1 T_2$ in integral form and to reduce equation (3) to the integral one which after some algebra can be in turn reduced to the Riemann problem

$$\Psi^+(\lambda) \sin \lambda a - \frac{e^{i\lambda a}}{2} \Phi^-(\lambda) = F(\lambda), \qquad -\infty < \lambda < \infty \tag{5}$$

in the class of the exponential-type function with known right side $F(\lambda)$.

Solution of this new type of Riemann problem obtained here is:

$$\Psi^+(\lambda) = \Phi^+(\lambda) \cos(ec\lambda a) - \frac{1}{a} \sum_{n=-\infty}^{\infty} \frac{\Phi^+\left(\frac{n\pi}{a}\right)}{\lambda - \frac{n\pi}{a}}, \tag{6}$$

where

$$\Phi^+(\lambda) = \frac{1}{2}F(\lambda) + \frac{1}{2\pi i}\text{v.p.}\int_{-\infty}^{\infty} \frac{F(a)}{a-\lambda}da. \quad (7)$$

Solving in the same way Dirichlet problem for the source $u_0(x, y, -2a - x', y')$ and the above-mentioned Neumann problems we obtain the desired solution for the slot.

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Waves produced by a traveling line current pulse with high-frequency filling

Andrei B. Utkin

INOV - Inesc Inovação, Rua Alves Redol 9, Lisbon 1000-029, Portugal

Generation of electromagnetic waves produced by current pulses of finite duration with high-frequency filling propagating along a straight segment is investigated.

The filling (modulation) term is represented in the form $\exp[ik(vt \mp z)]$. Here t is the time variable, z the direction of current pulse propagation, and $k = \omega/v$, where ω is the angular modulation frequency while v represents the phase velocity, which can be subluminal, luminal, or superluminal $(0 < v < \infty)$. Solutions are constructed by solving the system of inhomogeneous Maxwell equations in the space-time domain with the help of V.I. Smirnov method of incomplete separation of variables [1], using the electromagnetic field representation via the Whittaker-Bromwich potential [2].

For arbitrary slowly varying envelope of the current pulse, a closed-form quadrature expression is obtained for the magnetic component of the field, which enables the entire field to be reconstructed at long distances from the source. In contrast to the general consideration [1], separation of the modulating factor from the envelope enable one to illustrate and analytically describe the following phenomena:

- Directionality of the emanated waves.
- Transformation of the frequency of the electromagnetic wave carrier with respect to the initial frequency of the source current modulation, which takes place for certain wave regimes and manifests itself as the red or ultraviolet shift for the modulation factors $\exp[ik(vt - z)]$ and $\exp[ik(vt + z)]$ correspondingly.
- In certain space-time domains, the waves of two different frequencies, fundamental and shifted, are excited, which leads to formation of beating-type interferential patterns. The structure of these beatings become more and more complicated as the current pulse velocity tends to the velocity of light.

As far as in some intermediate stage the solving scheme reduces the electromagnetic problem to a scalar problem containing the wave equation, results obtained may be readily generalized to the case of scalar waves, the wave process being completely described by the wavefunction in both near and far zones.

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Estimating velocity model with multicomponent seismic data

Ekaterina A. Vsemirnova

State University of Aerospace Investigations,
St.Petersburg, Russia

With the aim of analyzing multicomponent data from complex regions a new algorithm for modeling based on traveltime computation is developed.

The numerical approach is used for independent analysis of P- and S-waves velocity. Recent developments in multicomponent sea-floor recording have refocused interests on converted shear-waves. Multicomponent seismic data can be used to derive P- and S-wave velocity structures of subsurface, which can be further used to estimate geological properties. The ability to construct reliable velocity model of the shear velocities of marine sediments is important in a number of disciplines. For exploration seismologists, for example, these models would help to improve the shear-wave static correction needed in oil and gas exploration. Images of deep shear-velocity horizons are sometimes better than those achievable with P-wave particularly in and around gas clouds and beneath high-velocity layers, and together with P-waves provide estimates of Poisson ratio which is used as a proxy for porosity.

P-wave analysis of Ocean Bottom Seismometer data acquired in Barents-Kara region is used for improving S-waves data sets for following interpretation and estimation Poisson ratio. The modeling shows a flexibility and robustness of method presented.

Theoretical, numerical, and experimental evidence of superluminal electromagnetic and gravitational fields generated in the nearfield of dipole sources

William D. Walker

Electronics Lecturer, Örebro University, Sweden
e-mail: `william.walker@tech.oru.se`

Theoretical and numerical wave propagation analysis of an oscillating electric dipole is presented. The results show that upon creation at the source, both the longitudinal electric and transverse magnetic fields propagate superluminaly and reduce to the speed of light as they propagate about one wavelength from the source. In contrast, the transverse electric field is shown to be created about 1/4 wavelength outside the source and launches superluminal fields both toward and away from the source which reduce to the speed of light as the field propagates about one wavelength from the source. An experiment using simple dipole antennas is shown to verify the predicted superluminal transverse electric field behavior. Broadband analysis of a dipole source is also presented which shows that the superluminal effects can be extended. In addition, it is shown that the fields generated by a gravitational source propagate superluminally and can be modeled using quadrapole electrodynamic theory. The phase speed, group speed and information speed of these systems are compared and shown to differ. Provided the noise of a signal is small and the modulation method is known, it is shown that the information speed can be approximately the same as the superluminal group speed. According to relativity theory it is known that between certain reference frames, superluminal signals can propagate backward in time enabling violations of causality. Several explanations are presented which may resolve this dilemma.

Resonant scattering of waves by the layer and grate a Kerr-like dielectric nonlinearity

Vasyl V. Yatsyk
Usikov Institute for Radiophysics and Electronics NASU
12 Ac. Proskura Str., Kharkov, 61085, Ukraine

On an example of the open nonlinear electrodynamic systems - transverse non-homogeneous, isotropic, nonmagnetic, linearly polarized, nonlinear (a Kerr-like dielectric nonlinearity) dielectric layer and grate, the algorithms of solution of the diffraction problems of a plane wave on the nonlinear objects and the results of the numerical analysis of the nonlinear problem are shown. In particular for a layer are found out: effect of non-uniform shift of resonant frequency and effect of increase of the angle of the transparency of the nonlinear layer at growth of intensity of the field.

The nonlinear diffraction problems reduced the solutions of the non-homogeneous nonlinear integrated equations of the second kind. For the layer this nonlinear integrated equation concerning a full field of diffraction. In case of the grate the appropriate system of the nonlinear integrated equations for Fourier harmonics of a full field of diffraction manages to be reduced to the nonlinear one-dimensional integrated equation of an operational (matrix) kind. That is, the system nonlinear one-dimensional (concerning cross-section coordinate) the integrated equations of the second kind (the one-dimensional nonlinear integrated equation is written down in the operational form) concerning of the Fourier amplitudes of a full field of diffraction in a nonlinear dielectric grate is received.

These integrated equations with application of the Simpson's quadrature method are reduced to the systems of the non-homogeneous nonlinear equations of the second kind. Algorithms of the solution of these equations are considered [1], [2]. These algorithms based on the iterative scheme. Also algorithms of the decision of nonlinear systems of the equations on the basis of the Newtonian method for the analytical a component of the required solutions and the Newtonian algorithm for the Teilor-series of the diffraction field are given.

The submitted algorithms have the following features. The iterative algorithms may be referred to direct numerical methods. It is simple in realization, but has lacks inherent in the iterative circuits based on quadrature forms. The following algorithms on the basis of a method of Newton, numerically analytical - has good (quadric) convergence.

By numerical methods are found out: effect of irregular change of resonant frequency characteristics of a nonlinear problem of diffraction; effect of increase in a corner of a transparency of a nonlinear layer with growth of intensity of a field. These effects are connected to resonant properties of a nonlinear dielectric layer. They are caused by change (increase) in dielectric permeability of a layer (a nonlinear component of dielectric permeability) at increase in intensity of a field of excitation of researched nonlinear object, see [1], [3].

The proposed algorithms for the solution of nonlinear diffraction problems and the results of the numerical analysis are applied: at investigation of processes of wave self-influence; at the analysis of amplitude-phase dispersion of eigen oscillation-wave fields in the nonlinear objects (the norm of own field is defined from the decision of a problem diffraction or excitation of a nonlinear layer), see [4]; development of the approach of the description of evolutionary processes near to critical points of the amplitude-phase dispersion of nonlinear structure (the case of a linear problem in [5], [6] is considered); at designing new selecting energy; transmitting, remembering devices; etc.

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Nonlinear interaction of electromagnetic waves guided by the dielectric slab in the anisotropic media

N. F. Yashina, T. M. Zaboronkova

Technical University of Nizhny Novgorod,
24 Minin St., Nizhny Novgorod 603950, Russia

We consider the parametric instability of low frequency electromagnetic surface waves guided by the plane dielectric slab in the anisotropic media (in particular, a magnetoplasma. It is assumed that superimposed static magnetic field is oriented parallel to the slab boundary and to be perpendicular to the propagation direction of the surface waves. The magnetoplasma is described by the dielectric permittivity tensor in the nondiagonal form. The elements of the dielectric tensor in the case of magnetoplasma are determined by the plasma parameters and given in, e.g., [1]. The nonlinear interaction of guided waves may arise due to the presence of a time-harmonic pumping electric field which is oriented perpendicular to the slab. This problem may be interest for the plasma diagnostic aims [2].

The nonlinear interaction can observed if the space-time synchronism of the surface waves and the incident electromagnetic wave takes place: $\omega_3 = \omega_1 + \omega_2$and $\vec{h}_1 + \vec{h}_2 = 0$ (here ω_3 is the frequency of the intense electromagnetic wave, $\omega_{1,2}$, $\vec{h}_{1,2}$ are the frequencies and propagation constants of the guided waves respectively). We consider the case when $\omega_1 = \omega_2$, and the surface waves propagate in the opposite direction. We have obtained the system of equations describing the behavior of the surface-wave amplitudes from the hydrodynamic equations and the system of Maxwell equations in the approximation of weak nonlinearity. Using the standard procedure we have found the expressions for the instability increment of the guided waves and the threshold value of the external electric field. The calculations are performed for the parameters corresponding the laboratory experiments [2].

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Plane wave diffraction by a semi-Infinite impedance sheet attached to an impedance wedge

N. Y. Zhu
Institut für Hochfrequenztechnik, Universität Stuttgart, Germany
e-mail: zhu@ihf.uni-stuttgart.de

It has been shown recently that plane wave diffraction by an impedance wedge whose exterior is bisected by a semi-infinite impedance sheet can be solved rigorously [Lyalinov, Zhu: Proc. R. Soc. Lond. A (2003), **459**, 3159-3180; Zhu, Lyalinov: IEEE Trans AP (2004), **52**, 2753-2758]. The exact solution has been enabled by a newly proposed efficient procedure for solving second order functional difference equations. The canonical wedge-shaped structure studied in this work consists of a semi-infinite impedance sheet attached to an impedance wedge; contrary to the previous works, the impedance sheet need not coincide with the bisector of the exterior of the impedance wedge. The solution procedure begins with the application of the Sommerfeld-Malyuzhinets technique to the original boundary value problem, resulting in a coupled system of difference equations for the spectral functions corresponding to angular regions above and below the impedance sheet. Eliminating the spectral function associated with the narrower angular region we obtain a second order difference equation for the other spectral function. By making use of the boundary condition on the respective wedge face, the generalised Malyuzhinets function $\chi_\Phi(\alpha)$ and the S-integrals, we arrive at an integral expression for an even and in the basic strip regular new spectral function. Now the integral depends upon the value of this new spectral function along a shifted imaginary axis in the complex plane, the shift being two times the angular deviation of the impedance sheet from the bisector of the wedge exterior. Precisely for points on this shifted imaginary axis we get a Fredholm equation of the second kind. Solving this integral equation numerically and making use of the integral representation, as well as analytical continuation if necessary, we determine one of the spectral fucntions; the remaining one is related to the first one and can be calculated accordingly. A first order uniform asymptotic solution has been derived and will be presented together with numerical examples.

Square integrable solutions of spheroidal Coulomb equation of the imaginary variable

Andrei M. Puchkov
Fock Institute of Physics, St.Petersburg State University
Stary Petergoff, 1 Ulianovskia St. 198904, St.-Petersburg, Russia

The boundary-value problem for the spheroidal Coulomb equation for pure imaginary variable with homogeneous boundary conditions is considered. It is important that singular points of the equation (-1 and $+1$) are laying outside the variable domain. This doesn't permit to use the standard method [1] for searching of the eigenfunction by power series. We suggest the generalized Jaffe-transformation. It leads to the recurrence relations for Poincare-Perron type expansion coefficients. The convergence of the expansions and asymptotic behaviour of eigenfunctions for large scaling parameter are discussed.

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Propagators weakly associated to a family of Hamiltonians

P. Št'ovíček
Department of Mathematics, Faculty of Nuclear Science,
Czech Technical University, Trojanova 13, 120 00 Prague, Czech Republic

We develop a new notion of a propagator $U(t,s)$ weakly associated to a time-dependent Hamiltonian $H(t)$. We indicate cases when the weak association can be verified while the usual relationship between a propagator and a Hamiltonian is unclear or even is not valid. Our approach is based on a theory due to Howland in which one relates to a propagator the so called quasienergy operator K in the extended Hilbert space $\mathcal{K} = L^2(\mathbb{R}, \mathcal{H}, \mathrm{d}t)$ with $\mathcal{H}$ being the original Hilbert space. This is done via the Stone theorem according to the prescription

$$\big(e^{-i\sigma K} f\big)(t) = U(t, t-\sigma) f(t-\sigma),\ \ \sigma \in \mathbb{R}.$$

Equivalently, K is defined by the equality

$$K = \mathfrak{U}(-i\partial_t)\mathfrak{U}^* \quad \text{where } \mathfrak{U} = \int_{\mathbb{R}}^{\oplus} U(t,0)\,\mathrm{d}t.$$

We say that a propagator $U(t,s)$ is *weakly associated* to $H(t)$ if

$$K = \overline{-i\partial_t + \mathfrak{H}} \quad \text{where } \mathfrak{H} = \int_{\mathbb{R}}^{\oplus} H(t)\,\mathrm{d}t.$$

Here we suppose that the intersection $\mathrm{Dom}(-i\partial_t) \cap \mathrm{Dom}\mathfrak{H}$ is dense in $\mathcal{K}$. For example, this is true in the case when the domain $\mathrm{Dom}H(t)$ is time-independent.

We show that at most one propagator can be weakly associated to a Hamiltonian. Furthermore, we indicate several sufficient conditions that guarantee the weak association. In particular, this is true when $H(t)$ has a time-independent domain and the relationship between $U(t,s)$ and $H(t)$ is the usual one. As an application of this concept we discuss a model describing a quantum particle which is moving in a plane under the influence of a constant magnetic field and driven by a slowly time-dependent Aharonov–Bohm flux. The known standard adiabatic results do not cover directly this model as the Hamiltonian has time-dependent domain and $H(t) - H(0)$ is not relatively $H(0)$ bounded.

These results were obtained jointly with J. Asch (Toulon) and I. Hradecký (Prague) and will be published in J. Math. Phys **46** (2005).

Scattering and resonances in leaky quantum wires

Pavel Exner
Department of Theoretical Physics, NPI, Czech Academy of Sciences
CZ-25068 Řež – Prague
e-mail: exner@ujf.cas.cz

We discuss a model quantum wires which takes tunneling into account, being formally described by Hamiltonians of the type $-\Delta - \alpha\delta(x - \Gamma)$ in $L^2(\mathbb{R}^2)$ where Γ is a graph. If the geometry of Γ is nontrivial, such systems can exhibit interesting spectral and scattering properties. We will analyze negative-energy scattering in the case when Γ is a local deformation of a straight line. We will also demonstrate that an approximation result which suggests existence of resonances due to the global geometry. Finally, we will present a solvable model describing Γ which consists of a line and a family of points.

Isoperimetric problems for δ interactions and mean-chord inequalities

Pavel Exner
Department of Theoretical Physics, NPI, Czech Academy of Sciences
CZ-25068 Řež – Prague
e-mail: exner@ujf.cas.cz

This talk is concerned with isoperimetric problems for generalized Schrödinger operators in $L^2(\mathbb{R}^2)$ describing "leaky" quantum wires. We will discuss two examples. In the first one two-dimensional point interaction of the same coupling are situated at vertices of an equilateral N-polygon and we ask about the configuration which maximized the ground-state energy; it is shown a regular polygon is a sharp local maximizer. In the second example we prove a similar result for the operator given formally as $-\Delta - \alpha\delta(x - \Gamma)$ with the interaction supported by closed loops Γ of a fixed length in the plane. We demonstrate that the problem reduces to an interesting family of geometric inequalities.

The distribution of gaps between prime numbers: physical approach

K. Oleschko
Centro de Geociencias, Universidad Nacional Autónoma de México (UNAM),
Apdo. Postal 1-742, Querétaro, Qro., 07600, México
F. Brambila
Facultad de Ciencias, Departamento de Matemáticas, UNAM, CU,
C.P. 04510, Coyoacan, México, D.F. México
R. Perez Pascual
Instituto de Física, UNAM, Apdo. Postal 20-364, CU,
C.P. 04510, Coyoacán, México, D.F., México
J.-F. Parrot
Instituto de Geografía, UNAM, Apdo. Postal, CU,
C.P. 04510, Coyoacán, México, D.F., México

The spatial distribution of gaps between prime numbers was analyzed across some deterministic and random fractal's images, and compared with the patterns extracted from the 50000 of the consecutive prime numbers, selected from two sets of natural numbers: from 2 to 611953, and from 1299721 to 2015177. The clear fractal behavior with fractal dimension close to 1.8 is documented for these nearest-neighbor gaps, independently on a nature and volume of analyzed sets. The strong antipersistent tendency and near to constant Hurst exponent (0.2) was shown for the gaps spatial geometry. The prime number distributions have very closed to random behavior with Hurst exponent near to 0.5. We speculate that the study of the prime number distribution in the real world will enrich the Number Theory with new ideas helpful for resolving the famous conjectures concerning the aforementioned gaps (specially the Twin Prime Conjecture).

Investigation of first-order antireflecting grating - computations and experiment

D. V. Myagkov, S. I. Nesterov, I. Gadjiev, E. L. Portnoi

Ioffe-Physical Technical Institute RAS, St.Petersburg, Russia

V. E. Grikurov

St.Petersburg University, St.Petersburg, Russia

Although a diffraction grating was developed and investigated more than hundred years ago, it still holds a great potential for new applications in many area of modern physics. One of them is antireflection structures based on zero-order diffractive gratings [1].

However, first-order diffractive gratings exhibiting antireflection properties are also of great interest as soon as it could be used for redirecting of transmitted light not only into the zero but also into the first diffracted order. The gratings have a great potential as light coupling devises in nanophotonics.

To investigate first-order diffraction gratings a reliable numerical model, which meets experimental data, for calculation of diffraction efficiency should be developed. Investigation of literature reveals the great number of rigorous models for calculation of diffraction efficiency of gratings [2], but most of them have restrictions on validity and convergence, which depends either on ratio of period and feature size to the incident light wavelength or light polarization or both. However, the desired numerical model has to provide rigorous calculation of diffraction efficiency for gratings of any profile with period and feature size of light wavelength scale, for any light polarization and incidence angle. The method developed in [3, 4] was chosen, as it meets all the above mentioned requirements.

Since after careful study of literature no papers on comparison of the method with an experiment were found, some experiments have been made to approve the validity of the model. Dependence of the diffraction efficiency of grating made by interference photolithography [5] on incidence angle of TE polarized light was measured and revealed suitable coincidence. Dependence of the diffraction efficiency on light wavelength was also measured and compared with calculations.

As the experimental data and calculations based on the method developed in [3, 4] are in satisfactory agreement, and as the method allows to investigate dependence of diffraction efficiency of grating on its geometry parameters and light wavelength in great size of changing, the method is found suitable for further investigation of first-order diffraction gratings.

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On intensity of high-frequency surface waves in anisotropic elasticity theory. The energy approach

Z. A. Yanson

POMI RAN. St.Petersburg. Russia

The present paper is concerned with the solution of the transport equation, which was obtained in [1]. In [1, 2], individual modes, concentrated near the stress-free boundary of an anisotropic elastic body, can be thought of as generalized Rayleigh waves well-known in isotropic elasticity theory. The wave field of a surface wave (e.g., of a whispering gallery mode), is represented as space-time caustic expansion

$$\vec{u}(\vec{r},t) = \Big\{\vec{A}v(-p^{2/3}m) + ip^{-1/3}\vec{B}v'(-p^{2/3}m)\Big\}e^{ipl(\vec{r},t)}, \tag{1}$$

where $\vec{A}$ and $\vec{B}$ are asymptotic series in integer powers of $1/p$, $p \gg 1$, and v is the Airy function. As in the isotropic case, we assume $m(r,t) = \gamma$ =const on surface S. Parameter γ is small and discrete (it describes proximity of the caustic of the ray field to surface S). Radius-vector $\vec{r}$ for any point M near S is written via radius $\vec{r}_0$ of a point M_0 on S and the normal $\vec{n}$ to S

$$\vec{r}(M) = \vec{r}_0(M_0) + n\vec{n}(M_0). \tag{2}$$

where n is distance to S. Let $\vec{A}^{\,0}$ and $\vec{B}^{\,0}$ be the main asymptotic terms for series $\vec{A}$ and $\vec{B}$. The problem of constructing an asymptotics for Rayleigh type waves of form (1) is solvable if, and only if, $(\vec{A}^{\,0},\vec{n})$ is not zero on S (see [1]). Provided this condition holds, m, eikonal l, and the coefficients of ray series in (1) can then be found, in a recurrent procedure, as series in integer powers n and γ. In the first instance, these computations seek to determine the principle term of asymptotics $\vec{A}^0$ on S. In (1), the equation for $\vec{A}^0$ (transport equation) is obtained in invariant form both in terms of the coordinate system and the type of anisotropy of the elastic medium, with amplitude $\vec{A}^{\,0}$ being a complex value in a general case. Integration of this equation using the eikonal equation for space-time surface rays yields the phase $\varphi = \arg A^0$ (Berry's phase).

In order to find $|\vec{A}^0|$, we use trivial transformations to derive the transport equations for amplitudes $\vec{U}^0_+$ and $\vec{U}^0_-$ for the ray field near the caustic with eikonals

$$\tau_+ = l + 2/3m^{3/2} \quad \text{and} \quad \tau_- = l - 2/3m^{3/2} \tag{3}$$

for the rays approaching surface S and those reflected from it, respectively. We then take the energy approach by averaging the energy and the energy flux transported by the rays. The above equations are solved in standard ray form by parameterization of the rays using a system of ray coordinates that are defined, in a special manner, on the space-time caustic. Additionally, we introduce a semi-geodesic coordinate system by defining normal $\vec{n}_0$ to the caustic in space-time metric. Taking into account the validity of (1) both near surface S and the caustic we find the relationship between the normals $\vec{n}$ (to surface S) and $\vec{n}_0$. We use the relation

$$|\vec{A}^{\,0}|^2 = 1/2m^{1/2}\Big\{|\vec{U}^{\,0}_{\,+}|^2 + |\vec{U}^{\,0}_{\,-}|^2 - 2m^{1/2}|\vec{B}^{\,0}|^2\Big\}. \tag{4}$$

Then by moving point M onto surface S and setting $m = \gamma = 0$ (i.e., bringing the point of tangency of the ray and caustic onto surface S), we arrive to the final formula for $|\vec{A}^0|$ (intensity).

The expression for $\vec{A}^0$ thus obtained takes the form of a modified ray method formula (for a field of space-time rays on surface S) which is analogous to the known expressions for $\vec{A}^0$ in the cases of isotropic and transversely isotropic elastic media (see [1]).

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The unidirectional stability of passive ring optical resonators

A. M. Radin, V. N. Kudashov

St. Petersburg State University of Low-Temperature, St. Petersburg, Russia
e-mail: `amradin@mail.ru`

A. B. Plachenov

Moscow institute of a radio engineering, electronics and automatics (Technical university), Moscow, Russia

It is shown, that passive ring optical resonators with small losses can have the unidirectional stability. It means, that in the resonator the mode of own oscillations is possible, when from two counter propagating waves , one is concentrated near the optical axis of cavity of the resonator, and another concentrated is not. A field of the concentrated wave decreases in cross concerning an optical axis direction, and field of not concentrated counter wave does not decrease and oscillate in all cavity of the resonator.

Earlier in our works [1] - [3] was it is shown, that active ring optical resonators (that is containing amplifying elements), can have the unidirectional stability.

To prove, that the unidirectional stability this one of the general properties of ring resonators, in the given work it is shown, that property of the unidirectional stability can have and passive ring optical resonators, that is not containing the amplifying elements, but containing only small, for example, ohmic losses. Presence losses necessarily. Ideal resonators with unidirectional stability does not exist, they either are steady, or unstable simultaneously. The physical reason of it It is extraordinary simple. In ideal resonators a fields of counter propagating waves are always mutual. Counter propagating waves have identical cross distribution of a field in everyone section perpendicular an optical axis. In resonators with losses (amplification) it not so. Fields of counter propagating waves can have differing cross section distributions and parameters of the resonator can be picked up so, that one of waves will lose stability, while the counter wave will remain steady. Such ring optical resonators we have named resonators with unidirectional stability. It is shown in our works [1]- [3]. Unfortunately, in these works we were limited, as it seemed to us more interesting, demonstration examples, following from the general theory, only for models active resonators, also have not resulted examples for models passive resonators. In given article we consider model passive the ring passive optical resonator having property unidirectional stability.

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