DAY ON DIFFRACTION'97

INTERNATIONAL SEMINAR

Saint Petersburg, June 3–5, 1997

ABSTRACTS

UNIVERSITAS
PETROPOLITANA
MDCCXXIV

ORGANIZING COMMITTEE

Professor V.M.Babich (co-chairman)

Professor V.S.Buldyrev (co-chairman)

Dr. I.V.Andronov (secretary)

Dr. V.E.Grikurov

Dr. M.A.Ljalinov

Prof. N.Ya. Kirpichnikova

Dr. N.S.Zabavnikova

Dr. Z.A.Janson

Prof. A.P.Kiselev

ADDRESS:

Prof. V.M.Babich
Mathematical Institute,
Fontanka quay 27, St.Petersburg 191011
Russia

e-mail: `babich@pdmi.ras.ru`

Prof. V.S.Buldyrev
Institute in Physics, St.Petersburg University
1 Ulianovskaja Ave., Petrodvoretz 198904 Russia
e-mail: `bvs@mph.niif.spb.su`
Fax: 7 (812) 428-7240

Dr. I.V.Andronov e-mail: `andronov@mph.niif.spb.su`

Home page : `http://snoopy.phys.spbu.ru/~grikurov/dd97/dd97.html`

The seminar is sponsored by
Russian Foundation for Basic Research
and
IEEE ED/MTT/AP St.Petersburg Chapter

FOREWORD

The Seminars "Day on Diffraction" are annually held in late May or early June by the St.Petersburg Branch of the Steklov Mathematical Institute of the Russian Academy of Sciences, St.Petersburg State University and by the joined recently Euler International Mathematical Institute.

The Seminars "Day on Diffraction" have been held since late 1960-s. Originally these seminars summed up the results of the famous Leningrad Diffraction Theory School headed by V.A.Fock and V.I.Smirnov. The works of V.I.Smirnov and S.L.Sobolev on functionally-invariant solutions of wave equation in the beginning of 30-s, the method of partial separation of variables suggested by V.I.Smirnov in 1937, widely known works of V.A.Fock on diffraction of radio-waves around the surface of the Earth of 40-s laid the cornerstone of Leningrad—St.Petersburg school. The main body of the diffraction school in 60-s – 70-s was constituted by the students of G.I.Petrashen' who proceed with and developed the subjects of diffraction after the World War II.

The idea of organizing annual seminars on diffraction theory and wave propagation has arisen in 1968, and the first seminar was held in June 1968. The next day after the Seminar its participants, which were young in those times, went to Komarovo on the bank of the Gulf of Finland, played volleyball, football on the beach, drank tea. All the sessions since have always ended with friendly picnic parties. Year by year the Soviet scientists from other cities began to participate and the "Day on Diffraction" became an All Union Seminar. Since 1986 the Seminar occupied two days: the first was organized in Leningrad, the second was at the faculty of Physics in Peterhoff. Nevertheless the name "Day on Diffraction" preserved. Since 1991 the sessions of the Seminar occupy 3 days. The number of participants is about 30. Within the last decade the Seminar became international. This year marks the 30-th anniversary of the Seminar.

The Day on Diffraction '97 Seminar will be opened on 3 of June 1997 in St.Petersburg. We are as always pleased to see active researchers in the field of Diffraction Theory from all over the world.

Organizing Committee

List of talks

A. M. Akhmeteli
Diffraction of a Gaussian Electromagnetic Beam on a Thin Conducting Cylinder and an Efficient Method of Heating of Small Targets . . . 7

Alexander G. Alenitsyn
Stoneley's Double Waves, II. . . . 8

Ivan V. Andronov
Scattering by a Narrow Crack in Fluid Loaded Elastic Plate . . . 8

V.M.Babich, K.D.Cherednichenko
Fock's Ansatz and its Generalizations . . . 10

A.V. Badanin
On the Quasimomentum of the Operator $d^4/dx^4 + q(x)$ with a Periodic Potential . . . 10

Habib Ammari, Natacha Béreux, Jean Claude Nédelec
Resonances for Maxwell's Equations in a Periodic Structure . . . 11

Victor V. Borisov, Irina I. Simonenko
Transient Electromagnetic Waves Produced by a Source on the Expanding Circle . . . 11

Serguei Yu. Dobrokhotov
Ray Method, Hugoniot-Maslov Chains for Trajectories of Point Singularities of the "Shallow Water" Equations, the Hill Equation and Typhoon Eye Propagation . . . 12

V.A.Borovikov
Singularities of the Green Function for Non-strictly Hyperbolic Equations . . . 13

A. B. Golovin, A. I. Khizhnyak, L. V. Kreminskaya
Diffraction Excitation of TEM_{00} Mode of Solid State Ring Laser . . . 14

V.E. Grikurov
Perturbation of Unstable Solitons for Generalized NLS with Saturating Nonlinearity . 15

Andrzej Hanyga
Fermat's Principle in Anisotropy . . . 16

E. M. Gromov, V. V. Tutin
Frequency Modulated Stationary Waves in a Third-order Nonlinear Schrödinger Equation 17

A. I. Khizhnyak, L. V. Kreminskaya
Interference Nature of the Transverse Mode Selection in Resonator with Sagnac Interferometer . . . 18

Sergei Kanaun
A Variational Principle of the Electromagnetic Wave Diffraction Problem and its Applications . . . 19

Aleksei P. Kiselev, Zigmund M. Rogoff
Diffraction by the Jump of Curvature by k^{-1}-scaling . . . 19

Aleksei P. Kiselev, Maria V. Perel
Quasiphotons and Nondispersive Waves . . . 20

Yurii V. Kiselev, Vladimir N. Troyan
Estimation of Elastic Parameters in Diffraction Tomography 21

N. Ya. Kirpichnikova
The Second Approximation of the Ray Method for Calculation of the Intensity of the Wave Front in Isotropic Nonhomogeneous Elastic Media 22

Yu. A. Baloshin, A. V. Kostin
Diffraction at Perfectly Conducting Rough Surfaces under Grazing Incidence 22

Evgeni Korotyaev
Inverse Problem and the Double-side Estimates for the Hill Operator 23

D. P. Kouzov
On the Possibility of Regularization of Contact Integrals 24

Victor F. Kravchenko, George A. Nesenenko
Mathematical Problems of Diffraction 26

Yu. A. Lavrov
On Natural Frequencies of Thin Cylindrical Shell with Free Edges 27

A. A. Lazarev
Acoustic Wave Radiation During Relative Sliding of Two Bodies 27

A. M. Lerer, S. M. Tsvetkovskaya
Method of Regularization in Two-Dimensional Problems of Narrow Electromagnetic Pulse Diffraction 28

V.Yu.Gotlib
Helmholtz equation solutions concentrated near periodical boundary 28

V.D. Lukjanov, G.L. Nikitin
Scattering of a Plane Acoustic Wave on an Elastic Plate Joined with a Periodic Set of Acoustic Resonators 29

M.A. Lyalinov
Diffraction by a Slowly Expanding Cylinder 30

James Lawry
Complex Ray Analysis of the S^* Wave 32

G. I. Mikhasev
Travelling Wave Packets in Thin Elastic Shells 32

G. L. Nikitin
Radiation of Acoustic Waves by the Flat Waveguide into a Half-space through a Thin Elastic Plate 33

Oleg V. Motygin
Trapped Modes in the Linearized Problem of a Potential Flow about Semisubmerged Bodies 35

Olga A. Obrezanova, Vladimir S. Rabinovich
Propagation of Sound Generated by a Moving Source in an Oceanic Waveguide with an Obstacle 36

S. V. Fedorov, G. V. Khodova, N. N. Rosanov, A. G. Vladimirov
Localized Structures in Active Nonlinear Waveguides and Bistable Lasers 37

I.T. Selezov
Modeling Heart Pulse Propagation in Jointed Blood Vessel 38

Andrey V. Shanin
Excitation of Wave Field in a Triangle Area 38

H. Schanz, H. Primack, I. Ussishkin, U. Smilansky
Penumbra Diffraction in the Quantization of Concave Billiards 39

E. V. Chernokozhin, Yu. V. Shestopalov
Scattering Frequencies of Cylindrical Slotted Resonators 39

Hiroshi Shirai
Transient Impulsive Response from an Interface between Two Media 40

E. Akkermans, R. Narevich, D. Spehner
Quantum Dynamics of an Electron in Magnetic Billiards with Hard Boundaries 40

A. S. Starkov
The Integral Equation in Multidimensional Spectral Inverse Problem 42

A. V. Krotov, P. E. Tovstik
Asymptotically Double Natural Frequencies of Thin Elliptical Cylindrical Shell Vibrations 42

T. I. Khabakhpasheva, I. V. Sturova
Diffraction of Internal Waves by a Submerged Circular Cylinder at Forward Speed in Two-layer Fluid 44

A. V. Kudrin, E. Yu. Petrov, T. M. Zaboronkova
Theory of Strip Loop Antennas in a Resonant Magnetoplasma 44

Matthias Zillmer, Dirk Gajewski, Boris M. Kashtan
Plane Wave Reflection Coefficients for Two Weakly Anisotropic Half-spaces in Welded Contact 46

V. V. Zalipaev
Plane Wave Scattering by a Step Discontinuity in an Impedance Plane 47

N. Ya. Kirpichnikova, V. B. Philippov
Behavior of surface creeping waves at transition through a junction line on the boundary of a conducting body 47

I. D. Abrahams
Matrix Wiener-Hopf Factorization with Application to Diffraction of Elastic and Acoustic Waves 48

V.S. Buldyrev, N.G.Gelfreikh
The Head Waves for the Problem of Diffraction on Obstacles with GIBC on their Surface 50

Diffraction of a Gaussian Electromagnetic Beam on a Thin Conducting Cylinder and an Efficient Method of Heating of Small Targets

A. M. Akhmeteli

Russian Federation, 121609 Moscow, Rublevskoe 38-2-139,
e-mail ipakhmam@redline.ru

In many applications, such as electromagnetic pumping of active media of short-wavelength lasers, it is necessary to achieve high specific input of power of an electromagnetic beam. As the diffraction constraints make it difficult to focus an electromagnetic beam on a very small target, this work presents the conditions (confirmed by exact solution of the Maxwell equations) that provide the absorption of a significant part (tens per cent) of the power of a free electromagnetic beam in a conducting cylinder with a diameter much less (by several orders of magnitude) than the transverse dimensions of the beam. In applications fast heating usually changes the electric properties of the target, so nonresonant conditions are of great interest. Such conditions were found for the longitudinal geometry (a free Gaussian beam propagates along the axis of a cylinder with transverse dimensions much less than those of the beam) and for the transverse geometry (an axially symmetric beam converges on a cylinder with transverse dimensions much less than the wavelength).Let the complex permittivity and the radius of the cylinder be $\varepsilon = \varepsilon' + i\varepsilon''$ and a correspondingly, the system of units is such that the wave vector in vacuum is equal to unity modulo. In the longitudinal geometry the incident electromagnetic field is described by the electric and magnetic Hertz vectors with zero x- and y-components and z-components equal to $\mathrm{Re}(\Pi(r,\varphi,z,t))$ and $\mathrm{Re}(-i\Pi(r,\varphi,z,t))$, correspondingly, where

$$\Pi(r,\varphi,z,t) = \delta^2 \exp(i(\varphi - \omega t)) \int_0^1 \exp(-\frac{\alpha^2\delta^2}{2} + i\sqrt{1-\alpha^2}z) J_1(\alpha r) d\alpha.$$

If $\delta \gg 1$ (the beam's waist's radius is much greater than the wavelength), this field is approximated by a Gaussian beam with great accuracy. The two most interesting parameter domains that provide efficient absorption may be roughly described as follows: 1) the skin-depth δ_s, the cylinder diameter, and the wavelength are of the same order of magnitude; 2)the cylinder diameter is of the same or greater order of magnitude as/than the skin-depth and the wavelength, and $\frac{2a}{\delta_s} \sim \frac{\sigma}{\omega}$, where σ and ω are the conductivity and the frequency, correspondingly ($\varepsilon'' = \frac{4\pi\sigma}{\omega}$). These parameter domains are nonresonant, so inside these domains the conductivity may typically vary by an order of magnitude without significant loss of efficiency of absorption. In [1] these parameter domains are described more precisely, and an asymptotic formula for the part of the incident beam power absorbed in the cylinder is given. In this work the results of [1] are generalized for the case of a superposition of TM and TE modes, and the calculations were carried out for the incident beam that is much better approximated by a Gaussian beam, and the efficiency of heating by a Gaussian beam was found to be even higher than in[1].

The analysis shows that these domains do not allow efficient heating of cylinders that are much thinner than the wavelength. Such possibility was found in the transverse geometry. If the electric field in the incident wave is directed along the axis of the cylinder, $a \ll 1$ (the cylinder radius is much less than the wavelength), $|\ln a| \gg 1$, $\varepsilon'' \gtrsim |\varepsilon'|$, $\varepsilon'' \sim \frac{2}{a^2|\ln a|}$ (roughly speaking, the skin-depth is of the same order of magnitude as the cylinder diameter), then the part of the power absorbed in the cylinder $\eta \sim \frac{\pi}{|\ln a|}$ is quite significant. The length of the region under heating may be of the order of the wavelength.

References

[1] Akhmeteli A.M., Day on Diffraction'96, Int-l Seminar, Saint Petersburg, June 4-6, 1996, Book of Abstracts, p.7

Stoneley's Double Waves, II.

Alexander G. Alenitsyn

St.Petersburg University
e-mail: alenicyn@onti.niif.spb.su

Sound waves on the interface of a liquid layer and an elastic solid half-space are discussed. The 1st part of the work was reported on the Day of Diffraction in 1991. It was shown that at certain conditions there can exist two waves of the Stoneley type which have close phase speeds, and that this results in a specific spatial oscillation of the sound field. Now we develop and complete the investigation. In particular, the group speeds as well as the phase ones are considered. The influence of attenuations is also taken into account.

Scattering by a Narrow Crack in Fluid Loaded Elastic Plate

Ivan V. Andronov

Institute in Physics, St.Petersburg University, Russia

When considering scattering by point models of cracks, stiffeners and other defects in fluid loaded elastic plates, the dimensions of the scatterer are involved only in the coefficients of the contact conditions. This permits to obtain simple solutions in the form of integrals or series. However the question of applicability of such models appears. In [1] the correction taking into account the height of a protruding stiffener was derived. In [2] the scattering by a narrow but finite width crack was examined. In particular it was shown there that for the case of thin plate the classical [3] model of point crack is valid only for exponentially narrow cracks.

The zero-range potential technique [4] permits to extend the class of explicitly solvable point models of defects in elastic plates [5]. In this paper the numerical dependencies computed for the scattering characteristics by a modified model of point crack are presented and discussed.

The field in the problem of diffraction by elastic plates with point-wise inhomogeneities can be constructed in two steps. First the so called "general" solution U is constructed. This field corresponds to passive point sources applied to the plate and satisfies the equations

$$\Delta U + k^2 U = 0, \qquad y > 0, \tag{1}$$

$$\left[\left(\frac{d^4}{dx^4} - k_0^4\right)\frac{\partial}{\partial y} + \nu\right] U\bigg|_{y=0} = \sum_{j=0}^{3} C_j \delta^{(j)}(x). \tag{2}$$

Here k is the wave number in fluid, k_0 is the wave number in the isolated elastic plate and the parameter ν characterizes the "strength" of fluid loading. The magnitudes C_j of these passive sources are determined at the second step of the procedure from the contact conditions

$$\frac{\partial^3}{\partial y\,\partial^2 x} U(\pm 0, 0) = 0, \qquad \frac{\partial^4}{\partial y\,\partial^3 x} U(\pm 0, 0) = 0. \tag{3}$$

Here we present the physical interpretation of the conditions that are mathematically rigorously derived with the help of the operator extensions theory (For 3-D case see [5]). The idea is to add to the mechanical sources presented in the right-hand side of the equation (2) an additional acoustic source in the right-hand side of (1)

$$\Delta U + k^2 U = c\delta(x)\delta(y). \tag{4}$$

The right-hand side is interpreted as a limit. First the source is placed in some point $x = 0, y = y_0$ and the limit as $y_0 \to 0$ is taken for the solution U.

If $c \neq 0$ the solution U does not satisfy the Meixner condition, its asymptotics is

$$U \sim a\,\pi^{-1}\ln(r) + b + o(1), \qquad r \to 0. \tag{5}$$

The Meixner condition means that $a = 0$ in (5). In this case the classical point model [3] appears.

For determining the coefficients C_j presented in the equation (2) and the coefficient c in (4) the contact conditions (3) are not sufficient. One more condition that replaces the Meixner condition appears by means of zero-range potentials technique in the form

$$a = Ab. \tag{6}$$

Here a and b are the coefficients of the asymptotics (5) and to guarantee uniqueness and reciprocity principle the parameter A should be real.

The main problem that appears is the choice of parameter A. The zero-range potential technique gives no tool to solve this problem. One can compare far field asymptotic derived in [2] with the far field asymptotics in the problem of diffraction by the model (3, 6). For a crack of width L this comparison yields $A = -\pi/\ln(L/4)$. It is important to note that the parameter A does not depend on the incident field, its frequency and on the parameters of the plate and fluid. The same value of A characterizes a narrow slot of width L in a rigid screen, when the condition (2) is replaced by Neumann boundary condition and contact conditions (3) are discarded.

The numerical dependencies of the effective cross-section (in dB) on frequency and angle of incidence are presented on Figs. for the case of steel plate of thickness 1cm in water and $L = 1$cm. The dotted lines correspond to classical point model of crack and solid lines to the modified model.

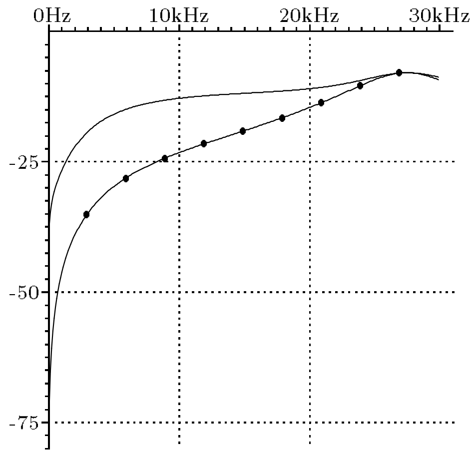


Fig. 1. Angle of incidence 20°.

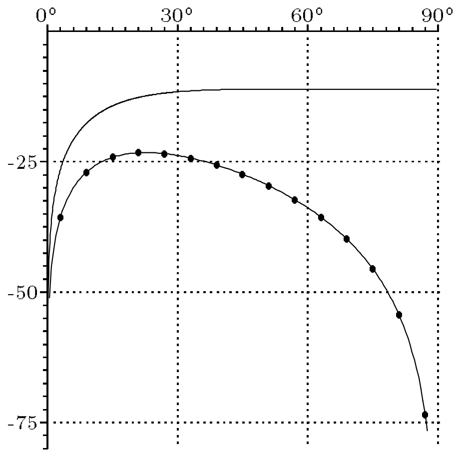


Fig. 2. Frequency 10kHz.

References

[1] B.P.Belinskiy "Sound radiation from a plate with protruding stiffener", *Soviet Physics Acoustics* **24**, 183–187 (1978)

[2] I.V.Andronov, B.P.Belinskiy, J.P.Dauer "Acoustic scattering by a narrow crack in an elastic plate", *Wave Motion* **24**, pp. 101–115 (1996)

[3] D.P.Kouzov "Diffraction of a plane hidroacoustic wave on a crack in an elastic plate", *PMM USSR* **27**, 1037–1043 (1963)

[4] B.S.Pavlov *Uspehi Mat. Nauk* **42**, No. 6, pp. 99–132 (1990)

[5] I.V.Andronov "Application of zero-range models to the problem of diffraction by a small hole in elastic plate", *J. of Math. Physics* **34**, No. 6, pp. 2226–2241 (1993)

Fock's Ansatz and its Generalizations

V.M.Babich

Steklov Mathematical Institute, Fontanka 27,
191011, St.Petersburg Russia, e-mail: babichpdmi.ras.ru

K.D.Cherednichenko

St.Petersburg State University, Bibliotechnaya 2,
198904, Staryi Peterhoff, St.Petersburg Russia,
e-mail: C42-12@fs1.niimm.spb.su

In 1943 V.A.Fock deduced an asymptotics of Legendre functions $P_{i\tau-\frac{1}{2}}(\cos\theta)$, $Q_{i\tau-\frac{1}{2}}(\cosh\alpha)$ ($\tau \to +\infty$, $0 \leqslant \theta \leqslant \pi$, $\alpha \geqslant 0$) from their integral representation. His approach gives the possibility to obtain the asymptotics of the associated Legendre functions $P^{\mu}_{\nu-\frac{1}{2}}(\cos\theta)$ and $\mathfrak{Q}^{\mu}_{\nu-\frac{1}{2}}(\cos\theta \pm i0)$ $\nu \to +\infty$. It is important for applications that this form of asymptotics is valid in a neighbourhood of critical points of Legendre equation: $\theta = 0$, or $\theta = \pi$. The Fock's asymptotics suggests to use a new ansatz for the construction of an uniform asymptotics of solutions of equations with large parameter and singular coefficients:

$$y'' + \frac{p(x)}{x}y' + \left(k^2 - \frac{q(x)}{x^2}\right)y = 0 \tag{1}$$

$$0 \leqslant x \leqslant a, \; k \to +\infty$$

where $p(x)$ and $q(x)$ are even smooth functions. This ansatz is not identical to classical Olver's and Cherry's ones. These ansatzes are not able to provide asymptotics of singular solutions of the equation (1), but the new ansatz leads to asymptotical expansion of such solutions. The proof of the smallness of the remainder terms was obtained by integral equations.

The research was supported by INTAS-RFBR grant 95-0012

On the Quasimomentum of the Operator $d^4/dx^4 + q(x)$ with a Periodic Potential

A.V. Badanin

AGTU, Arkhangelsk

We consider an equation

$$Ly = y'''' + q(x)y = \lambda^4 y$$

with the real 1-periodic potential $q(x), q(x) \in W_2^1([0,1])$. We suppose that the lower bound of the spectrum is 0. We show that the spectral theory for this equation may be formulated by analogy with the spectral theory for the Hill's equation. We introduce the Lyapunov function (discriminant). We prove that it is an analytic function of the first order on the two-sheets Riemann surface. It has the asymptotic

$$F(\lambda) = \cos\lambda + \frac{1}{4}Q\frac{\sin\lambda}{\lambda^3} + O(\lambda^{-4}),$$

as $|\lambda| \to \infty$ on one of the sheets. Here $Q = \int_0^1 q(x)dx$. We prove that the quasimomentum function $k(\lambda) = \arccos F(\lambda)$ performs the conformal mapping from $\mathbf{C}_+$ on the "comb"

$$K_+ = \mathbf{C}_+ \backslash \bigcup_n \Gamma_n, \qquad \Gamma_n = \{\mathrm{Re}k = \pi n, |\mathrm{Im}k| \leq h_n\}, \quad h_n \geq 0, \quad n \in \mathbf{Z}, h_0 = 0.$$

This fact allows to obtain some estimates of lengths of the spectral gaps by analogy with the Hill's case.

Resonances for Maxwell's Equations in a Periodic Structure

Habib Ammari, Natacha Béreux, Jean Claude Nédelec

Center of Applied Mathematics, CNRS URA 756
Ecole Polytechnique, 91128 Palaiseau cedex, FRANCE

We consider here the diffraction of an incident harmonic plane wave $\mathcal{E}^{inc}$ with frequency ω by an infinite multislot grating. The grating is periodic in the x-direction with period d and invariant in the z-direction. It consists in a perfectly conducting ground plane ($y = 0$) coated with an homogeneous dielectric layer of thickness h and covered by another perfectly conducting plane ($y = h$). The upper plane ($y = h$) is cut along z by infinitely long slots of width 2δ at periodic intervals. The half width of the slot, denoted by δ, is a small dimension in our problem.
This structure can be viewed as a perturbation of the following reference structure : a perfectly conducting plane ($y = h$) considered as a periodic obstacle with the same period d replaces the conducting strips. The resonant frequencies of the reference structure are exactly the poles of the periodic Green kernels for the Helmholtz problem set in the domains $0 < y < h$ and $y > h$. We show that the occurrence of small slots in the conducting plane moves some of the resonant frequencies on the real line and the others in the lower complex half plane.
To obtain these results, we reduce the diffraction problem to a system of integral equations written on the dielectric slot of the cross section of a unit cell of the grating. These integral equations involve two integral operator-valued functions whose characteristic values are exactly the resonant frequencies we look for. Following [4], we use [2] to show the existence of resonances in small neighbourhoods of the resonances of the reference structure. Then we use the explicit expression of the Green kernels to localize them in $\mathbb{C}$.

References

[1] H. Ammari, N. Béreux and J.-C. Nédélec, *Résonances pour les équations de Maxwell dans une structure périodique,* Technical Report, Ecole Polytechnique, September (1996).

[2] I. TS. Gohberg and E. I. Sigal, *Operator extension of the logarithmic residue theorem and Rouche's theorem,* Math. USSR Sbornik, 84, No.4, (1971) 607-642.

[3] J.C. Nédélec and F. Starling, *Integral equation methods in a quasi-periodic diffraction problem for the time-harmonic Maxwell equations,* SIAM J. Math. Anal., 22 (1991), pp. 1679-1701.

[4] V. P. Shestopalov and Yu. V. Shestopalov, Resonant scattering from open surfaces, IEEE Publications (1996).

Transient Electromagnetic Waves Produced by a Source on the Expanding Circle

Victor V. Borisov, Irina I. Simonenko

Institute of Physics, St.Petersburg University,
Ulyanovskaya 1, Petrodvorets, St.Petersburg, 198904, Russia
Fax: (++7) (812) 4287240 E-mail (Internet): boris@phim.niif.spb.su

The goal of the present report is to construct the explicit solution of the initial value problem to the inhomogeneous Maxwell equations in the space-time representation. The specific radial current is distributed on the circle, which lies on the surface of a sphere expanding with the velocity of light, and, therefore, the circle has a variable radius. The time shape of the current pulse is arbitrary. Earlier the transient electromagnetic field generated by the source on the circle has been constructed for the case of the source on the moving circle and the immovable circle with a constant radius [1].

In the spheric coordinate system, the radial current density has radial nonzero component only. Therefore, expressing the components of the electromagnetic field vectors and the variables in the spheric coordinate system, one can reduce the vector problem to the scalar one [2]. We reduce the obtained scalar equation to the wave equation, which is solved in the variables of the cylindric coordinate system. We obtain the solution of the above equation by using the incomplete separation of variables, the Riemann formula and the relations containing three Bessel functions. With the help of the derived solution and using the variables of the cylindric coordinate system we express the components of the electromagnetic field in the spheric coordinates and find the novel representation of the above components in terms of the Fourier series expansion only.

The obtained expressions differ from the traditional representations of the field vectors in terms of the spheric harmonics and allow us to give the adequate description of the peculiarities of the space-time structure of the electromagnetic waves produced by the current pulse, accompanying the absorption of the divergent hard radiation. Using the obtained relations, we discuss the specific features of the excited waves and compare the space-time structure of the constructed expressions with the alternative expansion in terms of the spheric harmonics [3].

References

[1] V.V. Borisov and I.I. Simonenko J. Phys. A 1994 **27**, 6243.

[2] A. Nisbet Proc. R. Soc. A 1955 **231**, 250-263.

[3] V.V. Borisov, A.V. Manankova, A.B. Utkin J. Phys. A 1996 **29**, 4493 -4514.

Ray Method, Hugoniot-Maslov Chains for Trajectories of Point Singularities of the "Shallow Water" Equations, the Hill Equation and Typhoon Eye Propagation

Serguei Yu. Dobrokhotov

Institute for Problems in Mechanics (Moscow)
prosp. Vernadskogo 101, Moscow 117526, Russia;
e-mail: dobr@ipmnet.ru

According to V.Maslov idea, many 2-D quasilinear hyperbolic systems of PDE posses only 3 types of singularities in general case with properties of "structure" self-similarity and stability. They are shock waves, "narrow solitons and "square root" point singularities (solitary vortices). The nonlinear analog of the ray method of expansions with respect to "unsmoothness" leads to the description of their propagations by means of infinite chains of ODE, which are generalization of the well known Hugoniot conditions for shock waves.

Unexpected, but probably not accidental result is that after some reasonable closing of this chain for solitary vortices ("point" singularities) of the "Shallow water" equations we obtain the nonlinear system of 16 ODE, which is exactly equivalent to the family of (linear) Hill equations with a periodic potential. It means that in some approximation the trajectory of solitary vortex can be described by the Hill equation. The further consideration shows that the existence of physically reasonable "point" singular solutions requires the stability of this family of the Hill equations and the presence of the Coriolis force. Also in the case when the trajectory of vortex is represented by more or less smooth curve, the explicit formula for the trajectory via trigonometrical functions is given.

These results can be used for prediction of the future trajectory of the centre ("eye") of typhoon via its observable part.

Singularities of the Green Function for Non-strictly Hyperbolic Equations

V.A.Borovikov

Inst. for Problems of Mech., Russian Acad. Sci., Moscow, Russia,
Fax (7-095) 938-20-48, e-mail borovik@ipmnet.ru
Now: Benemerita Universidad Autonoma de Puebla, Puebla, Mexico.
Fax (52 22) 332 403, e-mail borovik@fcfm.buap.mx

We study the singularities of the Green function $G(t,\mathbf{x})$; $\mathbf{x} = (x_1,\ldots,x_m)$ for the linear hyperbolic equation $L(\partial/\partial t, \partial/\partial x_1,\ldots,\partial/\partial x_m)G(t,\mathbf{x}) = \delta(t,\mathbf{x})$; $G = 0$ for $t < 0$. Here $L(\tau,\xi)$ is a homogeneous polynomial of degree n such that for any $\boldsymbol{\xi} \neq \mathbf{0}$ equation $L(\tau,\boldsymbol{\xi}) = 0$ has n real roots $\tau_1,\ldots,\tau_n$. The problem is well known for the strictly hyperbolic operators when all roots $\tau_1,\ldots,\tau_n$ are distinct for any $\boldsymbol{\xi} \neq \mathbf{0}$. The slowness surface $S : L(1,\boldsymbol{\xi}) = 0$ in ξ-space don't has singular points in this case and the Green function has singularities on the wave surface (otherwise known as the characteristic cone) consisting of all points $t > 0, \mathbf{x}$ such that the plane in ξ-space $t + \mathbf{x}\cdot\boldsymbol{\xi} = 0$ is tangent to the slowness surface. When seeking high-frequency asymptotics of fields governed by the hyperbolic equations and excited by oscillating sources full description of these singularities is necessary. The behaviour of the Green function near a regular point of the characteristic cone is known.

In this report we discuss the case of non-strictly hyperbolic equations when the equation $L(\tau,\boldsymbol{\xi}) = 0$ has for some $\boldsymbol{\xi}$ double roots and the slowness surface has singular points. Then new singular surface Σ of the Green function arise. To define this surface denote by Ξ the set of all singular points of slowness surface $L(1,\boldsymbol{\xi}) = 0$ at which grad$S = 0$, but $\det|\partial^2 L/\partial\xi_i\partial\xi_j| \neq 0$. The new singular surface Σ is the ruled surface. Each generator Σ_0 of Σ corresponds to the some point $\xi_0 \in \Xi$. To define Σ_0 let V be the hyperplane consisting of all $\mathbf{x}$ such that the plane $t + \mathbf{x}\cdot\boldsymbol{\xi} = 0$ hits the point ξ_0 and is tangent to Ξ. Let U be the subset of V consisting of all points $\mathbf{x}$ such that the plane $t + \mathbf{x}\cdot\boldsymbol{\xi} = 0$ is tangent to the slowness surface $S = 0$ at point ξ_0. Then Σ_0 is the part of V bounded by U. The problem is to find uniform asymptotics describing the behaviour of the Green function in the vicinity of Σ_0 including its boundary U.

We show that if the codimension of Ξ is odd then these asymptotics are similar to expressions describing the wave field inside the penumbra which is due to scattering of a plane acoustic wave by a smooth cone. If this codimension is two then these asymptotics are similar to expressions describing the wave field inside the penumbra which is due to diffraction of a plane acoustic wave by a wedge. If the codimension of Ξ is even and greater than two, then the Green function (or the Green's matrix) has no singularities at Σ_0.

The concept of a hyperbolic system can be naturally extended to cover systems of equations. Here the role of the Green function is played by the Green matrix. The equations of crystallo-optics and equations of elastodynamics for some transversally isotropic solids and for solids with cubic symmetry are the examples of non-strictly hyperbolic systems. We describe in report the behaviour of the Green matrix in the vicinities of new singular surfaces.

Diffraction Excitation of TEM_{00} Mode of Solid State Ring Laser

A. B. Golovin, A. I. Khizhnyak, L. V. Kreminskaya

International Center "Institute of Applied Optics", National Acad. Sci. Ukraine,
10G, Kudryavskaya St., 254053, Kiev, Ukraine,
Fax: 380 (44) 212 48 12, E-mail: golovin@marion.iop.kiev.ua

Hard copy full page.

Perturbation of Unstable Solitons for Generalized NLS with Saturating Nonlinearity

V.E. Grikurov

Dept. Math. & Comp. Phys., Institute on Physics, St.Petersburg Univ., 198904 Russia
e-mail: Valery.Grikurov@pobox.spbu.ru

We consider the generalized nonlinear Schrödinger equation (gNLS)

$$\imath\psi_t = -\nabla^2\psi + F\left(\mid \psi \mid^2\right)\psi \tag{1}$$

which is frequently utilized as a model for laser propagation through nonlinear optic materials, Langmuir waves in a plasma, etc. Under certain (nonrestrictive) conditions on the function $F(\xi)$ [1] this equation admits a family of localized solutions $\psi(t,x) = \exp(\imath\alpha t)\ u_\alpha(x)$, $\lim_{|x|\to\infty} u_\alpha(x) = 0$, which are called (ground) solitons.

The stability properties of these solutions depend on the nonlinearity $F(\xi)$: the general stability criteria is well known and may be formulated in terms of conserved quantity $N(\alpha) = \int \mid u_\alpha(x) \mid^2 dx$ of the equation (1): inequality $N'(\alpha) > 0$ yields the stability (see, e.g., [3] and references therein for review of the stability analysis; scattering properties of the stable soliton have been examined in the paper [2]). As an example, for the power law nonlinearity $F(\xi) = -\xi^p$ the stability criteria in d-dimensions is $pd < 2$. If $pd \geq 2$ (so called supper-critical case) then the typical behaviour of solution is the collapse, i.e. an infinite growth of amplitude for finite time.

¿From the another hand, if function $F(\xi)$ is not too rapidly goes to minus infinity for large ξ: $F(\xi) \geq -C\xi^p$, $p < 2/d$, then any solution with smooth initial data is known to be a global one. However, the instability condition $N'(\alpha) < 0$ may nevertheless be valid for some α-s: in the sequel, we assume that function $N(\alpha)$ has one minimum at some point α_* (by another words, the equation admits the existence of both unstable and stable solitons). An example is so-called saturating nonlinearity $F(\xi) = -\xi^p/(1+\gamma\xi^p)$ or its polynomial approximation

$$F(\xi) = -\xi^p + \mu\xi^q, \quad \mu > 0, \quad q > p. \tag{2}$$

So the question is: **what is the long-time behaviour of the unstable ($\alpha < \alpha_*$) soliton under small perturbation for such system?**

The problem in question is popular object for numerical simulations. However, known experiments were mostly devoted to the investigation of the collapse behaviour; saturating nonlinearity was partially considered in [4].

Due to numerics, perturbation of unstable soliton (even numerical noise may be sufficient) generally displays the following scenario. After some time of "stable" propagation (the only this time depends on a perturbation) the solution rebuilds in a short time to a higher amplitude level (which is only weakly depends on perturbation); the process of upswitching is concomitant by formation of rapid radiation; the post-upswitching behaviour is the relaxant oscillation of the amplitude (each period of oscillation also gives rise to some radiation); the long-time solution is close to another soliton corresponding to the parameter $\alpha_\infty > \alpha_*$. The scale and the strength of post-upswitching oscillations depend on the coefficient μ in (2): smaller μ-s corresponds to higher oscillations (provided the initial α is fixed).

In one dimension (note that $p \geq 3$ for supper-critical case) there exists an exact particular solution $U_\alpha(z;x)$ of the equation (1) which plays an important role in understanding of such a behavior. This solution is constructed as a series

$$U_\alpha(z;x) = \sum_{k=0}^{\infty} u_k(x) z^k, \quad z = \pm\exp\left(\lambda_\alpha(t+t_0)\right), \quad \mathcal{L}_\alpha u_k = \mathcal{M}_{k-1}(u_0, u_1, \ldots, u_{k-1}), \quad u_0 \equiv u_\alpha, \tag{3}$$

where operator $\mathcal{L}_\alpha$ is the linearization operator around the soliton $u_\alpha(x)$ ([2, 3]). The equation on $u_1(x)$ is uniform, thus $u_1(x)$ is an eigenfunction of the operator $\mathcal{L}_\alpha$. Under condition $N'(\alpha) < 0$ this operator is known (e.g., [2]) to have positive eigenvalues, and the eigenfunction $u_1(x)$ is chosen to correspond the maximum λ_α of such eigenvalues.

The main goal of our investigation is to derive the long-time asymptotics of the solution $U_\alpha(z;x)$. However, this work is not finally completed and besides the numerics we will discuss the computation of radiation part of solution $U_\alpha(z;x)$ and the relation between the series (3) and self-similar solution [4] of NLS with power-law nonlinearity.

References

[1] W.A.Strauss. Comm.Math.Phys. (1977), vol. 55, pp.149–162.

[2] V.S.Buslaev and G.S.Perelman. Algebra and analysis (1992), vol.4, pp.63–102.

[3] J.L. Bona and A. Soyer. J.Nonlinear Sci., (1994),vol.4, pp.449–470.

[4] B.J.LeMesurier, G.Papanicolaou, C.Sulem and P.L.Sulem. Physica D (1988), vol.31, pp.78-102.

Fermat's Principle in Anisotropy

Andrzej Hanyga

Institute for Solid Earth Physics University of Bergen
N2007 Allègaten 41 NORWAY

Fermat's principle is usually stated as a fundamental property of rays that needs no proof. It is often formulated sufficiently vaguely to mask the inherent problems.

Dropping reference to minima and saddles Fermat's principle can be formulated in terms of stationary points of a Lagrangian functional, which extends its applicability to new areas, such as complex rays [1].

Serious problems arise when it is realized that Fermat's principle is a consequence of the Hamiltonian theory of the eikonal equation, which is the primary result of applying asymptotic analysis to the wave equation. The deduction of Fermat's principle and the associated Lagrangian function from the Hamiltonian theory involves the Legendre transformation or some substitute [3].

It can be shown that the Legendre transformation in its generalized forms [2] and its substitutes break down if the slowness surface for a given wave mode is not convex. The second-order derivatives of the Lagrangian have singularities at the cusps and cuspidal edges of the wavefront. As a result, (1) the classical Lagrangian theory breaks down, (2) the Euler-Lagrange equations involve singularities in the highest order terms, (3) applications of Fermat's principle in linearized tomography are illegitimate.

Fermat's principle is always applicable to the fastest wavemode and hence it can be used to construct viscosity solutions of the eikonal equation [4].

References

[1] V. M. Babich Application of complex rays in calculation of nongeometric waves. *Mathematical problems of the theory of wave propagation*, 20:00–00, 1994.

[2] M. Epstein and J. Śniatycki. Fermat's principle in elastodynamics. J. Elasticity, 27:45–56, 1992.

[3] A. Hanyga, E. Lenartowicz, and J. Pajchel. *Seismic Wave Propagation in the Earth.* Elsevier, Amsterdam, 1984.

[4] P. Podvin and I. Lecomte Finite difference computation of travel times in very contrasted velocity models: a massively parallel approach and its associated tools. *Geophys. J. Int.,* 105:271–284 , 1991.

Frequency Modulated Stationary Waves in a Third-order Nonlinear Schrödinger Equation

E. M. Gromov, V. V. Tutin

Institute of Applied Physics, Russian Academy of Sciences
46 Ul'janov Str., 603600, Nizhny Novgorod, Russia;
e-mail: gromov@hydro.nnov.su

In the present paper we consider exact analytical solutions in form of stationary waves in a general type equation of the third-order approximation of the nonlinear dispersion wave theory (third-order nonlinear Schrödinger equation (TONLS)) [1]

$$2i\left(\frac{\partial\psi}{\partial z}+\beta\left|\psi\right|^2\frac{\partial\psi}{\partial\tau}+\mu\psi\frac{\partial\left|\psi\right|^2}{\partial\tau}\right)+q\frac{\partial^2\psi}{\partial\tau^2}+2\alpha\left|\psi\right|^2\psi+i\gamma\frac{\partial^3\psi}{\partial\tau^3}=0. \tag{1}$$

Rigorous analysis of Eq. (1) by the inverse scattering method [2] including finding exact N-soliton solutions has been carried out in cases for $\alpha=q=0$ and real function ψ (modified Korteveg-de Vries equation) - in Refs. [3,4]; for $\mu=0$ and $q\beta=3\gamma\alpha$ (to so-called Hirota equation) - in Ref. [4]; for $\alpha=\gamma=0$ and $\beta=\mu$ (so-called derivative NLS) - in Ref. [5], where algebraic soliton was also obtained. Another exact analytical method based on reducing TONLS to the ordinary differential equations (ODE's) including deriving exact one-soliton solutions has been used in Refs. [6,7,8]. Solitons with time-independent frequency shift were found in the following cases: in the TONLS at the zero dispersion point in Ref. [6], in the TONLS for its arbitrary coefficients in Ref. [7], in the TONLS for $\mu=0$ and $q\beta=3\gamma\alpha$ with linear profile of a inhomogeneous potential in Ref. [8]. Frequency modulated soliton in the TONLS by neglecting the third-order linear dispersion term, $\gamma=0$, was derived in Ref. [4].

In the present paper we consider frequency modulated unlocalized nonlinear stationary waves $\psi(z,\tau)=A(\vartheta=\tau-\sigma z)\exp\left(ihz+i\varphi(\vartheta)\right)$, moving with velocity $1/\sigma$, in a TONLS with its arbitrary parameters and with polynomial dependence of additional frequency shift $\Omega=d\varphi/d\vartheta$ on the wave amplitude, $\Omega=pA+S$, by means of reducing TONLS to the three ODE's of the same type

$$\frac{d^2A}{d\vartheta^2}+a_0+Ab_0+A^2c_0+A^3d_0=0. \tag{2}$$

Values a_0, b_0, c_0, d_0 depend on TONLS parameters (α, β, μ, γ, q). Parameters of the additional frequency and inverse velocity, σ, are the following

$$p^2=\frac{1}{30\gamma}\left(5\beta+22\mu\right),\ S=\frac{q\left(\beta+6\mu\right)+2\alpha\gamma}{\gamma\left(5\beta+18\mu\right)},\ \sigma=\frac{1}{8}\left(14qS-21\gamma S^2-\frac{q^2}{\gamma}\right).$$

Sech-like and algebraic soliton solutions of the Eq. (2) are following:

$$A=\frac{c_0}{3d_0}\pm\frac{\sqrt{-4c/d_0}}{\cosh\left(\vartheta\sqrt{-2c}\right)},\ c=\frac{1}{2}b_0-\frac{c_0^2}{6d_0}$$

$$A=\frac{c_0}{3d_0}+\left(\frac{b}{2d_0}\right)^{1/3}\left[1+\frac{-4}{1+\vartheta^2\left(2b^2d_0\right)^{1/3}}\right],\ b=a_0-\frac{c_0b_0}{3d_0}+\frac{c_0^3}{12d_0^2}.$$

The obtained exact stationary solitary waves can be useful for optic fiber communications lines.

This work was supported by the Russian Foundation for Basic Research (project N 96-02-19609).

References

[1] G. P. Agraval, Nonlinear Fiber Optic (Academic, Orlando, Fla., 1989);
A. Hasegava, Optical Solitons in Fibers (Springer-Verlag, Berlin, 1989).

[2] V. E. Zakharov, A. B. Shabat, Sov. Phys. JETP 34 (1972) 62.

[3] M. Wadati, J. Phys. Soc. Jap. 32 (1972) 1681.

[4] R. Hirota, J. Phys. Soc. Jap. 33 (1972) 1456.

[5] D.J. Kaup and A.C. Nawell, J. Math. Phys. 19 (1978) 798.

[6] D. J. Frantzeskakis, K. Hizanidis, C. Polymilis, J. Opt. Soc. Am. B9 (1995) 687.

[7] E. M. Gromov, V.I. Talanov, JETP 83 (1996) 73.

[8] E. M. Gromov, Propagation of short nonlinear wave packets and solitons in smoothly inhomogeneous media, Phys. Lett. A (1996), to be published.

Interference Nature of the Transverse Mode Selection in Resonator with Sagnac Interferometer

A. I. Khizhnyak, L. V. Kreminskaya

International Center "Institute of Applied Optics", National Acad. Sci. Ukraine,
10G, Kudryavskaya St., 254053, Kiev, Ukraine,
Fax: 380 (44) 212 48 12, E-mail: vmarkov@iao.freenet.kiev.ua

We report transverse mode selection in the resonator with Sagnac interferometer (RSI) inside. In the set of experiments with RSI it was observed the stable generation of the high directivity without special transverse mode selection. We have shown theoretically that transverse mode selection in the RSI is caused by two-beam interference of counter-propagating cavity waves. In the double-beam interferometers (like Michelson or Max-Zhender type) the different phase climb in their arms appears thanks to the geometrical path difference. As for Sagnac interferometer the phase climb of counter-propagating waves is arising when we bring inside it the phase or amplitude diffuser. The value of phase climb depends on the type of the diffuser and its location inside Sagnac interferometer. Commonly the ABCD wave matrix of the one clockwise round of RSI determines light modulation of Sagnac interferometer. The integral equation of complex field amplitude of RSI is:

$$\gamma\ u(x,y) = \frac{e^{ikL}}{i\lambda B}\iint dx_1 dy_1\ u(x_1,y_1)\exp\left\{\frac{i\pi}{2\lambda B}\left[(A+D)\left(x_1^2+y_1^2+x^2+y^2\right)-4\left(xx_1+yy_1\right)\right]\right\}\times$$
$$\cos\left\{(A-D)\left(x_1^2+y_1^2-x^2-y^2\right)\pi/(2\lambda B)\right\}.$$

Here γ is eigenvalue, $u(x,y)$ is eigenfunction of RSI, λ is wavenumber; (x,y) and (x_1,y_1) are coordinates on output and input of RSI. The kernel of integral equation contains the new modulation factor, which depends on type and location of the diffuser. When we bring our diffuser in the middle of Sagnac interferometer ($A=D$) the RSI leaves its selective properties and becomes "antiresonant ring-mirror configuration".

We made numerical simulation for the case of RSI with great Fresnel number and various types of transparent inside: hard diaphragm, focusing, defocusing lenses and object with Gaussian phase transparency, which is closer to real active medium. When we move the lens from the output mirror to the center of RSI the dependence of light modulation of Sagnac interferometer became the broadest. It is important that RSI oscillate without additional energy losses. The diffraction losses of the fundamental mode of RSI decrease under corresponding lens bias. We have provided such calculations for the linear resonator of equal length and same lens inside, when the lens was placed in the linear resonator center, it became confocal. Under some lens location the diffraction losses of the fundamental mode for RSI became less than for linear resonator ones.

In conclusion, transverse fundamental mode selective properties are peculiar to resonator with Sagnac interferometer. It is possible to direct parameters of resonator with Sagnac interferometer in wide range.

A Variational Principle of the Electromagnetic Wave Diffraction Problem and its Applications

Sergei Kanaun

ITESM-CEM, Mexico; IPM-RAN, St Petersburg

A new variational formulation of the diffraction problem of plane monochromatic electromagnetic waves on an isolated anisotropic inclusion in a homogeneous isotropic dielectric medium is considered. The proposed variational functional has a clear physical meaning: it is proportional to the forward amplitude of the scattering field and its imaginary part coincides with the scattering cross-section of the given inclusion. An approximate solution of the diffraction problem for a spherical anisotropic inclusion is obtained with the help of this variational formulation. It is assumed that the wave field inside the inclusion is a plane wave with unknown amplitude and wave number. The latter are found from the stationarity of the variational functional of the problem. The comparison of the exact and approximate solutions in case of an isotropic spherical inclusion allows to estimate the region of the application of this approximation.

The other area of application of this approximation is self-consistent schemes of the solution of the many-particle problems. In this work the problem of plane electromagnetic wave propagation through dielectric media with isolated inclusions is considered. The phase velocities and attenuation factors of such waves are calculated in the framework of the effective medium method. The proposed approximation is used for the solution of the corresponding one particle problem. The comparison of the results of application of the effective medium method on the base of the exact and approximate solutions of the one particle problem for inclusions of various properties is presented.

Diffraction by the Jump of Curvature by k^{-1}-scaling

Aleksei P. Kiselev

Russia

Zigmund M. Rogoff

UK (visiting St.Petersburg)

Diffraction of short waves by a piecewisely smooth contour with a jump of curvature was approached by several authors e.g. [1-4]. However a complete solution still remains unavailable. Considering the 2-dimensional Helmholtz equation with wavenumber k we undertake a systematic analysis of the very small neighbourhood of the singular point. We employ the scaling of coordinates $X = kx$ and $Y = ky$, first introduced in [2], and later applied successfully in other problems, [3]. Assuming non-tangent incidence we find the wave diffracted from the singular point, where its description involves certain Fresnel-type integrals. The inner solution is then matched with the representations of surface waves to find the excitation coefficients.

References

[1] A. V. Popov, Trudy 5 Vses. Sympos. Diffr. Rasprostr. Voln, 1970.

[2] L. Kaminetsky, J. B. Keller, SIAM J. Appl. Math., 22(1), 1972.

[3] V. B. Philippov, Wave Motion, 3(1), 1981.

[4] N. Ya. Kirpichnikova, V. B. Philippov, Zap. Nauchn. Semin. POMI, 230, 1995.

Quasiphotons and Nondispersive Waves

Aleksei P. Kiselev
Institute of Mechanical Engineering, St.Petersburg
e-mail: kiselev@amath.usr.saai.ru, apk@niif.spb.su
Maria V. Perel
Physics Faculty, St.Petersburg University; e-mail: perel@mph.niif.spb.su

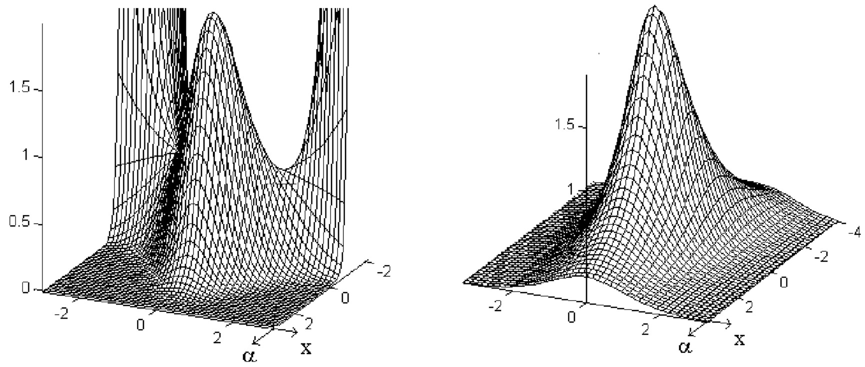
Quasiphotons first discussed in [1] are asymptotic solutions of hyperbolic equations which are localized in a small neighbourhood of a point moving with the velocity of wave propagation. They are described by a kind of complex space-time ray theory. The localization is ruled by a large parameter $p \gg 1$ in the ray ansatz $u = \exp(ip\tau) \sum_j p^{-j} u_j$, with $\tau = \tau(x, y, z, t)$ and $u_j = u_j(x, y, z, t)$. We discuss what are the explicit solutions of the wave equation $u_{xx} + u_{yy} + u_{zz} - c^{-2} u_{tt} = 0$, with $c = const$ of which the quasiphotons are the asymptotics. We consider the solutions from the so called Bateman class known also as 'nondispersive waves' [2,3]. These are

$$u = \frac{1}{\beta - i\epsilon} f(\theta), \quad \beta = z + ct, \quad \epsilon = Const,$$

where f is an arbitrary function of one complex variable and θ is a certain solution of the equation of the Geometrical Optics $\theta_x^2 + \theta_y^2 + \theta_z^2 - \frac{1}{c^2}\theta_t^2 = 0$. Namely

$$\theta = \alpha + \frac{x^2 + y^2}{\beta - i\epsilon}, \quad \alpha = z - ct.$$

We consider particular solutions from the Bateman class satisfying the first-order theory given in [1] i. e. the ones having the prescribed quadratic terms in the exponent. The absolute values of the solutions for $f(\theta) = \exp(ip(A\theta + B)^2)$ and $f(\theta) = \exp(ip(\sqrt{D(\theta + E)} + F))$ are presented on the figure.



One of our sample 'quasiphotons' is actually localized while another grows exponentially.

This research was supported by INTAS-RFBR grant 95-0012.

References

[1] V. M. Babich , V. V. Ulin, Zap. Nauchn. Semin. LOMI, 117, 5-11 (1981) (English translation: J. Sov. Math., 24(3), 1984)

[2] P. Hillion , Acta Mat. Apl., 30, 35-45 (1993); Wave Motion, 16, 81-87 (1992)

[3] H. Bateman, The Mathematical Analysis of Electrical and Optical ... , Dover, (1955)

[4] A. P. Kiselev, M. V. Perel, Zap. Nauchn. Semin. POMI, 238, 117-122 (1996) (English translation to appear in J. Math. Sci.)

Estimation of Elastic Parameters in Diffraction Tomography

Yurii V. Kiselev
St.Petersburg State University, Institute of Physics,
Ul'yanovskaya 1, St.Petersburg 198904, Russia
Vladimir N. Troyan
St.Petersburg State University, Universitetskaya nab. 7/9, St.Petersburg 199034, Russia

Solution of forward and inverse problems of theory of wave propagation includes the question concerning accuracy and correctness of one or other approximative methods. The main goal of this work consists of validity estimation of Born approximation for recovery of parameters of 'small', comparable with wavelength, local inhomogeneity (inhomogeneities) located inside basic (reference) medium which assumes to be piecewise-homogeneous with plane interfaces. Numerical modeling for forward and inverse problems solution is used.

The forward problem is solved by the use of finite difference method [1] and inverse problem is solved by diffraction tomography method [2],[3] in time domain [4]. Testing of the forward problem solution for layered homogeneous medium is implemented by comparison with numerical results given by the contour integral method [5]. The accuracy of diffraction on local inhomogeneity is checked by inner convergence.

In the case of 2-D SH-problem the wave field $\delta\varphi$ scattering by local inhomogeneity by using Born approximation and with help notion of tomography functional (p) [4] can be written, for example, for a case of mass density perturbation $\delta\rho$, as following

$$\delta\varphi(\vec{x}_s,\vec{x}_r,t)=\int\limits_V p(\vec{x},\vec{x}_s,\vec{x}_r,t)\delta\rho(\vec{x})d\vec{x}, \quad p(\vec{x},\vec{x}_s,\vec{x}_r,t)\equiv p=\int\limits_0^\infty \varphi_{out}(\vec{x},\vec{x}_r,t-\tau)\frac{\partial^2}{\partial\tau^2}\varphi_{in}(\vec{x},\vec{x}_s,\tau)d\tau, \quad (1)$$

where $\varphi_{in}(\vec{x},\vec{x}_s,t)$ and $\varphi_{out}(\vec{x},\vec{x}_r,t)$ are the wave fields at point $\vec{x}$, which are created by the seismic source located at the source point $\vec{x}_s$ and by the 'artificial' source, with temporary dependence $\delta(t)$, located at the observation point $\vec{x}_r$. After digitization the expression (1) can be presented as a system of linear equations

$$P\vec{\delta}_\rho=\vec{\delta}_\varphi, \quad (2)$$

where in the right hand side there is the measured $(\vec{\delta}_\varphi)$ values and in the left hand side there are the tomography functional matrix P and the vector of unknown values $\vec{\delta}_\rho$. The system of linear equations (2) is solved by method of least squares and with the help of different regularizing procedures.

Numerical modeling is realized for the cases of mass density (ρ) and joint mass density and shear module (μ) types of local inhomogeneities with sizes $\sim$ 1-1.5 wavelength. One or two inhomogeneities with central symmetry are used. For example, if deviation of $v=\sqrt{\mu/\rho}$ relatively to reference medium is about 20%, then with the help of three source-receiver pairs located at the free interface is possible to restore the inhomogeneity with relative velocity error $\sim$ 7-25% and in the case of the contrasting inhomogeneity (v deviation $\sim$ 80%) with relative error $\sim$ 40-50%.

The work was supported by Russian Foundation of Fundamental Research (project 96-05-65904).

References

[1] Kelly K.R., Ward R.W., Treitel S., Alford R.M. *Synthetic seismograms: a finite difference approach.* Geophysics, **41**, 2-27, 1976.

[2] Natterer F. *The mathematics of computerized tomography.* Chichester, John Willey & Sons, 1986.

[3] Devaney A.J. *Reconstructive tomography with diffracting wave fields.* Inverse Problems, **2**, 161-183, 1986.

[4] Ryzhikov G.A., Troyan V.N. *Tomography and inverse sounding problems.* St.Petersburg, St.Petersburg University, 1994 (in Russian).

[5] Petrashen G.I., Kashtan B.M., Kiselev Yu.V. *The quantitative investigation of nonstationary interference wave fields in layered-homogeneous elastic media with plane-parallel interfaces.* Zapiski nauchnykh seminarov POMI, **214**, 7-186, 1994 (in Russian).

The Second Approximation of the Ray Method for Calculation of the Intensity of the Wave Front in Isotropic Nonhomogeneous Elastic Media

N. Ya. Kirpichnikova

V.A.Steklov Mathematical Institute, St.Petersburg

This paper continues the series of the precedent papers devoted to elaboration of algorithms for calculation of the second and next terms of the ray series for the vector of longitudinal displacements in isotropic nonhomogeneous elastic media in the case of the dimension 2.5.

This method is based on expansions of the amplitudes, the eikonal and the given task parameters into series with respect to coordinates transversal to the direction of propagation of the considered waves. The comparison of the methods of computation of the correction terms of the ray expansions for the vector of longitudinal displacements is carried out.

The research described in this publication was made possible in part by Grant N 95-0-13.1-66 from the State Committee of Russian Federation on Higher education.

Diffraction at Perfectly Conducting Rough Surfaces under Grazing Incidence

Yu. A. Baloshin, A. V. Kostin

St. Petersburg Institute of Fine Mechanics and Optics (Technical University)
Sablinskaya 14, 197101, St. Petersburg, Russia

The problem of diffraction at rough surfaces under grazing incidence is very actual to various fields of physics and engineering. In particular, this problem occurs in radio wave location of objects situated near ocean surface. In dealing with diffraction from surfaces with characteristic dimensions much larger than the wavelength of incident field, approximated methods such as the Kirchhoff approximation are widely used. However, the Kirchhoff approximation fails under grazing incidence and rigorous approaches, for instance the method of integral equations, are to be explored. An implementation of the method of integral equations for perfectly conducting plane surface with a finite perturbation has been discussed in [1,2], where integral equations on unbounded domains for surface current density were formulated. Unfortunately, in order to solve these equations their regions of integration must be truncated, what gives rise to some hardly predictable errors in scattering amplitude calculations.

We invoke the results of potential theory to deduce the integral equations for densities of double (the Dirichlet boundary condition, TE polarization) and single (the Neumann boundary condition, TM polarization) layer potentials. We describe a rigorous formalism for reducing these equations to the Fredholm equations of the second kind to be solved numerically. Solutions of these equations are surface densities distributed within the perturbation, which are sufficient to calculate the complete surface density and, consequently, scattering amplitude.

Examples of calculations of diffraction for both polarizations under grazing incidence from different perturbations such as a smooth hump or a hump modulated by resonance asperities are presented. The optical theorem for testing the accuracy of calculations is employed. It is found that the approach [1,2] fails under some circumstances whereas ours provides stable computings and yields accurate numerics.

References

[1] D. Maystre *IEEE Trans. Antenn. Propag.,* **AP–31**, 885 (1983).

[2] D. Maystre *J. Optic (Paris)*, **15**, 43, (1984).

Inverse Problem and the Double-side Estimates for the Hill Operator

Evgeni Korotyaev

St.Petersburg, Russia

We consider the Hill operator $T = -d^2/dx^2 + q(x)$, acting on $L^2(\mathbf{R})$, where q is a 1-periodic real potential and $q \in H \equiv \{L^2(0,1), \int_0^1 q(x)dx = 0\}$. The spectrum of T is absolutely continuous and consists of intervals separated by the gap $\gamma_n = (a_n^-, a_n^+), n \geq 1$. Let $\mu_n, n \geq 1$, be the Dirichlet eigenvalue of the equation $-y'' + qy = \lambda y, y(0) = y(1) = 0$. Introduce the Lyapunov function $\Delta(\lambda, q)$ for T and note that $\Delta_\lambda(\lambda_n, q) = 0$ for some $\lambda_n \in [a_n^-, a_n^+]$. Let $\varphi(x, \lambda, q)$ be the solution of the equation $-\varphi'' + q\varphi = \lambda\varphi,\ \lambda \in \mathbf{C}$, satisfying $\varphi(0, \lambda, q) = 0, \varphi_x(0, \lambda, q) = 1$.

We consider the mapping $h : q \to h(q) = \{(2\pi n)h_n\}_1^\infty$, where

$$h_n = (h_{cn}, h_{sn}), \quad h_{sn} = \log[(-1)^n \varphi_x(1, \mu_n, q)], \quad h_{cn} = ||h_n|^2 - (h_{sn})^2|^{1/2} \mathrm{sign}(\lambda_n - \mu_n),$$

and $|h_n|$ is defined by the equation $\cosh |h_n| = (-1)^n \Delta(\lambda_n, q) \geq 1$. Using nonlinear functional analysis in Hilbert space, we prove that the mapping $h : H \to \ell^2 \oplus \ell^2$ is the real analytic isomorphism and

$$\|q\| \leq 3\|h\|\sqrt{6 + \sup |h_n|}, \qquad \|h\| \leq 2\|q\|(1 + \|q\|^{1/3}). \tag{1}$$

The continuous isomorphism was proved by Marchenko and Ostrovski (1975). We prove that this mapping is real analytic, we give the new proof and do not use their results.

We consider the second mapping $g : q \to g(q) = \{g_n\}_1^\infty$, where the vector $g_n = (g_{cn}, g_{sn}) \in \mathbf{R}^2$ has components

$$g_{cn} = (a_n^+ + a_n^+)/2 - \mu_n, \quad g_{sn} = ||\gamma_n|^2/4 - g_{cn}^2| \mathrm{sign} h_{sn}.$$

Using nonlinear functional analysis in Hilbert space, we prove that the mapping $g : H \to \ell^2 \oplus \ell^2$ is the real analytic isomorphism and the estimates are fulfilled:

$$\|q\| \leq 4\|g\| \max\{1, \|g\|^{1/3}\}, \qquad \|g\| \leq \|q\| \max\{1, \|q\|^{1/3}\}. \tag{2}$$

Moreover, the trace formula for $q \in L^2(0,1)$ is proved.

Remark: i) the second estimate in (2) is absolutely new. ii) The Gelfand - Levitan - Marchenko equation and the trace formula are not used. iii) The same estimates are obtained for the Dirac operator. iv) The double-side estimates are also found for the smooth potentials. v) We have the estimates for the effective masses, action - angle variables and so on.

References

[1] E. Korotyaev, The Propagation of Waves in Periodic Media at large time. to be published in Asympt. Anal.

[2] P. Kargaev, E. Korotyaev, Inverse Problem for the Hill operator, the Direct Approach. Inventiones Math. , to be published

[3] E. Korotyaev, Metric properties of conformal mappings on the complex plane with parallel slits. Inter. Math. Reseach. Notes. 1996, 10, 493-503.

[4] E. Korotyaev, Estimaites of periodic potentials in terms of effective masses. Commun. Math. Phys., 1997, 183, 383-400

[5] E. Korotyaev, Inverse Problem for the Hill operator, I, Inter. Math. Reseach. Notes. 1997, 3, 113-125

On the Possibility of Regularization of Contact Integrals

D. P. Kouzov

St.Petersburg Marine Technical University

In [1] the condition was formulated that ensures the boundary contact integrals in the problem of diffraction of acoustic cylindrical wave by a junction of two semi-infinite plates admit regularization. The same condition was noted (see e.g. [2, 3]) to be sufficient for the regularization in the boundary-contact value problems with other geometries. In this report it is shown that that condition is also the necessary.

Let us consider the simplest boundary-contact value problem: the diffraction of hydroacoustic wave by an infinite plate, the elastic properties of which are broken along straight line. We consider the 2D case. Let the plate be placed on the surface $\{y = 0\}$ of liquid $\{y > 0\}$, the origin $(0, 0)$ be in the point where the elastic properties are broken. The boundary conditions for the acoustic pressure $P(x, y)$ can be written in the form

$$L\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right) P(x,0) = \sum_{n=0}^{N-2} (-i)^n a_n \delta^{(n)}(x), \qquad (1)$$

where

$$L\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right) = m_1\left(-\frac{\partial^2}{\partial x^2}\right)\frac{\partial}{\partial y} + m_2\left(-\frac{\partial^2}{\partial x^2}\right).$$

Here m_1 and m_2 are polynomials, those particular expressions depend on the mechanic model of the plate theory. The order of the differential operator L is N. Equation (1) sets the concentrated defect of the plate (crack, rigid fastener, etc.) as a passive source of perturbations (i.e. this source radiates only under the action of the incident field). The constants a_n are unknown in this statement and are subject to $N-1$ boundary-contact conditions, which define the mechanical conditions in the point where the plate's elastic properties are broken. The general form of such a boundary-contact condition is the following

$$R_j^+ P + R_j^- P \equiv \lim_{x\to+0} R_j^+\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right) P(x,0) + \lim_{x\to-0} R_j^-\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right) P(x,0) = 0, \qquad (j = 1, 2 \ldots N-1).$$

Here $R_j^{\pm} = s_{j1}^{\pm}\left(-i\frac{\partial}{\partial x}\right)\frac{\partial}{\partial y} + s_{j2}^{\pm}\left(-i\frac{\partial}{\partial x}\right)$, $s_{j1}^{\pm}$ and $s_{j2}^{\pm}$ are polynomials.

The acoustic field in the described model is represented as a sum of two terms

$$P = P_0 + Q, \qquad Q = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{l(\lambda)} \sum_{n=0}^{N-2} a_n \lambda^n e^{i\lambda x - \gamma y} d\lambda$$
$$l(\lambda) = -m_1(\lambda^2)\gamma + m_2(\lambda^2), \gamma = \sqrt{(}\lambda^2 - k^2) \qquad (2)$$

Here P_0 is "classical", and Q is "boundary-contact" parts of the field. The field P_0 is the field that would be generated by the given sources in the supposition that the elastic properties are not broken. Assuming that the source is not in the origin, P_0 and all its derivatives can be shown continuous in the origin. The function Q represents the field diverging from the point of defect and contains the jumps of the total field derivatives in the origin. The path of integration in (2) is the real axis only for the case absorption in the medium. If $\mathrm{Im}k = +0$ it avoids the singularities of the integrated function according to the limiting absorption principle. The time dependence is assumed in the form of factor $e^{-i\omega t}$, which is omitted. The application of the boundary-contact operators to the classical part of the field P_0 meets no difficulties, but the same procedure with the field Q yields, generally speaking, the diverging integrals. We restrict ourselves to consideration of the right limit $R_j^+ Q$. Indexes j and $+$ are omitted below.

$$RQ = \lim_{x\to+0} R\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right) Q(x,0) = \sum_{n=0}^{N-2} a_n J_n,$$

where J_n are called to be boundary-contact integrals

$$J_n = \lim_{x\to+0}\lim_{y\to+0}\frac{1}{2\pi}\int\limits_{-\infty}^{\infty}\frac{r(\lambda)}{l(\lambda)}\lambda^n e^{i\lambda x-\gamma y}, \qquad r(\lambda) = -s_1(\lambda)\gamma + s_2(\lambda). \tag{3}$$

The first passage to the limit in (3) for positive values of argument x can be performed directly under the sign of integral, if the contour is moved to the upper half-plane (without crossing the singularities of the integrated function). Passing to the integration along the cut, that starts at the branch point k, and taking into account the residues yields

$$RQ = 2\pi i\sum_{Im\lambda>0}\mathrm{Res}\lambda^n\frac{r(\lambda)}{l(\lambda)} + \frac{1}{2\pi}\int\limits_{k}^{k+i\infty}\lambda^n\frac{r(\lambda)l^0(\lambda)-r^0(\lambda)l(\lambda)}{l(\lambda)l^0(\lambda)}e^{+i0\lambda}d\lambda, \tag{4}$$

where

$$l^0(\lambda) = m_1(\lambda^2)\gamma + m_2(\lambda), \qquad r^0(\lambda) = s_1(\lambda)\gamma + s_2(\lambda).$$

represent the functions $l(\lambda), r(\lambda)$ on the second sheet of the Riemann surface of λ. The integrals in the right-hand side of (4) converge for all $n \le N-2$, if

$$r(\lambda)l^0(\lambda) - r^0(\lambda)l(\lambda) = O(\lambda^N), \qquad \lambda\to\infty \tag{5}$$

The conditions (5) was noted in [1]-[3] to be sufficient for the regularization of all boundary-contact integrals in the corresponding problem. Let us show that in our case it is also the necessary condition, i.e. its violation brings to irregularizable boundary-contact integrals for some n.

Consider the function

$$f(z) = \frac{1}{2\pi}\int\limits_{k}^{k+i\infty}\Phi(\lambda)\frac{e^{i\lambda z}}{\gamma}d\lambda$$

of complex variable z, where $\Phi(\lambda)$ is the rational function. We assume that the contour avoids the pols of the integrated function. For real z the integrated function decreases exponentially. One can find that $f(z)$ has the branch point in the origin. Going around it in the positive direction the function $f(z)$ takes a new value $f^+(z)$. We have

$$f^+(z) = f(z) + 2\pi i\mathrm{Res}_{\lambda=\infty}\Phi(\lambda)\frac{e^{i\lambda z}}{\gamma}$$

The second term denoted below as $\varphi(z)$ is the entire function of complex variable z. Thus we have the representation

$$f(z) = \psi(z) + \frac{1}{2\pi i}\varphi(z)\ln z$$

where $\psi(z)$ has no branch point in the origin. Consider again the formula (4)

$$\Phi(\lambda) = \frac{1}{\gamma}\frac{rl^0 - r^0 l}{ll^0}\lambda^n$$

If (5) is not satisfied then it is always possible to find such n, that $\mathrm{Res}_{\lambda=\infty}\Phi(\lambda^2) \ne 0$. For such n

$$\lim_{x\to+0}\int\limits_{k}^{k+i\infty}\lambda^n\frac{rl^0 - r^0 l}{ll^0}e^{i\lambda x}d\lambda = \infty$$

due to term that contains logarithm. Thus the regularization of the corresponding boundary-contact integral appears impossible.

References

[1] Kouzov D.P. Diffraction of a cylindrical hydroacoustic wave at joint of a semi-infinite plates. J. appl. math. R. 1969, v. 33, N 2, pp. 225-234.

[2] Kouzov D.P., Pachin V.A. Diffraction of acoustic waves in a plane semi-infinite waveguide with elastic walls. J. appl. math. R. 1976, v. 40, N 1, pp. 89-96.

[3] Levitskii L.A. Diffraction of plane hydroacoustic wave on the open end of plane semi-infinite waveguide with thin elastic walls. J. appl. math. R. 1980, v. 44, N 2, pp. 291-295.

Mathematical Problems of Diffraction

Victor F. Kravchenko

IREE RAS, Moscow

George A. Nesenenko

Bauman Moscow State Technical University, Moscow

In the report a method is suggested and justified through which the focus effect of solutions of singularly perturbed boundary value problems for equations of the parabolic type in field with moving boundaries is investigated.

From geometrical theory of the diffraction of wave fields, it is known that the combination of methods of the applied catastrophes theory and saddle-point method makes it possible to classify types as caustic, and to find asymptotic wave fields near the caustic with the help of adequate reference integrals. This is an Airy integral and a Pierce integral [1, 2].

The authors established what conditions must be satisfied singularly perturbed boundary value problems posed for equations of the parabolic type, in order that in boundaries focusing as analogous focusing of wave fields become available. For the description of such focusing, it is suggested that the new model integral be used

$$I_\nu^\mu(z) = \int_0^\infty x^{\nu-1} \exp\left(x - \frac{z}{x^\mu}\right) dx, \qquad \mu = 1, 3, 4,$$

which can be regarded as integral analogs of Airy and Pierce.

With the help of these model integrals it was found that the focus effect of solutions of the explored boundary value problems on cusp catacaustic, embedded in boundary layers of moving boundaries, is greater than on smooth catacaustic.

References

[1] Babich, V.M. and Buldyrev, V.S., Asymptotical Methods in Diffraction Problems of Short Waves, Moscow: Nauka, 1972.

[2] Poston, T. and Stewart I., theory of Catastrophes and Its Applications, Moscow: Mir, 1980.

On Natural Frequencies of Thin Cylindrical Shell with Free Edges

Yu. A. Lavrov

St.Petersburg University of Means of Communications,
9, Moskovskiy str., 190031, St.Petersburg, RUSSIA

The method of construction of the approximate formulae for natural frequencies of a closed cylindrical shell of finite length is suggested. The oscillations of the shell are described by the Flugge equations. The edges of the shell are free. the characteristic size of the resonator is its length. The cases of small relative thickness of the shell and large relative radius are considered. The compact equations and formulae for natural frequencies that approximate exact frequency equation are derived. The obtained outcomes are numerically tested, are correlated with outcomes for other conditions of edges fastening and are compared to conclusions in literature.

Acoustic Wave Radiation During Relative Sliding of Two Bodies

A. A. Lazarev

Mechanical Engineering Research Institute
Russian Academy of Sciences
Belinskogo, 85, 603024 N. Novgorod, Russia

In order to describe the sound radiation from the contact region of two sliding bodies we use an interesting approach suggested in [1]. We consider a rigid body which moves with a constant velocity above the ideal elastic half space. The body surface consists of periodical roughnesses. The body and the half space are separated by a small distance due to surface (for example, molecular) repulsion forces. Besides, we assume that the moving body oscillates simultaneously perpendicularly to the sliding plane. The external stresses, arising on the surface of the half space due to the action of surface forces, generate elastic waves travelling into the half space. The process depends upon two space coordinates. We have found exact expressions for the body waves propagating perpendicularly to the sliding plane and for inhomogeneous waves, and for the body waves entering obliquely into the half space. Using these expressions for the elastic waves we have obtained the exact formula describing the wave radiation power dependence upon the elasticity constants, the kind of the surface forces, and the sliding velocity and surface profile of the moving rigid body. It is shown that the dependence of the radiation power on the sliding velocity is complicated and nonmonotonic. The energy loss caused by the radiation of the body waves propagating perpendicularly to the sliding plane depends no upon the sliding velocity. These waves are generated by the oscillating body even without sliding. The inhomogeneous waves take no away energy from the contact zone at all. The energy loss depends on the sliding velocity due to the radiation the body waves entering obliquely into the half space. If the sliding velocity of the body is small enough, only the body waves propagating perpendicular to the half space and the inhomogeneous waves are generated. If the sliding velocity is sufficiently large, the inhomogeneous waves convert into the body waves entering obliquely into the half space. We have found the critical sliding velocities corresponding to the converting the inhomogeneous waves into the body waves. If the sliding velocity is increased from the critical velocity, the angle of the entering of corresponding body wave into the half space is increased from zero, tending to 90 degrees. It is shown that the dependence of the wave radiation power upon the sliding velocity has the picks in the regions of the small angles of the body wave entering into the half space. The critical velocities and the picks in number, the heights of the picks in the radiation power dependence on the sliding velocity are determined by the surface profile of the sliding body.

References

[1] E. Adirovich, D. Blokhinzev. On the forces of dry friction, J. of Physics (Russia), 1941, 7, 29-36

Method of Regularization in Two-Dimensional Problems of Narrow Electromagnetic Pulse Diffraction

A. M. Lerer, S. M. Tsvetkovskaya

Rostov State University, Institute of Physics,
194 Stachky street, 344090 Rostov-on-Don, Russia.
Tel. (863)2280858. Fax (863) 2285044. E-mail: lerer@riphys.rnd.su

The conventional approach to the solution of problems of distribution and EMP diffraction is based on the solution of these problems for a monochromatic electromagnetic wave with the subsequent application of inverse Fourier transformation. This approach is rather fruitful for long EMP, but for picosecond pulses collides with series of difficulties such as sharp increase of volume of calculations, toughen requirements of accuracy of the solution in the spectral domain because of error accumulation going over the time domain and etc. Therefore in last years there were works in which Maxwell equations for complex microwave structures are all at once solved in the time domain.

To solve electrodynamic boundary value problems in the spectral domain various numerical - analytical methods based on solution of integral equations (IE) are widely applied. There are various modifications of these methods. The most effective ones in our opinion contain two main stages of the solution:

-In the first stage a key structure and corresponding boundary value problem is extracted. The key structure being geometrically the most similar to researched one contains all main their feature. For example, for numerous problems of diffraction by two-dimensional metal bodies with edges, by discontinuities in waveguides a key problem may be a problem of diffraction by a metal strip and by slot in a metal screen.

- In the second stage the solution of key boundary value problem is reduced to the solution of operator equations, for example, integral ones. To solve the operator equations a regularization is made use. The regularization takes into account analytical properties of the operators.

This approach used in the time domain are presented in this work.

The problem is reduced to the solution of integral and integro-differential equations. To solve these equations it is proposed to use Galerkin method both with respect to space coordinate x and with respect to the time t. With respect to x the basic functions are chosen in the terms of weighted Chebyshev polynomials which correctly describe a behavior of electromagnetic fields near to metal edges. With respect to the time t the basis functions are chosen in the terms of weighted Hermite polynomials. Application of a Galerkin method results sets of linear algebraic equations (SLAE) of second kind with fast convergence. All matrix elements of SLAE are obtained in a analytical form. Due to that computation using personal computer take deals of a second. The work contains computed results of fields diffracted by a slot for EMP with a duration up to 2 picoseconds. The solution of these key problems permits to construct effective numerical-analytical algorithms of mathematical modeling of EMP diffraction by complex cylinders with edges, by numerous discontinuities in rectangular waveguides, as well as EMP distribution in planar transmissions lines.

Helmholtz equation solutions concentrated near periodical boundary

V.Yu.Gotlib

St-Petersburg branch of P.P.Shirshov Oceanological Institute RAS
V.O., 1 Line, 30, St-Petersburg Russia; e-mail: gotlib@io.spb.su

The lecture is concerned to a question of existence of periodical solutions of Helmholtz equation exponentially decaying outside the vicinity of periodical boundary. These solutions can exist in the case of Dirichlet or Neumann boundary conditions but the form of boundary surface is quite different in both cases.

The investigations were supported by grant of INTAS RFBR 95-0012.

Scattering of a Plane Acoustic Wave on an Elastic Plate Joined with a Periodic Set of Acoustic Resonators

V.D. Lukjanov, G.L. Nikitin

The Defensive Constructional Engineering School
22 Zakharyevskaya street
St.-Petersburg, Russia

Scattering of a plane acoustic wave on a thin elastic plate jointed with periodic set of rectangular acoustic resonators is considered. The plate serves by one of the walls for each resonator, whereas the other three walls of each resonator are perfectly rigid. We assume that the plate is capable only to flexural oscillations and the displacements of plate are described by the Kirchhoff equation with regard of a contact of the plate with fluids.

The plate is rigidly connected with the walls of resonators, i.e. in the points of a fastening the displacements and the turn angles of the plate vanish. In the neighbourhoods of the points of fastening of the plate with resonators the Meixner condition should be fulfilled.

Acoustic field scattered on the plate should consist only of outgoing waves, which we select in accordance with the principle of the limiting absorption.

In order to solve the formulated problem, we first in view of periodicity of model on a variable x by using the Floquet theorem, reduce problem for the whole half-space to a problem for the strip $-H < x < H, -L < y < +\infty$ with the quasiperiodic boundary conditions on its lateral faces at $x = \pm H$ and $y > 0$.

The acoustic pressures $P_1(x, y)$ in a half-space $y > 0$ and $P_2(x, y)$ in resonators at $-L < y < 0$ satisfy the homogeneous Helmholtz equations and can be found as the following series expansions

$$P_1(x,y) = \varphi_0(x)e^{i\lambda_0 y} + \sum_{n=0}^{+\infty} r_n\varphi_n(x)e^{i\lambda_n y}, \qquad P_2(x,y) = \sum_{n=0}^{+\infty} \beta_n\psi_n(x)\cos\mu_n(y+L),$$

where r_n is the reflection coefficient ($n = 0$) or the transformation coefficient ($n \geq 1$) of incident wave into the orthogonal mode with number n : $\varphi_n(x) = \sqrt{\rho_1/\lambda_n}e^{-ip_nx}$, $p_n = k\cos(\theta) + \pi n/H$, $q_n = \pi n/(2H)$, $\lambda_n = \sqrt{k_1^2 - p_n^2}$, $\mu_n = \sqrt{k_2^2 - q_n^2}$, $\psi_n(x) = \cos(q_nx + \pi n/2)$, θ is the angle of incidence (measured from the plate) of a plane wave.

We seek the flexural displacements of the plate in the form [1]

$$\xi(x) = \sum_{n=1}^{4} A_n\chi_n(x) + \sum_{n=-\infty}^{+\infty} B_n\varphi_n(x) + \sum_{n=0}^{+\infty} C_n\psi_n(x),$$

where $\chi_1(x) = \cos(æx)$, $\chi_2(x) = \sin(æx)$, $\chi_3(x) = \cosh(æx)$, $\chi_4(x) = \sinh(æx)$ are the particular linearly independent solutions of the homogeneous Kirchhoff equation.

By substituting the above expansions into the boundary conditions on the plate we obtain an infinite system of linear algebraic equations for unknown amplitudes. It can be shown, that this system allows to get the solution by the truncation method.

Results of numerical computation of a reflection coefficient on energy of a plane wave as a function of frequency of an incident field are shown in fig. 1 for the case when the steel plate is in contact with water ($H = 0.25m, L = 0.25m$, and the thickness of plate $h = 0.01m$). The curve 1 corresponds to the case of the normal incidence of the plane wave, whereas the curve 2 corresponds to the inclined incidence.

At inclined incidence of a plane wave the frequencies at which the damping orthogonal modes turn into the traveling waves can be found from the relation $kH(1 \pm \cos\theta) = \pi n$, $(n = 0, \pm 1, \pm 2 \ldots)$. Thus the number of traveling modes and consequently of channels of scattering in the upper half-space grows with increase of frequency and with diminution of the angle of incidence of the plane wave.

At normal incidence of the plane wave only the symmetrical (about middle of resonators) forms of oscillations are excited in the resonators. At inclined incidence both symmetrical and antisymmetric forms of oscillations are excited in the resonators. Therefore the number of resonance frequencies in this case is more than in previous.

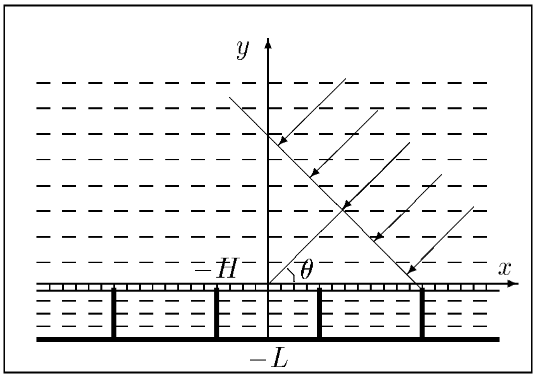


Fig. 1

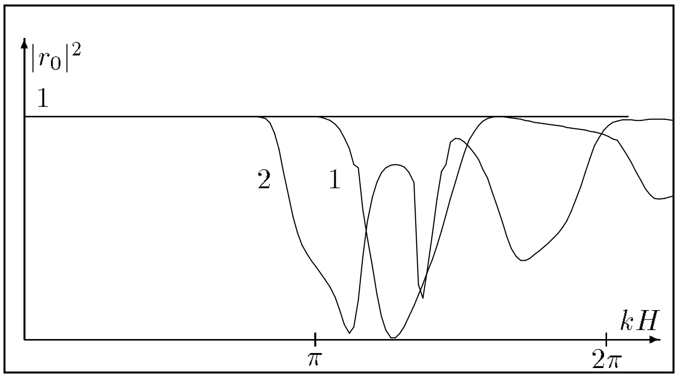


Fig. 2

The greater number of channels of scattering and resonant frequencies in resonators at inclined incidence explains the comparative complexity of the frequency dependencies of the scattering data in comparison with the case of normal incidence.

The work is supported by the RFFI, grant 95-02-04346.

References

[1] Lukyanov V.D., Nikitin G.L. Scattering of acoustic waves by an plate separating two different fluids in a waveguide. // Soviet Physics-Acoustics, 1990, v. 36, No. 1, p. 68-75.

Diffraction by a Slowly Expanding Cylinder

M.A. Lyalinov

Department of Mathematical Physics, Institute of Physics,
University of S.Petersburg, Ulianovskaja 1-1, 198904, Russia
e-mail: Lyalinov@onti.niif.spb.su

The explicit solution in the problem of diffraction by a circular cylinder with uniformly increasing radius is discussed in the paper [1]. In the present report we consider the problem of wave scattering by a uniformly extending cylinder with the Dirichlet boundary condition on its surface. The cylinder of an arbitrary cross-section instantly arises at the initial moment of time and, then, expands with the constant speed. The interaction of the unperturbed wave field, which is the plane wave in our case, leads to spreading the perturbation wave. The front of this wave propagates with the wave speed. The solution has a weak singularity on the front, which means that the solution is continuous on the front and its normal derivative has the discontinuity of the first kind. Since we assume that the incident wave does not depend on the coordinate along the cylinder, the perturbation wave does not depend on this coordinate too. Effectively, we have the problem with the two space variables and time. In Minkovski space time M^3 the position of the scatterer at all moments of time is represented by the cone K with the boundary S. The cross-section of the cone by a plane $\tau = ct_0 = \text{const}$ is the closed smooth curve. The projection of this curve in R^2 is called the scattering contour. If the scattering contour expands slowly in comparison with the wave speed c, the cone K is narrow. In this case, we construct an asymptotic solution of the problem in the form of asymptotic series. This formal asymptotic solution has different behavior in three subdomains D_0, D_1, and D_2 inside the region $G \setminus K$ (G is the interior of the light cone): in the vicinity D_0 of the tip of S; in the small vicinity D_1

of K and beyond D_0; in the region D_2 beyond the domains D_0 and D_1. In the transition regions between these domains the local asymptotics should be matched. The leading terms of the asymptotics depend upon the integral characteristic of the scattering contour, namely, on Wiener capacity of the corresponding region.

We also construct the model problem for the wave equation in the cone G with the effective boundary condition at the points of the semiaxis $\Lambda = \{(x,y,\tau) : \tau > 0,\ x = y = 0\}$. This problem represents a model for the limit case of the original problem if the cone K degenerates into the semiaxis Λ. The model is constructed using the well known procedure of the self-adjoint extensions of symmetric operators. The parameter of self-adjoint extension is determined by means of comparison with the asymptotic solution of the original problem. These approach is analogous to those in the theory of zero range potentials.

The perturbation field outside the narrow vicinity of the cone K is represented by the leading term of the asymptotics. Let β be the parameter connected with the relative speed of extension of the cylinder. The leading term of asymptotic solution in D_2 takes the form ($\tau > \beta^{1-\delta}$, $\delta > 0$)

$$v = \sum_{m=0}^{\infty}(-1)(-ikr)^m/m!\,\frac{Q_m(\cosh\theta)}{(W_{\beta/2}-\Psi(m+1)-C)}(1+O(\beta/ln(\beta)), \tag{1}$$

where $W_{\beta/2} = W - ln(\beta/2)$ is the Wiener capacity of the exterior of the small domain in R^2 which is defined as the cross-section of K and the plane $\tau = 1$, Q_m is the Legendre function, Ψ is the diagram function and C is the Euler constant. The coordinates (r, θ, φ) are the pseudospheric coordinate in G.

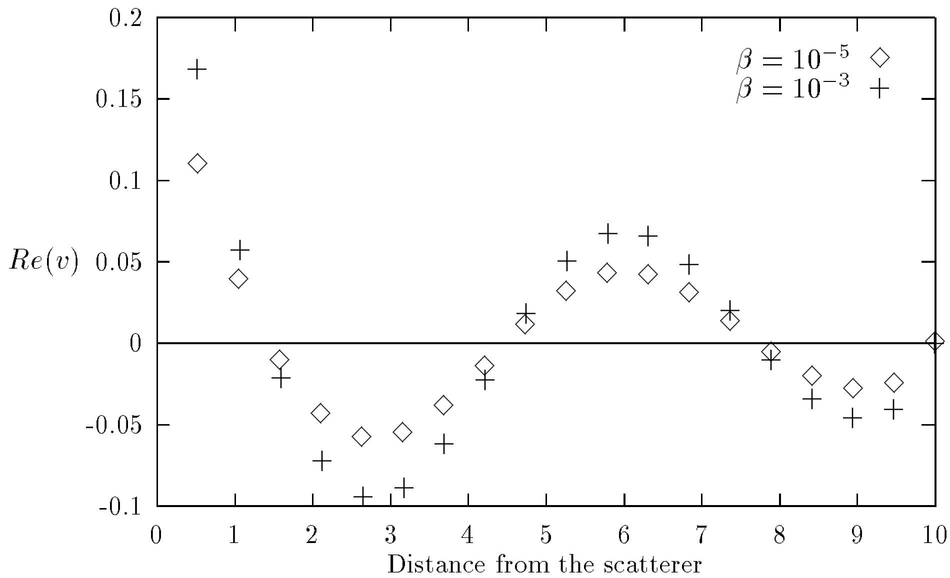
In engineering practice, when β is extremely small and $\mid ln\beta \mid \gg 1$, formula (1) can be additionally simplified. We formally neglect $-\Psi(m+1) - C$ in comparison with $W_{\beta/2}$ in the denominator of expression (1) and after summation obtain

$$v = -\frac{1}{W_{\beta/2}}\int_0^{\xi(x,y,\tau)}\exp(-ik(\tau-\sqrt{x^2+y^2}\cosh\psi))d\psi(1+O(1/ln(\beta))\,. \tag{2}$$

In expression (2) we have used the notation $\xi = 1/2\log[(\cosh\theta+1)/(\cosh\theta-1)]$ and the obvious equalities $\cosh\theta = \tau/\sqrt{\tau^2-x^2-y^2}$, $r = \sqrt{\tau^2-x^2-y^2}$, where τ, x, y are the Cartesian coordinates. In Figure the distribution $Re(v)$ of the perturbation field (2) for a fixed moment of time ($\tau = 10$) is shown for two relative speeds of extension of a circular cylinder.

References

[1] M.A.Ljalinov, Scattering of a plane wave by a circular cylinder with uniformly increasing radius, IEEE Trans. AP-42, no.11, pp. 1574-1577, 1994.



Complex Ray Analysis of the S^* Wave

James Lawry

Mathematical Institute, Oxford University
24-29 St Giles, Oxford OX1 3LB, United Kingdom

When a high-frequency time-harmonic line source is placed in a scalar wavefield adjacent to a flat interface with a halfspace having a slower wavespeed, a contribution to the transmitted wavefield is detected that cannot be described asymptotically by real geometrical rays. This field was detected numerically (Hron & Mikhailenko 1981) and found to be confined to the exterior of a sector in the slower halfspace.

The field corresponds to a complex saddle in the integral form of the exact solution, and correspondingly may be described by rays which emanate from the source at complex angles into a complex coordinate space, and are refracted back into the real domain from the analytic continuation of the boundary. The complex ray method enables us to calculate both the asymptotic field and the Stokes curves forming the boundary of the region in which it appears, on the basis of the ray data alone and without reference to the exact solution. This makes the method suitable for use in more general problems, such as curved interfaces, for which analysis of the exact solution is not possible. The structure of the complex rayfield reveals interesting features arising from the fact that the data on the interface has branch points in complex space.

Travelling Wave Packets in Thin Elastic Shells

G. I. Mikhasev

Belarus, Vitebsk University

Three kinds of problems on the propagation of wave packets (WP) in thin elastic shells are examined.

The first problem is concerned with packets of short bending waves travelling in the circumferential direction in non-circular medium length cylindrical shells with sloping edges [1]. A solution of the linear governing equations is constructed in the form of superposition of N wave packets

$$W = \sum_{n=1}^{N} W_n^*(s,\, \xi_n,\, t,\, \varepsilon)\, exp\,\{i\,\varepsilon^{-1} S(\xi_n,\, t)\}, \tag{1}$$

$$W_n^* = \sum_{m=0}^{\infty} \varepsilon^{m/2} w_{nm}(s,\, \xi_n,\, t),\;\; S = \int_0^t \omega_n(\tau)d\tau + \varepsilon^{1/2} p_n(t)\,\xi_n + \frac{1}{2}\,\varepsilon\, b_n(t)\,\xi_n^2,$$

where ε is a small parameter; $\xi_n = \varepsilon^{-1/2}[\varphi - q_n(t)]$ is a local coordinate connected with the packet center $\varphi = q_n(t)$; t is time; φ and s are coordinates on the shell surface in the circumferential and axial directions, respectively; w_{nm} are polynomials in ξ_n satisfying boundary conditions for any t; $\omega_n(t)$ is the instantaneous frequency of the shell vibration; $p_n(t)$ determines the variability in the direction of φ and $b_n(t)$ characterizes the rate of decay of the wave amplitude as the distance from the centre $\varphi = q_n(t)$ increases.

The second problem deals with WP running in the axial direction in an infinite shell of revolution [2] and cylindrical shell with variable parameters [3]. A solution of the initial system of equations is found in the form of superposition of the packets (being similar to (1)) of bending, longitudinal and torsional waves.

At last, the third problem is devoted to travelling two-dimensional WPs in an infinite shell of an arbitrary form.

References

[1] Mikhasev G.I. *Localized families of bending waves in a non-circular cylindrical shell with sloping edges* // Prikl. Mat. Mekh. 1996. Vol. 60. No. 4. P. 635-643.

[2] Mikhasev G.I. *Localized wave forms of motion of an infinite shell of revolution* // Prikl. Mat. Mekh. 1996. Vol. 60. No. 5. P. 826-834.

[3] Mikhasev G.I. *On the wave forms of motion of an infinite cylindrical shell with variable parameters* // Izv. Akad. Nauk. Mekh. Tverd. Tela. 1995. No. 6. P. 129-137.

Radiation of Acoustic Waves by the Flat Waveguide into a Half-space through a Thin Elastic Plate

G. L. Nikitin

The Defensive Constructional Engineering School
22 Zakharyevskaya street
St.-Petersburg, Russia

Consider the stationary scattering of acoustic waves on a thin elastic plate which separates the flat semi-infinite waveguide from a half-space. Acoustic pressures in the waveguide and in the half-space satisfy the homogeneous Helmholtz equations (their acoustic parameters may be different). We assume that the flexural oscillations of the plate are governed by the Kirchhoff equation. In addition the ends of the plate are rigidly fixed by walls of the waveguide.

Let the wave fields are generated by one of the natural waves of the waveguide. Then the acoustic field reflected back into the waveguide and the field transmitted into the half-space should consist only of outgoing waves, which we select in accordance with the principle of the limiting absorption. Finally, the full acoustic field is subject to the Meixner condition in neighbourhoods of contact points of the plate with the waveguide.

We seek the solutions of the Helmholtz equations as series expansions in terms of the natural waves (inside the waveguide) and in terms of elliptical cylinder functions (in the half-space)

$$P_1(x,y) = \varphi_l(y)e^{i\lambda_l x} + \sum_{n=0}^{\infty} D_n\varphi_n(y)e^{-i\lambda_n x}, \qquad P_2(x,y) = \sum_{n=0}^{\infty} C_n ce_{2n}(\eta,q)He_{2n}(\xi,q).$$

Here: $\varphi_n(y) = \cos p_n y$, $p_n = \pi n/H, \lambda_n = \sqrt{k^2 - p_n^2}$, the elliptical coordinates ξ, η are introduced by the formulas $x = H\sinh\xi\sin\eta$, $y = H\cosh\xi\cos\eta$, $ce_{2n}(\eta,q) = (1/\sqrt{2})B_0^{2n} + \sum_{m=1}^{\infty} B_{2m}^{2n}\cos 2m\eta$ are the periodic elliptical cylinder functions, $He_{2n}(\xi,q){=}He_{2n}(\eta,q) = (1/\sqrt{2})B_0^{2n}H_0^{(1)}(kH\cos\xi) + \sum_{m=1}^{\infty} B_{2m}^{2n}H_{2n}^{(1)}(kH\cos\xi)$ are the radial elliptical cylinder functions, which are chosen as well as in [1, 2] in accordance with a principle of the limiting absorption.

We seek the flexural displacements of plate as

$$u(y) = A\psi(y) + \sum_{n=0}^{\infty} B_n\varphi_n(y), \qquad \psi(y) = \left(\frac{\cosh æy}{\sinh æH} + \frac{\cos æy}{\sin æH}\right).$$

From the boundary conditions on the plate (equation of motion of the plate, kinematic condition, and conditions on the ends) we obtain a system of linear algebraic equations, which may be written in the matrix form as follows

$$\begin{pmatrix} cth æH + ctg æH & (-1)^{n+1} & 0 \\ \rho\omega^2 t_m & L_m\delta_{mn} & i\lambda_m T_{mn}He_{2n}(0,q)/\varepsilon_m \\ -\rho\omega\sum\sum_{n=0}^{\infty} T_{nm}t_n\dfrac{1}{\varepsilon_n} & -\rho\omega^2 T_{nm} & \delta_{mn}He_{2n}'(0,q) \end{pmatrix} \begin{pmatrix} A \\ B_m \\ C_m \end{pmatrix} = \begin{pmatrix} 0 \\ 2i\lambda_l\varepsilon_m\delta_{lm} \\ 0 \end{pmatrix},$$

where $T_{mn} = \int_0^H \cos(\pi m \cos\eta) ce_{2n}(\eta, q) \sin\eta d\eta$ is element of the transformation matrix to other basis, $t_m = (1/H\varepsilon_m)\int_0^H \psi(y)\varphi_n(y)dy = (4/H\varepsilon_m)(-1)^m$ æ^3 (æ$^4 - p_m^4)^{-1}$ is the Fourier coefficient of the special solution $\psi(y)$ of the homogeneous Kirchhoff equation, $L_n = g(p_n^4$-æ$^4)i\lambda_n + \rho\omega^2$ is the symbol of the plate operator.

The matrix of the coefficients of this system has three diagonal blocks, therefore after an obvious change of variables the matrix equation can be transformed to a kind of $(\mathbf{I}+\mathbf{V})\mathbf{X} = \mathbf{F}$. It can be shown, that this equation is equation with the completely continuous operator $\mathbf{V}$ in the space of square-integrable sequences. Hence, the solution of such system can be obtained by the truncation method.

On large distances from the beginning of coordinates the field scattered in the half-space acquires character of an outgoing cylindrical wave, i.e. asymptotic formula

$$P_2(x,y) = \sqrt{\frac{2\pi}{kr}} e^{ikr - i\pi/4}\Psi(\varphi)(1 + O(1/kr))$$

takes place at $kr \to \infty$. Here r, φ are the cylindrical coordinates, the function $\Psi(\varphi)$ is the directivity diagram of the scattered field.

By using the asymptotic formulas for the Hankel functions $H_{2n}^{(1)}(z)$ and also the approximate relations $r' \simeq H\cos(\xi)$, $\varphi \simeq \eta$ valid far from the beginning of coordinates, for the directivity diagram we obtain the following expression

$$\Psi(\varphi) = \frac{1}{\sqrt{2\pi}} \sum_{n=0}^{\infty} (-1)^n C_n ce_{2n}(\varphi, kH).$$

On the basis of the above procedure and of formula for the directivity diagram we carry out the numerical analysis of reflection coefficients of natural waves in dependence on frequency (Fig. 1) and present the directivity diagrams for typical and distinctive frequencies (Fig. 2).

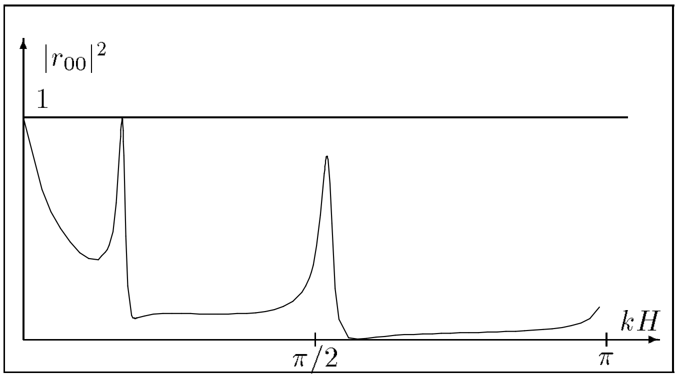


Fig. 1

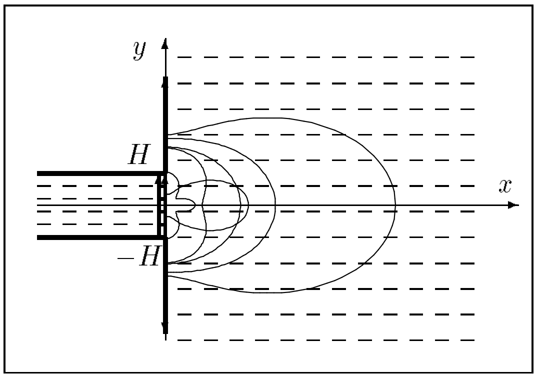


Fig. 2

For calculations we consider the model, in which the steel plate ($E = 1,96 \cdot 10^{11} n/m^2$; σ =0,3; $H = 1m; h = 0,03m$) is in contact with water ($\rho = 10^3 kg/m^3, c = 1500 m/\sec$), which filled the waveguide and half-space. Fig. 1 shows the dependence of the modulus squared of the reflection coefficient r_{00} of the zero-order natural wave, as a function of the dimensionless frequency kH.

The work is supported by the State Committee of Russian Federation, grant 95-0-2.1-135.

References

[1] Morse F.M., Fashbach G. Methods of theoretical physics, vol 1, Moscow, 1958.

[2] Morse F.M., Fashbach G. Methods of theoretical physics, vol 2, Moscow, 1960.

Trapped Modes in the Linearized Problem of a Potential Flow about Semisubmerged Bodies

Oleg V. Motygin

Institute of Problems of Mechanical Engineering
V.O. Bol'shoy pr., 61
199178, St-Petersburg, Russia
e-mail: mov@snark.ipme.ru

The 2D problem, describing the forward stationary motion of surface-piercing bodies in an ideal, incompressible, heavy fluid of infinite depth is considered. Linearization of the dynamic and kinematic conditions in the free surface in the framework of the theory of waves with small amplitude leads to the so-called Neumann-Kelvin problem for a velocity potential. In papers by N.E. Kochin, B.R. Vainberg and V.G. Maz'ya this problem was proved to be well-posed in the case of totally submerged bodies. At the same time, for surface-piercing bodies this statement allows existence of singularities of the velocity field in the corner point, where the free surface meets the contour of body. It results in subdefiniteness of the initial set of equations which is needed to be augmented by supplementary conditions. Some versions of such conditions were proposed in papers by F. Ursell, K. Suzuki, M. Lenoir, V.G. Maz'ya and N.G. Kuznetsov. We use the statement of the paper [1] delivering the so-called "least singular" solution. The velocity field of such potential is bounded in the corner points.

The "least singular" statement was proved to be uniquely solvable for all values of the forward velocity with possible exception for a sequence tending to zero. Our purpose is to demonstrate that these irregular values do exist. Following [2] where a non-uniqueness example was obtain for the 2D sea-keeping problem, we use the so-called inverse procedure for simultaneous construction of surface-piercing bodies and of the potential of mode with finite energy, trapped by these bodies. Namely, we fix a value of the forward velocity and construct a potential as a sum of source and sink of the same strength placed in the free surface and separated by a distance of some wavelengths. The special choice of distance guarantees absence of waves at infinity for the potential and, hence, the finiteness of energy. Investigation of potential's streamlines shows that there is a pair of them with the both ends in the free surface and enclosing sources inside. These streamlines are interpreted as body contours. Thus, a geometry is obtained for which the homogeneous problem has a nontrivial solution. The trapped modes are supposed to be related to resonances in some more general initial value problem (see [3]).

In [2] example was only constructed with two bodies. Indeed, more that two homotopical families of contours can be obtained in enlarging the distance between singularities. We prove that for any fixed value of forward velocity and a caravan, consisting of two and more bodies, the geometry of these bodies can be fixed so that the homogeneous Neumann–Kelvin problem has a nontrivial solution with finite energy. Proof of the assertion is based on description of the streamlines of level zero, which separate homotopical families of streamlines, and on a relationship connecting values of the stream function in the fluid and in the free surface at fixed horizontal coordinate.

References

[1] Ursell F. Mathematical notes on the two-dimensional Kelvin-Neumann problem. In: Proc. 13th Sympos. Naval Hydrodynamics, Tokyo: Shipbuilding Res. Assoc., 1981, pp. 245–251.

[2] McIver M. An example of non-uniqueness in the two-dimensional linear water wave problem. J.Fluid Mech., 1996, Vol. 315, pp. 257–266.

[3] McIver M. Resonance in the unbounded water wave problem. In: Proc. of 12th Workshop on Water Waves and Floating Bodies, Marseille, France, 1997.

Propagation of Sound Generated by a Moving Source in an Oceanic Waveguide with an Obstacle

Olga A. Obrezanova, Vladimir S. Rabinovich

Dept. of Mechanics and Mathematics, Rostov State University
5, ul. Zorge, Rostov-na-Donu, 344104, Russia

Propagation of electromagnetic and acoustic waves generated by stationary sources in waveguides with inhomogeneities is a well-known problem for which many numerical and analytical methods have been developed. Also there are many papers discussing wave scattering from moving obstacles. The scattering problems for fields produced by moving sources are less investigated although the significance of ones is obvious. The only paper known to authors is [1] where the phase characteristics of the field generated by a moving source in a media with an obstacle are obtained. It should be noted that scattering problem with a moving source does not reduce to one with a moving obstacle.

We consider 3D acoustic waves scattering problem for a field generated by a moving source in a stratified fluid semi-infinite waveguide containing an arbitrary convex obstacle. We suggest an approach for deriving the modal asymptotic representation for the direct and scattering fields for narrow-band sources moving with small acceleration at large distances from the receiver and obstacle.

To obtain the asymptotics we use the method described in papers [2, 3, 4] discussing the propagation of sound generated by moving sources in stratified oceanic waveguides without obstacles. It is based on the representation of the acoustic field in the form of repeated integral in time and frequency variables in terms of the stationary Green function associated with monochromatic point static source. We derive asymptotic formulas for the field valid for large values of the asymptotic parameter characterizing both smallness of acceleration, narrow-bandness of amplitude of the source and largeness of distances between the source and receiver and between the source and obstacle.

To obtain the Green function corresponding to the scattering field we use the modified boundary integral equation method which provides the uniqueness of solution of integral equation for every fixed frequency. It is represented in the form of combination of double and single layer potentials generated by the Green function of unperturbed stationary problem. We obtain the asymptotic modal representation of the Green function corresponding to the scattering field valid for large distances between the source and obstacle in comparison with the size of the obstacle and the wave length.

Using the asymptotics of the Green function, by the two-dimensional stationary phase method, we derive formulas for the direct and scattering fields generated by the moving source for the receiver situated both at short and large distance from the obstacle. The asymptotic formulas are expressed in terms of retarded times and Doppler-shifted frequencies. Scattering coefficients are evaluated in terms of solutions of stationary integral equations on fixed frequencies which are to be solved numerically.

References

[1] N.S. Grigor'eva, "Influence of the source motion to phase characteristics of wave reflected from arbitrary surface", *Acoust. Phys.*, 34, 64–67 (1988).

[2] V.I.Il'ichev, V.S.Rabinovich, E.A.Rivelis and U.V.Hoha, "Acoustic field of a moving narrow-band source in oceanic waveguides", *Dokl.Akad.Nauk SSSR*, 304, 1123–1127 (1989).

[3] S.M.Grudsky, O.A.Obrezanova and V.S.Rabinovich, "Propagation of sound from a moving airborne source in an ice-covered ocean ", *Acoust.Phys.*, 42, 166–171 (1996).

[4] O.A.Obrezanova and V.S.Rabinovich, "Wave propagation from moving sources in the ocean", in Proceedings of the Third ECCOMAS Computational Fluid Dynamics Conference, Paris, 754–759 (1996).

Localized Structures in Active Nonlinear Waveguides and Bistable Lasers

S. V. Fedorov, G. V. Khodova, N. N. Rosanov

Institute of Laser Physics,
Research Center "S.I. Vavilov State Optical Institute"
St. Petersburg, Russia
E-mail: rosanov@ilph.spb.su

A. G. Vladimirov

St. Petersburg University, St. Petersburg, Russia

Localized (soliton-like) structures in dissipative (with gain and losses) optical systems are in the lowest order described by the generalized complex Ginzburg-Landau equation for the field slowly varying envelope [1,2]. Depending on scheme, geometrical dimensionality of the problem D varies from 1 to 3. For greater D variety of localized structures is more rich and their study is more complicated, involving combination of rigorous and approximate analytical and numerical approaches. Taking into account of such important factors as finite relaxation time of nonlinearity and frequency dispersion needs further generalization of the initial Ginzburg-Landau equation. In this report we review the basic features of the localized dissipative optical structures in laser schemes of different geometrical dimensionality D.

The case $D = 1$ corresponds to (i) waveguides with gain and nonlinear losses and (ii) bistable wide-aperture lasers with single-mode operation in one of the two transverse directions. Here we present results of bifurcation and stability analysis of stationary localized structures. A discrete spectrum of single and coupled localized structures with different shape of the envelope is found. They are stable within some interval of the control parameter – pumping. With increase of pumping the Andronov-Hopf bifurcation takes place, stationary structures become unstable, and pulsing localized structures arise. Because the Ginzburg-Landau equation in the limit of fast nonlinearity has symmetry connecting with Galileian transformation, there is degeneracy of transverse velocity of localized structures (they can move with arbitrary transverse velocity). This degeneracy is broken for nonlinearity with finite relaxation time and finite width of the medium spectral line. The results depend on the type of scheme; motionless localized structures exist within some interval of parameters transforming into moving structures outside the interval. Interaction of localized structures depends strongly on their relative velocity (which gives an effective time of interaction) and may result in change of their total number.

For $D = 2$ (bistable wide-aperture lasers) we demonstrate new types of stable localized structures with wavefront dislocations of different order, and structures with axially asymmetric intensity distribution rotating with constant angular velocity. Their interaction may be also weak or strong depending on their relative velocity. An approximate method of momenta (similar to the variational approach) shows also existence of 3D-structures – "laser bullets" – in continuous medium with frequency dispersion, nonlinear gain and losses.

These investigations were supported by the Russian Foundation for Basic Research (grant 95-02-03830a) and the International Science and Technology Center (project 666).

References

[1] N.N. Rosanov. *Transverse Patterns in Wide-Aperture Nonlinear Optical Systems.* Progress in Optics (ed. E. Wolf), 1996, **35**, 1-60.

[2] N.N.Rosanov. *Optical Bistability and Hysteresis in Distributed Nonlinear Systems.* Moscow, Nauka, Fizmatlit, 1997 (in Russian).

Modeling Heart Pulse Propagation in Jointed Blood Vessel

I.T. Selezov

Department of Wave Processes, Institute of Hydromechanics, Nat. Acad. of Sci., Ukraine
8/4 Sheliabov Str., 252057 Kyiv, Ukraine; e-mail: selig@selig.nimetz.kiev.ua

In this paper a new physical-mathematical model is presented for the heart pulse propagation in the presence of jointed vessels of different radiuses a, thicknesses h, Young modulus E and densities ρ. The corresponding initial boundary value hydroelastic problem is stated for the heart pulse initiated from the input vessel cross-section and propagated through a joint. The vessel motion is governed by the equations of the theory of shells, the blood motion by the quasi-one-dimensional model for the velocity and pressure averaged over the cross-section. It should be noted that in the case of jointed vessels not only convenient circumferential stresses have to be considered, but the bending stresses must be taken into account, unlike an uniform vessel. The problem is solved on the basis of the Laplace transform approach with a consequent numerical inversion. It follows from calculations that for differences of properties up to 10% the bending stresses can be the same order as circumferential stresses of the static prestressed state.

The problem considered is of great interest for more and more often arising situation associated with the organ implantation. In this case the conjunction of donor and host vessels results to appearing a new jointed vessel of different geometrical, mechanical, biophysical and other properties. Therefore, it can be expected that in the neighbourhood of a joint the sharp concentration of fields above mentioned will take place. Moreover, the equilibrium state between the donor and host vessels can not be reached fundamentally.

Excitation of Wave Field in a Triangle Area

Andrey V. Shanin

Moscow State University, Physics department; e-mail: shanin@ort.ru

The problem of excitation of wave field in a flat equilateral triangle area with impedance boundaries is studied. The triangle can be a cross-section of a waveguid or resonator. The Helmhotz equation is defined in the triangle:

$$\Delta u + k_0^2 u = 0,$$

and boundary conditions on the sides are

$$\partial u/\partial n - iuk_0 \sin\beta = \Phi$$

where n is the internal normal, $\sin\beta$ is parameter, Φ is given function on the sides. The functional equation is derived to solve the problem for each of three different types of symmetry of the field (for $\nu = 0, 1, -1$):

$$(\sin\varphi + \sin\beta)\hat{u}(\varphi) + e^{-i(k_0 L\cos\varphi - 2\pi\nu/3)}(\sin(\varphi - \frac{2\pi}{3}) + \sin\beta)\hat{u}(\varphi - \frac{2\pi}{3}) +$$
$$+e^{-i(k_0 L\cos(\varphi - pi/3) + 2\pi\nu/3)}(\sin(\varphi + \frac{2\pi}{3}) + \sin\beta)\hat{u}(\varphi + \frac{2\pi}{3}) =$$
$$= \hat{p}(\varphi) + e^{-i(k_0 L\cos\varphi - 2\pi\nu/3)}\hat{p}(\varphi - \frac{2\pi}{3}) + e^{-i(k_0 L\cos(\varphi - \pi/3) + 2\pi\nu/3)}\hat{p}(\varphi + \frac{2\pi}{3})$$

where $$\hat{u}(\varphi) = \int_0^L u(l)e^{-ik_0 l\cos\varphi}dl, \qquad \hat{p}(\varphi) = \frac{i}{k_0}\int_0^L \Phi(l)e^{-ik_0 l\cos\varphi}dl.$$

The integration is performed over a side of the triangle. Thus, the dependent variable in the equations is the Fourier transform of the field on a side of the triangle that is taken with tree different values of spectral variable.

The functional equation can be solved explicitly and the solution is presented in the form of infinite sum over the poles of a rational expression. It is important to note that the solution does not use directly the method of reflection that makes possible to find the solution in the triangle with ideal boundaries.

Penumbra Diffraction in the Quantization of Concave Billiards

H. Schanz
Max-Planck-Institut für Physik komplexer Systeme
Bayreuther Str. 40, Haus 16, 01187 Dresden, Germany
holger@mpipks-dresden.mpg.de
H. Primack, I. Ussishkin, U. Smilansky
The Weizmann Institute of Science, Rehovot 76100, Israel

The semiclassical description of billiard spectra is extended to include the diffractive contributions from orbits which are nearly tangent to a concave part of the boundary. The leading correction for an unstable isolated orbit is of the same order as the standard Gutzwiller expression itself. The importance of the diffraction corrections is further emphasized by an estimate which shows that for any large fixed k almost all contributing periodic orbits are affected. The theory is tested numerically using the annulus and the Sinai billiard.

Scattering Frequencies of Cylindrical Slotted Resonators

E. V. Chernokozhin, Yu. V. Shestopalov
Moscow State University, 119899, Moscow, Russia

We consider a cylindrical slotted resonator with the 2-D cross section $\Omega = \{(x, y) : x^2 + y^2 < R^2\}$ and permittivity ϵ_2 of the dielectric medium filling the domain Ω. The boundary is assumed perfectly conducting and infinitely thin. The slot $S = \{(r, \phi) : r = R, \phi < l\}$ connects the cylinder with the external domain having the permittivity ϵ_1.

Using the methods [1] of analytical operator-functions and the perturbation theory, we derive the formulas for the complex scattering frequencies of this resonator as functions of the angular size l of the slot taken as a small parameter. The imaginary parts of the scattering frequencies are separated and proved to be negative.

The cases of the H- and E-polarization, as well as of the symmetric and antisymmetric fields (with respect to the axis of the cross section), are considered separately. For small widths of the slot, we show that the scattering frequencies are located near the eigenfrequencies of the closed circular resonator, but the deviations from the eigenfrequencies strongly depend on the type of polarization and the symmetry properties of the oscillations. In the case of symmetric H-polarized oscillations, the deviations are of an order of $\beta = \pi / \ln \frac{1}{l}$, in the cases of H-polarized antisymmetric and E-polarized symmetric oscillations, of an order of l^2, and in the case of antisymmetric E-polarized oscillations, of an order of l^4. In all cases under consideration, the imaginary part of the scattering frequency is proportional to the square of the distance between the real part and the corresponding eigenfrequency.

Similar methods are applied to the analysis of the problem of diffraction of a plane wave by this slot resonator. Formulas for the scattered field are obtained. It is shown that if the frequency of the incident field lies outside a small neighbourhood of an eigenfrequency, then the slot has a small influence on the scattered field, and this influence tends to zero when the slot vanishes. On the contrary, if the (real) frequency of the incident field is close enough to the real part of any scattering frequency of the slot resonator, then the influence of the slot on the scattered field is of the same order of magnitude as the field scattered by the closed cylinder and tends to a finite nonzero limit when the slot vanishes.

References

[1] Shestopalov, V.P. and Shestopalov, Yu.V., *Spectral Theory and Excitation of Open Structures*, London: Peter Peregrinus, 1996.

Transient Impulsive Response from an Interface between Two Media

Hiroshi Shirai

Department of Electrical and Electronic Engineering
Faculty of Science and Engineering, Chuo University
1-13-27 Kasuga, Bunkyo-ku, Tokyo 112 Japan.

Recently, as the transmission via high speed digital transmission channels becomes proliferated, the precise propagation analysis of the transient electromagnetic wave in a transmission channel such as an optical fiber has becomes necessary. While one knows quite well the total reflection and Goos-Hänchen shift in the frequency domain, the corresponding effects in the time domain are not discussed in detail yet. Therefore, it is an interesting problem to analyze the fundamental propagation characteristics of a transient electromagnetic wave. Especially, the analysis of the time transient field propagation excited by an impulsive source inside a dielectric waveguide such as an optical fiber is important for many practical applications. Based on the above background, the transient electromagnetic field excited by an impulsive line source near a dielectric half space has been rigorously formulated here.

Two approaches have been tried to derive the closed form representations. One is the Cagniard-deHoop (CH) method, which is essentially a frequency domain approach. Starting from the deformation of the contour of the corresponding time harmonic plane wave spectral integral representation of each wave constituents, CH method enable us to extract out the desired time domain result from the definition of the Laplace transform integral. The other method is called the Spectral Theory of Transients (STT). In this formulation, the field is expressed directly in the time domain as an angular (spectral) integral of time dependent plane wave, henceforth termed the STT integral. Evaluation of this integral via function theoretic techniques yields exact closed form expressions for the field, that are found to be exactly the same as those derived from the CH method. One of the main advantage of the STT is that it is a strictly time domain technique, so that one can readily explain the structure of the time dependent field by the (time varying) spectral singularities in the spectral plane. The loci of the time varying spectral singularities coincides with the corresponding deformed contour of the CH method. The STT can also be extended to evaluate the propagation and scattering of pulsed beam (PB) fields, while the CH method cannot be used.

Wavefront approximations for the direct, reflected and head waves are also derived and compared with exact one. Several numerical results as well as the three dimensional wave distribution will be prepared to clarify the wave propagation characteristics of the direct, reflected, transmitted and head wavefronts, and the evanescent field near the dielectric interface.

Quantum Dynamics of an Electron in Magnetic Billiards with Hard Boundaries

E. Akkermans, R. Narevich, D. Spehner†

Department of Physics, Technion, 32000 Haifa, Israel.
†Present address: Institut de Recherche sur les Systèmes Atomiques et Moléculaires Complexes (IRSAMC), Université Paul Sabatier, 118 route de Narbonne, 31062 Toulouse, France.
E-mail: spehner@irsamc2.ups-tlse.fr

We present some asymptotical methods for solving the Schrödinger equation and studying the magnetic response of non-interacting electrons on a billiard submitted to a perpendicular uniform magnetic field.

Historically, this problem traces back to the Bohr-van Leeuwen theorem [1] stating the absence of classical orbital magnetism. Later on, Landau showed the existence of a nonzero magnetization in his quantum approach [2]. An extension to finite systems was provided by Teller [2] who compared the bulk and edge contributions to the magnetism. This problem was more recently shown to be relevant to various situations in condensed matter physics. In the low magnetic field regime, defined by the condition $\Phi \ll \Phi_0$, where Φ is the magnetic flux through the system and $\Phi_0 = \frac{hc}{e}$ is the quantum flux, recent experiments performed on small metallic systems did show how important is the effect of the boundary. It determines the nature of the classical motion (integrable or chaotic). On the theoretical side, by using numerical and semiclassical methods the magnetic susceptibility was shown to be reduced and to present large fluctuations in the chaotic case, whereas it is larger and varies slowly with the field in the integrable case [3]. In the opposite limit $\Phi \gg \Phi_0$ of high fields, we are in the so-called integer quantum Hall effect regime, where the edge states associated with the boundary play a prominent role and are likely not to be strongly affected by the presence of disorder in the system [4]. In this work, we shall concentrate on the problem of non-interacting electrons in a half-plane and in a disc with Dirichlet boundary conditions.

The classical dynamics allows for a natural distinction between the bulk and edge motion: the former consists of complete circles and is not influenced at all by the boundary, whereas the later consists of skipping trajectories with specular reflections on the boundary. The usual Einstein-Brillouin-Keller quantization rules preserve this splitting between bulk and edge states by giving different quantization rules for each of them. Therefore a more careful semiclassical analysis is needed in order to describe the continuous increase of the energy of the electronic states as their average distance from the boundary decreases. This can be done by taking the coalescence of caustics with the boundary into account, which is due to those edge orbits completing nearly full circles before undergoing reflections. The spectrum of the half-plane and of the disc is computed by using respectively the usual WKB method and the more general 'comparison equation method' (see for example [5]). We find in a limiting case and by comparison with the numerical results that these solutions approximate very well the exact spectrum even for the lowest energy levels. Moreover, they are valid for any strength of the magnetic field, thus providing a way to study both the high and the low field regimes in more detail.

In order to describe the orbital magnetism, it is more convenient to study spectral quantities as the heat kernel. Using a method inspired from the work of Stewartson and Waechter [6], we find the exact and the small time asymptotic expansion of the heat kernel for the half-plane. This method is powerful enough to determine in principle all the asymptotic series. The result is compared with that obtained by another method, the Balian-Bloch multiple scattering expansion [7], which is shown to be less powerful for practical calculations in this situation. The perimeter correction to the Landau diamagnetism is calculated by assuming it is the largest correction for a generic billiard with smooth boundaries at sufficiently high temperature. Our result differs from a previous result obtained by Robnik [8].

References

[1] van Leeuwen J H 1921 *J. Phys. (Paris)* **2** 361

[2] Landau L 1930 *Z. Phys.* **64** 629; Teller E 1931 *Z. Phys.* **67** 311

[3] for a review see Richter K, Ullmo D, Jalabert R A 1996 *Phys. Reports* **276** 1 and references therein.

[4] MacDonald A H and Středa P 1984 *Phys. Rev.* B **29** 1616; Halperin B I 1982 *Phys. Rev.* B **25** 2185

[5] Berry M V, Mount K E 1972 *Rep. Prog. Phys.* **35** 315

[6] Stewartson K and Waechter R T 1971 *Proc. Camb. Phil. Soc.* **69** 353

[7] Balian R and Bloch C 1970 *Ann. Phys. (N.Y.)* **60** 401, 1971 **64** 271

[8] Robnik M 1985 *J. Phys. A: Math. Gen.* **19** 3619

The Integral Equation in Multidimensional Spectral Inverse Problem

A. S. Starkov

Department of Higher Math., Academy of Criotechnology,
Lomonosova 9, 191002, St-Petersburg, Russia

Let $M \subset R^n$ be a bounded domain with a boundary $S \in C^\infty$ and ρ be a positive infinitely smooth function (" density"), $\rho \in C^\infty(M)$. Consider a spectral family of the Dirichlet problems for operator $A{:} = \rho^{-1}\triangle$

$$-\rho^{-1} \triangle u - \lambda u = 0 \quad u|_S = f \tag{1}$$

Let $\{\lambda_k\}_{k=1}^\infty$; $0 < \lambda_1 < \lambda_2 \leq \ldots$ be its spectrum and $\{\phi_k\}_{k=1}^\infty$ be the eigenfunctions: $A\phi_k = \lambda_k \phi_k$. We denote by $\psi_k{:} = \lambda_k^{-1}\partial_n \phi_k|_S$ the trace on the boundary of the outer normal derivatives of ϕ_k.

The spectral inverse problem is to recover ρ using the given set $\{\psi_k\}_{k=1}^\infty$.

Our considerations are based on the integral identity

$$\int_M \rho h_1 h_2 dx = \int_{S^2} R(s_1, s_2) h_{1S}(s_1)\, h_{2S}(s_2) d^2 S,$$

where h_1, h_2 are smooth harmonic functions, h_{1S}, h_{2S} are their traces on S, and

$$R(s_1, s_2) = \sum_{k=1}^\infty \psi_k(s_1)\psi_k(s_2), \qquad \text{or} \qquad R(s_1, s_2) = \partial^2_{n_x n_y} A^{-1} G(x, y, \lambda)|_{S,\lambda=0}$$

is known kernel. Here $G(x, y, \lambda$ is Green function for the problem (1). Taking into account the integral representation of any harmonic function h

$$h = \int_S \partial_{n_y} G(x, s, o)\, h_S\, ds$$

we obtain integral equation

$$\int_M K(x, s)\rho(x)\, dx = R(s_1, s_2) \tag{2}$$

with $K(x, s) = G(X, s_1, 0)G(x, s_2, 0)$. Equation (2) is Fredholm equation of the first kind and for its regularization we use the compactness of the set of admissible densities. Hence, it is possible to find ρ from the problem of minimization of the functional

$$||K\rho - R||$$

on some compact in $L_2(M)$. Here K is integral operator with kernel $K(x, s)$ and $||\cdot||$ means the norm in $L_2(S^2)$.

Asymptotically Double Natural Frequencies of Thin Elliptical Cylindrical Shell Vibrations

A. V. Krotov, P. E. Tovstik

St.-Petersburg University

The equations of the two dimensional shell theory contain the geometric small parameter ε (connected with the relative shell thickness) which is used for asymptotic expansions. In some cases the eigenfunctions of the linear boundary value problems are localized near the weakest lines of the shell middle surface [1] and they are similar to the functions of "jumping ball" type in acoustics [2]. Here we discuss the asymptotically double eigenvalues [3] in shell vibration problems.

We study low-frequency vibrations of the thin shell of the form of an elliptical cylinder with the freely supported edges. After separation of variables $w(x,y) = w(x)\sin(\pi y L^{-1})$ and after some simplifications the problem is reduced to the ordinary system of the eighth order

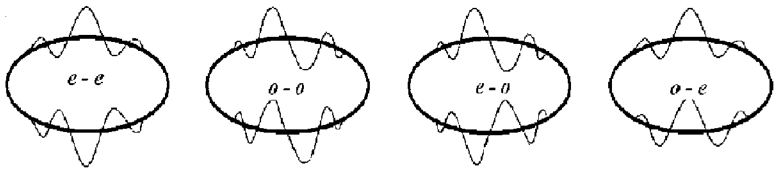
$$\varepsilon^4 \frac{d^4 w}{dx^4} - \lambda w + k(x)\Phi = 0, \qquad \varepsilon^4 \frac{d^4 \Phi}{dx^4} - k(x) w = 0, \tag{1}$$

where $k(x)$ is the curvature of the ellipse shell cross-section, λ is the unknown frequency parameter.

The eigenfunctions are localized near two generatrices which cross ellipse in points $x_*^{(1,2)} = \pm\pi/2$ with the minimal values of curvature $k(x)$. The asymptotic expansion of eigenfunctions near the weakest point x_* is

$$w(x,\varepsilon) \simeq \sum_{k=0}^{\infty} \varepsilon^{k/2} w_k(\xi) \exp\left[i(\varepsilon^{-1/2} p\xi + a\xi^2)\right], \quad \xi = \varepsilon^{-1/2}(x - x_*), \tag{2}$$

where w_k are polynomials in ξ, constant $p > 0$ is real, and the condition $\operatorname{Im} a > 0$ gives the decreasing of function (2) away from $x = x_*$. As the $\operatorname{Re} w$ so the $\operatorname{Im} w$ are asymptotic expansions of the real eigenfunctions. As we have two weakest points $x_*^{(1,2)}$ there are four eigenfunctions with the same expansions $\lambda^{(j)} \simeq \lambda_0 + \varepsilon\lambda_1 + \ldots$ of eigenvalues $\lambda^{(j)}$, $j = 1,2,3,4$. These eigenfunctions are even and/or odd with respect points $x = 0, \pi/2, \pi, 3\pi/2$ and they are shown schematically in figure.



The corresponding exact eigenvalues λ_{ee}, λ_{oo}, λ_{eo}, and λ_{oe} slightly differ from each other. These eigenvalues are found by numerical solution of system (1). From numerical results it follows that for small ε the eigenvalues make two groups: $\{\lambda_{ee}, \lambda_{oo}\}$ and $\{\lambda_{eo}, \lambda_{oe}\}$. We study difference δ_0 of eigenvalues between groups and their differences δ_1 and δ_2 within groups

$$\delta_0 = 1/2[\lambda_{ee} + \lambda_{oo} - (\lambda_{eo} + \lambda_{oe})], \qquad \delta_1 = \lambda_{ee} - \lambda_{oo}, \quad \delta_2 = \lambda_{eo} - \lambda_{oe}. \tag{3}$$

The numerical results show that $|\delta_0| \gg \{|\delta_1|, |\delta_2|\}$ for small ε. The difference δ_0 oscillates and the more small differences δ_1 and δ_2 do not change their signs as $\varepsilon \to 0$

$$\delta_0 \sim e^{-\alpha_0/\varepsilon} \sin(\beta_0/\varepsilon + \gamma_0), \quad \alpha_0 > 0, \qquad \delta_j \sim e^{-\alpha_j/\varepsilon}, \quad \alpha_j > \alpha_0, \quad j = 1,2. \tag{4}$$

We try to find theoretical explanation of numerical results (4).

References

[1] Tovstik P.E. *Stability of thin shells.* – "Nauka", Moscow, 1995, 320 p. (Russian)

[2] Babich V.M., and Buldyrev V.S. *Asymptotic methods in short wave diffraction problems.* – "Nauka", Moscow, 1972, 456 p. (Russian)

[3] Tovstik P.E. *Asymptotically double eigenvalues in the problems of shell dynamics and buckling //* Int. seminar "Day of Diffraction'96". Abstracts. St.-Petersburg. 1996. P. 33.

Diffraction of Internal Waves by a Submerged Circular Cylinder at Forward Speed in Two-layer Fluid

T. I. Khabakhpasheva, I. V. Sturova

630090 Novosibirsk prosp. Lavrentyeva 15
Lavrentyev Institut of Hydrodynamics
E-mail: sturova@hydro.nsc.ru

The sea propagation in the presence of different underwater obstacles and diffraction effect are the matters of great importance for water engineering and underwater navigation. The problem of wave scattering by a submerged body has been investigated in detail for regular linear surface waves, however, significant scattering of internal waves may occur when underwater objects are placed near the region of high gradient density.

The simplest example of stratified fluid is the two-layer fluid. A particular case is a homogeneous fluid with free surface, as the air density is assumed negligible compared with the water density. The linear problem of surface wave diffraction was studied both for a restricted and uniformly moving submerged body. This problem is a part of linear theory of seakeeping.

In the present work diffraction of internal waves by a submerged body in an uniform current of a two-layer fluid is considered. The layers are infinitely deep, and the flow are two-dimensional. The linearized potential theory is used for the inviscid and incompressible fluid. The explicit solution for the circular cylinder, which is either below or above the interface, is given in the form of rapidly converging series. The flow domain is mapped conformally onto a circle and the solution of the Laplace equation for complex potential is expressed in the form of expansion by fundamental solutions satisfying the boundary condition on a cylinder. The interface condition leads to the recursive relation allowing the coefficients of the expansion to be updated. The main difficulties of constructing the solution are associated with determining first several terms of series in a such manner that the radiation condition at infinity is satisfied. By some transformation this problem reduced to the partial differential equation, which may be solved in terms of special functions. The practical computation of the velocity potential is reduced almost to hand calculations.

Numerical results are provided for the exciting forces, wave resistance and lift which may be useful in testing numerical methods used for the study of internal wave diffraction by submerged body of arbitrary form.

Theory of Strip Loop Antennas in a Resonant Magnetoplasma

A. V. Kudrin, E. Yu. Petrov

University of Nizhny Novgorod, 23 Gagarin Ave., Nizhny Novgorod 603600, Russia

T. M. Zaboronkova

Radiophysical Research Institute, 25 B. Pecherskaya st., Nizhny Novgorod 603600, Russia
Fax: +7 8312 369902; E-mail: zabr@nirfi.sci-nnov.ru

Many papers have dealt with the behaviour of dipole and loop antennas in magnetized plasmas. However, a satisfactory solution to the current distribution and input and radiation characteristics of a loop antenna in a resonant magnetoplasma have not been found yet. By resonant magnetoplasma we mean a cold, magnetized plasma in which the refractive-index surface of one of the principal modes extends to infinity. In majority of papers, when dealing with loop antennas in such plasmas, the electrically small radiators are usually taken that allows using a uniform current distribution along the antenna wire. Recently Kondrat'ev et al. (1992) and Zaboronkova et al. (1993) have shown that such a distribution is a somewhat degenerated case since any, even rather small, violation of the current uniformity in a thin loop antenna influences appreciably its radiation characteristics under the resonant-plasma condition.

This work is devoted to determining the current distribution, impedance, and radiation pattern of a strip-loop antenna in a resonant magnetized plasma. We consider the loop antenna consisting of an infinitely thin, perfectly conducting strip coiled into the ring. The antenna is oriented perpendicularly to an ambient magnetic field and excited by the given EMF whose field $\mathbf{E}^{ext}$ has the only azimuthal component E_φ^{ext} distinct from zero at $\rho = a$, $|z| \leq b$ (here ρ, φ, z are the cylindrical coordinates, a is the loop radius, and b is the half width of the strip). Expanding formally the only non-zero (azimuthal) component j_φ of the surface current in the loop into a series

$$j_\varphi(\varphi, z) = I_0(z) + \sum_{m=1}^{\infty} \Big(I_{m,c}(z) \cos m\varphi + I_{m,s}(z) \sin m\varphi \Big), \tag{1}$$

we employ the boundary conditions

$$E_\varphi = -E_\varphi^{ext}, \;\; E_z = 0 \;\; (\rho = a, |z| \leq b) \tag{2}$$

for the tangential components $E_{\varphi,z}$ of the electric field excited by the current (1) to obtain the integral equations for the quantities I_0, $I_{m,c}$, and $I_{m,s}$. The boundary condition for the E_φ-component yields the equations in the form

$$\begin{aligned}
&\int_{-b}^{b} \mathcal{K}_0^{(0)}(z-z')\, I_0(z')\, dz' = A_0^{(0)}, \\
&\int_{-b}^{b} \Big[\mathcal{K}_m^{(1)}(z-z')\, I_{m,c}(z') - \mathcal{K}_m^{(2)}(z-z')\, I_{m,s}(z')\Big]\, dz' = A_m^{(1)}, \\
&\int_{-b}^{b} \Big[\mathcal{K}_m^{(2)}(z-z')\, I_{m,c}(z') + \mathcal{K}_m^{(1)}(z-z')\, I_{m,s}(z')\Big]\, dz' = A_m^{(2)}, \qquad |z| \leq b,
\end{aligned} \tag{3}$$

where the constant quantities $A_m^{(i)}$ are to be determined from expanding E_φ^{ext} into a series similar to (1). We have shown that the kernels $\mathcal{K}_m^{(i)}$ of the above equations may be divided into the singular, $K_m^{(i)}$, and nonsingular, $F_m^{(i)}$, terms in the manner $\mathcal{K}_m^{(i)} = K_m^{(i)} + F_m^{(i)}$, $i = 0$ $(m = 0)$, $i = 1, 2$ $(m = 1, 2, \ldots)$ and found that, for a rather narrow strip ($b \ll a$), the singular terms are of the logarithmic character: $K_m^{(i)}(\zeta) \sim \ln(|\zeta|/a)$. We first solve eqs.(3) for some special cases and then verify that the solutions do satisfy the latter boundary condition for E_z (see (2)).

Based on the results obtained, we have analysed for resonant plasmas how the current nonuniformity influences the antenna characteristics, particularly the radiation resistance, the radiation pattern, and the distribution of the radiated power of the spatial spectrum of emitted waves. Some results of concrete computations will be reported.

Acknowledgments: This work was supported in part by the grant No. 96-02-18666 from the Russian Foundation of Fundamental Investigations and in part by the INTAS grant 93-2492-ext within the ICFPM research program.

References

[1] I. G. Kondrat'ev, A. V. Kudrin, T. M. Zaboronkova, *Radio Sci.* **27**, No. 2, pp. 315–324 (1992).

[2] T. M. Zaboronkova, I. G. Kondrat'ev, A. V. Kudrin, *Radiotekhnika i Electronika* **38**, No. 8, pp. 1451–1460 (1993).

Plane Wave Reflection Coefficients for Two Weakly Anisotropic Half-spaces in Welded Contact

Matthias Zillmer, Dirk Gajewski

Institute of Geophysics, University of Hamburg
Bundesstr. 55, 20146 Hamburg. Germany

Boris M. Kashtan

Institute for Physics, St. Petersburg State University,
Ulyanovskaya 1, Petrodvorets, St. Petersburg 198904, Russia

The interaction of plane waves with a plane boundary between two weakly anisotropic elastic half-spaces is investigated. The anisotropy dealt with is of a general type.

Classical perturbation theory [4] is applied on a 6×6 non-symmetric real eigenvalue problem to calculate first-order corrections for the polarization and stress of the plane waves [7].

Explicit expressions for qP- and qS-wave reflection coefficients are derived. These relations are simplified for the case of a weak contrast interface between two halfspaces with similar elastic properties. They represent a generalization of previously obtained coefficients valid for isotropic [1] or transversely isotropic [2,6] media.

The derived formulae are applied on the problem of elastic waves reflected at a boundary separating an isotropic and a hexagonal halfspace. The hexagonal halfspace represents a system of aligned cracks which is arbitrarily oriented with respect to the receiver profile. The reflection coefficients depend on the orientation of the crack system, the crack parameters [3] and the Poisson ratio of the isotropic matrix of the fractured rock. An investigation restricted to normal incidence of shear waves has been given in [5].

References

[1] Aki, K. & Richards, P.G., 1980, *Quantitative Seismology, Vol.1,* San Francisco

[2] Banik, N.C., 1987, An effective anisotropy parameter in transversely isotropic media, *Geophysics,* **52**, 1654–1664

[3] Hudson, J.A., 1980, Overall properties of a cracked solid, *Math. Proc. Camb. Phil. Soc.* , **88**, 371–384

[4] Landau, L.D., & Lifschitz, E.M., 1958, *Quantum Mechanics - Non Relativistic Theory* , Pergamon Press, London

[5] Li, X.-Y. & Crampin, St., 1993, Variation of reflection and transmission coefficients with crack strike and crack density in anisotropic media, *Geophys. Prosp.* , **41**, 859–882

[6] Thomsen, L., 1993, Weak anisotropic reflections: in *Offset–dependent Reflectivity* , eds. Castagna,J.P. & Backus,M.M., SEG, Tulsa

[7] Zillmer, M., Gajewski, D. & Kashtan, B.M., 1997, *Geophys. J. Int* , **129**, in print

Plane Wave Scattering by a Step Discontinuity in an Impedance Plane

V. V. Zalipaev

V. A. Steklov Mathematical Institute, St.Petersburg

Diffraction by edges is one of the most important aspects of the scattering theory of electromagnetic, acoustic and seismic waves. Uniform geometrical theory of diffraction UGTD provides effective asymptotic solutions described the wave fields diffracted by wedges, half-planes and for the other related problems in the case of perfectly conducting boundaries. It is clearly that an extension of UGTD to describe the scattering on obstacles with impedance boundaries which absorb energy of incident waves would be considerable advance in developing the scattering theory. The Maluzhinets solution for the problem of plane wave reflection from a wedge with impedance boundaries (see [1]) enables to perform such an extension. One of the examples with a more complex geometry of reflecting boundary is a step discontinuity on a impedance plane. This scattering problem for the perfectly conducting boundary has been under consideration (see [2]).

In this report a high-frequency diffraction of a plane wave by 90-degree step discontinuity in an impedance plane boundary (the height of step is far greater than the wavelength) is analysed with the help of UGTD starting from the Maluzhinets solution. The time-harmonic scattering problem is two-dimensional. The total wave field as a scalar complex function satisfies the Helmholtz equation, impedance boundary conditions with different impedance coefficients corresponding to the cases of transverse electric and transverse magnetic polarization. The diffracted field must satisfy the principle of limiting absorption and Meixner condition at the edges of the boundary. The principal term of asymptotic solution to the problem which is uniform with respect to the angle of the plane wave incidence and angle of observation is derived. The excitation of primary, doubly and triply diffracted fields radiated from the upper edge as well as surface waves are considered (the lower edge does not radiate any cylindrical or surface waves owing to the 90-degree step). For simplicity the analytical details are given here for the 90-degree step discontinuity. A similar procedure is followed in the other examples with a more complex geometry. Numerical results are presented which demonstrates the importance of including multiple reflection and diffraction of surface waves for such a model. The comparison for the perfectly conducting and impedance step boundaries with same geometry are discussed.

References

[1] G. D. Maluzhinets // *Ann. d. Phys.*,**vol. 6**, 107 (1960)

[2] E. V. Jull // *Geophysics*, **vol. 55**, no. 6, 749 (1990)

Behavior of surface creeping waves at transition through a junction line on the boundary of a conducting body

N. Ya. Kirpichnikova, V. B. Philippov

V.A.Steklov Mathematical Institute, St.Petersburg

A conducting body with a smooth boundary is considered. The boundary of the body, which consists of a smooth cylindrical surface and a half-plane, has a continuous tangent plane, but the curvature of the normal section of the boundary has a discontinuity of the first kind at each point of the junction line. The behavior of two kinds (corresponding to Dirichlet boundary conditions and corresponding to Neumann boundary conditions) of creeping surface waves at transition through the junction line is studied.

Consider a convex homogeneous conducting body $\Omega \subset \mathbf{R}^3_+ = \{(x,y,z)|z \geq 0\}$ bounded by a surface $\partial\Omega$. The surface $\partial\Omega$ consists of a smooth cylindrical surface $\partial\Omega^-$ (with $z \geq 0$, $x \leq 0$)

with generator parallel to the y-axis and of the half-plane $\partial\Omega^+$ ($z = 0$, $x \geq 0$). The junction line $l_0 = \{(x, y, z) | x = z = 0\}$ coincides with the y-axis. The plane tangent to the surface $\partial\Omega$ is continuous on the junction line, but the curvature of the normal section of the surface $\partial\Omega$ has a discontinuity of the first kind at each point of the y-axis. The value of the discontinuity depends on the y-coordinate.

Let L be a ray lying in a plane coplanar to the x, z plane and belonging to the surface $\partial\Omega$. By definition, rays are extremals of the Fermat functional

$$\int \frac{ds}{c},$$

where ds is an element of the arc length of the curve along which the integration is carried out, and c is the velocity of propagation of the waves outside of Ω.

In this paper, we consider high-frequency asymptotic expansions of the solutions of the Heimholtz's equation concentrated in the vicinity of the boundary $\partial\Omega^-$. The problem of diffraction of creeping wave incident from the convex part of the smooth cylindrical surface to the half-plane is research. In conclusion, we note that the authors have applied a similar approach to the following quasi-two-dimensional problems:

(1) a problem with a jump of curvature in the case where the boundary $\partial\Omega^+$ is also curvilinear and the body is convex;

(2) the problem of diffraction of a creeping wave by the junction line of positive curvature and of negative curvature.

This research was partly supported by the Russian Foundation for Basic Research (Grant No. 96-01-00666).

Matrix Wiener-Hopf Factorization with Application to Diffraction of Elastic and Acoustic Waves

I. D. Abrahams

Department of Mathematics, Keele University
Keele, Staffs; ST5 5BG, U.K.

The Wiener-Hopf technique has, since its invention in 1931, proved an immensely important method in mathematical physics. It offers one of the very few approaches to obtaining an exact solution to a class of integral equations. Fields of application include diffraction of acoustic, elastic and electromagnetic waves, fracture mechanics, flow problems, diffusion models, geophysical applications, to name but a few. The solution method, although initially proposed to follow via construction of the integral equation, now usually proceeds directly from the boundary value problem to an equation in the complex plane defined in an infinite strip. This is accomplished by Fourier transformation (or other appropriate transform) or Green's theorem, and the essential physical details are manifested in the singularity structure of the Fourier transformed difference kernel often called the Wiener-Hopf kernel, $K(\alpha)$ (Noble [4]), where α is the transform parameter. This kernel is usually arranged to be singularity free in a finite-width strip of the complex α-plane which contains the real line as $|\alpha| \to \infty$. Further, this strip need not necessarily be straight (i.e. enclose the whole real line) and is denoted henceforth as $\mathcal{D}$. Solution of the Wiener-Hopf equation is straightforward once $K(\alpha)$ is decomposed into a product of two functions $K^+(\alpha)$, $K^-(\alpha)$, where $K^+(\alpha)$ is regular and zero-free in the region above and including the strip $\mathcal{D}$, denoted $\mathcal{D}^+$, and $K^-(\alpha)$ is regular and zero-free in the region below and including $\mathcal{D}$, denoted $\mathcal{D}^-$. Further, $K^\pm(\alpha)$ must have at worst algebraic growth as $|\alpha| \to \infty$ in $\mathcal{D}^\pm$, respectively. Cauchy's integral theorem provides a convenient method for obtaining the explicit sum factorization of a function, e.g. of the form $g(\alpha) = g^+(\alpha) + g^-(\alpha)$ where $\pm$ denote the above analyticity properties, and so by exponentiation we deduce (see theorem C of Noble [4]),

$$K^\pm(\alpha) = \exp[g^\pm(\alpha)] = \exp\left\{\frac{\pm 1}{2\pi i}\int_{-\infty}^{\infty} \frac{\log K(\zeta)}{\zeta - \alpha}\, d\zeta\right\}, \tag{1}$$

where ζ is a contour in $\mathcal{D}$ and α lies above (below) ζ for $K^+(\alpha)$ $(K^-(\alpha))$.

For problems in which $K(\alpha)$ is a scalar function, then, apart from some technical difficulties associated with computing integrals of the above form, the Wiener-Hopf procedure goes through without hitch. However, in more complex boundary value problems, it is not unusual to obtain a matrix functional Wiener-Hopf equation with a square matrix kernel $\mathbf{K}(\alpha)$. This presents several potential obstacles to allow the solution scheme to go through. It is the aim of this talk to tackle one such difficulty, namely the factorization procedure (1). For any matrix function, $\mathbf{K}(\alpha)$, the logarithm $\mathbf{G}(\alpha) = \log(\mathbf{K}(\alpha))$ can be defined, for example by its power series expansion $\log[\mathbf{I} + (\mathbf{K}(\alpha) - \mathbf{I})] = \mathbf{K}(\alpha) - \mathbf{I} - \frac{1}{2}(\mathbf{K}(\alpha) - \mathbf{I}) + \ldots$ where $\mathbf{I}$ is the identity matrix, or otherwise. Hence the sum split of $\mathbf{G}(\alpha)$ is accomplished by the Cauchy type integrals acting on each element of the matrix. However, the final step to obtain the product factors, i.e.

$$\exp\{\mathbf{G}^+(\alpha) + \mathbf{G}^-(\alpha)\} = \exp\{\mathbf{G}^+(\alpha)\}\exp\{\mathbf{G}^-(\alpha)\} \tag{2}$$

is true if and only if $\mathbf{G}^+(\alpha)$ and $\mathbf{G}^-(\alpha)$ commute. General matrices $\mathbf{K}(\alpha)$ will not have commutative factors, and although no procedure is presently available for tackling such arbitrary matrix Wiener-Hopf problems, the existence of these factors has been established in all cases. Fortunately, many physically interesting problems naturally give rise to kernels which although intrinsically matrix in form, have a commutative factorization or can be reworked (by pre- and/or post-multiplication by suitable matrices) into such form (Khrapkov [3]).

The aim of this talk is to demonstrate a procedure for obtaining non-commutative matrix factors which have algebraic growth. Usually matrix kernels yield non-commutative factorizations because they contain multiple branch cut functions, infinite sequences of poles or zeros, or a combination of both of these singularity structures. The new procedure enables explicit but approximate non-commutative matrix factors to be derived. It is applicable to many kernels of this, up-to-now unsolved, matrix class, and is based on the replacement of certain components of the matrix function by Padé approximants. (A detailed discussion is to be found in Abrahams [1].) This allows the level of approximation to be increased to very high accuracies whilst at the same time offering a surprisingly simple factorization form. The approach also preserves the singularity structure of the component matrix factors which is essential for a uniformly valid solution.

For ease of exposition the method will be illustrated by way of a simple but non-trivial example (see Abrahams [2]): a linear elastic half-space, occupying the region $y > 0$, where (x, y, z) are cartesian coordinates, has a free boundary on $x > 0$, $y = 0$, and has imposed time-harmonic displacements on $x < 0$, $y = 0$. Via the substitution of a scalar function in the kernel by its Padé approximant, an approximate solution to this boundary value problem is obtained explicitly for three different forcing cases. The approximate factorization technique described herein is simple to apply and a maximum error bound is easily obtained. Numerical evaluation of the explicit results reveal that convergence to the exact solution is extremely rapid (i.e. only a small number of poles and zeros are required in the Padé approximant), which is confirmed by a global energy balance calculation, and so numerical calculations are extremely rapid.

References

[1] I. D. Abrahams, *Radiation and scattering of waves on an elastic half-space; a non-commutative matrix Wiener-Hopf problem*, J. Mech. Phys. Solids, 44(12) (1996), pp. 2125–2154.

[2] I. D. Abrahams, *On the solution of Wiener-Hopf problems involving noncommutative matrix kernel decompositions*, *SIAM J. Appl. Math.* 57(2) (1997), pp. 541–567.

[3] A. A. Khrapkov, *Certain cases of the elastic equilibrium of an infinite wedge with a non-symmetric notch at the vertex, subjected to concentrated forces*, Appl. Math. Mech. (PMM), 35(4) (1971), pp. 625–637.

[4] B. Noble, *Methods Based on the Wiener-Hopf Technique*, Second ed., Chelsea Press, New York, 1988.

The Head Waves for the Problem of Diffraction on Obstacles with GIBC on their Surface

V.S. Buldyrev, N.G.Gelfreikh

St.Petersburg University

The high-frequency stationary plane diffraction problem with Generalized Impedance Boundary Conditions (GIBC) is investigated. GIBC include not only the function and its normal derivative but also second and fourth order derivatives along the direction tangent to the obstacle's boundary. First we consider the model problem for diffraction on a circle. Depending on the coefficients in GIBC scattered wave field besides the reflected wave can include the head wave. We have found the conditions for GIBC coefficients under which the head waves do exist. The high-frecuency asimptotic for the head waves is derived. There are two head waves simultaneously in some special cases. This possibility is described in the paper too. All results are generalised for an arbitrary convex obstacle.

Author index

Abrahams I.D., 48
Akhmeteli A.M., 7
Akkermans E., 40
Alenitsyn A. G., 8
Ammari H., 11
Andronov I.V., 8

Babich V.M., 10
Badanin A.V., 10
Baloshin Yu. A., 22
Béreux N., 11
Borisov V. V., 11
Borovikov V.A., 13
Buldyrev V.S., 50

Cherednichenko K.D., 10
Chernokozhin E. V., 39

Dobrokhotov S.Yu., 12

Fedorov S. F., 37

Gajewski D., 46
Golovin A. B., 14
Gotlib V.Yu., 28
Grikurov V.E., 15
Gromov E. M., 17

Hanyga A., 16

Kanaun S., 19
Kashtan B. M., 46
Khabakhpasheva T.I., 44
Khizhnyak A. I., 14, 18
Khodova G. V., 37
Kirpichnikova N. Ya., 22, 47
Kiselev A.P., 19, 20
Kiselev Yu.V., 21
Korotyaev E., 23
Kostin A. V., 22
Kouzov D.P., 24
Kravchenko V. F., 26
Kreminskaya L. V., 14, 18
Krotov A. V., 42
Kudrin A. V., 44

Lavrov Yu.A., 27
Lawry J., 32
Lazarev A. A., 27
Lerer A. M., 28
Lukjanov V.D., 29
Lyalinov M.A., 30

Mikhasev G. I., 32
Motygin O.V., 35

Narevich R., 40
Nédelec J. C., 11
Nesenenko G. A., 26
Nikitin G.L., 29, 33

Obrezanova O. A., 36

Perel M.V., 20
Petrov E. Yu., 44
Philippov V. B., 47
Primack H., 39

Rabinovich V. S., 36
Rogoff Z.M., 19
Rosanov N. N., 37

Schanz H., 39
Selezov I.T., 38
Shanin A. V., 38
Shestopalov Yu. V., 39
Shirai H., 40
Simonenko I. I., 11
Smilansky U., 39
Spehner D., 40
Starkov A. S., 42
Sturova I.V., 44

Tovstik P. E., 42
Troyan V.N., 21
Tsvetkovskaya S. M., 28
Tutin V. V, 17

Ussishkin I., 39

Vladimirov A. G., 37

Zaboronkova T. M., 44
Zalipaev V. V., 47
Zillmer M., 46