DAYS ON DIFFRACTION'2007

INTERNATIONAL CONFERENCE

Saint Petersburg, May 29 – June 1, 2007

ABSTRACTS

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FOREWORD

The Seminars/Conferences "Days on Diffraction" are annually held since 1968 in late May or in June by the Faculty of Physics of St.-Petersburg State University, St.-Petersburg Branch of the Steklov's Mathematical Institute and Euler International Mathematical Institute of the Russian Academy of Sciences.

This booklet contains the abstracts of 125 talks to be presented at oral and poster sessions in 4 days of the Conference. Author index can be found on the last page.

The full texts of selected talks will be published in the Proceedings of the Conference. The texts in LaTeX format are due by September 30, 2007 to e-mail `iva---@list.ru`. Format file and instructions can be found on the Seminar Web site at `http:/math.nw.ru/dd07/rules.html`. The final judgement on accepting the paper for the Proceedings will be made by the Organizing Committee following the recommendations of the referees.

We are as always pleased to see in St.-Petersburg active researchers in the field of Diffraction Theory from all over the world.

Organizing Committee

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List of talks

The diffraction and dispersion of waves in semiconductor-diffraction grating structure

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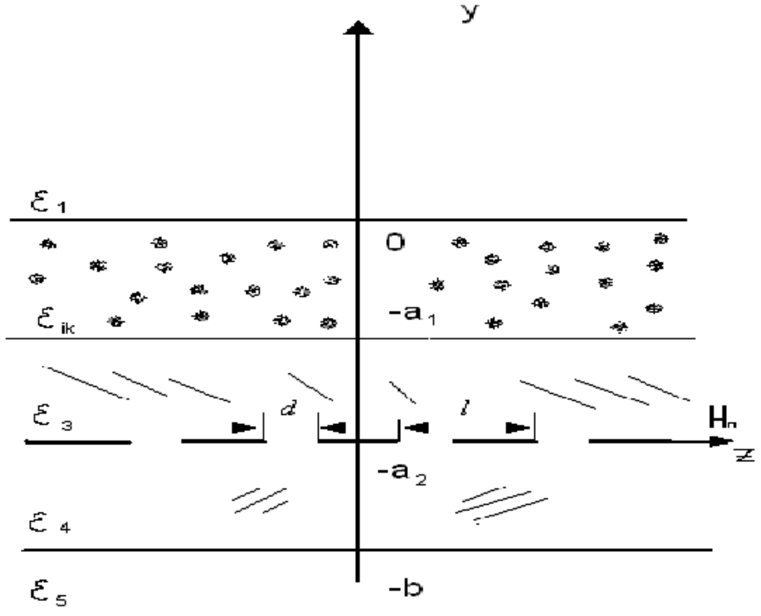
The wave phenomena in semiconductor structures have aroused considerable interest among physicists, mathematicians and practicing specialists. The close relationship existing between the wave phenomena and the fundamental properties of solids has spurred this interest, for the knowledge of the kind can contribute a lot to the improvement of the active and passive devices [1].

1. In the report, the electromagnetic wave diffraction by a diffraction grating closely spaced with a semiconductor layer backed by a dielectric substrate is considered relying upon Foigt's geometry. The structure of interest is shown in the figure. The semiconductor field is obtained from the simultaneous solution of Maxwell's equations and the hydrodynamic equations of the semiconductor plasma. The plasma electrical properties are described by the tensor ε_{ik}



$$\varepsilon_{ik} = \left\{ \begin{array}{ccc} \varepsilon_{xx} & \varepsilon_{xy} & 0 \\ \varepsilon_{yx} & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{array} \right\}$$

After the fields have been subject to boundary conditions at all the structure interfaces, one gets the functional equation system reducible to the problem of inhomogeneous conjugation of a unit circle in the complex plane [2], which the Riemann-Hilbert problem is. The Fourier coefficients of the obtained analytical function come from the infinite set of inhomogeneous linear equations [3]. In view of the problem parameter smallness, it is substituted by the reduced system which yields the unknown coefficients of field expansion to furnish us with the reflection and transmission factors.

2. Having known the structure fields, one gets the dispersion equation from the boundary value problem. The dispersion equation solution yields the propagation constants of plasma polaritons in either isotropic or magnetoactive semiconductor layer closely spaced with the diffraction grating. The behavior of the dispersion curves has been examined depending on the structure parameters.

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Wigner distribution function and intensity integral moments of Hermite–Laguerre–Gaussian beams

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Hermite–Laguerre–Gaussian (HLG) beams are recently introduced light beam family that contains Hermite–Gaussian and Laguerre–Gaussian beams as particular representatives. The Wigner function and some other integral characteristics for HLG beams are found.

The Wigher distribution function plays an important role in quantum mechanics and optics. It is used for optical signals analysis and light fields characterization [1, 2].

In paraxial approximation the most famous examples of light fields with structural stability under propagation in Fresnel zone are Hermite–Gaussian (HG) and Laguerre–Gaussian (LG) beams. The Wigner functions for these fields were found in [3] and [4] respectively.

Recently [5] a unity of HG and LG beam families was proposed by introducing an additional parameter α. The family of Hermite–Laguerre–Gaussian (HLG) beams $\mathcal{G}_{n,m}(x, y \,|\, \alpha)$ keep many important properties of HG and LG beams. In particular, for any fixed α the family of HLG beams is an orthogonal basis of the space $L_2(\mathbb{R}^2)$.

In this work the Wigner function for HLG beams is found with the help of generating function technique. Using this result, it is shown that Fourier transform of an HLG mode intensity may be presented in terms of Laguerre polynomials:

$$\iint_{\mathbb{R}^2} e^{i(x\xi+y\eta)} \left|\mathcal{G}_{n,m}(\xi, \eta \,|\, \alpha)\right|^2 d\xi\, d\eta = \\ = E \exp\Bigl(-\frac{x^2+y^2}{8}\Bigr) L_n\Bigl(\frac{x^2\cos^2\alpha + y^2\sin^2\alpha}{4}\Bigr)\, L_m\Bigl(\frac{x^2\sin^2\alpha + y^2\cos^2\alpha}{4}\Bigr), \tag{1}$$

where $E = \frac{\pi}{2}\, 2^{n+m}\, n!\, m!$ is the energy of an HLG mode $\mathcal{G}_{n,m}(x, y \,|\, \alpha)$, or the norm square of the mode in the space $L_2(\mathbb{R}^2)$.

By differentiation of eq.(1), the intensity integral moments are found:

$$\langle x^N y^M \rangle = \iint_{\mathbb{R}^2} x^N y^M \left|\mathcal{G}_{n,m}(x, y \,|\, \alpha)\right|^2 dx\, dy.$$

For example, for $n \leqslant 2$ and $m \leqslant 2$ the moments are

$$\langle 1 \rangle = E,\ \langle x \rangle = \langle y \rangle = \langle xy \rangle = 0,\ \langle x^2 + y^2 \rangle = \tfrac{n+m+1}{2} E,\ \langle x^2 - y^2 \rangle = \tfrac{n-m}{2} E \cos 2\alpha.$$

The above formulae are used for the calculation of some characteristics of HLG modes. Namely,

- beam quality factor M^2 (see definition in [6]): $M^2\bigl[\mathcal{G}_{n,m}(x, y \,|\, \alpha)\bigr] = n + m + 1$;
- phase space area (see definition in [7]): $\mathrm{PSA}\bigl[\mathcal{G}_{n,m}(x, y \,|\, \alpha)\bigr] = \frac{1}{4}(2n+1)(2m+1)$.

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Derivation of eigenvalues for the Sturm–Liouville boundary value problem with interior singularities

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Asymptotic solutions for the Sturm-Liouville boundary value problem with interior singularities were obtained using asymptotic forms of the Whittaker functions for higher order modes and Titchmarsh-Weyl m-functions for low order modes. However, these split interval techniques do not readily provide the eigenrelations for low order modes. For the first time, with minimal constraints, the eigenvalues for the Sturm-Liouville eigenproblem are obtained when the Titchmarsh-Weyl m-function technique is employed.

Plane shear wave diffraction in a composite elastic medium with partially bonded elastic strip

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A composite elastic medium which consists of an elastic strip ($|x| < \infty, |y| \leq h, |z| < \infty$) with the width $2h$ and two identical half-spaces ($y \leq -h, y \geq h$) is considered in a Cartesian coordinate system $0xyz$. The strip and the half-spaces have different elastic properties. It is assumed that along the half-panes ($x \geq 0, y = \pm h$) the strip and the half-space are bonded (full contact) and along the half-panes ($x < 0, y = \pm h$) there is no contact between them and the half-panes are free of stresses (crack). In the elastic half-spaces the plane shear wave

$$u_z^{(\infty)}(x, y) = \exp\left(-ikx\cos\theta - iky\sin\theta\right)$$

is incident from infinity at angle θ, ($0 \leq \theta \leq pi$), where $k = \omega/c$ is the wave number and $c = \sqrt{\mu/\rho}$ is the velocity of shear waves in the half-spaces. It is assumed that the medium is in the state of anti-plane deformation and the task is to investigate the diffracted wave field in the strip and in the half-spaces.

Representing the solution of the problem in the form of a sum of its even and odd parts and using the generalized Fourier transformations the problem is brought to solving two independent Wiener-Hopf functional equations.

Analytical expressions are obtained for the amplitudes of the displacements in each part of the composite medium. Asymptotic formulas are presented expressing displacements in the far field and the distribution of stresses in the neighborhoods of the tips of contact zones (tips of the cracks). Explicit expressions are obtained for the coefficients of transmission to the free part of the waveguide.

Application of Abel's integral transform to solving the electromagnetic wave diffraction by a thin disk

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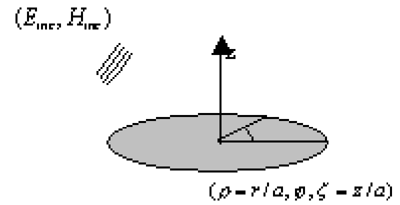
In this paper, we consider the diffraction of arbitrary electromagnetic wave by thin PEC, resistive, and dielectric disks and propose way to build a rigorous mathematical model of this group of problems. We start with Maxwell's equation, generalized boundary conditions on the median cross-section plane of the disk,

$$\left[E_{tg}^{+}+E_{tg}^{-}\right]/2=Z_0 R\cdot\vec{n}\times\left[H_{tg}^{+}-H_{tg}^{-}\right],$$
$$Z_0\left[H_{tg}^{+}+H_{tg}^{-}\right]/2=-S\cdot\vec{n}\times\left[E_{tg}^{+}-E_{tg}^{-}\right],$$

and radiation and edge conditions. Electrical and magnetic resistivities for a thin dielectric disk are given by



Disk geometry

$$R=i/2\,Z\cot\left(\sqrt{\varepsilon_r\mu_r}k\tau/2\right),\quad S=i/2\,Z^{-1}\cot\left(\sqrt{\varepsilon_r\mu_r}k\tau/2\right),\qquad |\varepsilon_r\mu_r|\gg 1,\quad \tau\ll\lambda_0$$

We represent the normal to the disk scattering field components in terms of integral Hankel transform image-functions:

$$E_{sc,z}^{\pm}(\rho,\varphi,\zeta)=\sum_{m=-\infty}^{\infty}e^{im\varphi}\int_0^{\infty}e^{\pm i\gamma(\kappa)(\zeta-\delta_l)}\left(\kappa e_{m,z}^{\pm}(\kappa)J_m(\kappa\rho)\right)d\kappa,$$
$$Z_0H_{sc,z}^{\pm}(\rho,\varphi,\zeta)=\sum_{m=-\infty}^{\infty}e^{im\varphi}\int_0^{\infty}e^{\pm i\gamma(\kappa)(\zeta-\delta_l)}\left(\kappa h_{m,z}^{\pm}(\kappa)J_m(\kappa\rho)\right)d\kappa.$$

On substituting the tangential field components into the boundary conditions, we obtain a set (or two independent sets, in the case of dielectric disk) of coupled dual integral equations. For the PEC disk diffraction problem they are

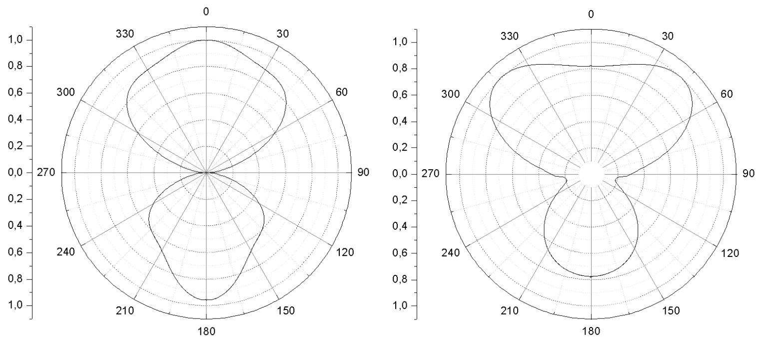
$$\begin{cases}\int\limits_0^{\infty}\left(i\gamma(\kappa)\left[e_m(\kappa)+e^{i\gamma(\kappa)|\delta-\zeta_0|}e_m^0(\kappa)\right]\frac{\partial}{\kappa\partial\rho}J_m(\kappa\rho)-ka\left[h_m(\kappa)+e^{i\gamma(\kappa)|\delta-\zeta_0|}h_m^0(\kappa)\right]\frac{m}{\kappa\rho}J_m(\kappa\rho)\right)d\kappa=0\\ \int\limits_0^{\infty}\left(-\gamma(\kappa)\left[e_m(\kappa)+e^{i\gamma(\kappa)|\delta-\zeta_0|}e_m^0(\kappa)\right]\frac{m}{\kappa\rho}J_m(\kappa\rho)-ika\left[h_m(\kappa)+e^{i\gamma(\kappa)|\delta-\zeta_0|}h_m(\kappa)\right]\frac{\partial}{\kappa\partial\rho}J_m(\kappa\rho)\right)d\kappa=0\end{cases}\quad(\rho<1)$$

$$\begin{cases}\int\limits_0^{\infty}\left(ika\,e_m(\kappa)\frac{\partial}{\kappa\partial\rho}J_m(\kappa\rho)-\gamma(\kappa)h_m(\kappa)\frac{m}{\kappa\rho}J_m(\kappa\rho)\right)d\kappa=0\\ \int\limits_0^{\infty}\left(ka\,e_m(\kappa)\frac{m}{\kappa\rho}J_m(\kappa\rho)+i\gamma(\kappa)h_m(\kappa)\frac{\partial}{\kappa\partial\rho}J_m(\kappa\rho)\right)d\kappa=0\end{cases}\qquad(\rho>1)$$

We integrate and decouple these equations and introduce two (or four) constants of integration. By using Abel's integral trans-form technique for solving "canonical" dual integral equations with the Bessel-function kernels,

$$\begin{cases}\int\limits_0^{\infty}\kappa u(\kappa)J_\alpha(\kappa\rho)\,d\kappa=\int\limits_0^{\infty}\kappa f(\kappa)J_\alpha(\kappa\rho)\,d\kappa\quad(\rho<1)\\ \int\limits_0^{\infty}u(\kappa)J_\alpha(\kappa\rho)\,d\kappa=0\qquad(\rho>1)\end{cases}\qquad\begin{cases}\int\limits_0^{\infty}v(\kappa)J_\alpha(\kappa\rho)\,d\kappa=\int\limits_0^{\infty}f(\kappa)J_\alpha(\kappa\rho)\,d\kappa\\ \int\limits_0^{\infty}\kappa v(\kappa)J_\alpha(\kappa\rho)\,d\kappa=0\end{cases}$$

we reduce our equations to a set of vector integral equations of the Fredholm second kind and add equations for finding the con-stants of integration. The features of the Fredholm second-kind



equations guarantee the existence of solution and numerical con-vergence of the algorithm based on any reasonable discretization scheme.

As an example, we show total far field radiation patterns for the PEC disk excitation by a horizontal electric dipole. Here the normalized disk radius is $ka = 3$, normalized distance from the dipole to the disk is 0.73, and radiation patterns are plotted in the planes of $\varphi = 0$ and $\varphi = 90°$.

Features of propagation and interaction of one-dimensional topological solitons in crystals

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The interaction of two analogous topological solitons (the Frenkel-Kontorova dislocations) at joint motion with similar velocities has been investigated in some specified velocity interval, and the features of their motion have been studied as well depending on the height of periodical potential barrier of lattice. Based on a numerical experiment for soliton solutions of the nonlinear sinus-Gordon wave equation, the following results were obtained: the interaction between the dislocations proved important at low velocities; the displacement field of some given dislocation was strongly distorted under the action of neighboring dislocation. In our opinion, in this case a transverse slide of the dislocation in question is observed. The interaction force between these dislocations decreases with increasing velocity and in the limit when the dislocation velocity tends to the velocity of sound, the interaction between the dislocations tends to zero. Having in view the interpretation of obtained results, we have studied the distribution of stresses around the dislocations in the velocity interval at issue. According to results of numerical experiment, as the velocity increases the stress field is contracted in the direction of slide and grows in the normal direction, by converging to δ. One may assume, thus, that the interaction force between the dislocations is determined by the distribution width of stresses of affecting dislocation. It was then obtained that the behavior of dislocation motion depended upon the height of periodical potential barrier of crystal lattice. In case of low barriers the motion is translational. In the course of slide the dislocation width is periodically changed. At an increase of barrier height the nature of motion changes. In high barrier crystals the dislocation as if executes oscillating motion in the direction of slide. The tail-end ranges of dislocations are distorted. The width of dislocation periodically changes remaining all the time less than that in low barrier crystals. When the allowance for the frictional force is made, the motion of dislocations slowed down, the amplitude of oscillations in the sliding direction is notably reduced.

When the allowance for the frictional force is made, the motion of dislocations slowed down, the amplitude of oscillations in the sliding direction is notably reduced.

The change of shape of dislocations in the course of slip has been studied for both the high and low barriers. For that, in the numerical experiment described above the distribution of displacement derivatives was given for fixed instants of time. It was obtained that in the same time interval this function periodically changed, the amplitude of variation at higher barriers being more than for lower ones. Respectively, the shape of dislocations for higher barriers is distorted stronger, than for lower ones, that is especially notable in the tail-end regions.

In all cases we give detailed graphical results of the numerical experiment, as well as made numerical estimates.

Smoothed Wigner Transforms and homogenization of wave propagation

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The Wigner Transform (WT) has been extensively used in the formulation of phase-space models for a variety of wave propagation problems including high-frequency limits, and the homogenization of linear, nonlinear and random waves. The WT however is well known to present significant difficulties in the formulation of numerical schemes, due to its so-called 'interference terms'.

In this connection, we propose the use of the smoothed Wigner Transforms (SWT), and derive new, *exact* equations for them, covering a broad class of wave propagation problems. The smoothing, i.e. the joint spatial-spectral resolution at which we choose to reformulate the original problem, now becomes part of the data of the phase-space equations. Equations for spectrograms and Husimi Transforms are included as a special case.

As an application, a semiclassical numerical method is developed and analyzed making use of the SWT. It allows for the efficient and accurate recovery of *ε-dependendt slow-scale observables* (e.g. the envelopes) in problems of semiclassical (i.e. high-frequency) wave propagation.

Working with concrete examples for the semiclassical linear Schrodinger equation it is seen that the SWT approach is indeed significantly cheaper to work with than full numerical solutions of the original equation in the semiclassical regime, as well as than using the WT without smoothing. Comparisons with exact and (independent) numerical solutions are used to keep track of numerical errors.

The analytic and numerical solution of second thrith, fourth and sixth order Wienner-Hopf system for mixed boundary elasticity dynamic problems

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The antiplane plane and three-dimensional unsteady elastic mixed problems for semi-infinite cracks in elastic plane and space and for stamps applied to elastic halfplane or halfspace are solved. The solution is obtained by integral transformations methods Laplace on time and Fourier by coordinates. The systems of Wienner-Hopf equations are obtained from boundary conditions, are brought to Hilbert problem for vector-functions of corresponding order, which by method of Plemeli-Hilbert-N.P. Vekua is reduced to solution of the same order Fredholm integral equations system. In mentioned problems corresponding matrix has singularities on infinity, which are avoided and are obtained corresponding Wienner-Hopf, Hilbert and furtehermore Fredgolm systems with continuous matrix. The last ones are solved numerically, which gives the technic of factorization of complex matrices. Also the inverse integral transformations are carried out and in all problems are obtained effective forms of Smirnov-Sobolev for stresses on corresponding parts of boundaries. The stress intensity coefficients near edges of cracks and stamps are derived and calculated numerically.

Periodic diffraction boundary-value problems

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We consider a wave diffraction problem by a periodic strip grating. Two boundary-value problems are studied in detail for an arbitrary geometry of the grating: the oblique derivative and the classic Neumann boundary-value problems. After we formulate these problems as convolution type operators acting on Bessel potential periodic spaces, associated operators acting on spaces of matrix functions defined on composed contours are derived. Within the last setting, Fredholm properties and Fredholm indices for the oblique derivative and the Neumann boundary-value problems are given. Sufficient conditions for the invertibility of the Fourier symbols of the associated operators are studied.

Subwavelength imaging at microwave, terahertz and infrared frequencies

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We study both theoretically and experimentally subwavelength imaging by a slab of le wire medium, two-dimensional electromagnetic crystal formed by parallel conducting wires [1]. WM supports so-called transmission line modes [1], which have completely flat isofrequency contours and exist at very low frequencies as compared to the periods of the lattice. In order to achieve subwavelength imaging, we do not involve negative refraction and amplification of evanescent modes. We propose to transform the most part of the spatial spectrum of the source radiation into the propagating transmission line modes of the crystal having the same group velocity (directed along wires and across the slab, respectively) and the same longitudinal components of the wave vector. The spatial harmonics produced by a source (propagating and evanescent) refract into the crystal eigenmodes at the front interface. These eigenmodes propagate normally to the interface and deliver the distribution of near-field electric field from the front interface to the back interface without disturbances. This way the incident field with subwavelength details is transported from one interface to the other one. The problem of strong harmful reflection from the slab is solved by choosing its thickness appropriately so that it operates as a Fabry-Perot resonator. In our case the Fabry-Perot resonance holds for all incidence angles and even for incident evanescent waves [2]. We call the described regime as canalization with subwavelength resolution [3]. The initial results on subwavelength imaging by wire medium were published in [2]. In the present work, we present recent results regarding bandwidth of operation and resolution of the imaging system [4]. We demonstrate transmission of images with l/15 resolution to the 3-4l distances. Also, we discuss opportunity to scale the structure so that it can operate at terahertz and infrared frequencies [5].

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Integrable models of the longitudinal motion of electrons in curved 3D-nanotubes

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In the framework of the adiabatic approach [1, 2], we find several model examples of curved 3D-nanotubes and configurations of a homogeneous magnetic field for which the one-dimensional matrix equation can be integrated exactly on a subregion of dimensional quantization. For periodic tubes (with the Born-Karman conditions), we obtain explicit (analytic) formulas of the electron spectrum and the time-domain Green function of the corresponding initially boundary-value problem. In the electron spectrum, we distinguish the corresponding contributions of the Aharonov–Bohm and Berry phases and the Maslov "geometric" potential.

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Semiclassical solyton-type solutions of the Hartree equation

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Concentrated asymptotic solutions in the form Haus packages are based on the ideas of complex WKB-Maslov method and covariant approach in the semiclassically approximation for the Hartree equation with smooth translation-invariant potential of self-action. These decisions are natural for understanding as semiclassical solytons for convex potentials in not resonant case (not blurring wave packages Haus structure).

Prediction of the trajectory of typhoons and the Maslov decomposition

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We use the Maslov decomposition for the solution of quasi-linear hyperbolic equations and the numerical integration of the Shallow Water equations for isolated vortices to show how to make a forecast of the trajectory of typhoons starting from the knowledge of the first three positions of the center [1]. We will show the results of the systematic application of this method to the a wide class of typhoons, from the tropical typhoons to the midlatitude ones.

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Kohonen neural networks and genetic classification

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High throughput technologies, such as genome sequencing and DNA microarrays are changing the face of biology. These recent advancements are producing an enormous amount of data from which biologically meaningful features need to be extracted and interpreted. However, because of the large number of genes and complexity of biological networks, it is difficult to interpret the resulting mass of data; so the clustering techniques become essential in data mining processes for identifying interesting distributions and patterns in the underlying data. Clustering algorithms have simplified the grouping of genes with similar biological expression. Co-expressed genes found in the same cluster suggest functional similarities. There is a large literature on cluster analysis and its application to genetic classification; numerous approaches were proposed on the basis of diRerent quality criteria and not all the algorithms are well founded. In addition the results of the algorithms depend strongly on many arbitrary choices, for example on the initial conditions and the value of the threshold. Therefore it is important to have a good control on the properties of clustering algorithms. We have investigated the efficacy of Kohonen algorithm to cluster transcription profiles. This algorithm was chosen since its properties are well known and can be controlled by the user. In fact we have clarified the state of the art of the convergence property of the algorithm considered as a stochastic process and proved rigorously that the rate of decay of the learning parameter which is most used in the application is a sufficient condition for almost sure convergence and we have confirmed it numerically. Many clustering algorithms are unable to produce an unique final result (e.g. hierarchical clustering, kmean, graph-based clustering, multidimensional SOM, etc.); this problem has been solved in our implementation of mono-dimensional Kohonen algorithm. We apply our theorem and considerations to the case of genetic classification which is a rapidly developing field.

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Dynamical behavior of a large complex system

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The problem of synchronization of N neurons interacting through a random gaussian matrix is discussed. The synchronization property is formulated in a weaker sense, via a generalized definition of the stability of the dynamics. This property is demonstrated by means of limit theorems of probability and the self-averaging property of disordered systems.

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Non-uniform grid (NG) based compression of the method of moments matrices

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The method of moments (MoM) discretization of the boundary integral equations of computational electromagnetics conventionally requires roughly ten unknowns per linear wavelength. Even higher density of discretization is needed in the quasi-static regime or for non-smooth geometries. On the other hand, only two unknowns per wavelength are sufficient to describe the number of degrees of freedom of the scattered or radiated field. The contrast between the local sampling requirements and the radiated field properties that affect the interaction between spatially separated domains can be exploited to reduce the complexity of solving the MoM equations. An efficient compression technique based on the construction of radiating and non-radiating basis and testing functions has been proposed in [1]. In this work, we propose to construct locally and globally interacting basis and testing functions (similar to those in [2]) via a numerically efficient algorithm based on the non-uniform grid (NG) sampling and interpolation of radiated fields [3]. As the first step, the scatterer is decomposed into subdomains of roughly equal size. Then, for each subdomain, an NG is constructed in a way that allows the computation of the field (produced by the currents confined to that subdomain) at any point of the scatterer by interpolation. This entails that the local and global radiating functions can be constructed based on a rank revealing factorization of the matrix describing field computation at the NG points. The size of this matrix is much smaller than the whole block of the conventional MoM matrix describing the interaction of a given subdomain with the rest of the scatterer. Following the basis transformation, the unknowns corresponding to the local basis functions are expressed in terms of the global ones and effectively eliminated from the system of equations to be solved using the Schur complement procedure.

For arbitrary geometries, the asymptotic computational complexity of the direct solver using the NG-based compression remains of $O(N^3)$, where N is the number of unknowns, however, the constant multiplier implicit in such complexity estimates is greatly reduced. In fact, thanks to the proposed matrix compression the complexity multiplier is much smaller than unity. In addition, the proposed technique is geometrically adaptive and very high compression and truly fast direct solvers are possible for geometries of reduced dimensionalities.

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Optimization of paraxial region for quasi-optical electron accelerator

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For the version of an electron-positron collider suggested in [1], the inner surface of accelerating structure is optimized to enlarge the accelerating gradient, the RF field being below the surface disruption threshold. Assuming the radial channels essentially narrower in comparison with the period of the structure, it is possible to consider an axially symmetric metallic pipe with a corrugated surface as an approximate model of the system. Inside the accelerating channel an electromagnetic field represents the axially symmetric π-mode of E-type. In the case considered the Maxwell equations for the axially symmetric π-mode are reduced to the Helmholtz equation with respect to the azimuth component of a magnetic field. To find the solution of the problem, the method of discrete sources is applied.

The following functionals are calculated: accelerating gradient, "coupling resistance", maxima of the electric and the magnetic fields on the metallic surface, normalized by the accelerating gradient. Dependence of the functionals on the geometric parameters of the electrodynamical system is investigated. The results are mainly consistent with conclusions of [2].

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The transient waves produced by hyperbolic motion of Gaussian's transverse sources

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The goal of the present report is to construct two axisymmetric explicit solutions of the inhomogeneous wave equation in the case of sub-superluminal hyperbolic motion of the Gaussian transverse sources. Possibilities of an adequate description of the generated waves due to the above solutions are discussed. Application of the obtained wave structures to the description of the electromagnetic waves is also investigated.

Note that kindred solutions are given for the pulse sources starting at a fixed instant of time and moving along a straight line with an acceleration (see [1] and [2] for an extended consideration).

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Sound field, excited by flexural oscillations of elastic plate with round inclusion (part 2)

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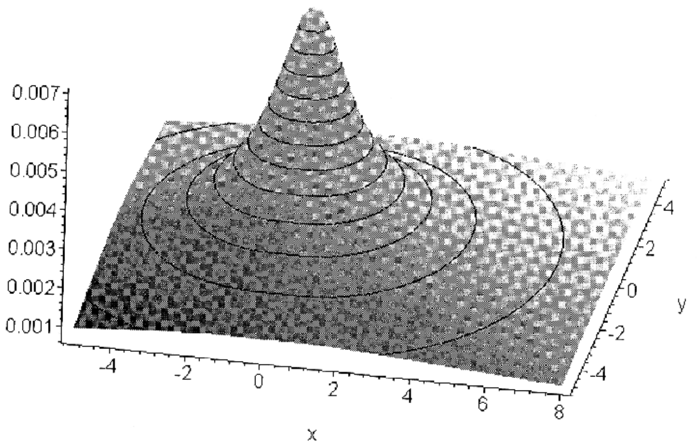
This problem was considered in the talk of V.A.Borovikov, A.L.Popov and A.L.Timofeev [1]. Its importance is connected with the mathematical modeling of non-contact non-destructive method of testing of tubes. This method consists of excitation of flexural oscillations of tube which in their turn excite the sound field outside of tube. Local in-clusions in tube lead to perturbations of this field which can be detected.



In this talk we present the results of numerical calculations of these perturbations. The example is shown on the figure where the sound pressure in Pa/m at distance 100 cm above the steel plate with thickness 10 mm is presented. The sound field is excited by the incident the plane flexural wave at round fixed inclusion with radius 25 mm. The frequency of this wave corresponds to wavelength 50 cm. Since the sound wave length for this frequency is larger than 50 cm, non-perturbed sound field exponentially decays while moving from the plate. Therefore all sound field which is shown on the figure is due to the presence of inclusion; its value is sufficiently large to be detected in experimental measurements.

The second item of our talk is the influence of the insulation layer (watertight coating) on the propagation of flexural waves in thin elastic tubes and envelopes and on excited sound field. The harmonic oscillations in such rubber-bitumen-like coatings are described usually by Helmholts equation with complex wave-number.

We consider the harmonic oscillations of thin elastic plate covered by liquid layer with thickness H and density ρ_l and seek for the first member of expansion in degrees of H of the equations of motion of this plate. In the absence of liquid layer this equation can be written in the form $a\Delta^2\xi - \rho h\omega^2\xi = 0$, where a depends on thickness h and elastic properties of plate and ρ is the density of plate so that the coefficient with $\omega^2\xi$ is the plate surface density. It turned out that the first approximation for small H of the equation of motion is $a\Delta^2\xi - (\rho h + \rho_l H)\,\omega^2\xi = 0$. In other words, in the first approximation the only influence of the thin liquid layer is the change of the plate surface density ρh by the total surface density $\rho h + \rho_l H$.

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Planar and three-dimensional chirality measure model

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During recent years progress of both microwave technology and photonics has been closely linked to the topic of electromagnetic meta-materials, structures with unique characteristics - amongst them the branch of planar and 3d-chiral materials. The authors of Ref. [1] found that the effectiveness of optical activity by planar chiral surfaces is several orders of magnitude larger than that of bulk chiral structures when operating in the proximity of resonances. Consequently, both planar chiral materials and monolayers consist of 3d-chiral elements have at-tracted much attention due to a number of new polarization phenomena associated with them [2-5].

The quantitative characteristic of the reason of these phenomena - chirality measure - is very important. It is obvious chirality measures can be introduced in a number of ways. Here, we suggest new algorithms, applicable to single chiral elements and their arrays returning a dimensionless valued chirality measure. It is scale-independent and, moreover, limited to the interval [-1; 1].

Here we provide algorithms for 2- and 3-dimensional chirality measure based on pure geometrical approach. They have been employed to calculate chirality indices of some elements and structures and both are simple in realization and converging stable and quickly.

Comparison of these chirality indices with known data about polarization transformation effects for light dif-fraction on some structures has been performed. Close fit has been observed. Further research is a current topic of presenter's work.

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Diffractive model of scattering by a rough surface of radiation

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The scattering of waves on a real surface is of interest for various areas of modern physics. Such practical problems as definition of intensity of the scattered field, computer modelling of illuminated objects visibility, measurement of object roughness parameters are permanent for physical optics.

Used mathematical model of the scattering [1-3], being synthesis of the theory of diffraction and mathematical statistics as well as application of the theory of indignations [4] do not provide adequate description of radiation scattered as a speckle-structure. In spite of the fact that the speckle-structure is a result of the scattering on random non-uniformities of the object (roughness, fluctuations of the refraction factor), numerous works cover only investigations of its statistical properties not related to parameters of phase non-uniformities. A diffractive model considering spatial distribution of the radiation scattered by the rough surface is similar to the distribution of the radiation in case of diffraction of the wave on an aperture with a shape and size coincident with the illuminated area is proposed in the submitted presentation. Intensity of the scattered field is a sum of intensities of field components, where the intensity of the first component is defined by a root-mean-square roughness at the base length equal to D/n (n - number of the diffractive order of diffraction on illuminated area corresponding to observation direction), the second component intensity - at length 2D/n, the third one - at length 3D/n and etc. The reason is that the waves propagated in the observation area have been diffracted on the structures of base lengths; multiple D/n and diffractive orders numbers divisible by n. Approximations of big and minor (compared to wavelength) roughness are not used in the proposed model.

There are numerous experimental validations for various wavelengths and objects types (solid and gaseous) [5].

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Two-body problem on pencil of lines

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Scattering theory for a nonrelativistic quantum mechanical 2-particle system on a quantum network with 2n open ends joined at a single vertex is considered. The scattering is decomposed into potential and geometrical contributions. The Green functions of the involved Hamiltonians are calculated. The wave operators and scattering matrices are obtained in the framework of stationary scattering theory. A detailed analysis is provided for inverse square particle interaction. Some applications to the quantum computing are discussed.

The work is supported by RFBR grant 05-03-32576.

Propagation of the wave packets in thin tubes with nonlinear integral potential

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We consider the problem of propagation of the Gaussian wave packets in quantum nonlinear wave guides (thin tubes) described by the 3D Hartree-type equation

$$i\Psi_t = \widehat{\mathcal{H}}_\Psi \Psi, \qquad \widehat{\mathcal{H}}_\Psi = -\frac{1}{2}\Delta + v_{\mathrm{int}}(x, \mathbf{y}) + \kappa \int_{\mathbb{R}^3} G(\mathbf{r}, \mathbf{r}')|\Psi(\mathbf{r}')|^2 d\mathbf{r}'. \tag{1}$$

The kernel $G(\mathbf{r}, \mathbf{r}')$ is a smooth function and the confinement potential grows up when $|\mathbf{y}| \to \infty$. The last property organizes the thin tube (wave guide) with so-called "soft walls". The well known fact is that wave packets spread during the motion along the wave guides. We show that integral nonlinearity can crucially change the situation and Gaussian wave packets can move along the wave guide preserving its form. We apply this result to the problem of the ballistic transport of perturbation in a long protein molecules. We compare the propagation of Gaussian packets with the propagation of so-called Davydov's solitons in long molecules.

This work was partially supported by RFBR-CNRS grant N 05-01-22002 and DFG-RAS project DFG 436 RUS 113/785.

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Functional integrals for Schroedinger equation on a Riemannian manifold via Smolyanov–Weizsaecker approach

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Abstract: We represent a new approach (introduced by Smolyanov and Weizsaecker) to get solutions of evolutionary equations on Riemannian manifolds with the help of functional integrals. We obtain solution of Cauchy problem for Schroedinger equation on a Riemannian manifold in the form of functional integrals with respect to Smolyanov–Weizsaecker surface measures.

Dependences of lasing thresholds for a layered structure with a quantum well on the mode symmetry

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Optical modes and linear thresholds of lasing for a quantum well (QW) sandwiched between two distributed Bragg reflectors (DBRs) are investigated as a specific eigenvalue problem with "active" imaginary part of the QW refractive index.

QWs embedded in epitaxially grown semiconductor microcavities are widely used in photonics [1,2]. We consider a layered structure consisting of the cavity whose width is w_c, sandwiched between two DBRs. In the centre of the cavity there is a QW (or active layer) of the width w_a (Fig. 1). We model the active layer by a complex-valued refractive index with a complex value $\nu = \alpha_c - i\gamma$, $(\gamma > 0)$ while adjacent layers have the same real values of this index, $\alpha_c = 3.42$, which corresponds to GaAS. For this structure we introduce a lasing eigenvalue problem (LEP) [3,4] and reduce it to the following equation:

$$e^{-i2\kappa(\alpha_c - i\gamma)(w_a/w_c)} = R_1 R_2,$$

$$R_{1,2} = \frac{R_{B,T} e^{i\kappa\alpha_c[1 \pm (2b \mp w_a)/w_c]} + R_a}{1 + e^{i\kappa\alpha_c[1 \pm (2b \mp w_a)/w_c]} R_{B,T} R_a}, \quad R_a = \frac{i\gamma}{2\alpha - i\gamma},$$

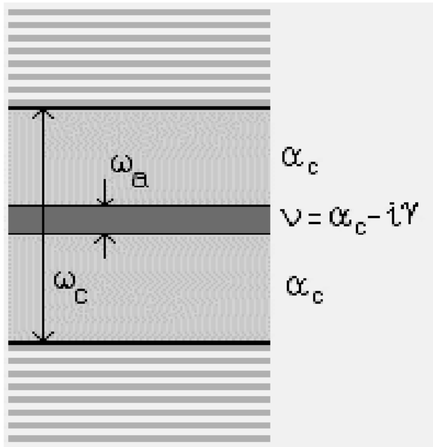

Figure 1: Sketch of layered structure

where $R_{B,T}$ are reflectivities of the bottom and top DBRs, respectively. We computed the LEP eigenvalues by iterations, starting from their values for the QW occupying entire cavity and using a two-parameter Newton method. Fig. 2 shows results of calculation of discrete pairs of the real-valued normalized frequency of lasing, $\kappa_0 w_c$, and associated threshold material gain, γ, for $w_c = w_a$ and $w_c = 0.1 \cdot w_a$, for a QW in the cavity located in free space, i.e. without DBRs. In the latter case several of the first odd modes are lost because their thresholds are too high. The thresholds of the odd modes are higher than those for even modes until the tenth mode and vice versa until the twentieth one, etc. We can suppose that periodicity in ten modes relates to the fact that QW is one tenth of cavity. Unlike thresholds, the lasing wavelengths are very stable with respect to the QW width. This is because they are determined mainly by the width of the cavity.

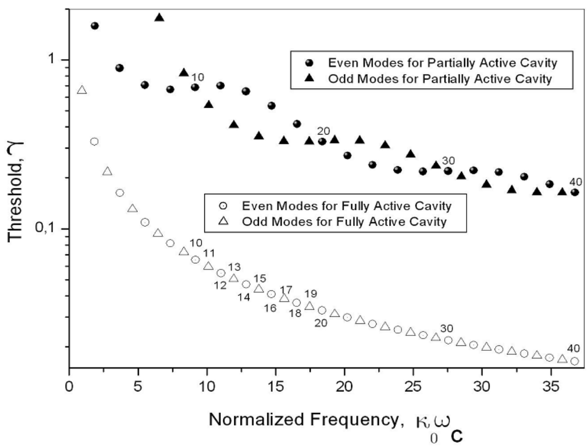

Figure 2: Thresholds of Lasing.

We discuss how placing the cavity equipped with QW between two DBRs effects the lasing wavelengths and thresholds. We also estimate the fractions of modal power leaking into the top and bottom media through the DBRs as a function of the layer pairs in reflectors.

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Effective Riemann space for wave propagation in nonlinear electrodynamics

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The characteristic equation for the nonlinear Born-Infeld electrodynamics is considered. This equation has the form of the characteristic equation for the linear electrodynamics in some effective Riemann space. The appropriate effective metric includes the energy-momentum tensor components of the electromagnetic field. The distortion of light beams by the action of some distant electromagnetic soliton solution is investigated in the framework of a perturbation method. This distortion corresponds to attraction with the solitons and looks like the gravitational distortion.

The effect under consideration is used for the possible solution of the problem for unification the electromagnetism and the gravitation. For more details see [1-5].

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Bessel Oscillator

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The approach to construction of oscillator-like systems connected with given family of orthogonal polynomials on the real axis, developed early by the authors, is extended to the case of Bessel functions. We construct a generalized oscillator related with the family $\{Z_{\nu+k}(x)\}_{k=0}^{\infty}$ of Bessel functions. We shaw that eigenvalue equation for the Hamiltonian of this oscillator is equivalent to the usual Bessel equation. We also discuss the possibility of construction of the family of coherent states for this oscillator. This investigation is partially supported by RFBR grant No 06-01-00451.

Waveguide structure containing metamaterial slab with resistive film

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The electromagnetic properties of artificial media have attracted a great deal of theoretical and experimental interest in recent years. Many authors have investigated the waveguide structures containing a slab of negative index material, whose both permittivity and permeability are negative [1, 2]. In this report we have considered theoretically the wave propagation in two-layered closed waveguide divided by a thin resistive film. On layer of thickness d_1 is a negative index material and the other one of thickness d_2 is a usual material. The thin resistive film is consider as the boundary condition for the tangential component of the magnetic field at the interface between adjacent media divided by this film [3]. The dispersion characteristics of TE_{0n} and hybrid modes are derived. The properties of eigenwaves in the waveguide are analyzed. The dependency of dispersion parameters on the thickness and the conductivity of the resistive film is examined. If the losses in the metamaterial slab increase, then the solution of the dispersion relation disappears. The considered waveguide structure can be applied as the filters with frequency response.

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One representation of localized functions via the Maslov canonical operator and its application to asymptotic solutions of linear hyperbolic systems

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We present a representation for the fast decaying functions having a form $V(x/h)$, $h \ll 1$, based on an integral of the Maslov canonical operator on some special Lagrangian manifolds. We apply it for a construction of asymptotic solutions to the Cauchy problem with localized initial data for a wide class of linear hyperbolic systems. Our main examples are the wave equations with variable velocity and the linearized Shallow water equations.

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Localized asymptotic solutions of 1-d wave equation with variable velocity implied by time-dependent source

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We construct effective formulas for asymptotic solutions of 1-D wave equation with variable velocity and with time-dependent space-localized right hand side localized (the time-dependent source). We consider the wide range of sources and show that their different dependence on time converts into different space-dependence of the wave profiles of the solutions.

This work was partially supported by RFBR grant N 05-01-00968 and Agreement Between University "La Sapienza", Rome and Institute for Problems in Mechanics RAS, Moscow.

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The problem of a perfect lens made of a slab with negative refraction

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Recently there has been growing interest in the creation of lenses with unusually sharp foci. Those lenses come in two types.
A.) The Veselago lens that is a slab of the material where due to some reason both electric permittivity and magnetic permeability are negative at some frequency. Such a medium is often called left-handed medium.
B.) The quasistatic lens where only one of those two is negative. Pendry [1] claimed that in the absence of absorption both types of lenses are perfect. It has been shown that the supersolution, proposed by Pendry does not exist for any lens with a real focus. After regularization by a small absorption the solution strongly depends on a distance from the source to the slab. In this presentation I show that for small wavelength the image is always controlled by diffraction. Mathematically this means that two limits, wavelength$\to 0$ and absorption$\to 0$ do not commute. In the first limit one gets diffraction controlled image that is independent of absorption at small absorption while in the second limit one gets absorption controlled image.

I propose also a lens with a virtual focus (VF) and show that in some sense this focus is perfect. The VF is located either in front of the slab or inside the slab at a distance 2d from the source. It is perfect in a sense that the radiation from the VF as it is observed behind the slab is exactly the same as radiation from the source but the VF is shifted with respect to the source. The slab does not introduce any distortions because of the amplification of the evanescent waves, exactly as Pendry proposed. However, in the case of the VF these arguments do not contradict to any general theorems because the fields do not have any singularity in the VF. The supersolution exists without any absorption. In fact, there is no maximum of the field in the VF itself, but this focus may be at the point that is very close to the rare interface of the slab, though still inside the slab. Then the field at the interface will have a very narrow (subwavelength) maximum. The influence of the small absorption inside the slab is considered.

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Diffraction of a perfectly conducting half-plane immersed in a gyrotropic medium

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The diffraction of an electromagnetic wave by a body immersed in a gyrotropic medium is a very complex problem even when the body has a simple shape, because gyrotropic media support two modes of propagation. Nevertheless, the diffraction problem of a perfectly conducting half-plane has been solved in particular cases, when one of the axes of the homogeneous gyrotropic material is a symmetry axis of the structure, for example it is parallel or orthogonal to the metallic half-plane [1-4]. Closed-form solutions are available also for plane waves at oblique incidence [3-4]. The formulation of these problems yields to Wiener-Hopf (WH) equations. By using the WH technique we are able to obtain closed form solutions since the involved kernels are scalars or matrices that can be explicitly factorized because of their particular form, in fact they commute with polynomial matrices [5]. In this paper we consider the general problem of the diffraction of a skew plane-wave by a perfectly conducting half-plane immersed in an arbitrary gyrotropic region. The WH formulation of this problem is very complex. Difficulties are overcome by introducing in the Fourier domain a circuit model given in terms of vector transmission lines. In this manner we establish a straightforward approach to obtain the two-times-two WH matrices to be factorized. The form of these matrices permits one to infer the cases when explicit factorization is possible. Whenever explicit factorization is not possible, or it is not pursued, we reduce the factorization problem to the solution of a Fredholm integral equation of the second kind [5]. Numerical results for an arbitrary lossless gyrotropic medium will be presented at the Conference. These results include also the evaluation of the diffraction coefficients for particular observation direction.

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Excitation of whistler modes guided by a lossy anisotropic plasma cylinder

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Whistler waves guided by cylindrical plasma structures aligned with an external dc magnetic field have attracted considerable interest in view of many applications, ranging from space plasma research [1, 2] to radio-frequency helicon plasma sources used in various plasma physics experiments [3]. Recently, several investigations have been made involving the influence of dissipative losses in a magnetoplasma on the characteristics of guided modes of a plasma cylinder located in free space, and some interesting and unexpected physical behaviors have been found for such modes [4]. In particular, conditions have been revealed under which allowance for comparatively small losses results in division of the guided waves into weakly and strongly damped modes with significantly different field distributions across the plasma channel. This should evidently lead to different excitation efficiencies of the corresponding modes due to a given source. It is the purpose of the present work to study how dissipative losses in a plasma channel can affect the source-excited field.

We consider the excitation of a lossy anisotropic cylindrical plasma column by a loop antenna with uniform current distribution placed coaxially with the column near its surface. Losses in the plasma are accounted for by introducing electron collisions. We calculate the total source-excited field and determine the contribution to it from the discrete- and continuous-spectrum waves. Conditions have been found under which the total field is dominated by the discrete-spectrum waves, i.e., eigenmodes of the plasma cylinder. It is shown for this case that in the presence of two groups of eigenmodes with essentially different damping rates, the amplitudes of weakly damped modes turn out to be notably greater than those of strongly damped ones. Despite this fact, the major part of the power spent by the loop antenna goes to modes with higher collisional losses, because their number considerably exceeds the number of modes with smaller damping [4]. As a result, the field near and far from the antenna is determined by different groups of modes, which stipulates the corresponding variations in the total-field distribution across the plasma column as a function of the distance from the source. The results obtained allow one to clarify factors responsible for the features of formation of the source-excited fields in lossy anisotropic plasma channels in the whistler frequency range.

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Investigation of multilayered and multislot resonant and periodical structures

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Resonant and periodical multilayered and multislot structures are used as antennas elements, frequency selective devices and special waveguides [1].

Cross section of investigated structure is shown at the figure 1. Slots have finite length in z direction. Structure may be periodical one with period d along z direction, may include magnetic or electrical walls at the $z = 0$, $z = d$, also may be infinite in z direction. Number of slots N and number of dielectric layers P are arbitrary values.

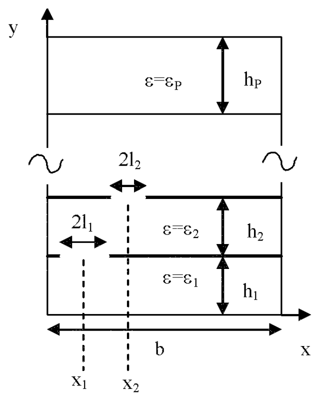

Figure 1

The system of pair sum equations in two dimensional Fourier transforms of magnetic current densities was obtained.

Electromagnetic field at the structure is superposition of slots fields. Field of each slot is determined in the situation of absence of all other slots. These slots are replaced by perfectly conducting screens. Magnetic current densities are unknown functions. To find them we satisfy continuity boundary conditions of tangential magnetic components at the surface of each slot $H_{x,z}(y+0) - H_{x,z}(y-0) = 0$. As a result we obtain $2N$ equations with $2N$ unknown magnetic current densities at the slots surface $I_{x,z}$.

Galerkin's method with basis in the form of Fourier transform of functions taking into account field singularities at the edges of rectangular area was used for solving obtained system of pair sum equations (SPSE).

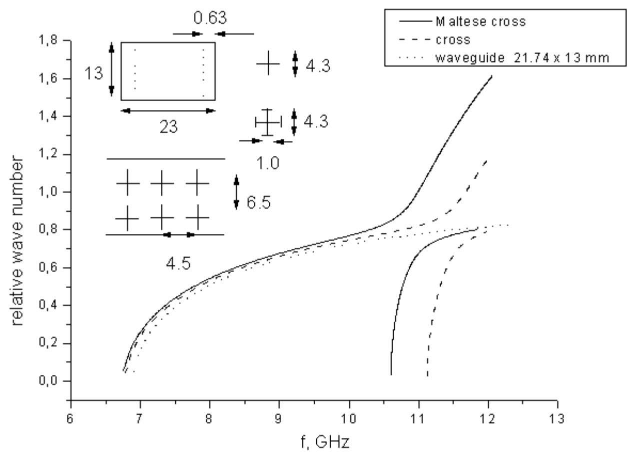

Figure 2

In SPSE the singular static part of kernel was extracted and transformed by analytical manner at the stage of numerical calculation.

Used basis functions provide fast inner convergence of the method. To make calculations with accuracy better than 0.1% it is necessary to take into account 1-6 basis functions for each current density component at each slot. Usually the number of sum terms after convergence improving not exceeds 20.

In the work was obtained following dependencies: dispersion curves of complex waveguides with insertions in the form of periodical slots of two kinds : "cross" and "Maltese cross" (Fig. 2); resonant curves of multilayered and multislot resonators, resonance curves of complex π-shaped slot resonators situated in below-cutoff waveguide.

Comparing of obtained results with known theoretical and experimental data for some particular cases of analyzed structures confirm accuracy of the investigation.

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Multiple diffraction loss calculation in micro-cell environments

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This paper will introduce a model for prediction diffraction loss from arbitrary numbers of equal height parallel buildings by using uniform theory of diffraction (UTD) in urban environments. Some recent work focused on plane wave approximation rather than spherical wave assumption, even though this may be correct in macro- cell, but in micro-cell since the transmitter and buildings are near each other, this approximation may not hold any more. Therefore in this paper we used spherical wave approximation. The present work takes into account, only single diffractions over buildings for calculations diffraction loss. Moreover because of simplicity of the model it can be applied to area coverage and interference predictions in the planning of cellular systems. For calculation the diffraction loss a typical urban cellular radio wave propagation is shown in Fig.1, as can be observed in this Figure, buildings could be seen as an array of wedges with interior angles of radian, joined two by two forming the flat roofed buildings and assumed to have the same height relative to the base station antenna height, same thickness v, and constant inter-building spacing w. The spherical source, which has an arbitrary height, is considered to be above or level with the building's height and located at a near distance from the mentioned buildings so that a spherical wave impinges on them with an angle of incidence . Using a straightforward technique based on the final UTD solution for the diffraction of plane waves by an array of dielectric wedges given in [1], a new expression in terms of UTD coefficients for the analysis of the diffractions produced by an array of dielectric buildings considering spherical-wave incidence is proposed. The total field at the observer point indicated in Fig. 1 is calculated using the summation of produced fields by single diffractions. If there will be only one building between transmitter and receiver, the received field at observed point is given:

$$E(1) = E_0[\frac{r_0' exp(-jk(r_1 - r_0'))}{r_1} + \sqrt{\frac{r_0'}{v(r_0'+v)}} D_x(\phi = \frac{3\pi}{2}, \phi' = \frac{\pi}{2} + \alpha, L = \frac{r_0' v}{r_0' + v}) \exp(-jkv)]$$

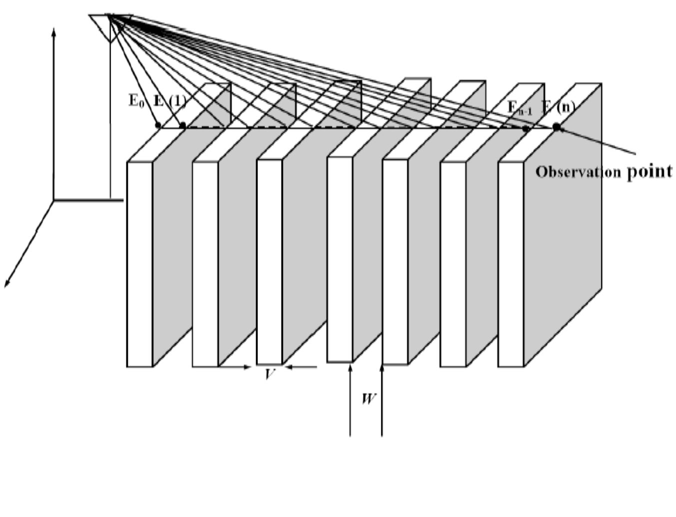

Figure 1: Scheme of the proposed environment.

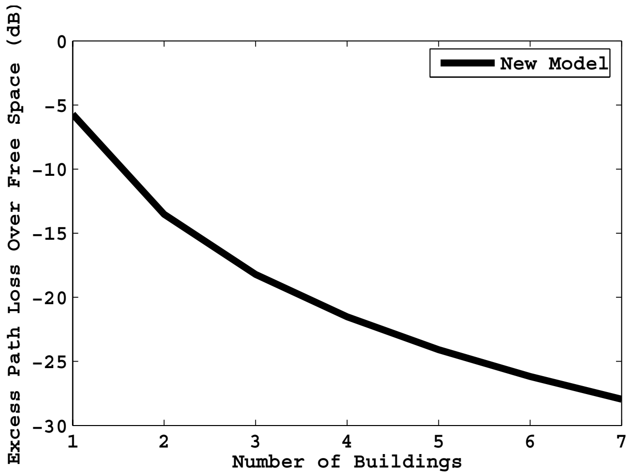

Figure 2: Excess path loss over free space, for single diffraction, $v = 22$m, $w = 33$m, $f = 922$ MHz, $\varepsilon_r = 5.5$ and $\sigma = 0.023$S/m.

Furthermore, for $n \geq 2$, $E(n)$ the following is denoted:

$$E(n)=\frac{1}{2n-1}\left\{\sum_{m=0}^{n-1}E_m\left[\frac{r'_m e^{-jk(r_n-r'_m)}}{r_n}+\sqrt{\frac{s'_y}{s_y(s'_y+s_y)}}D_y\left(\phi=\frac{3\pi}{2},\phi'=\frac{\pi}{2}+\alpha_y,L=\frac{s'_y}{s_y(s'_y+s_y)}\right)e^{-jks_y}\right]\right.$$
$$\left.+\sum_{p=1}^{n-1}Ep\left[\frac{r''_p e^{-jk(r_n-r''_p)}}{r_n}+\sqrt{\frac{s'_z}{s_z(s'_z+s_z)}}D_z\left(\phi=\pi,\phi'=\alpha_z,L=\frac{s'_z}{s_z(s'_z+s_z)}\right)e^{-jks_z}\right]\right\}$$

E_{m} is the field reaching the first corner of the roofs(as shown Fig. 1). Hence, for $m \geq 1$, E_{m} can be presented as:

$$E_m=\frac{1}{2m}\left\{\sum_{q=0}^{m-1}E_q\left[\frac{r'_q e^{-jk(r_m-r'_q)}}{r_m}+\sqrt{\frac{s'_w}{s_w(s'_w+s_w)}}D_w(\phi=\frac{3\pi}{2},\phi'=\frac{\pi}{2}+\alpha_w,L=\frac{s'_w}{s_w(s'_w+s_w)})e^{-jks_w}\right]\right.$$
$$\left.+\sum_{r=1}^{m}Er\left[\frac{r''_r e^{-jk(r_m-r''_r)}}{r_m}+\sqrt{\frac{s'_v}{s_v(s'_v+s_v)}}D_v(\phi=\pi,\phi'=\alpha_v,L=\frac{s'_v}{s_v(s'_v+s_v)})e^{-jks_v}\right]\right\}.$$

In these formulas r is distance between transmitter and diffraction point, also s is for calculation of distance parameter and spreading factor. The model was derived for 7 buildings with same heights (28m). It was assumed that the height of transmitter antenna is equal 30m and distance between transmitter and first building is 36m. The calculated results are shown in Fig. 2 The proposed model can find application in the development of theoretical models to predict more realistic path loss in microcells.

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Depolarization of shear waves in anisotropic heterogeneous media

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During propagating of shear waves through geological media an effect observed connected with the change of polarization degree of these waves has been observed. Such a change is accompanied by a buildup in the ellipticity of shear oscillations or by their depolarization. Following Aleksandrov [1], the polarization phenomenon is accounted by the scattering of shear waves from small- and large-scale inhomogeneities. His conclusions were corroborated by Crampin's experiments on inhomogeneous cracked media [2]. A different interpretation of the polarization phenomenon was proposed by Obolentseva [3]. According to her theory, depolarization occurs in some solids due to their gyrotropic or acoustical activity. When a shear wave propagates in such media, its polarization vector rotates through an angle proportional to the distance travelled.

To our opinion, the depolarization effect is also appreciably manifested as shear waves propagate in a medium consisting, for example, of differently oriented elastic anisotropic layers. We have analysed some theoretical aspects of the shear wave depolarization (SWD) effect, modeling results, and determinations of the effect in crystalline rock samples.

The modeling results are shown in the Figure. The acoustical polarization method [4] was used to measure the amplitude of linear polarized shear waves passing through two anisotropic plates. The model prepared consisted of two anisotropic plates of uniform thickness, composed of the same elastically anisotropic ceramic material (PZT-19). Acoustopolarigrams were obtained for different angles a between the elastic symmetry elements of the plates. According to Figure the VP diagrams show a gradual change-over from a four-petal pattern to a figure more and more resembling a circle, as the angle α increases. The diagram for $\alpha = 90°$ almost coincides with that of an isotropic medium.

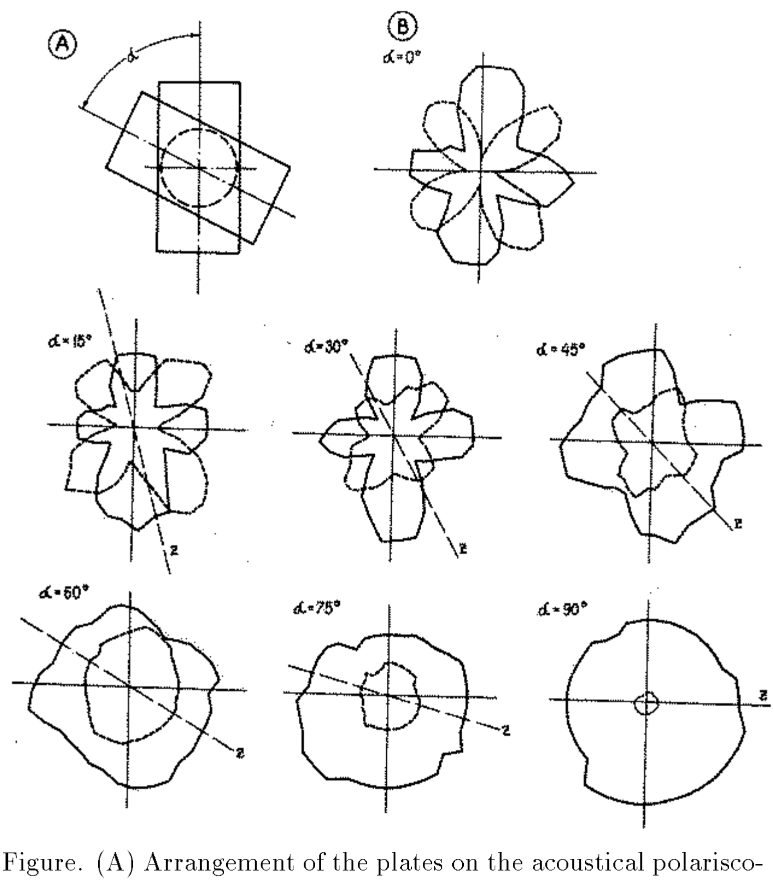

Figure. (A) Arrangement of the plates on the acoustical polariscope platform and (B) acoustical polarization diagrams obtained for different angles α between the elastic symmetry elements of the plates. Solid and dashed lines are for the parallel and crossed polarization vectors, respectively.

The obtained results [4] show that this type of the SWD effect is rather frequently observed in anisotropic crystalline rocks composed of layers or grains whose elastic symmetry is sufficiently maintained, for example, in two directions. It is necessary to note that the SWD effect manifests itself on the background of the usual change in the degree of ellipticity of shear waves while propagating in anisotropic media [5]. Consequently, the method used to detect the effect must not be sensitive to such a change. One such method is provided by the acoustopolariscopy.

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A modified Euler transformation for machine precision evaluation of potential integrals

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The superiority of the cancellation method with respect to the subtraction method to numerically compute singular and nearly singular potential integrals with $1/R$ singularities is discussed in [1]. A new numerical technique to compute to machine precision singular and nearly singular potential integrals by working directly in the parent reference-frame is presented here. Our integration scheme is based on a new rational expression for the integrands, obtained by a cancellation procedure and by use of a modified Euler transformation for the integration variables, as well as on special quadrature rules: Gauss quadrature for rational functions [2], together with classical Gauss-Legendre quadrature. The technique can deal with static and dynamic potentials on surface and volume elements. In particular, in the static case of polynomial source distributions, our new cancellation procedure allows for the exact integration of the potential integrals. The rules to establish the quadrature weights/points (including their number) to guarantee machine precision are reported in [3]. Several numerical results for potential integrals will be presented at the conference.

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Windowed oscillating integral: Gaussian beams method beyond its limitations

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High-frequency asymptotic methods in wave-propagation problems (quasi-classical in quantum mechanics) are known as physically-based approaches to obtain, in particular, numerical results for the range of parameters which are not accessible by finite-deference or finite-element techniques. However, despite of the physical clearness of asymptotic methods their implementation is not always straightforward: for example, in the simplest form known as the *geometrical optics* the approach is said to be *non-uniform* and fails on manifolds carrying the singularities of the geometrical objects connected with classical trajectories (rays).

Starting the middle of 20th century a lot of work was done to understand the uniform generalizations of the geometrical optics. The *Maslov's theory* (also known as the *canonical operator method*) provides the most common and uniform integral representation for a high-frequency wave field. However, this representation is not effective for numerics. Another uniform integral representation known as *Gaussian beams method* is highly promising because of its simplicity and computational efficiency but has certain internal limitations.

In the talk we suggest the new uniform representation entitled *windowed oscillating integral* (WOI). The approach preserves all attractive features of Gaussian beams method but can be applied beyond the limitations of the latter. WOI also is asymptotically equivalent to both Maslov's and Gaussian beams methods and thus serves as an "asymptotic bridge" between these two approaches. The numerical efficiency of the proposed approach will be demonstrated.

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Beam summation analysis of half plane diffraction

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Gaussian beam summation (GBS) formulations are an important tool in wave theory as they provide a framework for ray-based construction of spectrally uniform solutions in complex configurations. In these formulations, the field is expanded into a spectrum of collimated beam propagators that emanate from the source domain in all directions, and thereafter are tracked locally in the medium and summed up at the observation points (see [1] for a recent review).

In many applications, such as in indoor or urban propagation, the ambient environment involves edges and corners that cause the incident bream wave impinging near the edge to diffract in all directions. In order to be used self-consistently in GBS schemes, this scattered field needs to be expressed as a sum of GB's that emerge from the edge in all directions, thereby describing the edge by an equivalent GB-to-GB scattering matrix. Various solutions for beam scattering by edges have been derived in the past, yet none of them has addressed the problem described above, namely GBS representation of the edge diffracted field.

Following the discussion above we present a GBS representation for a half-plane diffraction of an incident beam wave. The scattered field is expanded into GB's that emerge from the edge with appropriate excitation amplitudes. Asymptotic expressions for the diffracted beams amplitudes are derived and compared to exact numerical calculations. Applications to GBS modeling of indoor propagation are discussed.

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A phase-space beam summation representation for 3D radiation from a line source distribution

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Beam summation formulations are an important tool in wave theory as they provide a framework for raybased construction of spectrally uniform local solutions in complex configurations. In these formulations, the field is expanded into a phase-space spectrum of collimated beam propagators that emanate from a set of points and directions in the source domain, and thereafter are tracked locally in the medium and their contributions at the observation point are summed up (see a recent review in [1])

There are essentially two classes of GBS schemes, for point sources and for distributed (aperture) sources. In the former, the field is expanded as a angular spectrum of beams that emerge from the source in all directions [2] whereas in the latter the field is described as a discrete phase space sum of beams that emerge from a set of points and directions in the source domain [3]. Here we introduce a hybrid scheme for radiation from a line source distribution, involving a phase phase-space decomposition of the source distribution along its axis, and an angular spectrum of GB's in the plane normal to the axis. Our motivation for this new expansion scheme has been the preparation of the analytical framework for a GBS representation of 3D edge diffraction, as an extension of the 2D expansion in [4]. The accuracy of this scheme and the choice of the expansion parameters are illustrated numerically

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The impact of the small scale fluctuations of the ionospheric plasma on the ultra wide band orbital ground penetrating radar measurements

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The propagation of the ultra wide band (UWB) signals of the ground penetrating radars (UWB) through the ionosphere with small scale irregularities is investigated. Signal distortions due to joint action of phase distortions in the regular layered dispersive ionospheric plasma and the scattering on the random small scale inhomogeneities is simulated numerically. The impact of the diffractive effects on the function of the phase correction algorithms [1] is investigated.

It has been stated that the plasma density fluctuations introduce significant distortions in the chirp signal, especially at the frequencies, close to the critical frequency of the ionosphere. Satisfactory quality of the signal compression can be achieved at frequencies, well exceeding the critical frequency. When the fluctuations are strong, the signal frequency must be quite high compared to the critical frequency of the ionosphere. This conclusion is in agreement with the published experimental results on planetary GPR sounding, known at present.

The operation of the algorithms of correction of phase distortions in the regular ionosphere is relatively little influenced be the plasma density fluctuations, at least when the fluctuations are not too strong. Thus the conclusion can be made is that the regular and random components if the ionospheric distortions of the UWB chirp signals are independent. The stability of the investigated phase correction algorithms with respect to the scattering of the signals on the small scale fluctuations is shown.

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An attempt to improve semiclassical calculations of wave transport in a quantum wire with Diffraction

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Recently, the systems of nanoscale wires made in semiconductor heterosurfaces are attracting much attension because of their potential application to electronic devices. They have interesting features of quantum e.ects such as quantization of conductance, conductance fluctuations etc. All these phenomena originate from the coherence of electronic quantum waves. In our previous presentations in the series of this seminar, we have shown an interesting phenomenon which had been found numerically and later explained semiclassically for coherent electronic transmission through a mesoscopic quantum wire with a step structure. The motion of free electrons in the wire is ballistic with specular reflections from the Dirichlet boundaries. The wire allows only a few modes for electronic transmission. This time, we report an attempt to improve the semiclassical calculations of wave transport in such a system.

Vibrations of inhomogeneous solid discs subjected to an inertial rotation

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The effect of a vibrating pattern's precession in the direction of inertial rotation of a vibrating ring was discovered by G. Bryan in 1890. This effect has several applications to navigational instruments, such as cylindrical, hemispherical and planar circular disc rotational sensors. A model of inhomogeneous structures composed of concentric thin circular discs vibrating along the plane of the disks and subjected to inertial rotation is considered. Dynamics of the disc gyroscope are considered in terms of linear elasticity. It is assumed that the angular rate of inertial rotation of the discs is constant and it has axial (tangential???) orientation. We also suppose that the angular rate is much smaller than the lowest eigenvalue of the discs and consequently any centrifugal effects that are proportional to the square of the angular rate are neglected. The model is formulated in terms of Novozhilov-Arnold-Warburton's theory of thin shells. The system of equations of motion of the discs are separated and transformed to a pair of wave equations in polar coordinates. A solution is obtained in terms of Bessel's and Neumann's functions. Various non-axisymmetric modes of the disc are considered and eigenvalues as well as Bryan's constant is investigated for imperfection of mass densities of the structure.

A probabilistic approach to bi-variate function approximation

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For an arbitrary x, the Gauss-Bernstein approximation for a function $f(x)$, known in the points $(x_j, y_j = f(x_j), j = 1, \ldots, n)$, is of the form $k(x) = \sum_{j=0}^{n} y_j p_j$, a finite weighted sum of y_j with weights p_j. The weights are probabilities determined by the Gaussian distribution with mean x and standard deviation $\sigma(x)$. It is also possible to derive a scheme of choosing $\sigma(x)$ so that the Gauss-Bernstein approximation of a straight line, passing through two points, is exact - that is the difference between $f(x)$ and $k(x)$ is guaranteed to be exactly zero. Ultimately it is then possible to uniformly approximate any function of bounded variation by piecewise straight line segments.

In this paper this idea will be extended to approximate a bi-variate function $f(x, y)$:

- If a function $z = f(x, y)$, is known in the points
$$(\{(x_i, y_j), z_{ij} = f(x_i, y_j)\}, i = 1, \ldots, n_1, j = 1, \ldots, n_2),$$
it will be shown that the Gauss-Bernstein approximation is
$$k(x, y) = \sum_i \sum_j z_{ij} p_{ij}$$
with p_{ij} probabilities from a bi-variate normal distribution with means (x_i, y_j), variances (σ_x, σ_y) and covariance σ_{xy}.
- A scheme to select the variances and covariance, and hence to calculate the probabilities, will be explored.
- Appling the technique to approximate a surface plane, it will be shown that this technique is free of the Gibbs effect.

Vibrations of a rotating solid elastic sphere filled with an inviscid fluid

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The dynamics of a solid elastic sphere, filled with a compressible inviscid fluid are considered in the frame of a three-dimensional theory. Equations of motion are solved by means of a four potential method. It is assumed that the system is subjected to a small inertial rotation with regards to inertial space. This slow rotation means that the maximum of the absolute value of the inertial angular rate is substantially smaller than the minimum eigenvalue of the system. Consequently it is possible to neglect effects (such as centrifugal forces, prestress of the sphere due to inertial rotation, et cetera) that are proportional to the square of the angular rate. It is shown that the gyroscopic effect of the vibrating pattern's precession depends on spheroidal modes of vibrations. The effect of inertial rotation of vibrating patterns in the direction of revolution that has a rate that is proportional to the inertial angular rate is considered. Various types of inertial rotation (with constant and radial dependence of angular rates) are examined. These investigations are tentatively submitted as tools to be used in order to understand some fundamental phenomena in seismology and both solar and stellar dynamics.

Dynamics of a vehicle-trailer system with a limited power supply

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A model of a vehicle-trailer system, having four degrees of freedom is considered. The linear motion of the vehicle is characterized by two degrees of freedom, one degree describes an angular location of the trailer and an additional degree of freedom is introduced to describe an engine, which is considered as a source with a limited power supply. This means that the torque, produced by the engine and transformed to a driving force could oppose a mean value for the frictional forces involved, but cannot oppose periodic vibrations created by the perturbed motion of the vehicle-trailer system. This is why a strong interaction between the engine and the vehicle-trailer system exists and this interaction has to be taken into consideration in order to produce a realistic description of the dynamics of the system. It is shown that a simplified model of the vehicle-trailer system can be described in terms of Mathieu's equation and by means of which the critical regimes are found. These regimes are then investigated in a complete system with four degrees of freedom by an approximate perturbation technique as well as by means of a numerical solution of nonlinear equations of motion using Runge-Kutta methods.

Quantization of surfaces in magnetic field

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We consider a point charge on a surface in a sufficiently strong magnetic field. The surface is two-dimensional, smooth, oriented and arbitrarily curved, as well the field is smooth and arbitrarily inhomogeneous.

In classical mechanics, this is an important example of a natural system with two degrees of freedom which involves metric and gyroscopic forces only. In quantum mechanics, such a system models the electron in a thin film (for example, on the surface of a nanotube) [1]. In this case, the surface curvature and the magnetic field inhomogeneity manifest themselves already at atomic scales. The basic quantum Hamiltonian is the usual two-dimensional Schroedinger operator with metric and magnetic field. If spin effects are taken into account then the spin-orbit and Pauli corrections are added to the Hamiltonian.

Our main goal is to demonstrate that, in the framework of the nano-momentum mechanics (i.e., on the class of states with nano-scale kinetic momenta), this nonintegrable system is asymptotically reduced to a system with one degree of freedom, which can be effectively analyzed in both the classical and quantum cases.

The original surface serves as the phase space for the reduced system. The Poisson bracket on this surface is determined by the inverse magnetic field tensor. In quantum case, the Poisson bracket is replaced by the commutator and, as a result, the original surface becomes a quantum surface. This is an inhomogeneous generalization of the well-known Heisenberg-type commutation relation between coordinates on the configuration plane, which was proposed by Peierls in 30th of the last century in describing the Fock level splitting of a charge in a homogeneous magnetic field and a perturbing potential. In the general case, studying arbitrary magnetic fields and curved surfaces, we obtain a rich family of examples of phase (quantum) manifolds of a complicated shape, in particular, like the sphere, the torus, etc. The symplectic area (i.e., the magnetic flux) around holes in the quantum surface must be multiple of the elementary London' flux. We stress that for the original problem such quantized surfaces are just the usual configuration spaces.

In the given model, the motion of the charge can be separated into oscillations of a "light" magnon (quantum vortex), similar to the spin or hydrodynamic vortex [2], and to dynamics of a "heavy" quasiparticle representing the motion of the vortex center along the quantum surface. The magnon energy levels are degenerate. Each of them is split into a cluster of energy levels of the quasiparticle. We note that the cause of this splitting is different from that in the well-known Peierls scheme (there is no perturbing potential in the given problem at all).

The Hamiltonian of the quasiparticle [3] is a function on the surface, which can be expressed in terms of the joint geometric invariants of the magnetic tensor and the metric. This Hamiltonian determines both the regular dynamics and the local instabilities, separatrices, which characterize zones of irregularity, quantum tunneling, etc. The behavior of the quasiparticle can be analyzed, in particular, by methods described in [4 - 7], which allows us to solve, in the nano-momentum approximation, the original problem about the quantum charge on a surface in a magnetic field.

We conclude that the surface in a sufficiently strong magnetic field is stratified by the quasiparticle orbits (actually each orbit is presented by a pair of nearby fermion suborbits if the spin is taken into account). In the stability domain, the orbits are closed and carry the discretized magnetic flux. Now, by considering a collective motion of electrons along the surface, we can state that a vortex-free flow of a quantum charged "liquid" of quasiparticles appears on the surface. The flow is generated by geometric objects (magnetic and metric tensors) only. This can be regarded as one of possible origins of the 2-dimensional superconductivity effect.

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Nonlinear waves in coupled waveguides

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A system of coupled Klein-Gordon equations (cKG) is proposed as a model for one-dimensional nonlinear wave processes in two-component media (e.g., long longitudinal waves in elastic bi-layers, where nonlinearity comes only from the bonding material). We discuss general properties of the model (Lie group classification [1], conservation laws, invariant solutions) and special solutions exhibiting an energy exchange between the two physical components of the system [2, 3]. To study the latter, we consider the dynamics of weakly nonlinear multi-phase wavetrains within the framework of two pairs of counterpropagating waves in a system of two coupled Sine-Gordon equations, and obtain a hierarchy of asymptotically exact coupled evolution equations describing the amplitudes of the waves. We then discuss modulational instability of these weakly nonlinear solutions and its effect on the energy exchange. Our methodology is generic and can be applied to other systems, and other multi-phase wavetrains.

The simple lattice model leading to the cKG equations has been used to study some features of propagation of nonlinear waves in bi-layers with delamination [4]. The model can be naturally modified to take into account nonlinear interactions in the layers and other degrees of freedom (for example, by considering chains of interacting mechanical dipoles [5] instead of chains of point masses). It can also be easily two-dimensionalised. We expect such models to be useful for the modelling of nonlinear waves in a large class of bi-layers [6]. We discuss some results in this direction.

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Singular nonlinear boundary value problem for bubble–type or droplet–type solutions in nonlinear physics models

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For a second–order nonlinear ordinary differential equation, a singular boundary value problem (BVP) is investigated which arises in hydromechanics and nonlinear field theory when the static centrally symmetric bubble–type or droplet–type solutions are sought. The main theoretical results of this talk are described in detail in [1]; for the models, see, e.g., [2], [3]. Posed and partially studied in [4], for a capillary fluid model with a simplified form of the chemical potential of the medium, this singular BVP is the same for the scalar Higgs field with two distorted vacua. In the normalized variables, the considered singular nonlinear BVP has the form:

$$\rho'' + \frac{N-1}{r}\rho' = 4\lambda^2(\rho+1)\rho(\rho-\xi), \qquad 0 < r < \infty, \tag{1}$$

$$|\lim_{r\to 0+}\rho(r)| < \infty, \qquad \lim_{r\to 0+} r\rho'(r) = 0, \tag{2}$$

$$\lim_{r\to\infty}\rho(r) = \xi, \qquad \lim_{r\to\infty}\rho'(r) = 0. \tag{3}$$

Here all magnitudes are real, N, λ and ξ are the parameters.

Eq. (1) has a regular singularity as $r \to 0$ and an irregular one as $r \to \infty$. We give the restrictions to the parameters for a correct mathematical statement of the limit boundary conditions (2), (3) and their accurate transfer into neighborhoods of singular points using some results for singular Cauchy problems and stable initial manifolds. Due to the approach of [5] and certain results of [6], the necessary and sufficient conditions for existence of bubble–type or droplet–type solutions are discussed (in the form of additional restrictions to the parameters) and some estimates are obtained. A priori detailed analysis of singular nonlinear BVP (1)–(3), including the representation of one–parameter sets of solutions in the neighborhoods of the singular points, leads to efficient shooting methods for the numerical solution of this BVP. Some results of numerical experiments are displayed and their physical interpretations are discussed. In particular, we give a comparison of different definitions of the bubble radius, which influence important characteristics of the bubble, such as the minimal nucleation radius, surface tension and interface thickness.

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The diffraction tomography and iterative approach to restore the elastic parameters and electrical conductivity

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The results of numerical simulation on restoration of electrical conductivity and elastic parameters (Lame parameters and mass density) of the local inhomogeneities correspondingly with the help of electromagnetic and elastic sounding signals are considered.

The direct problems for the Maxwell and Lame equations are solved by the finite difference method. Restoration of the desired parameters is implemented by the diffraction tomography method in the time domain [1], [2] using the first-order Born approximation. Restoration of the electrical conductivity is studied in the low frequency case.

We consider restoration of the inhomogeneity parameters with the help of the iterative procedure. Each step of the iterative procedure includes solution of the direct problem and correction of the desired parameters by the diffraction tomography method. For this purpose the algebraic methods with different regularization schemes are used.

The numerical simulation for 2-D problems in elastic and electromagnetic cases is implemented for the local inhomogeneities with a simple and complex geometry and with a contrast about 50 % relatively to the reference medium. Main attention is given to study the convergence and accuracy of the considered algorithms.

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The nonparaxial self-focusing of few-cycle light pulses in dielectric media with dispersion

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In the last decade, laser systems that allow pulses comprising only a few cycles of a light field to be obtained were created in many laboratories. The concept of an envelope loses its physical meaning for such extremely short pulses (ESPs); therefore, for describing propagation of these pulses, the equations of evolution of envelopes that are conventional in nonlinear optics, are no longer appropriate. The theory of nonlinear propagation of ESPs in various media is based, as a rule, on equations describing the dynamics of no the pulse envelope but its field directly. The rigorous theory of the self-action of ESPs in a bulk media should be nonparaxial, since, in the calculation of the space-time evolution of pulses with small longitudinal size, the possibility of occurrence of transverse inhomogeneities of the same scale in the pulse structure should also be taken into account.

In the present work the solutions of the equation describing the nonparaxial dynamics of the space-time spectrum of an ESP in a homogeneous isotropic dielectric medium with an arbitrary spectral dependence of the linear refractive index and nonresonant electron nonlinearity deduced in [1] is illustrated by numerical modeling.

The figure illustrates the temporal spectrum and the corresponding electric field of two-dimensional ESP of TE-polarized radiation at different distances passed by the pulse in the fused silica. The dispersion of the fused silica id described by the dependence $n(\omega) = N_0 + a\omega^2$, where $N_0 = 1.450$; $a\omega_0^2 = 0.007$; $\omega_0 = 2.4 \cdot 10^{15}\text{s}^{-1}$; $\lambda_0 = 2\pi c/\omega_0 = 0.78\ \mu$m.

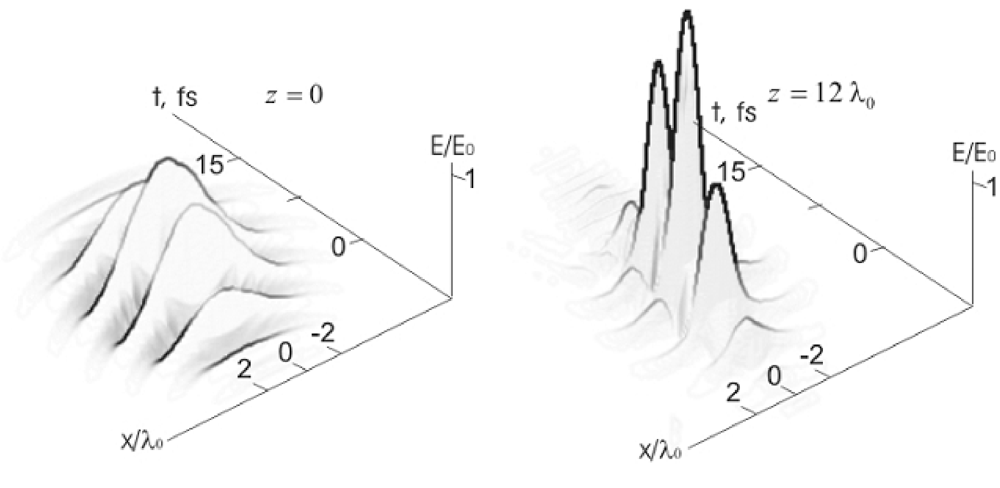

Figure : The dynamics of the temporal spectrum g and the electric field E of initially focused few cycle titanium-sapphire laser pulse with input peak intensity is $I = 8 \cdot 10^{13}\text{W/cm}^2$ in the fused silica.

The figure shows that during self-action of ESP the highly intensive light kern with the transverse size comparable with the central wave length is generated. As the figure indicates, the inhomogeneous broadening of the spectrum into the red and the blue regions, including the generation of tripled components, takes place mainly at the expense of the energy of the time spectrum at high spatial frequencies.

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Boundary value problems for the Helmholtz equation in domains bounded by closed curves and open arcs

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Boundary value problems for the Helmholtz equation are considered in planar domains with complicated boundary. The boundary of a domain consists of both closed curves and open arcs (cracks or cuts). Formulation of boundary value problems in these domains implies that the boundary condition is specified on the whole boundary (i.e. on both closed curves and open arcs). Boundary value problems in such domains were not actively treated before though they have many applications. Similar domains model cracked solid bodies or they model several obstacles and screens in a fluid. However problems in domains bounded by closed curves and problems in the exterior of open arcs were treated separately, because different methods were used in their analysis. In the present talk we consider both the Dirichlet problem and the Neumann problem for the Helmholtz equation in domains bounded by closed curves and open arcs. We study either the dissipative Helmholtz equation in interior and exterior domains or propagative Helmholtz equation in exterior domains. The rigorous mathematical formulation of the boundary value problems are given, the uniqueness theorem is proved. With the help of potentials the problems are reduced to the integral equation on the boundary of a domain. By means of some transformations this equation is reduced to the Fredholm integral equation of the second kind and index zero. It is proved that this integral equation is uniquely solvable in the appropriate Banach space. In this way, the classical solvability of the problems is proved, and the integral representation for a solution is obtained. The effective computational scheme for finding numerical solution is suggested. The Dirichlet and Neumann problems outside open arcs in a plane have been studied in [1,2]. The Neumann problem for the dissipative Helmholtz equation in both interior and exterior cracked domains has been studied with Neumann boundary condition in [3,4] and with Dirichlet boundary condition in [14,15]. The Neumann problem for the propagative Helmholtz equation in exterior of several obstacles and screens has been studied in [5,6] in case of non-resonance obstacles and in [7,8] in general case. The Dirichlet problem for the propagative Helmholtz equation in an exterior cracked domain has been studied in [12,13]. The Helmholtz equation outside cuts in a plane with the skew derivative boundary condition, generalizing the Neumann boundary condition, has been treated in [9-11].

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Semiclassic asymptotics for the vector Sturm-Liouville problem with parameters

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Consider asymptotics of linearly independent solutions of the matrix Sturm-Liouville equation in some vicinity $W_\varepsilon = \{x : |x - x_0| < \varepsilon\}$

$$\left(-ih\frac{d}{dx}\right)^2 y - A(x,u)y = Ey;\ x \in W_\varepsilon; \tag{1}$$

with respect to a small parameter h. Here $y \in \mathbb{C}^2; h \in (0, h_0]$, and $u = u_1, ..., u_n$ are some parameters. Let $u \in V_\mu := \{u : |u| < \mu\}$.The matrix Sturm-Liouville problem occurs in many physical problems [1]. Let $A(x,u)$ be a self-adjoint (2×2) matrix function of class $C^\infty(W_\varepsilon \times V_\mu)$. Suppose that the matrix $A(x,0)$ has the form $A(x,0) = \mathbb{U}_0(x)\,\|\lambda_j(x)\delta_{ij}\|\,\mathbb{U}_0^*(x)$, where $\mathbb{U}_0(x)$, $\|\lambda_j(x)\delta_{ij}\|$ are the $C^\infty(W_\varepsilon)$ matrix function, and the matrix $\mathbb{U}_0(x)$ is unitary .Consider the case when eigenvalues $\lambda_j(x)$ in the vicinity W_ε coincide only at the point x_0 and it holds that

$$\lambda_1(x) = \lambda_2(x) \text{ only at } x = x_0,\ \left.\frac{d[\lambda_1(x) - \lambda_2(x)]}{dx}\right|_{x=x_0} \neq 0,\ \inf_{W_\varepsilon}[E + \lambda_j(x)] > 0, \tag{2}$$

where $j = 1, 2$. It is proved that in the domain $W_{\varepsilon_1} \times V_{\mu_1}$ for sufficiently small μ_1, ε_1 the function $\det A(x,u)$ has the following representation

$$\det A(x,u) = \nu^2(x,u)[x - x_0(u)]^2 + |d(x,u)|^2\,|\delta(u)|^2\,, \tag{3}$$

where $\inf_{W_{\varepsilon_1}\times V_{\mu_1}} \nu^2(x,u) > 0$; $x_0(u), \delta(u) \in C^\infty(V_{\mu_1})$ and $x_0(0) = x_0, \delta(0) = 0$. We call $x_0(u)$ the turning point function, and the function $\delta(u)$ we call the efficient parameter. It is easy to verify that in the domain $\sqrt{[x - x_0(u)]^2 + |\delta(u)|^2} > h^{1/2-\gamma}$ for any fixed $\gamma > 0$ the application of WKB method provide asymptotic solutions of equation (1). We will construct the asymptotic expansion for the solutions of equation (1) in the domain

$$|\delta(u)| \leq h^{1/2-\gamma_1};\ |x - x_0(u)| \leq h^{1/2-\gamma_2} \text{ for some fixed } \gamma_1, \gamma_2 > 0 \tag{4}$$

in modified "p-representation", and obtain representations of these asymptotics as the WKB asymptotics near the boundary of this domain, i.e. for $\frac{1}{2}h^{1/2-\gamma_2} < |x - x_0(u)| < h^{1/2-\gamma_2}$. By this approach the connection formulas between WKB asymptotics of solutions of problem (1) in domain $x - x_0(u) < 0$ and WKB asymptotics of these solutions in domain $x - x_0(u) > 0$ are obtained.

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Asymptotic solution of linear system with a turning point of high order

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Linearizing a nonlinear hyperbolic system of partial differential equations (for example the system of equations of ideal compressible fluids, system of Maxwell equations with the current $J(\rho, E)$ non-linearly dependent on the charge density ρ and the electricity field E) one obtains a linear hyperbolic system with three or more coinciding characteristics at the points of multiplicity. One dimensional model of this case is the following system with a small parameter at the derivative:

$$-ih\frac{dy}{dt} + A(t)y + hB(t)\,y = 0, \tag{1}$$

where $y \in \mathbb{C}^{r+2}, r \geq 1$, and $h \in (0,1]$ is a small parameter. We consider the asymptotics of the linearly independent solutions of system (1) in a vicinity $U_\varepsilon = \{t : |t| < \varepsilon\}$. Let $B(t) \in C^\infty(U_\varepsilon)$ be a $(r+2) \times (r+2)$ matrix function, and $A(t)$ be a matrix function of the form:

$$A(t) = \begin{bmatrix} Ia(t) & 0 & 0 \\ 0 & b(t) & d \\ 0 & 0 & c(t) \end{bmatrix}, \tag{2}$$

where I is $(r \times r)$ identity matrix, $d \in \mathbb{R}^1_+$, and $a(t), b(t), c(t)$ are $\mathbb{C}^\infty(U_\varepsilon)$ functions such that $a(0) \neq b(0) \neq c(0)$. Above mentioned system can not be reduced to the Webber equation. In the present paper we construct the asymptotics of this problem. In the domain $|t| > h^{1/2-\delta}, 0 < \delta < 1/8$, the asymptotics of equation (1) can be constructed by the standard WKB method. In the domain $|t| < h^{1/2-\delta}, 0 < \delta < 1/8$, for the solution of problem (1) the asymptotics of the form

$$y(t,h) = \frac{1}{\sqrt{h}} \int_\gamma \exp\frac{i}{h}\left(tp - \int_0^t b(\tau)\,d\tau\right) w(p)\,\chi(p)\,dp \tag{3}$$

is obtained. Here $\chi(p)$ is a patch function, and γ is a contour that goes along the real axis and bypasses the point $p = 0$ from below over the circule of radius $h^{1/2+2\delta}$.

In the domain $|t| > h^{1/2-\delta}, 0 < \delta < 1/8$, an expansion of asymptotics $y(t,h)$ is obtained by the saddle point method.

Averaging and chaotic modes for nonautonomic nonlinear dynamic systems

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The Krylov-Bogolyubov averaging theorem has been considered, the local existence of periodical orbits in some tasks about forced oscillating of nonlinear nonautonomic dynamic systems has been determined with its help. Some general features, particularly, the existence of homo- and heteroclinic orbits have been determined. Such orbits can be found with the help of Melnikov method, when the task has the saddle connection. The discussion of resulting chaotic movement at availability of such orbits is given; the examples are given.

Diffraction of nonstationary wave with varying along the front amplitude by a soft wedge

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Exact analytic solution of new 2D problem of diffraction by a wedge with dirichlet boundary condition on its surface is found. The incident field is a plane wave with δ-type profile. The amplitude of this wave varies linearly along its front. The solution is constructed by Smirnov–Sobolev method (the method of functionally invariant solutions).

Spectra of modes of waveguides, loaded with wire media

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Wave propagation in rectangular waveguides, loaded with single and double wire media, is studied. Both TE and TM modes are considered. Propagation of TE modes is quite trivial because the spatial dispersion causes only reduction of the transversal wavenumber by the plasma wavenumber. TM modes possess more interesting dispersion including propagation below cutoff of an empty waveguide, existence of backward waves, [1], etc.

Following features of spectrum of modes, propagating in a waveguide, filled with double wire media, were discovered [2]:

- Dominant TE_{10} mode cannot propagate.
- The dominant mode of WMW can propagate even below cutoff of the TE_{10} mode of the empty waveguide.
- In pass band, occupied by TE_{10} mode in the empty waveguide, now propagates a backward wave.
- Both low-frequency forward and backward waves have polarization similar to TM_{11} mode of the empty waveguide.
- No waves propagate between high-frequency cutoff of this backward wave and TE_{11} (TM_{11}) modes, both lowest modes in WMW propagate in single-mode regime.

There is difference between geometries with connected and non-connected wires [2]. If to connect mutually perpendicular wires, the first, forward mode, disappears. The features, described above, take place independently, whether the plasma frequency is below or above the cutoff frequency of the empty waveguide. Decrease of the plasma frequency causes shift of the lowest pass band to lower frequencies.

The waveguides, filled with wire media, can find applications as size reduced transmission lines, mode filters and controllable devices if to load the wires by controlling elements.

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The number of energy levels in MQW-structure

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A formula for the number of energy levels of a quantum particle in one-dimensional piecewise constant potential field of certain kind is obtained. This field we call a MQW (multiple quantum well) structure. It consists of an arbitrary number (but more than 2) of layers, that is segments of constant potential. We treat a special type of MQW structures, consisting of wells and walls. The potential of each well is equal to 0, and the potential of each wall is equal to $U > 0$. The widths of wills and walls are arbitrary, besides external walls having infinity width. The deduction of the formula is based on the analysis of the recently obtained multilayer equation [1, 2], allowing to calculate eigenvalues of energy E of a quantum particle in MQW structure. The multilayer equation looks like $F_i(E) = 0$, where $F_i(E)$ is a rather complex function, built by a given MQW structure and depending on the index i of an arbitrary chosen (preferred) bounded layer. In the previous work the author proved that nontrivial zeroes of the function $F_i(E)$ are independent of index i of the preferred layer. The formula obtained for the number of energy levels is valid only for a "general position" case. It means that in some cases one can pick up widths of wells and walls in such way that the formula ceases to be valid. But this collections of widths are very rear!

Besides the problem treated here, the multilayer equation is extremely important in the theory of optical waveguides. It permits to calculate the eigenvalues of effective index of refraction for the multilayer waveguide. The formula discussed here can be rewritten to give the number of the eigenvalues of effective index of refraction. Note that the multilayer equation may be applied in the different situations [3]. This situations and problems now are of significant practical interest for nanotechnologies.

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Solution scattering problems of electromagnetic waves from inhomogeneously layered scatterers using pattern equation method

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This paper presents the generalization of the pattern equation method (PEM), which had been applied before to solve electromagnetic scattering problems for conducting objects coated with dielectric materials, utilized to solve scattering problems of electromagnetic waves from inhomogeneously layered dielectric scatterers. The core of layered object represents the dielectric obstacle.

By analogy with the previous works, the basic point of the PEM is to reduce an initial boundary problem to solving an infinite system of algebraic equations with respect to unknown expansion coefficients of the scattering pattern and also fields inside the dielectric layer and core of scatterer into series in terms of vector angular spherical harmonics that compose the orthogonal basis in the

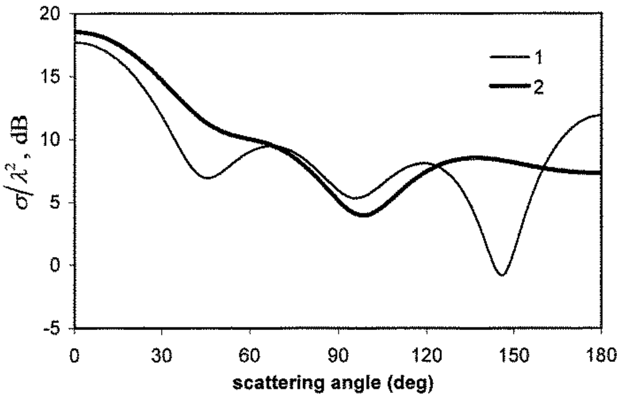

Fig. 1. Scattering pattern for sphere

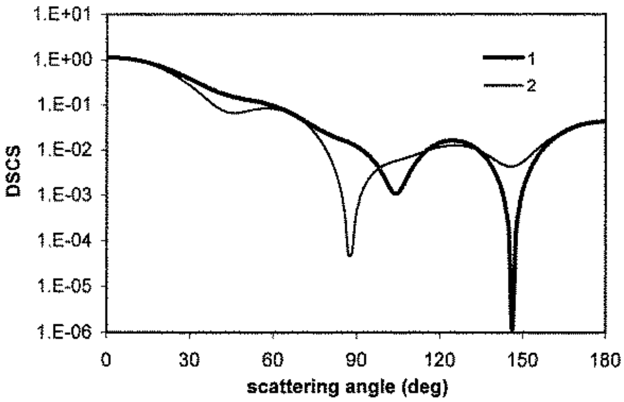

Fig. 2. Scattering pattern for spheroid

spherical coordinates. Under the certain restrictions on geometry of the problem, which can be strictly established, the received infinite linear system of the algebraic equations is solved by the method of a reduction. In the framework of the method, the analytical solution of the scattering problem for sphere as the Mie series with respect to the expansion coefficients of the scattering pattern has been obtained.

Under the developed numerical algorithm of PEM, we carried out researches of accuracy of numerical calculations for scattering characteristics. Moreover, we carried out comparisons of our results with the results obtained by PEM for impedance and dielectric scatterers, and by other methods also. The class of scatterer's geometry was limited to the bodies of revolution that significantly simplifies the numerical algorithm and reduces the surface integrals for matrix elements in the system of PEM to single integrals.

Some of the results obtained are shown here. We consider the axial incidence of a plane wave directed along the symmetry axis of the scatterer, which coincides with the z-axis. The bistatic RCS's σ/λ^2 of the dielectric sphere coated with a dielectric spherical layer are shown in Fig. 1. The geometry of sphere coincides that was presented in [2]. The internal sphere has a 0.8π radius and its coating spherical layer is of 0.2π thickness. There are two cases presented: in first case the materials of the internal sphere and the covering coincide (curve 1: relative permittivity and permeability $\varepsilon_r = 4$, $\mu_r = 1$); in second case materials of core and the covering are equal to $\varepsilon_{r2} = 4$, $\mu_{r2} = 1$ and $\varepsilon_{r1} = 1.5 - 0.009i$, $\mu_{r1} = 1$, respectively. Both curves completely are agreed to what are given in [2]. We consider also a dielectric prolate spheroid coated by dielectric material. One consists of two concentric spheroids with the semiaxes of internal spheroid $ka_1 = 3$, $kc_1 = 6$ and the external spheroid $ka_2 = 1.5$, $kc_3 = 3$. The medium in internal spheroid and its coating layer have the relative permittivities $\varepsilon_r = 3.24$ and $\varepsilon_r = 2.25$, respectively. The results plotted in Fig. 2 show the differential scattering cross sections (DSCS) normalized by πc_1^2 for the coated spheroid (curve 1: E-plane, curve 2: H-plane). DSCS is calculated from $\sigma = |\vec{F}^E|^2$, where $\vec{F}^E$ is a pattern of electrical field. Our numerical results agree very well with results that are obtained by the null-field method with discrete sources [3].

The method proposed here will be extended to the solution of the electromagnetic wave scattering problems by multilayered scatterers.

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Methods of the continued boundary conditions and the pattern equations

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For almost 15 years the pattern equations method (PEM) has been successfully applied to the solution of the broad spectrum of wave diffraction and propagation problems. However, essential limitation of the method is that in it's strict formulation it is not applicable to the solution of diffraction problems on bodies with non analytical (in particular, piece-smooth) boundary caused by divergence of Sommerfeld-Weil integral representation in singular points of a wave field. The method of continued boundary conditions (MCBC) suggested recently allows to overcome this limitation. The trick is that according to MCBC, the boundary condition is satisfied not on boundary S of scatterer, but on some surface S_δ, covering S and separated from it by some sufficiently small distance δ. It leads to the approximate statement of a problem, however, as a result all difficulties related to singular points of a wave field on scatterer boundary in case it has breaks, corners, edges, etc., as well as difficulties related to singularity of the corresponding integral equation kernels are removed. Computational algorithm thus becomes significantly simpler and practically universal. It has been shown [1], that in framework MCBC the boundary problem can be reduced to the solution of Fredholm integral equation of the 1^{st}, and 2^{nd} kind with smooth kernel. In particular, in case of Dirichlet boundary condition MCBC gives the following equation

$$\left.\frac{\partial u(\vec{r})}{\partial n}\right|_{S_\delta} = \left.\frac{\partial u^0(\vec{r})}{\partial n}\right|_{S_\delta} - \frac{1}{4\pi}\int_S \left.\frac{\partial u(\vec{r}')}{\partial n'}\right|_S \left.\frac{\partial}{\partial n}\frac{e^{-ikR}}{R}\right|_{S_\delta} ds', \quad R = |\vec{r}-\vec{r}'|, \tag{1}$$

where u^0 is a known function (an incident wave). By parametrizing surfaces S and S_δ and assuming that in points corresponding to the same parameter values, the derivatives $\partial u(\vec{r})/\partial n|_{S_\delta}$ and $\partial u(\vec{r}')/\partial n'|_S$ are equal (which is reasonable, given the small distance between S and S_δ) the equation (1) can be treated as Fredholm integral equation of the 2^{nd} kind relative to the unknown function $\partial u(\vec{r})/\partial n|_{S_\delta} \cong \partial u(\vec{r}')/\partial n'|_S$. By multiplying equation (1) by $e^{ik\rho_\delta(\theta,\varphi)[\sin\alpha\sin\theta\cos(\beta-\varphi)+\cos\alpha\cos\theta]}$, integrating on S_δ and taking advantage of generalized Sommerfeld-Weil representation for function e^{-ikR}/R, we obtain the following integrooperator equation of PEM relative to the scattering pattern g

$$g_\delta(\alpha,\beta) = g_\delta^0(\alpha,\beta) + \frac{1}{8\pi^2}\int_0^{2\pi}\int_0^{\pi}\int_0^{2\pi}\int_0^{\frac{\pi}{2}+i\infty}\Big[k^2\rho_\delta^2(\theta,\varphi)\hat{g}(\alpha',\beta';\theta,\varphi)\cos\alpha'\sin\theta - ik\rho'_{\delta\theta}\sin\theta\,\hat{g}'_\theta(\alpha',\beta';\theta,\varphi)$$
$$- \frac{ik\rho'_{\delta\varphi}}{\sin\theta}\hat{g}'_\varphi(\alpha',\beta';\theta,\varphi)\Big]\exp\left[-ik\,\rho_\delta(\theta,\varphi)(\cos\alpha' - \cos\gamma)\right]\sin\alpha' d\alpha' d\beta' d\theta d\varphi. \tag{2}$$

Here $\hat{g}(\alpha',\beta';\theta,\varphi) = -\frac{k}{4\pi}\int_S \frac{\partial u(\vec{r}')}{\partial n'}\exp[ik\rho(\theta',\varphi')\cos\hat{\gamma}]ds'$ is the generalized scattering pattern [2], and $\cos\hat{\gamma} = \sin\alpha\sin\hat{\theta}\cos(\beta-\hat{\varphi}) + \cos\alpha\cos\hat{\theta}$, $\cos\hat{\theta} = \sin\theta\sin\theta'\cos(\varphi-\varphi') + \cos\theta\cos\theta'$, $\sin\hat{\theta}\cos\hat{\varphi} = \sin\theta'\sin(\varphi'-\varphi)$, $\sin\hat{\theta}\sin\hat{\varphi} = \sin\theta\cos\theta' - \cos\theta\sin\theta'\cos(\varphi-\varphi')$. Generally speaking the equation (2) is approximate, since at its derivation it was assumed, that $g(\alpha,\beta) \cong g_\delta(\alpha,\beta)$. However this equation is now applicable to the diffraction problems on bodies with non-analytical boundary. If boundary S is analytical the obtained equation becomes exact. It is interesting to note, that integrooperator equation of PEM cannot be derived from standard current Fredholm integral equation of a 2^{nd} kind even for bodies with analytical boundary because of the simple layer potential normal derivative jump.

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Modeling the characteristics of scattering of electromagnetic waves by the bodies of complex geometry

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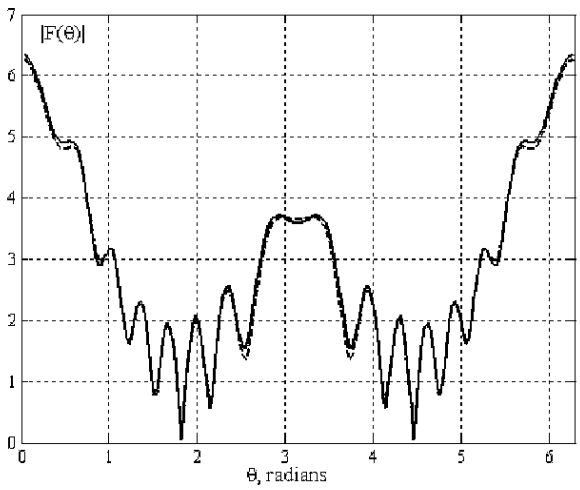
The pattern equations method has appeared to be the high efficient method on a series of diffraction problems, in particular at solving of an acoustic problem of diffraction for scatterers with a complex configuration [1]. In this paper, this technique is extended to an electromagnetic case.

Consider the problem of the waves scattering of primary monochromatic electromagnetic field $\vec{E}^0$, $\vec{H}^0$ on a scatterer of a complex geometry with latter to be presented as a combination of objects of the more simple structure. Let's consider two bodies case.

Let the impedance boundary conditions $\left(\vec{n}_j \times \vec{E}\right)\Big|_{S_j} = Z_j \left[\vec{n}_j \times \left(\vec{n}_j \times \vec{H}\right)\right]\Big|_{S_j}$ are set on surfaces S_j, $j = 1, 2$, $\vec{E} = \vec{E}^0 + \vec{E}_1^1 + \vec{E}_2^1$, $\vec{H} = \vec{H}^0 + \vec{H}_1^1 + \vec{H}_2^1$ is the total field, $\vec{E}_j^1$, $\vec{H}_j^1$ are secondary (diffraction) fields, which satisfies to a homogeneous system of Maxwell's equations everywhere outside of S_j, and also to a condition of radiation on infinity.

Reduction of an initial boundary problem to solving an algebraic system of equations is based on using the series expansion of the scattering patterns in vector spherical harmonics.

Examination of the velocity of the convergence of the proposed method for scatterers with an analytical (spheres) and a nonanalytic (cylinders) boundaries has shown, that the algorithm of PEM remains well converging when scatterers are coming close together up to their contact. It allows the extending the PEM to solving the problem of the wave diffraction for bodies with a complex configuration by their representation as a combination of objects with a simple geometry at minimum consumption of the computer resources.



We carried out the examination of this technique, based on comparison of scattering patterns of superellipsoids with scattering patterns of a group of two halves of these superellipsoids composing together the larger superellipsoid.

Figure shows an example of such a comparison for two perfectly conducting superellipsoids with parameters $ka_{1,2} = 2.5$, $kc_{1,2} = 5$ (larger semiaxis is along axis z) and a single superellipsoid with parameters $ka = 2.5$, $kc = 10$. The case of a longitudinal falling of a wave was considered. It is clear from figure, that the differences of these characteristics are rather insignificant.

One of the methods of a validity estimation of the solution of the problem of the diffraction is verifying of fulfillment of the optical theorem [2].

The results obtained are showed, that the accuracy of fulfillment of the optical theorem for bodies with an analytical boundary (spheres, spheroids) is very high (9-15 valid significant figures) irrespective sizes of scatterers, and the accuracy for the objects with a nonanalytical boundary (cylinders) is noticeably below (3-4 valid significant figures), but also is still acceptable.

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Modelling of scattering characteristics of electromagnetic waves by group of bodies using the modified method of discrete sources

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The modified method of discrete sources (MMDS), offered in work [1], subsequently has been applied to the solution of a wide class of problems of the theory of diffraction, and in all cases high efficiency of a method [2] has been shown. The main idea of the method consists in uniform way of construction of the carrier of discrete (auxiliary) sources by means of analytical deformation of border of a scatterer. Thus the a priori information on properties of analytical continuation of diffraction field inside the scatterer [3,4] is materially used.

Represents the big interest a question on how it is necessary to apply MMDS to the solution of problems of diffraction of waves on a group close located bodies of revolution. The problem at issue is that as a result of diffraction interaction of bodies the picture of arrangement of singularities of analytical continuation of wave field inside of each scatterer can differ from that which takes place in case of single body rather essentially. At a close arrangement of the scatterers singular points start "to be multiply" [4], that is the singularities inside of one body generate the singularities inside of another. We shall consider, for example, a problem of diffraction of waves on two bodies: flattened spheroid and smoothed double cone. Axis cross-section of the bodies is shown in Fig. 1. Here at a close arrangement of bodies the singular point in the "edge" of double cone is represented in a spheroid, generating an additional singular point, which would not be if the problem about diffraction of a plane wave on this single spheroid was solved. Standard technique MMDS does not consider this circumstance.

In the present work some modification of MMDS is realized, allowing to make this method effective at the solution of problems of wave diffraction on a group close located bodies. The essence of this idea consists that the carrier of discrete sources for each of the body is constructed using usual scheme of MMDS, but in addition to the basic sources, the sources surrounding singular points which appear because of interaction of the scatterers are added. As a result, as have shown researches, accuracy of calculations increases approximately for two orders at the same sizes of corresponding algebraic systems. This fact is illustrated in Fig. 2 in which the distribution of the residual on the contour of the cross-section of the flattened spheroid is shown (see the geometry in Fig. 1). In some cases coordinates of the images of singular points are found analytically [3], however, in the general situation it can be necessary to use numerical methods to search of these coordinates. For a finding of coordinates of singular points in this work the effective numerical algorithm based on a method of continuation on a parameter is offered. Efficiency of the method is illustrated by a lot of numerical results.

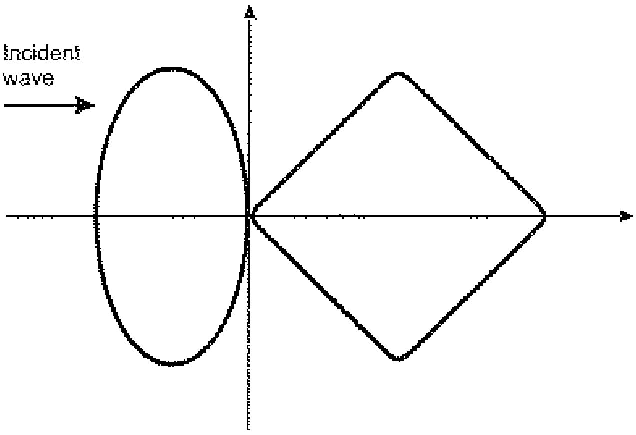

Figure 1.

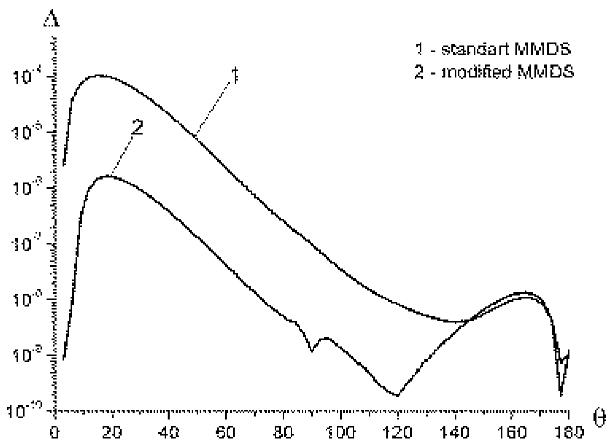

Figure 2.

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Berry phases for the Hartree type equation

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The Hartree type equation

$$\{-i\hbar\partial_t + \hat{\mathcal{H}}_\varkappa(t,\Psi)\}\Psi(\vec{x},t) = 0,$$
$$\hat{\mathcal{H}}_\varkappa(t,\Psi) = \mathcal{H}(\hat{z},s) + \varkappa\int_{\mathbb{R}^n} d\vec{y}\,\Psi^*(\vec{y},t)V(\hat{z},\hat{w},s)\Psi(\vec{y},t) \tag{1}$$

is considered. Here pseudo-differential operators $\mathcal{H}(\hat{z},s)$ and $V(\hat{z},\hat{w},s)$, being functions of non-commutative operators

$$\hat{z} = (-i\hbar\nabla_x, \vec{x}), \qquad \hat{w} = (-i\hbar\nabla_y, \vec{y}), \qquad \vec{x},\vec{y} \in \mathbb{R}^n,$$

depend on "slow" time $s = t/T$, where $T \gg 1$ is a period of adiabatic evolution. The operators $\mathcal{H}(\hat{z})$ and $V(\hat{z},\hat{w})$ are determined by their smooth symbols $\mathcal{H}(z)$ and $V(z,w)$, respectively. A function Ψ^* is complex conjugate to Ψ, $\varkappa$ is a real parameter, $\hbar > 0$ is a small parameter.

We construct semiclassical solutions of the Cauchy problem for Eq.(1) in adiabatic approximation, taking eigenfunctions of the nonlinear operator $\hat{\mathcal{H}}_\varkappa(0)$ as an initial condition. The solutions are obtained in the class of semiclassically concentrated functions [1], using the Maslov's complex germ method. The leading term of the asymptotics in the parameter $1/T$ is shown to remain an eigenfunction of the nonlinear operator $\hat{\mathcal{H}}_\varkappa$ at every time and only gains a phase factor. Following Berry [2], we decompose the phase into the dynamic and the adiabatic (Berry) phases. In the linear case ($\varkappa = 0$) the expressions obtained correspond to the well-known results.

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Factorization of Wiener-Hopf matrices using Fredholm integral equations

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The main problem in solving vector Wiener-Hopf equations is the factorization of the $n \times n$ matrix kernel $G(\alpha)$. This problem has been considerably studied in the past. Nowadays closed form factorizations for particular classes of problems are available in literature, but no method to factorize a general $n \times n$ matrix kernel is still known. Consequently it is necessary to develop approximate techniques of factorization. To this end all the powerful methods developed in functional analysis, i.e. iterative methods, moment methods, regularization methods and so on, can be effectively used for solving W-H equations or for obtaining approximate factorized matrices. Since it is possible to factorize rational matrices, the factorization problem could be faced by introducing rational approximants of the entries of the matrix kernel: for instance Pade approximants. These authors experienced all these technique [1]. Some of them are very cumbersome and perhaps not easy to be applied in engineering applications.

In the framework of approximate techniques these authors suggest a new efficient technique for the factorization of Wiener-Hopf matrices. This method is based on the reduction of the factorization problem to the solution of a Fredholm integral equation of the second kind. The aim of this paper is to discuss the theory of this method [1,2]. Several numerical applications concerning matrices of order two, three and four will be presented in the oral presentation and in the full paper.

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Model for the regular nanostructuring of transparent dielectrics and semiconductors by femtosecond laser radiation

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Experimental results about the interaction of linear polarized femtosecond pulses of laser radiation with wide-band dielectrics (quartz glass, calcium fluoride, zinc selenide) and silicon were analyzed. It is shown that at laser power density of the order 10^{12} W/cm^2 (τ 100 fs) the nonequilibrium plasma having concentration $n \geq 10^{21}$ cm^{-3} is produced and the nonthermal phase transition occurs. In this situation at the boundary nonequilibrium plasma-dielectric the conditions for surface plasmon-polaritons excitation by incident radiation arises. Mutual surface plasmon-polaritons interference and one with the incident laser radiation cause the standing interference fingers of nonequilibrium electron density formation. Then the dielectric volume breakdown occurred the near symmetric channel surface plasmon-polaritons interference is the cause of the volume nanostructures formation.

As a next step the different ways of the time-dependent evolution of modulated electron density into the remnant periodic nanostructures of material are considered.

The mechanism for nanostructures formation on the transparent semiconductor surface is the same.

Diffraction of 2D complex beams by a perfect conductor half-plane: a spectral approach

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The classical approach when tackling radiation and di.raction problems dealing with Complex beams [2] is to use an analytical continuation of the cartesian coordinates in the isotropic line source solution. Since this can be applied to the exact solution of a particular problem with the isotropic cylindrical wave, the obtained solutions are exact as well [4]. The analytical continuation of asymptotic solutions can be also performed but the region where the approximation is good can be different from that considered when the continuation is not carried out, as such in the radiation problem of a 2D Complex beam happens [3].

The diffraction problem of a 2D Complex beam by a half-plane made of perfect conductor is discussed in [5], by using the analytical continuation of the asymptotic solution of the cylindrical incident wave. The authors' proposal is di.erent from [5] since other approach to the problem is used. The way the problem is treated is similar to the isotropic line source in [1]: the solution is constructed by superposition of plane waves solutions of the half-plane problem. In order to set out the problem, the plane wave spectrum of Complex beams will be carried out.

To tackle the problem in this way provides a closed expression for the exact solution from which the asymptotic forms arise. Likewise, the asymptotic solutions are good enough in certain regions that will depend on parameters defined during the early steps taken in order to solve the problem. The paper will present a detailed derivation of the asymptotic expressions for the problem and a first characterization of the regions where asymptotic solutions are valid.

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Gap condition for nonlinear evolution equations

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Conditions of non periodical solutions existence for a class of nonlinear evolution equations have been considered. The conceptions of invariant stable and unstable manifolds, inertial manifolds are mainly used during studying their properties. The conditions, where inertial manifolds of finite fractal dimension exist in the task are determined; gap condition is the main of them. The examples of this condition for classic tasks of diffraction and also for transport equation are given.

The number of zeroes in invariant manifolds for evolution equations

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The properties of stable and unstable invariant manifolds for nonlinear evolution equations have been considered. As a model task one-dimensional evolution equation of parabolic type has been considered, where the main member is elliptic operator, the secondary member is nonlinear operator with certain conditions. Detailed properties, thus number of zeroes, number of laps, stable and unstable manifolds and also its dimensions and asymptotical properties (hyperbolic) solutions of original nonlinear task are determined. The examples of proposed approach application for more complicated diffraction tasks are given.

The comparison of fields excited by input grating coupler with those of SEW

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Surface electromagnetic waves (SEW) are the eigenmodes on interface of two media one of which has negative real part of dielectric constant (for example, metal) and the other has positive one (for example, vacuum or air). The electromagnetic fields associated with SEW are localized in the vicinity of the interface and damp out on both sides of it. As SEW propagates along the surface it's intensity decreases exponentially because of dissipative losses in metal. In the middle and far infrared the SEW propagation length is about some centimeters and it increases with wavelength. In this case the processes of SEW launching by incident radiation, free SEW propagation along the surface and SEW recovery back into bulk radiation may be separated. A great number of studies in the middle and far infrared SEW propagation along metal surface have been made. It was supposed that exactly SEW propagates along a free part of surface behind the input coupler. However we have discovered that if input grating coupler is used to excite SEW the generated wave is differed essentially from SEW [1]. This wave is conventionally named "pressed wave" by us.

We have calculated the fields generated by input grating coupler for the radiation $CO2$ laser. The metals used are copper, gold and aluminum. This selection of metals and wavelength permits us to compare calculated results with experimental ones. In this work we compare the fields of pressed wave with that of SEW. We examine the evolution of pressed wave field on the metal surface with distance from input grating coupler and compare that with analogous dependence for SEW. It is shown that unlike SEW the intensity of pressed wave decreases not exponentially while it propagates along the surface. The pressed wave behavior may be the reason to re-examine the results of early experiments of SEW's propagation lengths. Also we have calculated the pressed wave field as a function of distance from the surface for number positions along the wave propagation. We have got that unlike SEW the pressed wave field profile is modified as it propagates along the surface. It follows from our calculations that as the distance from the input grating is increased more and more the discrepancy between the fields of pressed wave and SEW decreases.

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On plasmonic and other modes of double metal-slab structures

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The electromagnetic modes of planar optical systems comprising two metal slabs bounded by dielectrics have been studied on several occasions in the past [1-3]. Original studies by Economou [1], considered implicit dispersion relations and schematic energy-versus-momentum plots for ideal metals in a range of configurations. The dispersion of the modes of a double-slab system were later studied by Stegeman and Burke [2], and more recently by Revelli and co-workerset al. [3] in the context of Organic Light Emitting Diode structures where the metal plates act as the electrodes. In the present work we re-examine and extend these analyses to consider a much broader range of system parameters and find some previously unreported features of the general system.

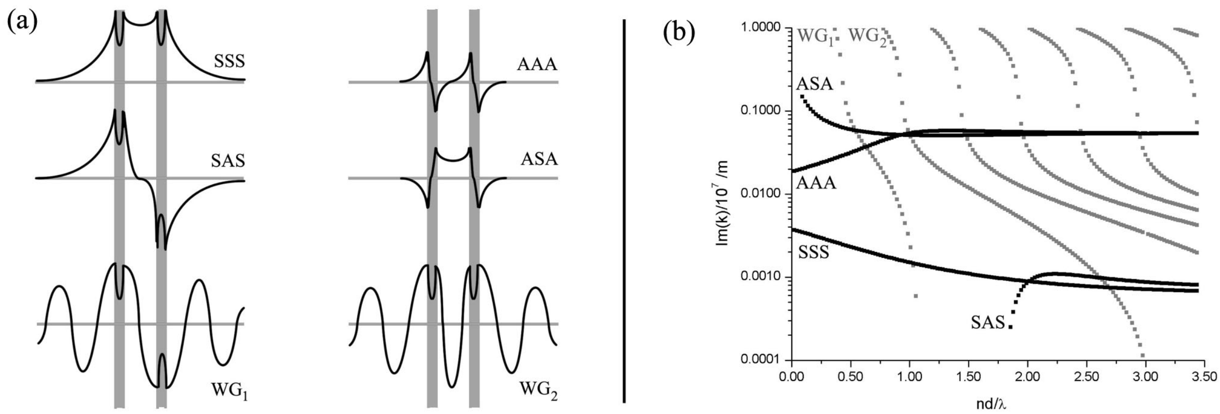

Figure : a) The geometry of the system and the schematic form of the electromagnetic modes. b) The imaginary part of the propagation constant of the electromagnetic modes of the fully-symmetric double Aluminium slab, as a function of slab separation divided by wavelength in medium.

In common with Stegeman, four surface-plasmon modes exist that correspond to the four possible symmetries (figure 1(b), black traces). There is, however, a spectrum of weakly-guided modes (grey traces), similar to parallel-plate waveguide modes, which can penetrate to great depths into the outer dielectric layers and which cut off at a particular maximum value of slab separation. As in the case of the plasmonic modes, the properties of these modes are very sensitive to system parameters. Furthermore, whereas Stegeman found that one of the surface-plasmon modes (SAS) exhibits a cut-off below a certain slab separation (as in figure 1(b)), we have found that for many cases, where the metal thickness is varied, the lowest order weakly-guided mode (WG1) evolves into the SAS mode (which has the same symmetry – figure 1(a)) and neither is cut off. The evolution of the modes can be best appreciated by mapping their loci in the complex plane, which for in some cases extends into other sheets of the Riemann surface. We show that for a range of permittivitypermittivities and thicknesses, the rather complicated behaviour can be rationalised and understood in terms of the basic symmetries. Some interesting features emerge. For example, it is possible by design to increase greatly the long-range plasmon propagation lengths by making very small changes ($< 5\%$) change in the permittivity of the central dielectric slab.

Our findings suggest that structures such as these provide many desirable features, which could be utilised in the design of nano-scale optical devices.

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Universality of Feigenbaum and dissipative microstructures for highly nonequilibrium nonlinear systems

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The discovered phenomenon of step like resonant microrelief period reducing of titanium surface under the action of series of femtosecond laser pulses is discussed. The physical model of phenomenon is proposed based on interference processes with surface plasmon-polaritons participation.

On the base of physical picture of the phenomenon the mathematical model is given involving logistical mapping. It is shown that functional dependence of total electromagnetic field intensity for considered system has quadratic dependence of the height of formed resonant microrelief. This functional dependence changes in the process of sequential irradiation of surface by series of femtosecond laser pulses allows to explain experimentally observed changes of microrelief period. Following the Feigenbaum the bifurcation diagram of cascade of the period doubling for quadratic mapping is created. So the simple example of highly dissipative nonlinear system with the transition from simple periodic to complex nonperiodic state by the reverse microrelief period doubling mode is demonstrated.

One more experimentally realized example of reverse cascade period doubling is analyzed for microrelief of quartz glass surface under the action of scanned linear polarized radiation of continuous wave CO_2 laser.

Biot's equations for multiphase media with different temperatures on the basis of generalized variational principle

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The system of generalized Biot's equations, describing waves propagation in multiphase or multicomponents media in presence of heat exchange between phases, is derived on the basis of generalized variational principle [Maximov G.A. DD2006, p.173-177]. It is shown that at presence of N phases there are $2N$ eigen modes in this system. At high frequencies N modes have the wave type of motion and N modes have the diffusive (heat) type of propagation. At low frequencies there is the single acoustical (wave) mode and the rest $2N-1$ modes possess the diffusion (heat) type of behavior. For the two component medium without temperature exchange the developed approach is reduced to the well know Biot's model. Account of temperature fields gives generalized Biot's model for two components medium.

Reflection from a half-space filled with wire medium: the auxiliary source method

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Solving wave reflection problems for slabs of materials that exhibit spatial dispersion is not trivial. A method to attack such problems will be presented. The method operates in wave-vector space and requires $\varepsilon(\omega, \mathbf{k})$ of the material to be known (some hints how to correctly define $\varepsilon(\omega, \mathbf{k})$ for spatially dispersive media will be also given). It will be shown that by introducing auxiliary sources into unbounded media it is possible to simulate finite slabs or material half-spaces. Earlier, Henneberger in [1] proposed a somewhat similar source-based method, however, his approach was incomplete and even erroneous, [2]. Another approach operating in the wave-vector space was developed in [3, 4].

We will apply our method to the problem of plane wave reflection at the interface of free space and the wire medium with wires oriented orthogonally to the interface. The problem will be the same as in [5] where it was first solved microscopically. In contrast to that, our solution will be completely macroscopic and will make use of the spatially dispersive permittivity of wire medium that was derived in [6]. It will be shown that the results of our method are in agreement with [5]. Another calculation of the reflection and transmission coefficients for slabs of wire media was performed in [7]. The authors of [7] used Henneberger's method directly and obtained results different from ours and of [5].

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On uniqueness in the two- and three-dimensional Neumann–Kelvin problem

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We consider the uniqueness question for the classical Neumann–Kelvin problem of the linear theory of ship waves. The problem describes motion of bodies in an ideal, incompressible, heavy fluid W with a free surface F. The ships move through the still water with a constant speed U along the horizontal x-axis, the coordinate system is attached to the bodies and y-axis is directed upwards. We consider both 2D and 3D problems and the case when the contours of ships S are totally submerged. The mathematical problem consists in finding the velocity potential u which is a harmonic function ($\nabla^2 u = 0$) in the unbounded domain W, satisfies the Neumann condition on S, and the boundary condition $\partial_x^2 u + \nu \partial_y u = 0$ on the unperturbed free surface $F = \{y = 0\}$, where $\nu = g/U^2$ and g denotes acceleration due to gravity. The problem is completed with the condition $\sup_W |\nabla u| < \infty$ and the condition at infinity upstream: $|\nabla u| \to 0$ (2D), $\int_{-\infty}^{+\infty} |\nabla u(x, 0, z)|^2 \mathrm{d}z \to 0$ (3D), as $x \to +\infty$.

A feature of the problem under consideration is that solutions to the homogeneous problem can have infinite energy due to wave propagation to infinity downstream. There are quite few uniqueness theorems, not assuming finiteness of energy ($\int_W |\nabla u|^2 \mathrm{d}V < \infty$): F. Ursell proved uniqueness for one circular cylinder (2D problem) (see e.g. [1, § 7.1.6]), B.R. Vainberg and V.G. Maz'ya (see e.g. [2], [1, ch. 7]) proved unique solvability of the problem for bodies of arbitrary shape and for all values of U except some finite set of values, which cannot be defined within the method (thus, the uniqueness cannot be guaranteed for any given interval of U).

In this work we prove a new uniqueness theorem, valid for bodies of arbitrary shape (we only demand $S \in C^{1,\alpha}$, $0 < \alpha < 1$), without any assumptions on finiteness of energy. For this we use a modification of the technique suggested in [3]. Being based on Green's identity, maximum principles, and study of solution's and Green function's behaviour at infinity, the approach leads us to the conclusion that if the inequality $\sup_{Q\in F}\big\{\int_S \big|\{\partial_{n(P)} G(Q, P)\}\big| \,\mathrm{d}s_{(P)}\big\} \le 1$ holds, then the homogeneous problem has only the trivial solution (G is the Green function of the problem).

The latter inequality allows us to find very simple bounds for the set of parameters, for which the uniqueness is guaranteed. For example, for the three-dimensional problem uniqueness parameters are given by the inequality:

$$\nu^2 \int_S \left[\left(\frac{128}{\pi^4} + \frac{16}{\pi^2} \right) (\nu y)^{-4} + \frac{3 \mathrm{e}^{2\nu y} (1 + 2\nu |y|)^2}{4\pi |\nu y|^3} \right]^{1/2} \mathrm{d}s_{(x,y,z)} \le 1,$$

and it can be easily deduced (also referring to [1, ch. 7] for the solvability arguments) that the problem has a unique solution for any system of bodies when they are sufficiently deeply submerged; the value of the depth can be found from the latter formula. Results of numerical computation of the bounds for various geometries will be presented.

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Electromagnetic properties of double-periodic planar chiral array

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We report the first numerical and experimental results on electromagnetic properties of a new planar chiral metamaterial. The metamaterial is formed by a periodic array of uninterrupted crossed wavy-shaped metal strips placed on a dielectric substrate (see inset to Fig.). The metal-dielectric periodic structure is a modification of a grating of wavy strips or a "fish-scale" structure [1, 2]. Crucial differences of the considered metamaterial are its 4-fold symmetry and a planar chirality due to an arrangement of nonchiral-shaped strips in the array. The inset to Figure shows one of two enantiomorphic array fashions.

Doubly-periodic planar structures of complex-shaped strips are attractive for microwave applications because of their resonance properties in the frequency band of a single-wave regime. In recent years, a great number of papers was published on arrays of separated metal strip elements. Nevertheless properties of arrays of uninterrupted complex-shaped crossed strips are not considered sufficiently.

Numerical analysis of electromagnetic wave diffraction by the structure was carried out by the integral equation method described in [1]. The metal pattern was treated as a perfect conductor, while the substrate was assumed to be a lossy dielectric. In the experiments we used the structure that was etched from a 35 μm copper film on a fiberglass PCB material. The overall size of the sample was approximately 220 mm $\times$ 220 mm. We found a very good agreement of the numerical and experimental results.

Because to a 4-fold symmetry of the array, the reflection and transmission properties of the structure don't depend on a polarization of normal incident plane wave. In contrast to arrays of separated metal particles, the structure is non-transparent in low frequencies. The first frequency of a high reso-

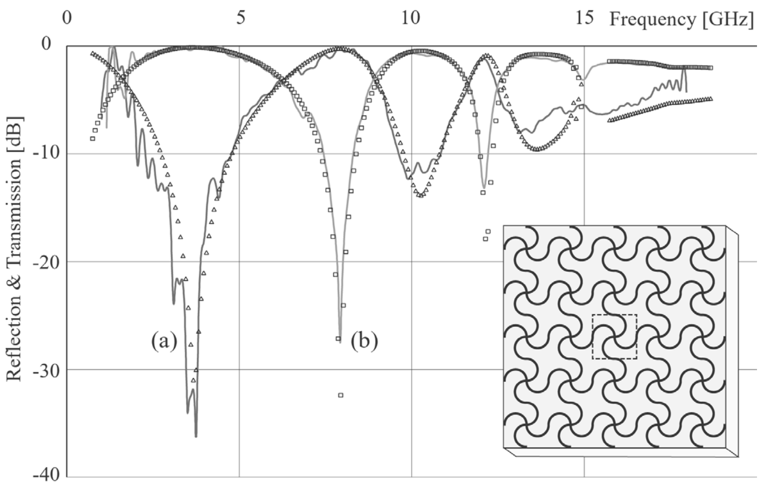

Figure : Normal incidence reflection (a) and transmission (b) of linearly polarized wave by the planar metamaterial. Solid lines and separate symbols denote the experimental and numerical results respectively. The inset shows a fragment of the array of crossed wavy-shaped copper strips patterned on a dielectric substrate. The dashed line box indicates a 15 mm $\times$ 15 mm elementary translational cell of the structure. Wavy-shaped strips consist of semicircles. The width of strips is 0.8 mm. The thickness of substrate is 1.5 mm. Substrate relative permittivity is 4.5-i0.08.

nance transparency is considerably lower than corresponding frequency of an array of straight crossed wires arranged with the same spacing. A planar chirality of the metamaterial leads to difference in diffraction of left and right circular polarized waves at an oblique incidence. This property may be useful for FSS applications of the metamaterial in setups operated with circular polarized wave beams. At last, control electron devices may be incorporated into structure at the cross points of strips and boundaries of translational cells in order to transform the array of uninterrupted strips into an array of separated gammadion-shaped particles and vice versa.

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The mixed value problem with the skew derivative for the 2-D Helmholtz equation outside cuts

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We study the mixed value problem outside cuts in a plane for the Helmholtz equation with attenuation. The similar problem for the Helmholtz equation, describing waves, has been studied in [1], [2] under different conditions.

By a simple open curve we mean a smooth nonclosed arc without self-intersections [3]. In the plane $x = (x_1, x_2) \in R^2$ we consider simple open curves $\Gamma_1, \dots, \Gamma_N$ of class $C^{2,\lambda}$, $\lambda \in (0, 1]$, so that they do not have common points. We put $\Gamma = \bigcup_{n=1}^{N} \Gamma_n$. We assume that each curve Γ_n is parameterized by the arc length s:

$$\Gamma_n = \{x : \ x = x(s) = (x_1(s), x_2(s))\}, \ s \in [a_n, b_n], \ n = 1, \dots, N,$$

so that $a_1 < b_1 < \dots < a_N < b_N$. Therefore points $x \in \Gamma$ and values of the parameter s are in one-to-one correspondence. Below the set of intervals $\bigcup_{n=1}^{N}[a_n, b_n]$ on the $0s$ axis will be denoted by Γ also.

The tangent and normal vectors to Γ at the point $x(s)$ are denoted by $\boldsymbol{\tau}_x = (x_1'(s), x_2'(s))$ and $\mathbf{n}_x = (x_2'(s), -x_1'(s))$ respectively.

We consider Γ as a set of cuts. The side of Γ which is on the left, when the parameter s increases, will be denoted by Γ^+, and the opposite side will be denoted by Γ^-.

We say, that the function $u(x)$ belongs to the smoothness class **K** if

1) $u(x) \in C^0(\overline{R^2 \setminus \Gamma}) \cap C^2(R^2 \setminus \Gamma)$, and u is continuous at the ends of Γ,

2) $\nabla u \in C^0(\overline{R^2 \setminus \Gamma} \setminus X)$, where X is a point-set, consisting of the end-points of Γ: $X = \bigcup_{n=1}^{N}(x(a_n) \cup x(b_n))$,

3) in the neighbourhood of any point $x(d) \in X$ for some constants $\epsilon > -1$, $c > 0$ the inequality $|\nabla u| \le c|x - x(d)|^{\epsilon}$ holds, where $x \to x(d)$ and $d = a_n$ or $d = b_n$, $n = 1, \dots, N$.

By $C^0(\overline{R^2 \setminus \Gamma})$ we denote functions, which are continuously extended on cuts Γ from the left and right, but their values on Γ from the left and right can be different, so that the functions may have a jump on Γ. Let us formulate the mixed value problem for the Helmholtz equation in $R^2 \setminus \Gamma$.

Problem **U**. Find a function $u(x)$, which belongs to the class **K**, obeys the Helmholts equation

$$\Delta u(x) + k^2 u(x) = 0, \quad x \in R^2 \setminus \Gamma; \quad \Delta = \partial_{x_1}^2 + \partial_{x_2}^2, \quad \arg k \in \{[0, \pi/2) \cup (\pi/2, \pi]\},$$

satisfies the boundary condition

$$u(x)\big|_{x(s)\in\Gamma^+} = F^+(s),$$

$$\left.\frac{\partial u}{\partial \mathbf{n}_x} + \beta \frac{\partial u}{\partial \boldsymbol{\tau}_x}\right|_{x(s)\in\Gamma^-} = F^-(s), \qquad \beta = i|\beta| = \text{const}, \qquad |\beta| < 1,$$

Zommerfeld radiation conditions if $\operatorname{Im} k = 0$ and the following conditions if $\operatorname{Im} k > 0$

$$|u(x)| = o\left(|x|^{-1/2}\right), \quad |\nabla u| = o\left(|x|^{-1/2}\right), \qquad |x| \to \infty.$$

All conditions of the problem must be satisfied in a classical sense.

On the basis of the energy equalities we can prove that the problem **U** has at most one solution. The problem **U** is studied by potential theory and the boundary integral equation method. The problem is reduced to the Cauchy singular integral equation and then to the Fredholm integral equation of the second kind, which is uniquely solvable. The integral representation for a solution of the problem **U** is obtained in the form of potentials.

Theorem. *If arcs* Γ *are of class* $C^{2,\lambda}$; $F^+(s) \in C^{1,\lambda}(\Gamma)$, $F^-(s) \in C^{0,\lambda}(\Gamma)$; $\lambda \in (0,1]$, *then the solution of the problem* **U** *exists, it is unique and is given in the form of potentials. The density in potentials is a unique solution of the Fredholm integral equation of the second kind and index zero.*

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Interaction of shock waves for genuinely nonlinear hyperbolic systems

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We consider 2×2 strictly hyperbolic systems of conservations laws,

$$\frac{\partial u}{\partial t} + \frac{\partial f(u)}{\partial x} = 0, \quad x \in \mathbb{R}^1, \quad t > 0, \tag{1}$$

where $u = (u_1, u_2)$, $f = (f_1, f_2)$. Let λ_i, R_i be the eigenvalues and the eigenvectors of the matrix $f'(u)$, $i = 1, 2$. The assumption of genuine nonlinearity means that $(\lambda'_{iu}, R_i) \neq 0$ uniformly in u. Let the initial data for (1) be a superposition of two jumps,

$$u|_{t=0} = u_0 + A_1 H(x - x_1^0) + A_2 H(x - x_2^0), \tag{2}$$

were u_0 and A_i are constants, $x_1^0 < x_2^0$ and $H(x)$ is the Heaviside function. We consider the problem (1),(2) for arbitrary values of the amplitudes A_i assuming that the initial jumps (2) generate two shock waves which remain stable until the time instant of the shock waves paths intersection.

To describe uniformly in time the process of the shock waves interaction we pass to a smooth regularization of the problem (1), (2) and use the weak asymptotics method. Since centered rarefaction waves can appear as a result of the interaction, the asymptotic anzatz contains many "free" parameters. Respectively, there appears a very complicated dynamical system to describe the time evolution of these parameters. However it is possible to avoid a detailed analysis of the system supplementing to the anzatz small in the weak sense corrections. Therefore we do not need to specify the nonlinearity.

We consider the uniform in time description of the centered rarefaction appearance phenomena as a most interesting result of this research.

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3D isotropic metamaterial based on dielectric resonant spheres

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Artificial metamaterials with negative permittivity and permeability (double negative material, DNG) are very promising for designing novel electromagnetic components with interesting and useful properties. Most of models introduced are anisotropic, however isotropic DNG-material should be more practical for many applications. Earlier in [1], the model of isotropic double negative structure was introduced. A structure considered consists of two sublattices of dielectric spherical particles of high permittivity and different radii embedded in a dielectric matrix of smaller permittivity. It has been shown that such a composite medium reveals properties of an isotropic double negative media (DNG) in a limited frequency range, when resonance oscillations of H_{111} mode in one kind of particles and E_{111} mode in another kind of particles are excited simultaneously.

Consideration of scattering of the plane electromagnetic wave on a sphere [2] allows introducing a diffraction model to define effective DNG medium parameters. Modules of fields inside and outside the spheres were analytically calculated. The calculation shows a sizeable attenuation of the electromagnetic field outside the spheres therefore in a first approach we can neglect mutual interactions between particles in case of appropriate distance between the particles and to consider each particle as a single independent sphere. Our next step was taking into consideration this interaction. In order to do that analytical diffraction model was improved. Firstly for calculating resonant frequencies of dielectric spheres, the real permittivity of the host material was used. In the next iteration we consider that spheres are placed in a host medium with effective frequency-dependent permittivity. The effective permittivity results obtained are close enough to the previously calculated. This implies the fact that the interaction between particles is weak enough.

In previous papers [1] the structure containing three single cells with 2 type of spheres with permittivity $\varepsilon = 400$, and radii $r_1 = 0.748$mm, $r_2 = 1.069$mm, was simulated by the full-wave analysis. More accurate analysis taking into account symmetry of the structure has been implemented. It has been shown that there is a stop band near the resonant frequency in case of negative permittivity or permeability only. But for mixture medium with both set of spheres, there is a narrow pass band near the resonant frequency. The frequency range of the electromagnetic wave transmission corresponds to the double-negative characteristics of the structure. Evidently, the resonance frequency is close to that calculated by the diffraction model. Pass band width is about 20 MHz. This value is close to the analytically calculated.

Also experiment has been performed with four samples with different radii: 1.3 mm, 1.305 mm, 0.902 mm, and 0.893 mm. These dimensions were selected to obtain the resonant frequency near 15 GHz. Ferroelectric spherical particles have been placed by turns into metallic waveguide connected with the network analyser. Firstly one waveguide port was shorted with a moveable electric wall. Moving the electric wall, we obtain the electric field distribution in the waveguide with its maximum in the place where the sphere is situated. That allows to get maximum reflection level from the sphere at the resonant frequency. In the case of bigger sphere inside the waveguide, we have a magnetic dipole resonance. That means that we should use a magnetic wall to get maximum reflection. With regard to impossibility to create a real magnetic wall we also use real electric wall placed at $\lambda/4$ distance from the plane where the virtual magnetic wall should be. In the next experiment, the waveguide with a matched termination have been used and the transmission coefficient was measured. Resonance nature of S_{11} and S_{22} parameters was observed near theoretically predicted resonance frequency.

Experimental verification proves validity of diffraction model. The results obtained allow to consider possible realization of artificial double negative material.

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PO/PTD near-field scattering and diffraction method for path loss prediction in urban mobile radio-systems

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The near/Fresnel-zone field calculation of radio-wave scattering and diffraction is very important for the accurate path loss prediction in cellular mobile radio systems, since in a typical urban street-grid environment the scattered and diffracted near/Fresnel-zone field from building walls and wedges occupies a large percentage of the surrounding area. In recent publications, we have developed a two-dimensional simulation model for the radiocoverage prediction in urban area sites. This model is based on the analytical methods of Physical Optics (PO) and Physical Theory of Diffraction (PTD) and takes into account the electromagnetic fields at the receiving antenna due to first and high order propagation mechanisms. In contrast to other published deterministic methods, where the radiocoverage analysis is limited to the far field scattering, this model includes near/Fresnel field calculations, since in a typical urban environment the near or Fresnel field scattering area covers most of the street canyons' region, due to the large dimensions of the building walls (relative to the wavelength of the radiation). Even though, this model was proved to have satisfactory prediction accuracy for 900 MHz and 1.8 GHz, as compared to other high frequency methods and measurement data, it is quite time-consuming, mainly due to the near field computations, which are based on numerical integration over the surface PO currents, and subsequent numerical differentiation of the vector potential. In this paper, a time-efficient method for the calculation of the scattered field from plane scatterers [rectangular plates for the 3-D case, and plane strips of finite length and infinite height for the 2-D case] in the near and Fresnel zone is presented. The method, which we call 'Near to Far Field Transformation Method', is based on the analytical method of Physical Optics (PO) and uses the technique of segmentation of the scattering surface into an appropriate number of small rectangles (cells). By such a division of the large scatterer, an observation point which is originally located in the near or Fresnel zone of the scatterer, is now relocated in the far zone of the smaller cells. As a result, the PO far-field approximations can be invoked in order to calculate the electromagnetic field at the receiver due to the induced PO currents at each cell. The total scattered field at the receiver's position is calculated by complex vector addition of all signal components received. The incorporation of the proposed method in the simulation code, by replacing the numerical integration and differentiation operations mentioned above, decreases drastically the calculation time, and at the same time provides even more accurate results, since it avoids possible numerical errors due to the differentiation of the vector potential. Finally, extensive research studies are currently undertaken in order to implement the 'Near to Far Field Transformation Method' for 3-D diffraction from the building wedges. In particular, the diffracted field, near the building wedges, is calculated by applying a combination of the proposed method described above, with the accurate 'Incremental Length Diffraction Coefficients' theory (PTD), which has been introduced by Mitzner for the 3-D diffraction problem in the far field area.

Two-layer spherical dielectric lens with reflector excited by a radial electric dipole

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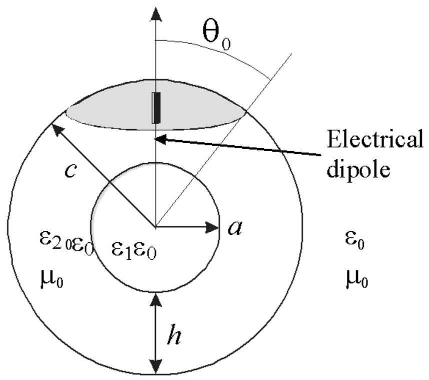
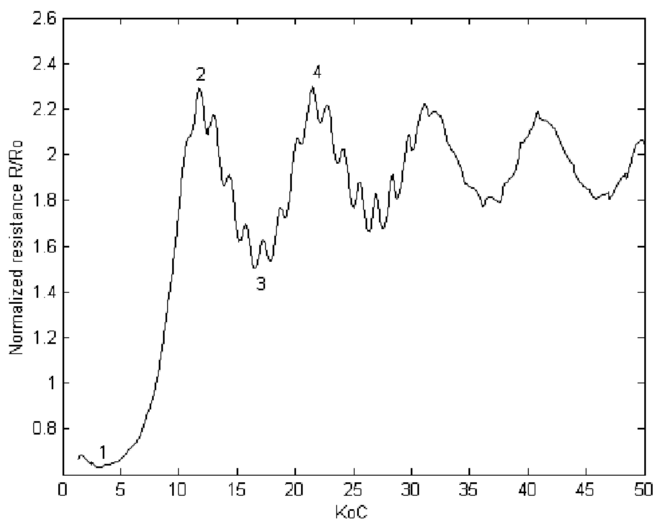
In this paper, we study the properties of the double layered spherical dielectric lens with reflector. It is considered that the layered sphere excited by a radial electric dipole located near the surface of the perfectly conducting zero-thickness reflector shaped as a spherical disk. These antennas are used for multi-beam scanning at millimeter and microwave frequencies in mobile communication systems, remote sensing and radio astronomy. In the geometrical-optics approximation, these types of lenses transform the feed's spherical wave front into a plane wave. The attractive features of these antennas are the operation over a broad band of frequencies and the spherical symmetry of the lens that allows multi-beam scanning by placing an array of feeds around the lens. There are many papers considering this type of lenses. Normally they are analyzed by the FDTD method, which has both advantages and disadvantages, such as accuracy loss in the presence of resonances. To overcome this problem we use well developed method of analytical regularization (MAR).



To formulate the electromagnetic wave radiation problem for a dipole and lens, we use all conditions necessary for the uniqueness theorem. From boundary conditions on the spherical surface we obtain the dual series equations in terms of the associ-ated Legendre functions and further use MAR, which is efficient for numerical analysis. It is based on the analytical inversion of the static problem for a perfectly conducting spherical disk in free space. The final result has a form of infinite set of linear alge-braic equations:

$$X_m - \sum_{n=1}^{\infty} X_n \varepsilon_n Q_{nm}^{(1)}(\theta_0) = \sum_{n=1}^{\infty} n(n+1)\alpha_n \left[\delta_n^m - Q_{nm}^{(1)}(\theta_0)\right]$$

This set is of the Fredholm second kind and therefore has guaranteed convergence and controllable accuracy even in the presence of resonances.



$a/c = 0.5$, $\theta_0 = 18°$, $s_2 = 2$, $s_1 = 1.1$

The main aim of our research is the lens properties study to optimize them in future. The plots of the radiation resistance, radiation pattern and directivity, in the wide band of parameters variation, are presented in this paper for different lens layers parameters and different angular sizes of reflector. Besides, we compare the properties of the layered dielectric lens in the presence of reflector and without it. As one can suppose, the presence of reflector decreases the level of the back-side lobes in the radiation pattern.

Stability of perturbation of flows and typhoon generation

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We analyze the velocity field of the atmospheric flows involved in the origin of a typhoon. According to a recent theoretical result(1) the local perturbation of a given flow conserves its form if the Cauchy-Riemann conditions of the velocity field of the main flow are verified. Thus we analyze the velocity fields in the region where local perturbations are generated starting from the velocity distribution of the wind measured in such situations and check these conditions.

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Interaction of dissipative fiber Bragg solitons

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The interaction of dissipative optical solitons in fibers with saturable gain and absorption and linear Bragg grating is investigated. The initial phase difference between solitons plays a crucial role in their interaction. Depending on this value, the solitons can attract or repel each other at the beginning stage of their interaction. As illustration, Figure shows the interaction of two dissipative Bragg solitons that are in-phase initially; then these initially motionless solitons transform into two solitons moving with fairly large speed in the opposite directions.

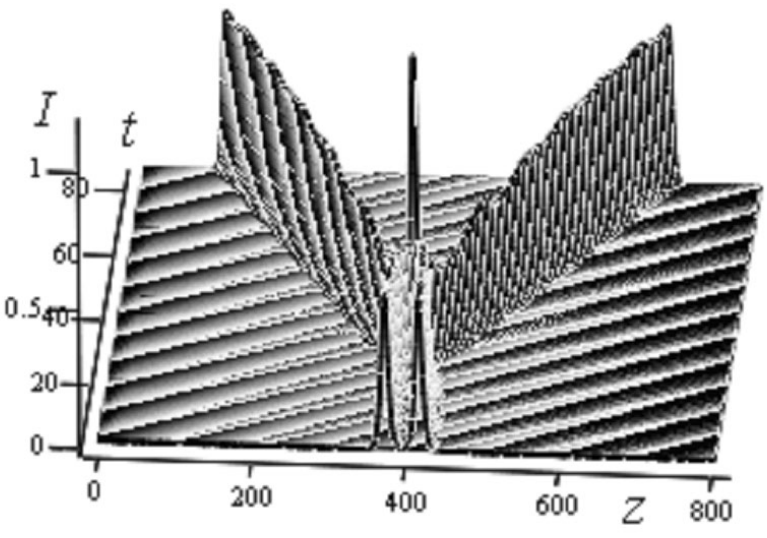

Figure: Interaction of two dissipative Bragg initially in-phase solitons.

Beyond the approximation of slowly varying amplitudes, we have found that single solitons are located near maxima of refractive index of Bragg grating. This feature results in the possibility of large number of coupled states of motionless dissipative Bragg solitons. We demonstrate stable bound states of two motionless dissipative Bragg solitons whose final phase difference is nearly equal to $\pi/2$ or $3\pi/2$, whereas similar pairs with phase difference near 0 and π are unstable.

Complexes of weakly coupled 3D-laser solitons

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We consider complexes of 3D-dissipative solitons in a continuous medium with frequency dispersion and nonlinear gain and absorption described by the generalized complex Ginzburg-Landau equation for the electric field envelope E:

$$\frac{\partial E}{\partial z} = (i+d_\perp)\left(\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2}\right) + (i+d_{||})\frac{\partial^2 E}{\partial \tau^2} + f\left(|E|^2\right)E, \qquad f\left(|E|^2\right) = -1 + \frac{g_0}{1+|E|^2} - \frac{a_0}{1+b\,|E|^2}.$$

Here we use the dimensionless Cartesian longitudinal coordinate z, transverse coordinates x and y, $\tau = t - z/v_g$, t is time, v_g is the group velocity, g_0 and a_0 are small-signal gain and absorption coefficients, respectively, b is the ratio of intensities of saturation for gain and absorption, non-negative coefficients d represent the medium dichroism ($d_\perp$) and spectral filtering ($d_{||}$). The governing equation has the most symmetric form in the special case $d_\perp = d_{||}$. In Fig. 1 we present some types of stable soliton complexes, and Fig. 2 demonstrates a variant of these complexes collision.

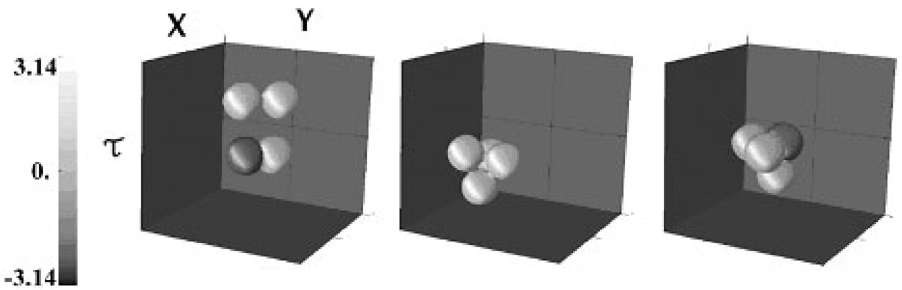

Fig. 1: Stable complexes of 3D-laser solitons: in-phase and antiphase pairs (left cube), in-phase motionless tetrahedron (centre), and moving in-line pyramid with one antiphase soliton in the vertex (right); the field phase is coded in grey scale on the left.

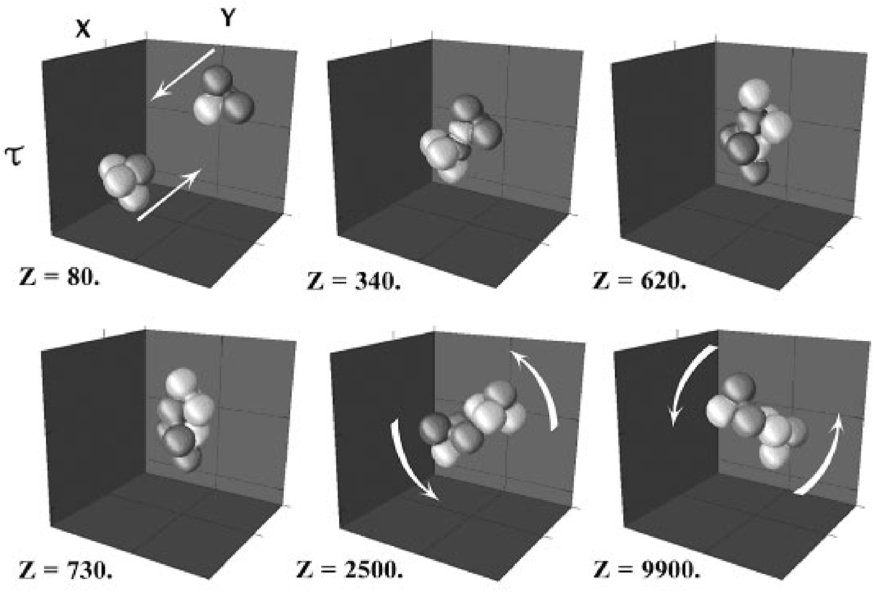

Fig. 2: Collision of two moving pyramid-like complexes and formation of a rotating 8-soliton complex.

Effect of Kerr nonlinearity on discrete dissipative optical solitons

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We consider propagation of intensive radiation in an array of weakly coupled single-mode fibers with fast nonlinearity of gain, absorption, and refractive index (Kerr nonlinearity). The goal is to analyze the effect of Kerr nonlinearity on characteristics of spatial discrete dissipative solitons in the array.

The analysis of one-dimensional spatial discrete dissipative solitons with weak coupling on the nearest neighboring fibers bases on the following equations of the coupled mode theory:

$$i\frac{\partial E_n}{\partial z}+C\Big[E_{n-1}+E_{n+1}\Big]+f\Big(|E_n|^2\Big)\,E_n=0,\quad f\Big(|E_n|^2\Big)=\gamma\,|E_n|^2-i\left(-1+\frac{g_0}{1+|E_n|^2}-\frac{a_0}{1+b\,|E_n|^2}\right).$$

Here z is the longitudinal coordinate, E_n is mode amplitude of the n-th fiber, C is coupling coefficient, γ is Kerr coefficient, g_0 and a_0 are small-signal gain and absorption coefficients, respectively, b is the ratio of intensities of saturation for gain and absorption. In the continual limit, $E_{n\pm1}=E_n\pm\frac{\partial E}{\partial x}h+\frac{1}{2}\frac{\partial^2 E}{\partial x^2}h^2$ where h is the spacing between the fibers, these equations come to the generalized complex Ginzburg-Landau equation, where dissipative solitons are well known in the literature. At the same time, more interesting are highly localized structures not described by the continual model.

Numerical simulation of radiation propagation along the array of fibers results in the following conclusions on the effect of Kerr nonlinearity, see Fig. Self-focusing nonlinearity ($\gamma > 0$) gives the ultimate narrow radiation localization and extremely stable discrete solitons. On the contrary, self-defocusing nonlinearity ($\gamma < 0$) results in soliton profile widening and decreases its stability.

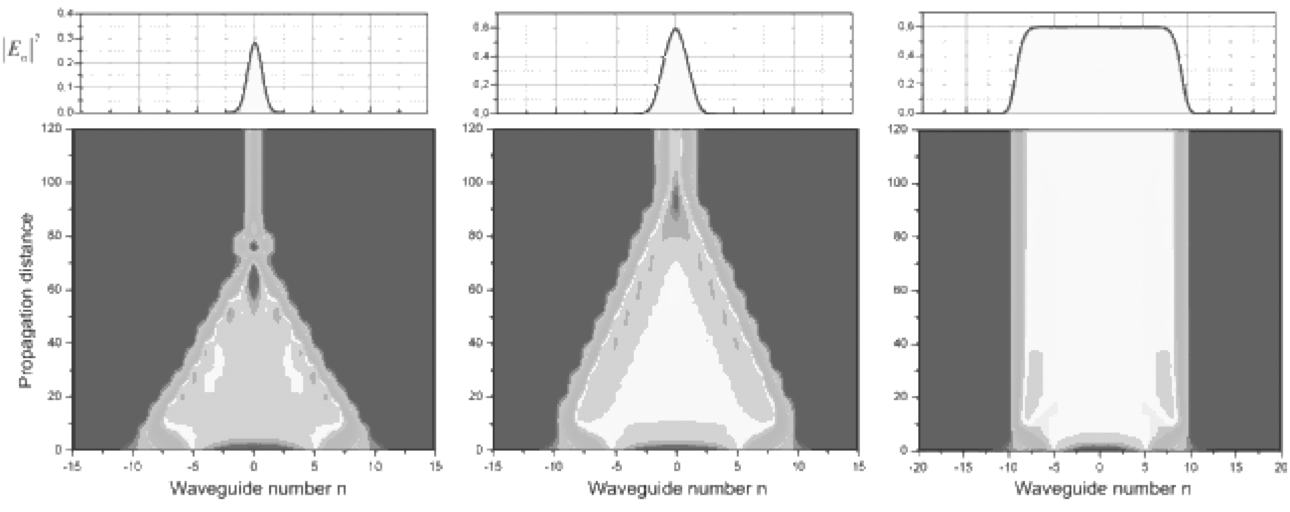

Figure : Evolution of intensity distribution in an array of weakly coupled single-mode fibers with nonlinear gain and absorption. Kerr nonlinearity type is self-focusing (left), zero (centre), and self-defocusing (right).

Simulations show also the possibility of formation not only transversely motionless, but also moving discrete dissipative solitons with profile defined by the system parameters. It is possible to control the soliton transverse motion introducing gain or absorption inhomogeneity. These regimes present additional tools for optical information processing.

Electromagnetic field transformation in circular waveguiding and resonant structures with time-varying permittivity

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This paper presents the analytic results of comparison of modes transformation in time-varying structures on examples of circular waveguide and circular whispering gallery mode resonator. The situation is that a natural mode propagates inside the structure at the moment when the refractive index changes abruptly in time inside it. A closed form solution is obtained for the electric field component by applying the Laplace transform directly to the wave equation and enforcing the corresponding initial and boundary conditions.

The basic interest is to gain insight into details of mode evolution during the transient period and in steady state regime. We reveal the common features that underlie these phenomena in both structures.

Time changing of the dielectric permittivity in unbounded space leads to change of a wave frequency [1] and conservation of a wave number. In the presence of boundaries this process is more complex and it depends qualitatively on configuration of the initial field. For the case of initial field without external source (e. g. mode of the resonator) changing of refractive index leads to the changing of frequency and wave number, but near field pattern conservation [2]. If the initial field has external source (e.g. point source) then the temporal change of dielectric permittivity inside the resonator leads to complicated transient process caused by excitation of all modes of the resonator in its new state and transformation of the near-field pattern. Possibility of the source near-field pattern control in 2D circular resonator by adjusting in time of media parameters is discussed.

The analytical solutions of circular waveguide modes transformation due to abrupt time change in dielectric permittivity is considered as well. It is revealed that initial mode evolves into "time transmitted" and "time reflected" modes of the structure in its new state. These modes have the same field pattern as initial one. Exception is the fast time decay of waveguide higher modes when dielectric permittivity in the core decreases.

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Solving the problems of diffraction by Wiener-Hopf-Fock method for finite fine structures

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This paper deals with further development of Wiener-Hopf-Fock method for solving the wave's diffraction on fine strip-slotted structures problems. The classical problem on diffraction of plane wave on a strip is offered as a key problem. The boundary value problem is consecutively solved by reducing to the system of singular boundary integral equations, then to the system of the second kind Fredholm equations. A few way constructing the analytic solutions of this system are proposed. Using the integral operator of diffraction from introduced edges sources, the solutions are presented in the form of diffraction series and also with use of special functions. From solutions for the strip-slotted structures the characteristic equations and resonance conditions are obtained.

On application of asymptotic solutions to calculation of tsunami piston model

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The conditions are derived at which asymptotic solutions of the Cauchy problem for the wave equation are applicable to calculation of the tsunami piston model in an uniform depth basin. It is done by comparing the results of numerical modeling with those obtained using the asymptotic solutions for the initial bottom disturbances in the form of the gaussian exponents. The question of the wave propagation is discussed.

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On the information transmission method using material objects' far-action reaction on the external electromagnetic unconverted influence

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The aim of this paper is to describe a possible method of one-bit information transmission using a far-action reaction (i.e. instantaneous action in distance) of material objects on external electromagnetic unconverted influence.

The initial information is created by means of simultaneous formation of an electromagnetic soliton and an electromag-netic instanton, respectively. Then, electromagnetic soliton and instanton are united into electromagnetic soliton-instantonic breather (SIB). After being formed, the SIB is forwarded to a certain first resonator and it excites the first resonator. Then the SIB located in the first resonator is exposed to the influence of the electromagnetic field with pre-set parameters to cause a soli-ton-instantonic tremor. The first resonator excitation by the electromagnetic breather results in induction of an extreme fast-decaying EMP in the second resonator. Both resonators should have a pass band not smaller than the SIB spectrum; in opposite case the breather will disintegrate into ordinary waves group.

The first and the second resonator can have different overall dimensions. In case the second resonator admits equal or larger amplitude values of power characteristics in comparison with the first resonator, then in the second resonator the EMP not larger than the first resonator EMP is inducted. In case the second resonator admits amplitude characteristics lesser than the first resonator signal amplitude characteristics, then in the second resonator the maximum possible EMP, but not larger than the first resonator EMP is inducted. The first and the second resonators are oriented in space accordingly one another.

In case the said conditions are violated, information transmission by use of the described method is impossible.

It has been pointed that, during the signal trip through dielectric media, in signal a reactive component appears, which is characterized by the presence of an imaginary component in analytical description of a non-linear signal. Thus, in case the soli-ton is formed with simultaneous propagation of this signal through dielectric media, the splitting of the signal to soliton and to instanton takes place. Both parts are shown as an intertwined structure - the soliton-instantonic breather. Therefore, in this case the supplementary unification of the soliton and the instanton to breather is not needed.

The said method of information transmission is a result of an actual transference of the first part of Einstein-Podolsky-Rosen paradox statements to macro-level, where the resonance systems are represented not by elementary particles, but by macro-resonators in singlet connection. In light of the above-stated, it is obvious that the singlet connection between macro-resonators is realized due to their coherence and correspondence, including identical orientation in space. Besides the breather tremor corresponds to the effect of parametric absorption of electromagnetic waves energy by material objects.

Consequently, the formation of solitons (non-linear signals that excite the first resonance system) is crucial for realization of the information transmission method using a far-action.

The linear signal represented by an electromagnetic sine wave oscillation or impulse signal is formed. The linear signal is fed to a non-linear body. In general the frequency of the linear signal carrier wave does not influence a possibility of a nonlinear signal formation with the given non-linearity type. In fact for every frequency range of a linear signal carrier wave, there is only one optimal choice of a non-linear body formation.

A non-linear body may be represented by a three-pole unit (two inputs, and one output). One of the 3-pole unit's inputs receives a linear signal that should be transformed into non-linear signal, and the other input receives a pumping signal, which can manage the non-linear parameters of a non-linear body. The output yields a non-linear signal, which is a result of parametric addition of a linear signal and a pumping signal in a non-linear body.

The non-linear parameters management is performed at the expense of the influence of a strong pumping signal and/or linear signal, since the non-linear parameters are a temporal functional of the pumping signal and/or linear signal variation law.

In whole the algorithm of the pumping signal and/or linear signal parameters definition is as follows: the type of the non-linearity of the resulting nonlinear signal is know a priori (is given). The equation is resolved with one or more unknown quantities, each of which is a time function that describes required signal or signals. The choice of a respective non-linearity type of a nonlinear signal influences the choice of pumping signal and linear signal parameters that forces us to a priori define the nonlinear properties (nonlinear parameters) of a nonlinear body.

The utilization of an artificial nonlinear body located in the zone of nonlinear signal propagation in the soliton formation device can be considered a realization of the described method of a nonlinear signal formation (a soliton). Such nonlinear signal is passed through a nonlinear body as a parametric converter. The nonlinear body receives a pumping signal. The resulting nonlinear signal will be a result of parametric addition of a linear signal and a pumping signal under conditions of a non-linear body. The required pumping signal parameters are defined as a result of a parametric addition equation solution according to above-mentioned algorithm of pumping and/or linear signal parameters definition.

According to the described method of the information transmission, which is based on the hypothesis of a far-action of material objects' reaction on unconverted external electromagnetic influence, an excitation of the first resonator is carried out under fixed small power expenditures. The second resonator(s) can be located at maximum distance. That is, utilization of the described method of the information transmission allows to increase the distance of information transmission under fixed small power expenditures, and also ensures the information transmission in complicated electromagnetic conditions, since according to the first part of the Einstein-Podolsky-Rosen paradox statements the resonance systems in singlet connection will react inde-pendently from the distance and electromagnetic conditions of the circumambient media.

Numerical study of diffraction phenomena in achitectural acoustics

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The aim of the research is to estimate the influence of typical diffraction phenomena on acoustical properties of rooms, halls etc. As the main property of a room we take its reverberation time. Thus, we develop a method of numerical estimation of the reverberation time accurate enough to contain the information on the diffraction effects.

To achieve this objective, we use averaging in frequency domain to define the reverberation time. This averaging enables us to suppress the influence of global interferention pattern. As a result, we introduce a family of dependencies of reverberation time vs. frequency having the bandwidth of averaging filter as a parameter. A proper choice of the bandwidth is discussed in the talk.

The program for numerical modeling in 2D case is described. The results are presented and discussed.

Singular solutions to systems of conservation laws connected with transportation and concentration processes

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There are "nonclassical" situations when systems of conservation laws admit δ- and $\delta^{(n)}$-shock type solutions. They are singular solutions such that their components may contain the Dirac delta functions and their derivatives up to the order n, $n = 1, 2, \dots$. For the theory of δ-shocks see [1], [2] and the references therein. An absolutely new theory of $\delta^{(n)}$-shocks was established in [3]–[5].

Using our previous results [1], [2] we study the Cauchy problems admitting δ-shocks for multidimensional zero-pressure gas dynamics systems in conservative form

$$\rho_t + \nabla \cdot (\rho U) = 0, \quad (\rho U)_t + \nabla \cdot (\rho U \otimes U) = 0,$$

and non-conservative form

$$\rho_t + \nabla \cdot (\rho\, U) = 0, \quad U_t + (U \cdot \nabla) U = 0.$$

Using our results [3]– [5] we study the Cauchy problems admitting δ'-shocks for the system

$$u_t + (f(u))_x = 0, \quad v_t + (f_1(u)v)_x = 0, \quad w_t + (f_{22}(u)v^2 + f_{21}(u)w)_x = 0,$$

where $f(u)$, $f_1(u)$, $f_{21}(u)$, $f_{22}(u)$ are smooth functions.

These types of solutions are related with *transportation and concentration processes*. We derive δ- and δ'-shock balance relations related with mass, momentum, and "surface" transportation to singular fronts.

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Self-focusing of few-cycle light pulses in transparent dispersive optical media with electronic and electronic-vibration cubic nonlinearities

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Optics of femtosecond (fs) pulses containing only few field cycles is a hot topic of a modern laser physics [1]. Here we analyze theoretically propagation of few-cycle wave packets in bulk optical media with normal dispersion and cubic nonlinearity of electronic and electronic-vibration (Raman) nature. For the purposes of this work, let us label pulses with ten or less field cycles as extremely short pulses (ESP).

Theoretical models of ESP propagation having been actively discussed in recent years differ from conventional models based on the slowly-varying envelope approximation which was initially suggested for quasimonochromatic pulses with many field oscillations [1]. Nonlinear equations of ESP propagation are written more and more often directly for the pulse electric field or its spectrum [2]–[4] because the envelope approach becomes unnecessary for few-cycle regimes both analytically and numerically. Following the technique suggested in [5] we have derived unidirectional equation for ESP spectrum G:

$$\frac{\partial G}{\partial z}+\frac{i\omega n(\omega)}{c}G+\frac{i\omega^2}{\pi c}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}\frac{\left[\chi_3^e+\chi_3^{e\nu}\gamma(\omega_\nu^2-\alpha^2+i\alpha/T_\nu)\right]G(z,\omega-\alpha)G(z,\alpha-\beta)G(z,\beta)}{\omega n(\omega)+(\omega-\alpha)n(\omega-\alpha)+(\alpha-\beta)n(\alpha-\beta)+\beta n(\beta)}d\alpha\, d\beta = -\frac{ic}{2\omega n(\omega)}\Delta_\perp G, \quad (1)$$

where z is the propagation coordinate, ω is the radiation frequency, c is the light velocity in vacuum, $\Delta_\perp=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ is the transverse Laplacian, and x, y, are transverse coordinates. Medium response is defined by the arbitrary dependence of the linear refraction index on frequency $n(\omega)$, cubic susceptibilities of electronic and electronic-vibration nature χ_3^e and $\chi_3^{e\nu}$, as well as parameters γ, ω_ν and T_ν, which quantitatively characterize dispersion of electronic-vibration nonlinearity. This is the most general equation of unidirectional paraxial evolution of pulses with continuum spectra in transparent media with electronic and electronic-vibration nonlinearities.

Equation (1) has been solved numerically using Fourier Split-Step Technique [6] for axisymmetric Gaussian-like wave packets with few-cycle duration and continuum spectra centered at the fundamental wavelength of a Ti:S laser and propagating in bulk fused silica. We demonstate regimes leading to the formation of ultrashort light structures like dumbbells or bubbles depending on the initial intensity. An example of dumbbell-like distribution is given in Figure. A "bubble", which is not presented here,

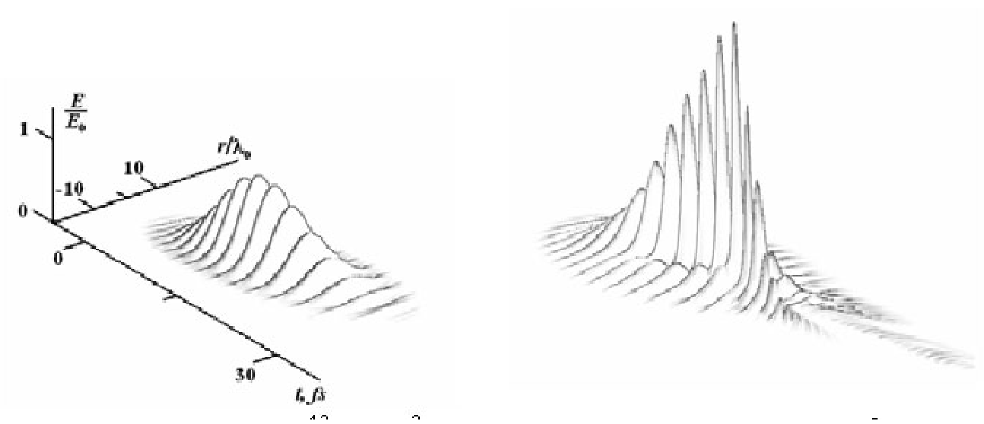

Figure : Self-focusing of 13-fs ESP with intensity $1.5\cdot 10^{13}$ W/cm^2 in fused silica: medium input $z=0$ (left), and $z=0.2$ mm (right).

includes a center area with "emptiness", where the field is practically nullified. We considered the process of the pulse tail steepening and subsequent turnover of the envelope shock wave and illustrated dramatic field changes and associated supercontinuum generation. This regime is beyond the scope of the envelope approach by definition.

It is shown that the account of electronic-vibration nonlinearity of fused silica does not change the selffocusing scenario qualitatively: the differences appear to be quantitative and better seen in the spectral domain. We found that the central frequency of generated supercontinuum is moved to lower frequencies due to Raman effect to compare to simulations were electronic-vibration nonlinearity is ignored. Formation of evident Stokes or anti-Stokes features has not been observed in computations. We believe that this is due to (1) consideration of ultrabroad spectra which far overlap Stokes and anti-Stokes shifts and (2) comparatively low combination activity of fused silica.

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The scattering operator in the stepwise waveguides

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The problem of modes transformation in non-homogeneous waveguides has the very long history, and extensive list of physics and mathematics publications was devoted to this problem during last century. Nevertheless, the well-known and widely used term "scattering matrix" seems to be not yet mathematically well-defined in the waveguide problems, and, correspondingly, the mathematical properties of the scattering operator in waveguide (in any sense) are not well described. Such description is, however, necessary to understanding the processes in the quantum, electrodynamics and acoustics waveguides, and, in particular, to developing good numerical algorithms for waveguides simulation.

I consider in this work the mathematical model of the stepwise waveguide and demonstrate that the investigation of this model is, in essential, the problem of the theory of the selfadjoint extensions of the symmetric operators in the Hilbert spaces. In such approach the scattering operator appears naturally as the parameter of the selfadjoint extensions of some symmetric operator and, hence, the scattering operator is, from the very beginning, the operator in appropriate Hilbert space, namely, the deficiency space of the initial symmetric operator. This allows us to investigate the properties of the "scattering matrix" in frame of the operator theory.

Besides the construction of the scattering operator, we obtain in this way the description of some of its important properties. Some of them seem to be rather unexpected, as, for example, the fact that this operator may be, in general, unbounded in the Hilbert space of the sequences of modes amplitudes with usual scalar product. Also it is interesting that, under some conditions, the scattering operator may be approximated (in some well-defined sense) by finite-dimensional operators, which is scattering operators for appropriate "finite dimensional waveguide". This approximation conserves most of important properties of such operators, such as the flow conservation low, and can be used to numerical calculation of the scattering matrix for various types of waveguides.

I consider also some algebraic properties of the family of scattering operators, which allow us to construct new operators from existing ones. These properties are well-known in the finite dimensional case, and are very useful for investigation of multi-step waveguides.

Explicit solutions to a singular differential equation with Bessel operator.

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Consider a problem of finding explicit solutions to a singular differential equation with the Bessel operator B_α and a potential function $q(x)$

$$B_\alpha u(r) - q(r)u(r) + \lambda u(r) = 0$$

via solutions of simpler homogeneous equation $B_\alpha v(r) + \lambda v(r) = 0$ in the next integral form

$$u(r) = v(r) + \int_r^\infty P(r,t)v(r)\,dr. \tag{1}$$

Here $P(r,t)$ is a kernel, $v(r)$ is the Bessel functions of some kind and B_α is Bessel operator of the form $B_\alpha u = u'' + \frac{2\alpha}{r}u'$, $\alpha \geq 0$, potential function is integrable at infinity and satisfies condition $|q(r+s)| \leq p(r)$, $0 \leq s \leq r$ with some nonnegative $p(r)$.

Theorem. Under above conditions the representation (1) exists with the next kernel function estimate

$$|P(r,t)| \leq \frac{1}{2}\left(\frac{t}{r}\right)^\alpha \int^\infty P_{\alpha-1}\left(\frac{y^2(t^2+r^2)-(t^2-r^2)}{2try^2}\right)|p(y)|\,dy\,\cdot$$

$$\cdot\exp\{\frac{1}{2}\left(\frac{t-r}{2}\right)\int_{\frac{t+r}{2}}^\infty P_{\alpha-1}\left(\frac{y^2(t^2+r^2)-(t^2-r^2)}{2try^2}\right)|p(y)|\,dy\},$$

here $P_{\alpha-1}(z)$ is the Legendre function.

If $q(r) \in C^1(0,\infty)$ then $P(r,t) \in C^2(t>r)$ and (1) is twice differentiable. For the kernel also explicit integral equations are found. Many important potentials such as strongly singular, Bargman and Jukawa are covered by this theorem. Equivalent problems are finding of transmutation (intertwining) operator such as $S_\alpha(B_\alpha - q) = B_\alpha S_\alpha$ or the generalized translation operator. The case with integrals in (1) over interval $(0, \frac{t+r}{2})$ is also studied. In special case $\alpha = 0$ we arrive at the explicit formula for the mixed problem to the equation

$$u''_{xx} - q(x)u + \alpha u = u''_{yy} - p(y)u.$$

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On continuous spectra of the elasticity problems

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It will be demonstrated that, for a peak-shaped solid with the surface free of traction, the spectrum of the operator becomes continuous and a certain information on the structure of the spectrum can be obtained. This observation leads to several conclusions on asymptotics of solutions to spectral elasticity problems for solids with singularly perturbed boundaries. In particular, concentration of eigenvalues in the ultra-low frequency range and localization of the corresponding eigenmodes were discovered for elastic bodies with blunted peaks and for heavy and hard elastic peak-shaped inclusions. As a result of these consideration, the following result is obtained: Under a few symmetry assumptions on a composite elastic cylinder, any preassigned number of linearly independent trapped modes (solutions to the homogeneous elasticity problem decaying at infinity) can be found on the continuous spectrum by varying the physical properties of inclusions. This result provides the effect of decrement (damping) of certain packets of elastic waves.

Anisotropic approximation of radially layered elastic media

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Approximate solution of elasticity equation for the cylindrical pipe consisting of multiple isotropic homogeneous layers was obtained.

Following L.Molotkov technique we replaced multilayered pipe by effective anisotropic pipe with radial anisotropy. Thus we got one equation for anisotropic pipe instead of several equations for each layer interconnected by boundary conditions. Then formal solutions for effective equations were found as series. All these considerations are valid only for low frequencies and under some geometrical conditions.

As the next step we conidered solutions obtained at zero frequency limit (only first terms of series were taken) and got formula for tube wave velocity in multilayered cylindrical pipe with water inside. Velocities computed by the formula obtained are in a good agreement with numerical computations.

Transient waves produced by a spherical source pulsating with different periods

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The solutions of the initial value problem to inhomogeneous wave equation are constructed in terms of spherical harmonics for sources distributed on a spherical surface periodically expanding with the velocity of light. The sphere has the fixed radius at the initial moment of time. The time of sphere expansion is fixed. The number of expantions is finite. Periods between two consequent expansions are not equal to the time of sphere expansion as it was considered in [1], [2]. The wavefunctions are constructed for the different periods with the help of the Smirnov method of incomplete separation of variables and the Riemann Formular [3]. Separating the polar-angle variable we represent the solution as the Legendre polynomial series whose coefficients are function of the radial and time variables. The analytical expressions for the above coefficients are obtained by means of the Riemann formula.

We apply the constructed solution in the cases when the source is distributed on the circle belonging to the pulsate sphere and moving point source on the circle. We investigate the space-time structure of formed waves in dependence of pulsation periods. Applicatiom of the scalar solution to the description of electromagnetic waves is also discussed.

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Analysis of the TE-wave propagation in nonlinear dielectric three-layer planar waveguides with non-Kerr nonlinearity

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Propagation of electromagnetic waves in linear media along three-layer planar dielectric waveguides is a relevant topic of classical electrodynamics [1, 2]. Cylindrical waveguide structures consisting of nonlinear media is also an object of intense studies [2–8]. However until recently any rigorous proofs of the existence of modes in nonlinear dielectric waveguides have not been obtained, as well as the correct mode classification. Planar structures consisting of several plane-parallel layers filled with nonlinear Kerr-law media is another well-studied family of waveguides (see [2]). Whereas, for example, in the special case of a three-layer planar waveguide with Kerr nonlinearity in the layers the field can be described in terms of elliptic functions [6].

In this work, we consider propagation of TE-polarized electromagnetic waves in a three-layer planar waveguide filled with a non-Kerr-type nonlinear, nonabsorbing, nonmagnetic, and isotropic dielectric. We look for time-harmonic propagating solutions and reduce the problem to a transmission and boundary value problem for quasilinear differential equations. The differential equations are solved analytically by elliptic functions. Numerical results are also presented.

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Analysis of the TM-wave propagation in nonlinear dielectric layer planar waveguides with Kerr nonlinearity

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Propagation of electromagnetic waves in linear media along three-layer planar dielectric waveguides is a relevant topic of classical electrodynamics [1, 2]. Waveguide structures consisting of nonlinear media is an object of intense studies [2–8]. However until recently any rigorous proofs of the existence of modes in nonlinear dielectric waveguides have not been obtained, as well as the correct mode classification. Planar structures consisting of several plane-parallel layers filled with nonlinear Kerr-law media is well-studied family of waveguides (see [2]). Whereas, for example, in the special TE-case of a three-layer planar waveguide with Kerr nonlinearity in the layers the field can be described in terms of elliptic functions [6].

In this work, we consider propagation of TM-polarized electromagnetic waves in a two-layer and three-layer planar waveguides filled with a Kerr-type nonlinear, nonabsorbing, nonmagnetic, and isotropic dielectric. We look for time-harmonic propagating solutions and reduce the problem for Maxwell's equations to a transmission and boundary value problem for quasilinear differential equations. Integration yields an equation so that one field component can formally be expressed in terms of the other, leading to the possibility of finding one field component by integration. Evaluation of the boundary conditions for the layer structure leads to the dispersion relation. The differential equations are solved analytically by elliptic and hyperelliptic functions. Numerical results are also presented.

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Cauchy problems in bounded domains, Feynman path integrals and difusions

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One investigates some relations between Cauchy problems in bounded domains, Cauchy problems in the whole space, Feynman path integrals over trajectories in Riemannian manifolds and diffusions on such manifolds.

Lasing spectra and thresholds of a circular microcavity laser embedded in an annular Bragg reflector

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Key elements of compact integrated circuits are microcavity lasers due to their small size and very low threshold operation. Ring and disk lasers use whispering-gallery (WG) modes that experience almost total internal reflection at the cavity rim. These modes operate under extremely low threshold pumping energy. Further reduction of microcavity size leads to the increasing of mode threshold as far as low-order lasing modes are non-WG modes. Here we analyze a circular microdisk laser embedded into an annular Bragg reflector (ABR) (Figure). This structure has potential to support low-threshold modes with low azimuth index as far as ABRs have the advantage of strong in-plane radial optical confinement [1]. In this paper we study 2-D model of microcavity and focus on the H_z polarized modes. Our study is based on the Lasing Eigenvalue Problem (LEP) [2], which enables one to find not only the lasing frequencies but the thresholds as well.

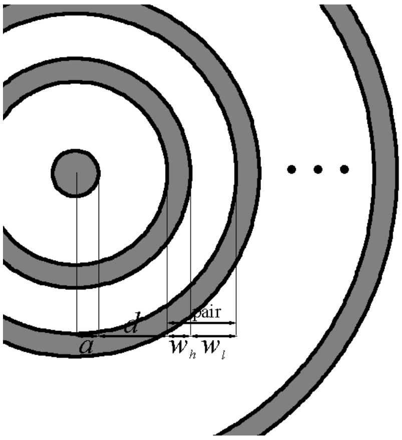

Figure: Microdisk inside ABR in 2D

Several factors bring fundamental influence on thresholds of lasing modes. The first is the distance from the active cavity to ABR, the second is the number of pairs of layers of lower and higher refractive indices in ABR, and the third is the thicknesses of pairs in reflector. We discuss a possible optimal configuration of the microlaser to achieve the lowering of the thresholds.

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The scattering of the localized surface wave by an elastic plate floating on shallow water of variable depth

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The wave forcing of a floating thin plate can be used to model a wide range of physical systems: for example, very large floating structures, sea ice floes and breakwaters. In mathematical modelling, such structures are often treated as thin elastic plates. At present, there have been considerable numbers of studies on the hydroelastic behavior of the floating elastic plate. Main interests of the previous researches have been confined in the hydroelastic response of a thin plate on flat sea-bottom. In reality, the seabed is not uniform in depth. To our knowledge, the consideration of a variable water depth was made only for diffraction problem, by solving the linear hydroelastic problem for a single frequency. Unsteady behavior of beam plate floating on shallow water of variable depth is considered in this paper. This problem has been chosen because the sea-bottom effects become more significant in shallow water, than that in deep water. Proposed method may be used for any unsteady 2D problem of linear shallow-water theory, but here the motion of the elastic beam plate is considered for a travelling localized surface wave.

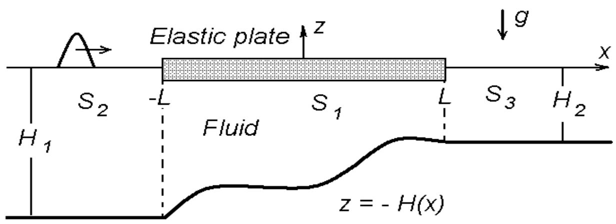

Figure :

An elastic beam of width $2L$ floats freely on the surface of an inviscid incompressible fluid layer. The surface of the fluid that is not covered with the plate is free. The fluid region S is divided into three parts: S_1 $(|x| < L)$, S_2 $(x < -L)$, S_3 $(x > L)$. Without the plate, the fluid depth is equal to $H(x)$ in S_1, and the fluid depths in the left and right hand domains of constant depth S_2 and S_3 are equal to H_1 and H_2, respectively. With the plate, the fluid depth in S_1 is equal to $h(x) = H(x) - d$, where d is the draft of the plate. It is assumed that the maximal depth of the fluid is small in comparison with the lengths of surface waves and flexural-gravity ones, and the shallow water approximation is used. The velocity potentials describing the fluid motion in the regions S_j are denoted by $\phi_j(x,t)$ $(j = 1, 2, 3)$. A deflection of an elastic plate $w(x,t)$ is described by the equation:

$$D\partial^4 w/\partial x^4 + m\partial^2 w/\partial t^2 + g\rho w + \rho\partial\phi_1/\partial t = 0 \quad (x \in S_1),$$

where D is the flexural rigidity of the plate; m is the mass per unit length of the plate; ρ is the fluid density, and g is the gravity acceleration. The draft of the plate is equal $d = m/\rho$.

According to linear shallow-water theory, the following relation is valid:

$$\partial w/\partial t = -\partial\big(h(x)\partial\phi_1/\partial x\big)/\partial x \quad (x \in S_1).$$

In the free-water regions, the velocity potentials $\phi_2(x,t)$ and $\phi_3(x,t)$ satisfy the equations

$$\partial^2\phi_2/\partial t^2 = gH_1\partial^2\phi_2/\partial x^2 \quad (x \in S_2), \qquad \partial^2\phi_3/\partial t^2 = gH_2\partial^2\phi_3/\partial x^2 \quad (x \in S_3).$$

The displacements of the free surface $\eta_2(x,t)$ and $\eta_3(x,t)$ are determined in the regions S_2 and S_3 from the relations

$$\eta_j = -g^{-1}\partial\phi_j/\partial t \quad (x \in S_j), \ j = 2, 3.$$

If $|x| = L$ the matching conditions (continuity of pressure and mass) should be satisfied. At the edges of the beam, the free-edge conditions are satisfied, which imply that the bending moment and shear force are equal to zero.

It is assumed, that at the initial time the plate and fluid in the regions S_1 and S_3 are at rest. In region S_2, the localized displacement of the free surface $\eta_0(x - \sqrt{gH_1}t)$ travels to the right. At $t = 0$ the displacement reaches the left edge of the plate and the plate begins to undergo a complex bending motion in response to the incoming wave. Consequently, the initial conditions have the form:

$$w = \eta_3 = \partial\phi_1/\partial t = \partial\phi_3/\partial t = 0, \quad \eta_2 = \eta_0(x), \quad \partial\phi_2/\partial t = -g\eta_0(x) \quad (t = 0).$$

The transient elastic deformations of a beam is computed by means of time-domain differential equation method, with elastic deflections expressed by a superposition of modal functions with time-dependent unknown amplitudes. The plate deflections and wave motions of the fluid for various bottom topographies have been calculated.

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The waves propagation in the composed rod

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In this paper we investigate the waves propagation in the composed materials. Let us consider the model problem deal with the one-dimensional longitudinal waves in the composed rod. We consider the composed rod as two connected rods and introduce the elastic and viscous linear interaction between points of rods. At the initial moment the left end of rods is under action the force impulse or the displacement impulse of this end is given. The right side of rods is free, we consider also the semi-infinite rods.

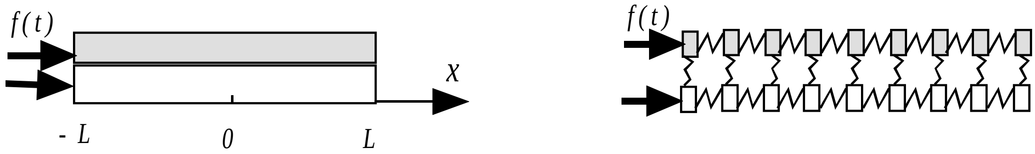

Figure : The model of the body.

Longitudinal displacements are described by the system of equations

$$
\begin{aligned}
E_1 S_1 \frac{\partial^2 u_1}{\partial x^2} &= \rho_1 S_1 \frac{\partial^2 u_1}{\partial t^2} + R(u_1 - u_2) + B\left(\frac{\partial u_1}{\partial t} - \frac{\partial u_2}{\partial t}\right), \\
E_2 S_2 \frac{\partial^2 u_2}{\partial x^2} &= \rho_2 S_2 \frac{\partial^2 u_2}{\partial t^2} + R(u_2 - u_1) + B\left(\frac{\partial u_2}{\partial t} - \frac{\partial u_1}{\partial t}\right),
\end{aligned}
\tag{1}
$$

where $u_i(x,t)$ — are the longitudinal displacements of the points of rods ($i = 1, 2$), E_i, S_i, ρ_i — are the parameters of rods (Young modules, transverse areas and mass densities), R, B — are the coefficients of the elastic and of the viscous interaction of rods. This problem is a model of the longitudinal wave propagation in a rod of sandwich type, consisting of the alternating hard and soft lowers.

In contrary to the single rod here the wave velocity depends on the wave length and therefore the dispersion equation is investigated. By using the Laplace transform in time the stress-strain state near the front of wave is investigated. For comparison the solutions are constructed by using the Fourier transformation and also by the finite elements method. The influence of the problem parameters (in partial the viscous forces on the motion is discussed.

Casimir effect and exactly integrable Hamiltonians

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Application of the dimensional regularization to a certain class of multi-dimensional manifolds with boundaries is considered. Simple formula for regularized vacuum energy is obtained. Examples for some integrable Hamiltonians are considered (including Hamiltonians constructed via Darboux transformation). The Casimir force results from the alteration by the boundaries of the vacuum energy. Applications to a certain class of integrable model of field theory is discussed. This approach can be useful in the numeric calculations.

Transformation of electromagnetic fields using metamaterials

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In this review paper we discuss various approaches to electromagnetic control and transformation of electromagnetic fields with engineered metamaterials, including a novel concept of field-transforming metamaterials. These artificial media change the fields in the volume occupied by the medium in a prescribed fashion. We show what electromagnetic properties of transforming medium are required if the transformed fields are arbitrary linear functions of original fields.

It has been recently realized that metamaterials – artificial electromagnetic materials with engineered properties – can be designed to control electromagnetic fields in rather general ways. More precisely, it has been shown that transformation of spatial coordinates (accompanied by the corresponding transformation of electromagnetic fields) can be mimicked by introducing electromagnetic materials with specific electromagnetic properties into the domain where the coordinates have been transformed [1, 2]. As an example of such "coordinate-transforming" device an "invisibility cloak" has been suggested. Not only spatial coordinates can be transformed and the result simulated by some metamaterial, but also the time coordinate can be transformed with a similar interpretation. It has been known for a long time that the material relations of an isotropic magnetodielectric transform into certain bi-anisotropic relations if the medium is moving with a constant velocity (e.g., [3]). It was suggested that also this effect of time transformation can possibly be mimicked by a metamaterial [4, 5, 6].

In this presentation we discuss opportunities for field control from a more general position, introducing the concept of field-transforming metamaterials. In this concept we start directly from a desired transformation of electromagnetic fields and find the material properties that are necessary to realize this transformation. We will show that an artificial medium with the bianisotropic constitutive relations of the form

$$\mathbf{D} = \epsilon\epsilon_0\mathbf{E} - \frac{j}{\omega G}\nabla G \times \mathbf{H}, \quad \mathbf{B} = \mu\mu_0\mathbf{H} + \frac{j}{\omega F}\nabla F \times \mathbf{H} \tag{1}$$

transforms electric and magnetic fields $\mathbf{E}_0(\mathbf{r})$ and $\mathbf{H}(\mathbf{r})$ into

$$\mathbf{E}(\mathbf{r},t) = F(\mathbf{r},t)\mathbf{E}_0(\mathbf{r},t), \quad \mathbf{H}(\mathbf{r},t) = G(\mathbf{r},t)\mathbf{E}_0(\mathbf{r},t). \tag{2}$$

Here ϵ and μ are the relative permittivity and permeability of the host medium, respectively.

Materials with the constitutive relations (1) belong to the class of moving omega media [7]. Of course, this material is physically at rest but the fields see it as a moving material. This is also a material which is required to physically realize the field transformation corresponding to transformations of time [6]. Possible realizations of such media will be discussed in the presentation.

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Pulsed radiation produced by a travelling exponentially decaying bipolar current pulse with high-frequency filling

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Investigation of electromagnetic radiation accompanying propagation of pulses with high-frequency filling moving along a line segment is stimulated by the problem of launching directional scalar and electromagnetic waves as well as by results of experimental investigation of superradiation waveforms (see, e.g., Egorov V.S. et al. 1986, Sov. J. Quantum Electronics 13, p.729). In this work the specific envelope function, an exponentially decaying bipolar pulse (a sum of two exponentially decaying pulses of opposite polarity), is considered as being one of the simplest but feasible approximation of many real source currents.

Generation of the electromagnetic waves is described by solving the inhomogeneous Maxwell equations directly in the space-time domain: first the electric and magnetic field vectors are expressed in terms of one scalar function and then the resulting hyperbolic-type PDE is solved using incomplete separation of variables and the Riemann formula.

Being an extension of a well-known description of the travelling-wave antenna, this model can explain peculiarities of some non-stationary electromagnetic waves produced by traditional artificial and natural line radiators, in particular, frequency transform and beatings. The choice of the specific envelope function allows deriving observable and easy-to-analyse expressions related to the space time structure of the emanated waves, which may be used for characterizing natural phenomena and optimizing parameters of man-made radiators.

One-dimensional nonlinear FPK equation with the potential of a special form in semiclassical approximation

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Solution of the Cauchy problem is constructed for the Fokker-Planck equation

$$\partial_t P(x,t) = \partial_x [U'(x,t) + \varepsilon \partial_x] P(x,t), \tag{1}$$

$$U(x,t) = \frac{x^4}{4} + (\theta - 1)\frac{x^2}{2} - \theta \langle x(t) \rangle x, \tag{2}$$

in semiclassical approximation accurate to $O(\varepsilon^{3/2})$. Here, prime denotes derivative with respect to x,

θ is a constant, ε is a small parameter. The moment $\langle x(t)\rangle$ is defined by expression

$$\langle x(t)\rangle = \int_{-\infty}^{+\infty} dx x P(x,t). \tag{3}$$

The solution of eq. (1) is constructed in the class of semiclassically concentrated functions [1] in the framework of the Maslov complex germ method. The nonlinear evolution operator is found in explicit form that admits to construct a parametric family of symmetry operators.

The results obtained are discussed in the context of solutions found in [2].

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Symmetry operators for the multidimensional FPK equation with quadratic nonlocal nonlinearity

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A method of construction of semiclassical asymptotics for for the Fokker–Planck–Kolmogorov equation (FPKE) of special form with a quadratic nonlocal nonlinearity based on the Maslov's complex germ theory is applied to the multi-dimensional FPKE with the quadratic potential

$$\Big\{ -\partial_t + \varepsilon\Delta + \partial_{\vec{x}}\Big(\vec{V}(\vec{x},t) + \varkappa \int_{\mathbb{R}^n} \vec{W}(\vec{x},\vec{y},t)u(\vec{y},t)d\vec{y}\Big)\Big\}u(\vec{x},t) = 0, \tag{1}$$

where

$$\vec{V}(\vec{x},t) = K_1\vec{x}, \quad \vec{W}(\vec{x},\vec{y},t) = K_2\vec{x} + K_3\vec{y}. \tag{2}$$

Here, $t \in \mathbb{R}^1$, $\vec{x} = (x_1, \dots, x_n)^\intercal \in \mathbb{R}^n$, $\vec{y} = (y_1, \dots, y_n)^\intercal \in \mathbb{R}^n$ are independent variables; $(x_1, \dots, x_n)^\intercal$ means a transpose to a vector or a matrix; $d\vec{x} = dx_1 \dots dx_n$; the dependent variable $u(\vec{x},t)$ is a real smooth function decreasing as $\|\vec{x}\| \to \infty$; K_1, K_2, K_3 are arbitrary constant matrices of order $n \times n$; ε and $\varkappa$ are real parameters; $\partial_t = \partial/\partial t$; $\partial_{\vec{x}} = \partial/\partial\vec{x}$ is a gradient operator with respect to $\vec{x}$; $\Delta = \partial_{\vec{x}}\partial_{\vec{x}} = \sum_{i=1}^{n} \partial^2/\partial x_i^2$ is a Laplace operator.

The Cauchy problem for the Fokker–Plank–Kolmogorov equation (1) is reduced to a similar problem for the correspondent linear equation. The relation between symmetry operators of the linear and nonlinear Fokker–Plank–Kolmogorov equations is considered. Illustrative examples of the one-dimensional symmetry operators are presented.

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Asymptotics for nonlinear wave equations with external source

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This paper considers the Cauchy problem in $\mathbf{R}^n$ for some nonlinear wave equations with external sources, for example, for sin-Gordon equation:

$$u_{tt} - \Delta u + a \sin u = \lambda f(x,t).$$

Such systems are important for many applications, for example, for physics, chemistry, phase transitions. In these applications one often observes wave solutions [2].

It is well known that for integrable systems ($n = 1$) solutions for small λ can be studied (see, for example, [3] and references in it).

Moreover, in general case $n > 1$ such asymptotics can be obtained under essential restrictions to initial data (when these data are close to a wave solution, for example, a soliton or a kink).

We study formal asymptotics for large λ, with a strong external source. Under some conditions on the initial data (they should be close to a wave solution of the usual linear wave equation), such asympotics can be obtained by standard linear methods [1]. However, justification of these formal solutions is a difficult problem.

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Fractional boundary conditions in diffraction problems on plane screens

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Last fifteen years N.Engheta developed the method of fractional operators in solving wide class of problems in electromagnetic theory [N.Engheta, "Fractional Paradigm in Electromagnetic Theory", a chapter in IEEE Press, chapter 12, pp.523-553, 2000]. Fractional derivative and integral, and also fractional curl operator are some of the examples of fractional operators. In this paper fractional derivatives defined by Riemann-Liouville integral are considered. N. Engheta introduced the concept of "Intermediate state in electrodynamics" — fractional paradigm. It means the following: if the function and its first derivative describes two canonical cases of some electromagnetic field, then fractional derivative of this function ${}_{-\infty}D_y^{\nu} f(y)$ $(0 < \nu < 1)$ corresponds to intermediate situation of the field between two canonical original cases. Engheta investigated such intermediate situations as new waves, intermediate between plane and cylindrical waves; new sources of field, intermediate between sheet and line sources, etc. In this paper, developing the fractional paradigm, new fractional boundary conditions (BC) are introduced, which act as intermediate between known BC of Dirichlet and Neumann. In some sense fractional BC can be treated as impedance BC. Utilizing fractional BC we solve diffraction problem on infinite thin strip with impedance of special kind. It is shown, that for $\nu = 0.5$ the solution can be expressed in analytical form for any values of frequency. For the strip with special impedance such physical characteristics as radiation pattern, cross sections are studied.

Metamaterials based on structures with embedded 3-D resonant inclusions

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A classical structure exhibiting properties of medium with negative permittivity and permeability within a limited frequency range was suggested in [1] and then experimentally examined in [2], where the negative refraction phenomenon was observed. The structure suggested in [1] is a combination of two lattices: a lattice of the split-ring resonators (SRR) and a lattice of infinitely long parallel wires. This structure is anisotropic and reveals the negative permittivity and permeability, if the propagation direction of the electromagnetic wave is orthogonal to the axes of wires. Though the theory and experimental investigation of such structures are still under interest of many scientists, isotropic DNG structures are very attractive for practical use. In this case, structures with embedded 3-D resonant inclusions are very promising.

In this presentation, different ways to create the 3-D isotropic and 2-DDNG medium based on a regular lattice of resonant inclusions are reviewed:

1. Single-negative or double-negative metamaterial formed by a rectangular lattice of isotropic cubic unit cells of particles. Each unit cell is formed by putting perfectly conducting resonant particles on the faces of a cubic unit cell [3-5]. Different particles are used: simple and complicated SRRs, W-particles, or combination of SRR and wire medium.
2. DNG medium formed by a regular lattice of spherical resonant inclusions, providing excitation of electric and magnetic dipoles at the same frequency [6-9]. In this case the magnetodielectric particles or dielectric particles in double-spherical structures are used.
3. DNG medium formed by an array of dielectric rods with different radii, which is 2D analogue of artificial media based on spherical particles [10].
4. Clustered dielectric particle metamaterials, which are constituted by the periodic repetition of a molecule-like cluster of dielectric atom-like particles [11].

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Resolvent operator of maxwell equations for 6-dimentional field vector

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Current development of technologies requires investigation of non-stationary electromagnetic fields in complex organic mediums (e.g. chiral mediums, moving mediums, artificial mediums, metamaterials etc.). Modeling of corresponding phenomena requires solution of complex initial-boundary electromagnetic problems with explicit accounting for time dependence of fields that is usually nonharmonic. Moreover, in such tasks it is necessary to account for the essentially vector structure of electromagnetic field. Furthermore it is often impossible to separate material equations into independent electric and magnetic components. In such cases it is necessary to consider essentially 6-dimensional structure of electromagnetic field.

Along with differential approach to description of electromagnetic field there exists a new powerful and rapidly growing tool for temporal modeling of electromagnetic processes in complex structures - integral equations. For their solution there are used analytical as well as numerical methods.

Regardless of solution method, formulating of integral equations in time domain require knowledge of explicit expression of space-time Green function. Such function is used in free-space problems as well as in problems for mediums: three-dimensional vector problems with complex mediums, anisotropic mediums, problems of multilayer dielectric structures, problems of structures with forbidden regions. Temporal Green function for Maxwell equations in unrestricted space allows solving many electrodynamic problems such as problems for non-stationary sources, problems of scattering in temporal Born approximation, problems of electromagnetic oscillations in photonic crystals etc. In work there was considered full Green function of free space for Maxwell equations in time domain. The Green function was obtained using method of propagation operators (propagators) that are common to quantum mechanics but quite seldom used in classical electrodynamics. There exist a limited number of works that use such approach e.g. temporal integral operators of propagation were obtained in work for investigation of pulses propagation in non-stationary uniform and isotropic dielectric and magnetic medium.

In this work there is given general approach to investigation of initial-boundary problem for Maxwell equations in uniform medium and in plasma in time domain by their reduction to integral Volterra equation of the second kind for 6-dimensinal field vector. This is attained using derived Green function in 6-dimensional formulation in time domain. The integral equation is equivalent to Maxwell equations and contains initial and boundary conditions. It gives unified definition of field in the whole space including non-stationary region of irregularity and ambient space. There was obtained resolving operator for this equation which was used for investigation of plane wave transformation and emission of lumped source in the medium with sharp time variation of parameters. Arbitrary non-stationary of the medium can be approximated by sequence of such sharp variations of its parameters and for each variation there is calculated exact analytical solution obtained using resolvent method.

Enhancement of magneto-optic effects with resonant structures

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Magnetooptical (MO) Faraday effect, which consists in rotation of polarization plane of transmitted wave in presence of magnetic field can be substantially amplified by means of different resonant structures. The examples are Fabry-Perot resonance in homogeneous layer, defect-mode of photonic crystal and Tamm (surface) state on a boundary of two photonic crystals with intersecting band gap. In all these cases a resonance is manifested as a peak in the transmission coefficient. In the presence of magnetization the peak splits into two ones, corresponding to both circular polarizations (Fig. 1, thick lines). The incident wave being linearly polarized, the transmitted one has the same polarization at frequency of crossing of the thick curves in Fig. 1. The Faraday rotation angle θ is determined as shown in Fig. 1.

We carry out an analysis of amplification of the Faraday rotation on the base of partial waves (Fig. 2). The large phase difference, which determines θ, is a result of different sign of phases of partial waves. This mechanism has nothing in common with multiple passing the MO layers.

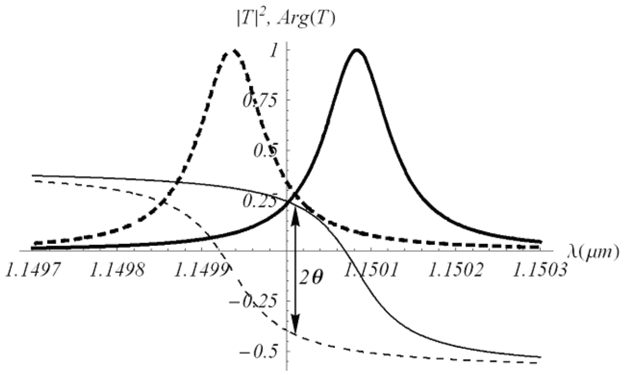

Fig. 1. Transmission coefficient (thick lines) and corresponding phases (thin lines) for two circular polarizations at the presence of magnetization.

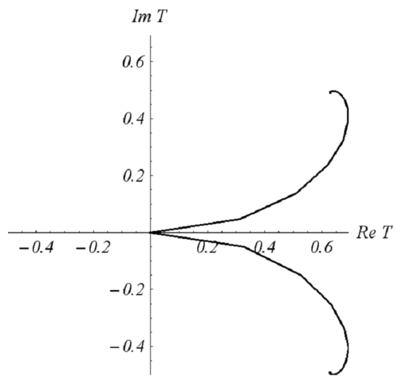

Fig. 2. Two circularly polarized waves presented as a sum of partial waves

Distinct feature of magneto photonic crystals on formation of the Yeh band gap

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Systems of anisotropic layers play a significant role in a number of applications (narrow-band Solc filter, multistage electro-optic modulators, and polarizer). We consider the effect of (natural and artificial) anisotropy and birefringence on magneto-optical (MO) properties of photonic crystals (PC).

It is well known that the natural anisotropy usually masks the magneto-optical effects in homogeneous media. Indeed, the difference in the values of wavenumbers of ordinary and extraordinary waves, as a rule, exceeds the difference appearing due to the external magnetic field. As a consequence, contrary to isotropic medium in naturally anisotropic media magneto-optics effects is quadratic in small magneto-optics parameter.

In PC, beyond the natural anisotropy there is anisotropy due to anisotropy of primitive cell. The later effect is more pronounced in 1D and 2D PC. Thus, in the general case, to observe linear in the magnetization MO effects one needs working with directions where the Bloch wavenumber of ordinary and extraordinary waves are identical [1]. Along such directions the PC is efficiently isotropic.

A Bloch wave vector is determined up to the vector of reciprocal lattice. Thus, there is a series of wavenumbers characterizing the same Bloch wave. Each wavenumber corresponds to one of the plane wave the Bloch wave is built of. These plane waves differ in wavenumbers as well as in amplitude, polarization and phase of their phasors. Considering MO effects we have to distinguish the case of coincidence of wavenumbers of ordinary and extraordinary Bloch waves depending on the rank of the corresponding plane wave in formation of the Bloch wave. If we deal with plane waves (harmonics) each of which gives the maximal contribution (having maximal amplitudes) to the corresponding Bloch wave it means that the Bloch waves belong to the bands with the identical order numbers (intraband). In opposite case the Bloch waves belong to the bands with different order numbers (interband case).

It shown that the Faraday effect can be observed in intraband case only [1]. If the ordinary and extraordinary Bloch waves belong to different bands then in the very direction of the optical axis the Yeh's band gap forms [1, 2, 3]. A significant difference in amplitudes as well as beamsteering effect make it difficult to observe MO effects. On the other hand the formation of the Yeh's band gap in 1D MPC under influence of external magnetic field, allows us to create new type of magneto-controlled devices [3].

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A comparative analysis of mixing formula for SNG and DNG media

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Russia

Recently a great deal of attention has been paid to metamaterials, which are, as a rule, composite materials consisting of ingredients with negative permittivity and permeability (double negative media DNG) or wit negative permittivity or permeability (single negative media SNG). In this connection the problem appears: how to treat mixing formula. The key moment is existence of the Bergman-Milton poles of the spectral function on real negative axis.

It is shown that in static case (when even DNG media can be treated as a SNG media) we should take care when extracting of the root of function of two complex variables. Firstly, we need make a proper cat of the complex plane to obtain a unique analytical branch. Secondly, we need factorization to reduce the function of two complex variables to two functions of single complex variable.

Dealing with calculation of wave number of a plane wave traveling through a DNG media we cannot choice a unique analytical branch of the root to treat all possible events. The reason is that in electrodynamics we deal with two quantities, namely with wave number and impedance. As is shown by our analysis calculation of the physically true values of quantities we need employ different branches of the root. Thus, we have to take addition assumption that, for example, a plane wave transferring energy from left to right has a positive value of real part of impedance. After doing that we can choice a proper branch of the root and determine the sign of the wave number.

Analisys of applicability ranges of exact light scattering methods using spherical basis

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The light scattering methods expanding fields in terms of wave functions are widely applied due to their high efficiency. We analyse some of these methods, namely the extended boundary condition (EBCM), separation of variables (SVM), and point matching (PMM) ones [1, 2], considering their theoretical and practical applicability. Though the methods look alike because of their use of the same expansions of the fields, it is found that these approaches differ in important aspects.

These approaches seem to be very similar as they search for the problem solution in the form of the *same* field expansions with the expansion coefficients being derived from solution of systems of linear algebraic equations. The main difference of the methods is their use of different problem formulation which leads as a result to different systems. In the EBCM the fields and Green function expansions are substituted into the boundary condition presented in a surface integral form; in the SVM the field expansions are substituted into the boundary condition written in a usual (differential) form; in the PMM any of the above forms can be used, but the system is derived from minimization of a residual of the boundary condition considered in selected points at the scatterer surface (see [2] for more details).

From the theoretical point of view applicability of the approaches is related with *convergence* of the field expansions and *solvability* of the systems used to derive the expansion coefficients. The convergence of the expansions of the scattered and internal fields depends on the field singularities [3]. It is shown that in the far-field zone EBCM can be formulated in terms of field patterns [2]. In this case its applicability is restricted only by the solvability condition, which is weaker than the convergence one. For the SVM, a similar analysis has not been performed, though at least the EBCM system can be obtained within the SVM [3]. The system arising in the PMM is always solvable, as it is positively determined (see, e.g., [2]).

For numerical analisys we have developed a homogeneous set of codes based on the methods under consideration. Their accuracy in the far field zone was studied by using the relative difference of the extinction and scattering cross sections $\delta = |C_{\rm ext} - C_{\rm sca}|/(C_{\rm ext} + C_{\rm sca})$ calculated for nonabsorbing particles.

For spheroids, where all the three methods are mathematically correct [2], the EBCM is generally preferable to other 2 methods. The situation is a bit different for axisymmetric Chebyshev particles. Our calculations well confirm that EBCM provides poor accuracy when its resolvability condition isn't met. However, SVM and PMM still coverge.

Thus, despite the large similarity of the EBCM and SVM (see the discussions in [2, 3]), their theoretical applicability conditions differ in principle as the solvability of the SVM is not determined by the EBCM solvability condition. Our preliminary results of similar comparison of the methods, when the spheroidal functions were used for the field expansions, led to the same conclusions.

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Numerical approach for analysis of surface wave field in the vicinity of caustics

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Due to lateral heterogeneity of the Earth surface wave paths deviate from great circle arcs, that results in appearance of caustics on the hemisphere opposite to the epicenter. The caustics consist of two branches forming the cusp in the point of contact. Between the branches three waves interfere: coming to and from one of the branches, and coming to another branch. Asymptotic representation of the wave field in the vicinity of such a caustic (at high frequencies) is expressed in terms of the Pearcey integral, which is a function of two variables, related to location of the point, where the field is considered, respectively to the cusp and to the caustic. Analysis of harmonic surface wave field in a simplest plane model on the basis of the Pearcey integral (Yanovskaya, 2004) has shown that wave amplitude oscillates within the cusp in two direction - along the caustic axis and in the direction perpendicular to the caustic. The wave phase varies along the caustic axis unevenly, that results in sharp variations of the phase velocity. However these results are true only for high frequencies, where the asymptotic approach can be applied. In the real Earth periods of surface waves may be rather big (up to 150–200 s) and correspondingly the wavelengths are big as well. In such a case the asymptotic approach for analysis of surface wave field cannot be applied. Besides this method can be realized for plane surface, and for an individual caustic, while the surface of the real Earth is spherical, and we frequently observe superposition of such caustics. Another difficulty is related to necessity to calculate travel times of the superimposed waves in the same point with very high precision that is necessary to calculate parameters of the Pearcey integral. At last, application of this method needs to extrapolate the Pearcey integral parameters outside the caustic. So this approach is valid only for drawing qualitative conclusions about behavior of the wave fields near the caustic.

In the present study we propose a numerical method for calculation of surface wave field in the region containing caustics. The wave field is considered as a superposition of the waves arriving from the circle dividing two hemispheres (centered in the epicenter and in the antipode). So for calculation of the wave field on the hemisphere centered in the antipode we can use the representation theorem. The factor in the expression for surface wave field describing variation of the field due to configuration of the rays on the surface (so-called 'geometrical factor') satisfies the wave equation with phase velocity of the waves $\Delta U = \omega^2 c^{-2}(\mathbf{r})$. We consider only the geometrical factor, for which the representation theorem becomes

$$U(\mathbf{r}) = \int\limits_S \left(\frac{\partial U}{\partial n} u_G(\mathbf{r}, S) - U(S) \frac{\partial u_G}{\partial n} \right) dS,$$

where $u_G(\mathbf{r}, \mathbf{r}')$ is the Green function. The geometrical factor U on the contour S (a circle bounding the hemisphere centered in the epicenter) is calculated on the basis of the ray theory, because the rays on this hemisphere are not intersected. The Green function on the opposite hemisphere is assumed the same as on a laterally homogeneous spherical surface. This limitation is not too substantial, because configuration of the rays on this hemisphere is mostly determined by the field on the circle. The integral in the representation theorem is calculated numerically. The result turns to be dependent on a choice of the limits of integration. A method is proposed for estimating the integral independent of these limits.

The results for some numerical models as well as for the model of the real Earth are presented. It is shown that in case of two superimposed caustics strong increase or decrease of amplitude may occur in the intersection of the caustic axes. Strong phase anomalies (forestalling of the phase) are observed outside the cusps that results from arrival of the waves from the parts of the contour neighbor to the caustic axis. The main conclusion from numerical modeling of surface waves on the real Earth is that in a fixed point we may observe different amplitude anomalies for different periods. This shows that spectrum of the wave may be strongly distorted.

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Electromagnetic surface waves guided by the metal-composite medium boundary

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Over the past decade the impressing success in the development, the producing and the testing of a composite materials has been achieved. In microwave range these materials demonstrate some qualities which are unusual for natural one. As it is known the composite material consists of a metal resonant elements placed in dielectric which sizes are smaller then wave lengths. It is possible to create isotropic as well as anisotropic magnetic media. These materials are used for nonreflecting coverage, frequency multipliers, transformers and so on. Electrodynamics of new composite materials for the Microwave technical equipment as well as propagation, reflection and refraction of electromagnetic waves have been discussed in the literature [1, 2].

It is the purpose of the present work to consider the possibility of existence of electromagnetic surface waves on the metal – composite medium boundary. Our investigations have shown that it depends on the sign of components of dielectric permittivity and magnetic permeability tensors. Anisotropic of any tensor leads to appearance of surface waves if one of diagonal components is negative. The dispersion properties of the surface waves are investigated for some cases of interest (bianisotropic, chiral medium and so on). We also considered the particular cases when parametric instability of surface waves guided by the planar metal – gyrotropic anisotropic medium boundary can arise due to the presence of intense external electric field into the medium.

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The problem of diffraction of a plane electromagnetic wave on a kerr-type nonlinear dielectric layered structure

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The problem of diffraction of a plane electromagnetic wave on transverse non-homogeneous, isotropic, not magnetic, linearly polarized, weakly nonlinear dielectric layer and layered dielectric structures (Kerr-like nonlinearity) in resonant area of frequencies are considered. The analysis of a problem of diffraction on weakly nonlinear layer (layered dielectric structures) without taking into account multiple the frequencies generated in considered open weakly nonlinear structure is lead [1].

Numerical analysis of the diffraction by the weakly nonlinear layer (layered dielectric structures) is performed in the resonant frequency range when the parameter of cubic susceptibility takes positive and negative values. It is shown that as the excitation field intensity increases the diffraction characteristics acquire essentially different properties for positive and negative values of susceptibility. For a layered structure consisting of layers with positive and negative susceptibility of the enveloping medium the effects inherent to environments with positive and negative value of susceptibility are observed in certain areas of variation of the excitation field intensity [2].

The proposed methods and results of computations can be further applied to the analysis of various physical phenomena including self-influence and interaction of waves; determination of eigenfields, natural (resonance) frequencies of nonlinear objects, and dispersion amplitude-phase characteristics of the diffraction fields; description of evolution processes in the vicinities of critical points; and to the design and modeling of novel scattering, transmitting, and memory devices.

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Asymptotic theory of the one dimensional linear scattering of the solitary waves on a beach

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We discuss the formulas which describes the amplitude of the solitary waves approaching a beach in the one dimensional case and in linear approximation. These formulas are derived using the asymptotic expansion, in some small parameter of physical relevance, of the solutions of the one dimensional wave equation with non constant velocity. The case of a beach with a linear bottom of constant slope is explicitly treated. The scattering of the solitary waves on the beach is discussed also in connection with the problem of the run-up in the tsunami events. The metamorphosis of the scattered solitary wave is shown. The event of the Algerian tsunami of 2003 will be used as an example.

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Reflection, refraction and transformation into photons of surface plasmons on metal wedge

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Maluzhinets theory developed in the 1950th for plasmons on wedge surface with a small impedance ζ is used for the calculation of surface plasmons reflection, refraction, and transformation into photons. The methodically simplified summary of Maluzhinets theory which is valid in the first order on ζ is presented. The approach is satisfactory for the well-conductive metals up to optic frequencies. Photons radiation takes place primary in the direction of their motion along the wedge surface. Angular distribution of the radiated photons has Lorenzian form with width $\sim \zeta$. For the rectangular wedge reflection, refraction coefficients and diffusion part of photons radiation are also $\sim \zeta$.

Diffraction field in wave zone:

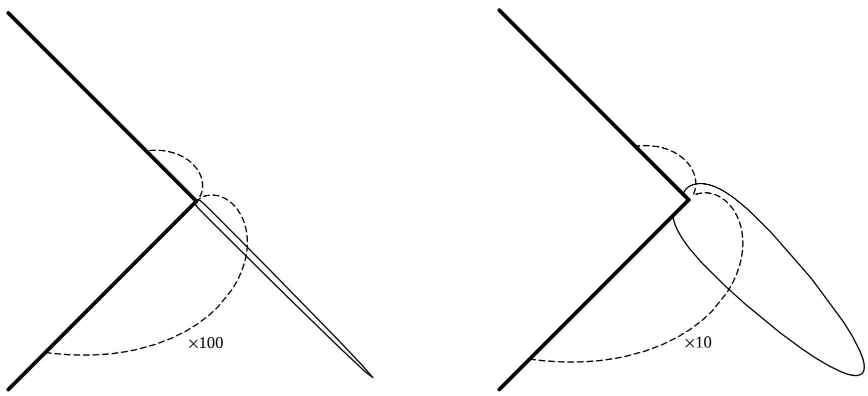

Figure : Angle distribution of radiation for rectangular wedge with impedance $\zeta = -0.015i$ and $\zeta = -0.15i$ (from left to right): solid curve is directed radiation, dashed curve is diffusion radiation.

Diffraction near-field:

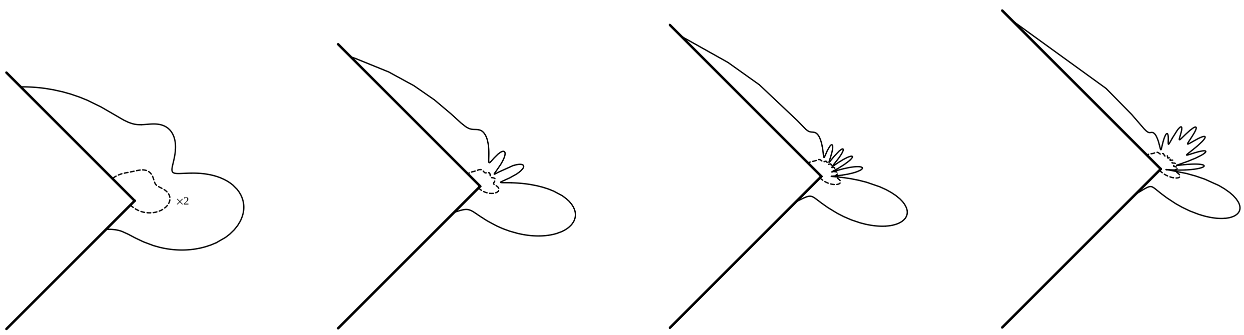

Figure : Near-field for copper at distances equal to λ, 2λ, 3λ, 4λ from the wedge edge (from left to right). Solid curve corresponds to $zeta = 0.0015 - 0.15i$ ($\lambda = 2244$Å), dashed curve corresponds to $\lambda = 0.0014 - 0.015i$ ($\lambda = 224$Å)

Whispering gallery eigenmodes of a surface with anisotropic surface impedance

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An algorithm of finding high frequency whispering gallery (WG) eigenmodes of a curvilinear surface with anisotropic surface impedance is proposed. It is based on the method of reference problem [1] modified to deal with three-dimensional problems.

Whispering gallery (WG) modes are the waves usually propagating along a concave surface and, according to geometrical optics (GO), repeatedly reflecting from it. It has been shown [1] that GO fails to describe such waves if the number of reflections exceeds predefined level. Instead, the method of reference problem [1] can be used for this purpose if the wavenumber k tends to infinity and can be considered as a large parameter. We consider the problem of finding asymptotic solutions of the Maxwell equations in complex form that are concentrated in a vicinity of a smooth surface and behave as WG modes. The boundary condition on the surface is given by $\vec{H}_\tau = \widehat{W}_s \cdot \vec{n} \times \vec{E}_\tau$, where $\vec{H}_\tau$, $\vec{E}_\tau$ are tangential to the surface components of the magnetic and electric field strengths, and $\widehat{W}_s$ is the surface impedance tensor. It is required that the permittivity and permeability of the outer medium to be slow variable on a wavelength real functions, and the effective radius of curvature to be positive. The problem is solved in coordinate system (n, s, t), where n is a distance from surface to the observation point, s is a natural parameter of a geodesics passing through foot of a perpendicular dropped from the observation point to the surface, and t is a natural parameter of a curve on the surface, perpendicular to the geodesics. The Maxwell equations have been reduced to system of the second order for the electric field strength. It has been shown [2] that the system can be reduced to three Helmholtz equations by perturbation method, and the Helmholtz equations can be solved by the method of reference problem. Each of the components of the electric field strength is obtained in the form of asymptotic series

$$E_j = \exp\left(\sum_{m=-3}^{M} \alpha_m(s,t,\nu)k^{-m/3}\right) v\left(\sum_{m=0}^{M} \beta_m(s,t,\nu)k^{-m/3}\right),$$

where M is an integer number depending upon required accuracy, $v(z)$ is the Airy-Fock function, $\nu = nk^{2/3}$, and α_m, β_m are known coefficients. Some numerical results of applying the described technique are presented and the properties of the solutions are discussed.

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Vibration of cylindrical shell partially submerged into the layer of liquid. The liquid is inside and outside the cylinder

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The problem of oscillations of elastic constructions partially submerged into liquid is one of the actual problems of modern techniques. It is important to estimate the amplitude of vibration and acoustical fields in such systems as ships, oil platforms, different supports, tubes protruding above the surface of water and so on.

The calculations of such bodies' vibrations are very complicated, need great computational resources and have low accuracy. So it is useful to explore the processes in these systems taking as an example more simple mechanical systems. The cylindrical shell is one of these often used models.

In this work the problem of cylindrical shell oscillations when the axis of the cylinder is orthogonal to the liquid's surface is considered. The liquid is inside and outside the cylinder. The problem of oscillations of the cylindrical shell partially submerged into the layer of liquid is considered in the rigorous mathematical statement. The exact analytical solution of the problem is constructed. The dispersion equation basing on the exact solution and the representation for the eigen functions are obtained.

Pulse shape of current, moving along a straight line with superlight velocity, and time derivative of Whittaker potential in far region

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The electromagnetic fields of sources moving with velocities (v) equal or higher than velocity of light in vacuum (c) have been discussed in connection with an interest to the problem of localized electromagnetic waves [1, 2] and Vavilov-Cherenkov radiation particularly [3]. One way to do this is to use potential of Whittaker.

In this work the calculation of derivative of Whittaker potential U with respect to τ was performed in the framework of linear current model. Here $\tau = ct$ is the time variable.

In the general case the $\partial U/\partial\tau$ is equal to the sum of several integrals depending on the interrelation of the following parameters: position of observation point (ρ, z), pulse duration, velocity of the running pulse and length of the line segment (L).

It was shown that correlation:

$$\frac{\partial U}{\partial \tau} = \int_0^L \frac{f(\beta(\tau - \sqrt{\rho^2 + (z-z')^2}) - z')}{\sqrt{\rho^2 + (z-z')^2}}\, dz'$$

holds for the Cherenkov angle $\vartheta = \arccos(c/v)$, where $f(\beta\tau - z)$ is a form of current pulse propagating along a straight line.

Results of computation have shown similarity between different forms of running pulse (Gaussian distribution, square pulse or distribution deduced from simulation by means of a computational experiment, where the code GAINT was used [4]) and derivatives of Whittaker potential $\partial U/\partial\tau$ in far region.

Good agreement between $\partial U/\partial\tau$ and $f(\beta\tau - z)$ is observed with β infinitesimally close to 1.

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Investigation of wave field in effective model of layered elastic medium with slide contact on interfaces

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For the medium consisting of two alternating elastic layers with slide contact, the effective model was constructed [1] and investigated [2]. In the wave field of this model there are leading, "triangular", and back concave fronts. In this model, the wave field exited by an impulse point source is represented in a half-plane as Fourier and Mellin integrals. We transform the expressions of the wave field by the methods used in [3]. In the Mellin integral we replace contour of integration by a stationary contours. In the obtained expressions, we rearrange the integrals and calculate the inner integral. The external integral is equal to two residues. The corresponding poles are roots of two equations of sixth order. These roots are situated at the right half-plane and can be complex or real. If these roots go out at the imaginary axis then the corresponding points of observation are on the fronts or inside of "triangular" and back concave fronts. In these cases the displacements are equal to zero. The obtained representation for the wave field corresponds to the expressions derived by the method of Smirnov-Sobolev. For calculation of the complex and real roots determining the wave field, we suggest an effective method.

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About light pressure in negative refraction materials

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As it was shown in [1], for material with negative refraction (NRM) many formulas of electrodynamics must be used very carefully. This in particular pertains to well-known formula

$$p = hk = h\omega n/c = h\omega/v_{ph} \tag{1}$$

linking value of momentum of photon p with its wave vector k and/or index of refraction n and phase velocity v_{ph}.

If formally follow this equation, one might conclude, that momentum of electromagnetic field in the NRM is directed opposite to flow of energy of field or group velocity v_{gr}. This means that in NRM instead of light pressure must exist the light attraction. However it is easy to show, following method, developed in [2], that requirement of conservation of the centre of masses of system, consisting of field, spreading in NRM, transmitter and receiver, can be not satisfied if acknowledge the equitable formula (1). For this case the conservation of the centre of masses position follows only from correlation

$$p = h\omega v_{gr}/c^2 \quad .$$

The details of contradiction between these formulas, are discussed in report.

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