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ABSTRACTS

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FOREWORD

The Seminars/Conferences "Days on Diffraction" are annually held since 1968 in late May or in June by the Faculty of Physics of St.-Petersburg State University, St.-Petersburg Branch of the Steklov's Mathematical Institute and Euler International Mathematical Institute of the Russian Academy of Sciences.

This booklet contains the abstracts of 154 talks to be presented at oral and poster sessions in 4 days of the Conference. Author index can be found on the last page.

The full texts of selected talks will be published in the Proceedings of the Conference. The texts in LaTeX format are due by September 15, 2010 to e-mail `iva---@list.ru`. Format file and instructions can be found on the Seminar Web site at `http://eimi.imi.ras.ru/~dd/submission.php`. The final judgement on accepting the paper for the Proceedings will be made by the Organizing Committee following the recommendations of the referees.

We are as always pleased to see in St.-Petersburg active researchers in the field of Diffraction Theory from all over the world.

Organizing Committee

OBITUARY

Vladimir Sergeevitch Buldyrev, whose role was essential in transforming the diffraction theory into an important force of mathematical physics in the second half of the 20th century, died on 5 April 2010, after a two-month illness. His 80th birthday we celebrated at "Diffraction Days 2009".

Vladimir Sergeevitch was a Professor in the Department of Higher Mathematics and Mathematical Physics, Faculty of Physics of the St. Petersburg State University, where his teaching career lasted for more than 60 years. He was the key figure in sustaining the department as a center for research and the training of several generations of mathematical physicists who were to go on to pursue successful research and teaching careers. It must be added that the initiation of the "Diffraction Days" conferences is undeniably connected with his name.

Vladimir Sergeevitch was the author and coauthor of many pioneering works in the field of the mathematical theory of diffraction. His studies of the 'whispering gallery' and 'jumping ball' modes had resulted in proposing the so-called 'infinitesimal ray method' and the technique of investigation of the ray stability based on the first approximation. Along with the boundary layer approach originated by V. A. Fock, these techniques opened a new era in the diffraction theory lasting up to the present.

More than twenty students got their PhD degree under Buldyrev's supervision; six of them became professors later on. He wrote four monographs and textbooks, the best known of which is "Asymptotic Methods in Short Wave Diffraction Problems" co-authored with V. M. Babich and translated into English. The State Prize of the USSR and the St. Petersburg University Prize are among of his awards.

THE 80TH BIRTHDAY OF V. M. BABICH

Vassily Mikhailovich Babich is a dominant figure in the St. Petersburg school of mathematical physics, not only for his outstanding original contributions to theory of wave propagation and asymptotic methods, but also for his personal influence in creating world famous centre for studies in the field of mathematical theory of wave phenomena. The diffraction theory community will celebrate his 80th birthday on 13 June 2010.

Almost 60 years ago, Vassily Mikhailovich graduated from the Leningrad State University, where his teachers were G. I. Petrashen' and S. G. Mikhlin, and in 1954 he began his own teaching career at his Alma mater (the Department of Mathematics and Mechanics, and the Department of Physics). Since 1967 V. M. Babich heads the Laboratory for Mathematical Problems in Geophysics at the Steklov Mathematical Institute in St. Petersburg (PDMI).

This year, Vassily Mikhailovich co-chairs his 43d "Days on Diffraction", and his name irrevocably connected with this annual conference. His weekly seminar at the PDMI has an undisputed reputation for high-level talks. Together with several his colleagues, V. M. Babich was awarded a Soviet State prize for a series of works concerned with the application of the ray method to propagation of seismic waves. In 1998, he was awarded the V. A. Fock prize for his studies of asymptotic methods in diffraction theory.

In order to characterise the level of scientific contributions made by V. M. Babich, we restrict ourselves to mentioning his brilliant paper on waves in anisotropic media (it was reprinted in a leading international geophysics journal 33 years after the original publication). It seems that this was the first use in mathematical physics of a technique based on application of the Finsler space.

We wish Vassily Mikhailovich good health and many happy returns of 13 June!

List of talks

The diffraction and dispersion of waves in the space-periodic structure with the 2-dimentional electronic gas

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The last years, characterized by increased attention to the various physical phenomena in the electron gas in semiconductor structures. The plasma oscillations in the semiconductor are propagated at frequencies of terahertz range[1-2].

1. The diffraction grating is needed to ensure a radiating mode. The problem of the electromagnetic wave on the semiconductor structure with a diffraction grating is solved in a rigorous formulation method [3].The field is obtained from the joint of Maxwell's equations and the hydrodynamic equations of the semiconductor plasma. Fields substituted into the boundary conditions on all interfaces. The system of functional equations obtained from the boundary relations. The system of linear algebraic equations was obtained from the application of the method [3].The amplitudes of the reflected and transmitted waves are obtained in analytical form by solving the reduced system of linear algebraic equations. The reflection (transmittion) coefficient signal is investigated in a wide frequency range of the parameters of the structure.

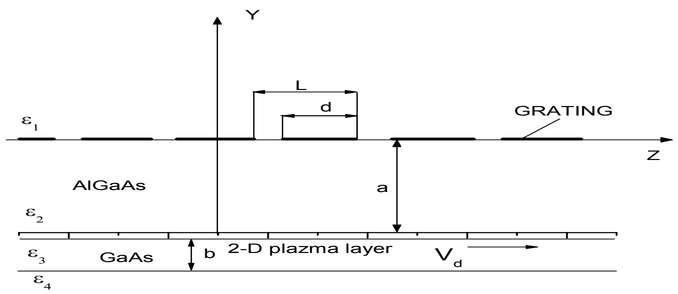

Figure : The electodynamic model under study.

2. Within the framework of general electrodynamical formalism of spatially bounded plasma media and the rigorous solution of the boundary problem using the method [3] there has been obtained a characteristic equation of the system.
$\det\left\{\frac{|n|}{n}L^n_m\xi_m-\delta_{mn}\right\}=0$, where m,n =0,$\pm1,\pm2,...$;$L^n_m=V^n_m-R_m\frac{V^n_\sigma}{R_\sigma}$; δ_{mn}- the Kroneker symbol ; $\xi_n=1+i\frac{|\chi\alpha+n|}{\chi\alpha+n}\cdot\frac{|n|}{n}\cdot\psi_{n1}\sqrt{\frac{\chi^2}{(\chi\alpha+n)^2}-1}$; $\chi=\frac{l}{\lambda}$;V^n_m,V^n_σ,R_m,R_σ, are defined in [3] ψ_{n1}-is function of the physical and geometrical parameters. The conductivity of plasmons can be obtained. $\sigma(\omega,k_{xn})=i\omega\nu\sigma_0\left(\omega'^2+i\nu\omega'-\omega_p^2+V_Tk_{nz}\right)^{-1}$;ω_p-plasma frequency, ν-is a phenomenological electron scattering rate. $\sigma_0=e^2N_s/m^*\nu$; N_s-is the areal density of electrons.

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New approach to solution of sine-Gordon equation with variable amplitude

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A new approach to solution of sine-Gordon (SG) equation with variable amplitude

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2 U}{\partial t^2} = n(x,y,z,t)\sin U \tag{1}$$

is represented. The SG equation appears in many branches of modern natural sciences. It describes deformation of a nonlinear crystal lattice, the orientation structure of liquid crystal (LC), surface metric, etc. In present time there are many effective methods for SG equation solution. However the main methods have been developed for case $n(x,y,z,t) = \text{const}$. It is strong limited the domains of the SG equation application. So in mechanics of nonlinear crystal lattice the case $n = \text{const}$ describes deformation of ideal lattice by a homogeneous stress field. The deformation of real lattice with structure defects (dislocations, disclinations) by nonhomogeneous stress field is described by SG equation with $n \neq \text{const}$. In mechanics of LC the case $n = \text{const}$ models of the axes orientation by a homogeneous electromagnetic field. For nonhomogeneous fields $n \neq \text{const}$. In differential geometry SG equation with $n = \text{const}$ describes the metrics of Chebyshev nets on a surface with constant curvature [1]. If the curvature is changing then $n \neq \text{const}$. It is clear from these examples, that the domains of SG equation applications are extended if one find the solutions of SG equation with a variable amplitude.

A new approach to SG equation with $n \neq \text{const}$ is found on the next results:

1. If function $\varphi(x,y,z,t)$ obeys simultaneously to equations

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2 \varphi}{\partial t^2} = 0\,, \quad \left(\frac{\partial \varphi}{\partial x}\right)^2 + \left(\frac{\partial \varphi}{\partial y}\right)^2 + \left(\frac{\partial \varphi}{\partial z}\right)^2 - \frac{1}{v^2}\left(\frac{\partial \varphi}{\partial t}\right)^2 = n(x,y,z,t)\,, \tag{2}$$

then $U = 4 \,\text{arc tg}\, e^{\varphi(x,y,z,t)}$ is a solution of (1).

2. If function $\varphi(x,y,z,t)$ obeys simultaneously to equations

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2 \varphi}{\partial t^2} = n(x,y,z,t)\,, \quad \left(\frac{\partial \varphi}{\partial x}\right)^2 + \left(\frac{\partial \varphi}{\partial y}\right)^2 + \left(\frac{\partial \varphi}{\partial z}\right)^2 - \frac{1}{v^2}\left(\frac{\partial \varphi}{\partial t}\right)^2 = 0\,, \tag{3}$$

then $U = 2 \,\text{arc tg}\, e^{\varphi(x,y,z,t)}$ is a solution of (1).

A solutions of Eqs. (2), (3) can be found by method of construction of functionally-invariant solutions of differential equations [2–5]. New solutions of SG equation with $n = \text{const}$ are found and the approach to constructing of SG equation solution with variable amplitude are represented.

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Transition radiation of a charge moving in a waveguide with semi-bounded cold plasma

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Review of papers into the problems of transition radiation in a waveguide shows that such study even in the case of relatively simple media has not been carried out sufficiently. Authors [1 - 3] mainly paid attention to formulation and research of the energy characteristics but the electromagnetic field structure has not been analyzed. However, information about this structure is of significance for the accelerator physics and other areas concerning the charge particles detection and diagnostic of the particles beams. In present paper the investigation of the electromagnetic field of a charge moving in the waveguide along its axis through the boundary between vacuum and cold plasma is under consideration. Such problem is of great interest because transition radiation can be studied per se in the absence of Vavilov-Cherenkov radiation in both media.

The analytical solution of this problem is found for the case of arbitrary homogeneous isotropic dispersive media. The electromagnetic field components in both media are presented as decompositions in infinite series of normal modes [4]. Each of modes is Fourier integral with respect to frequency. The electromagnetic field components have two summands: "forced" field being equal to the field of the charge in unbounded medium and "free" field connected with influence of the boundary (it includes transition radiation). The "free" field in vacuum and cold plasma is the main object of this research carried out with two methods: analytical and computational.

In the analytical way, asymptotic expressions for the electromagnetic field components of each mode are obtained with the steepest descend technique [5]. Such analysis is performed with methods of the function theory of a complex variable and drawn the conclusions concerning important physical phenomena. Studying of singularities of integrands in a complex plane educes the different structure of the electromagnetic fields in vacuum and cold plasma. In vacuum, along with the saddle points contribution, there are also the poles contribution and the cuts contribution which can be named "surface standing wave" and "lateral standing wave" correspondingly. Both these types of waves exist near the boundary only. In plasma, instead of the cuts contribution, there is another poles contribution which is "plasma oscillation" exponentially decreasing with the distance from the border. The saddle points contribution is given space transition radiation both in vacuum and in plasma.

In the second method, the exact integral representations are used. Efficient algorithm based on certain transformation of the initial integration path in the complex plane is developed (earlier such an algorithm was used for the "forced" field [6]). The field is computed both before and behind wave front for arbitrary distances. The behavior of the field components depending on distance and time is explored for different velocities of the charge motion and different radiuses of the waveguide. Some important physical effects are noted. So, considerable increasing and concentration of the field near the wave front in plasma is noted for the case of ultra-relativistic particle.

In conclusion, it should be noticed that procedures developed in present paper can be useful for investigation of the field structure in the waveguide with different media, e.g. media with negative refraction index (some energetic characteristics for such problem were considered in [7]).

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On the concept "pseudofunction" and its application to construct mathematical expressions for waves concentrated in small neighborhood of points, curves and surfaces

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Formal power series coefficients of which are smooth functions of some parameters are pseudofunction in our terminology. It is possible to develop some version of differential and integral calculus of pseudofunctions. This "calculus" gives the possibility to construct asymptotic expressions for high-frequency waves concentrated in small neighborhood of points, curves, surfaces. We shall consider Rayleigh waves concentrated in a small neighborhood of a moving curve as an example of the application of the "calculus".

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Spectral estimates for periodic fourth order operators

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We consider the operator $H = d^4dt^4 + ddtpddt + q$ with 1-periodic coefficients on the real line. The spectrum of H is absolutely continuous and consists of intervals separated by gaps. We describe the spectrum of this operator in terms of the Lyapunov function, which is analytic on a two-sheeted Riemann surface. On each sheet the Lyapunov function has the standard properties of the Lyapunov function for the scalar case. We describe the spectrum of H in terms of periodic, antiperiodic eigenvalues, and so-called resonances. We prove that 1) the spectrum of H at high energy has multiplicity two, 2) the asymptotics of the periodic, antiperiodic eigenvalues and of the resonances are determined at high energy, 3) for some specific p the spectrum of H has an infinite number of gaps, 4) the spectrum of H has small spectral band (near the beginner of the spectrum) with multiplicity 4 and its asymptotics are determined as $p \to 0, q = 0$.

Solving problems of elastic ring dynamics by the generalized method of eigenoscillations

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The ring resonator gyroscope (RRG) belongs to the class of Coriolis vibratory gyroscopes and is used as a sensitive element in navigational systems [1]. The principle of its operation is based on the use of inertial properties of standing waves excited in the elastic ring. Ring resonators dynamics is described by the partial differential equation of the 2nd order with respect to time and the 6th order with respect to the spatial variable (angle). For imperfect ring resonators the this equation has variable coefficients and it is impossible in the general case to obtain its solution analytically. Therefore, numerical approaches should be applied. For the first time, in this report the application of the generalized method of eigenoscillations (GME) [2] for solving differential equations of dynamics of perfect and imperfect elastic rings is proposed.

The GME, as applied to solving a wide class of internal and external diffraction problems, is a further development of the method of eigenfrequencies. Some modifications of the GME are known, namely, the k-method (the eigenfrequency method), the e-method (eigenvalue in the equation), the w-method (eigenvalues in the impedance boundary condition), the r-method (eigenvalues in the conjugation condition), and the s-method (eigenvalues in the infinity condition). From the point of view of numerical analysis, the GME is based on a solution of some auxiliary eigenvalue problem. Here, the unknown solution is expanded in a series with respect to a proper system of basic functions, whose undetermined coefficients are found by one of variational or projective methods (Ritz, Bubnov-Galerkin, the least squares, etc.). The basic functions must satisfy definite requirements, in particular, boundary conditions of the original problem (periodic boundary conditions in our case). Some novel numerical approaches for solving boundary-value problems in complex-shaped domains with using the GME are based on the R-functions theory [3].

Results of numerical experiments demonstrate effectiveness of application of the GME for solving some problems of elastic ring dynamics.

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Modeling of magnetic gap by energy balance method

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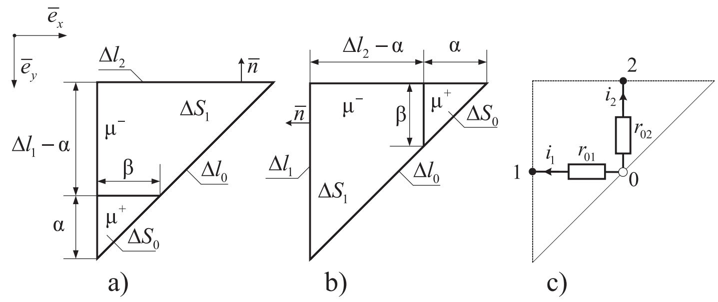
Kirchhoff electric circuit networks (KECN) are widely applied as mathematical models of electrotechnical devices. One of the most universal methods to construct KECN is the method of power balance [1]. In order in computer modeling the computation area is divided into elements, and then equivalent KECN is constructed for each element of discretization. On this element we suppose magnetic capacity is constant. In practice we often solve problems with small magnetic gaps. But we can solve such problems without increasing of discretization. In this case we have elements of discretization with nonconstant magnetic capacity $\mu = \begin{cases} \mu^- \text{ on } \Delta S_1 \backslash \Delta S_0; \\ \mu^+ \text{ on } \Delta S_0. \end{cases}$ We can modeling this elements by energy balance method.



In the case shown on fig.1,a we have $B_y = \text{const}$, $H_x = \text{const}$ on ΔS_1. By using ψ, $\bar{B} = [\text{grad}\psi, \bar{e}_z]$ we construct model of this element. The model is shown on fig.1,c, here

$r_{01} = \frac{\mu^-}{2}\frac{\Delta l_2}{\Delta l_1} - \frac{\alpha\beta}{2\Delta l_1^2}(\mu^- - \mu^+)$, $r_{02} = \frac{\mu^-}{2}\frac{\Delta l_1}{\Delta l_2} - \frac{\alpha\beta}{2\Delta l_2^2}(\mu^- - \mu^+)$,

$i_k = \int\limits_{\Delta l_k} \bar{H}\,[\bar{e}_z, \bar{n}]\, dl,\ k = 1, 2.$

In the case shown on fig.1,b we have $B_x = \text{const}$, $H_y = \text{const}$ on ΔS_1 and then doing the same operations.

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Dynamics of convecting elastic solids

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The dynamics of a class of convecting elastic media is considered. On the basis of an appropriate variational principle, the general field equation governing small oscillations is derived. The variational formulation demands (i) conservations of mass, (ii) conservation of energy, and (iii) conservation of the identity of particles. Of these, conservation of mass needs to be satisfied explicitly as a constraint. This is achieved by constraining the classical mechanical Lagrangian using a Lagrange multiplier with the continuity equation. Hamilton's principle modified for a control volume in this way then leads to the equation of motion for small oscillations of convecting gyroelastic solids.

The mathematical structure of the field equation thus derived is examined. The origins of the 'gyroscopic' and the 'centrifugal' effects are traced. These can be associated with various terms in the expression for the Lagrangian density. In particular, terms in the kinetic energy density that are independent the velocity field, those that are linear in the velocity field and those that are quadratic in the velocity field are associated with the centrifugal, gyroscopic, and inertia terms in the equation of motion respectively. A close mathematical analogy between the dynamics of this class of continua and the dynamics of discrete gyroscopic-centrifugal systems having fixed material particles is noted. The free vibration problem is posed in its generality. An appropriate Rayleigh quotient is defined. The stationarity associated with the quotient can potentially be used for computational work. Illustrative examples and applications are discussed.

Resonance mode patterns in the paraxial volume of a quasi-optical electron accelerator

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One of the versions of the accelerating structure intended for a future electron-positron collider comprises a periodic set of coaxial metallic discs with radial corrugation, which is exposed to a quasi-cylindrical electromagnetic wave convergent onto the structure axis [1]. Forming of a resonance electric field with the longitudinal component synchronous to accelerating particles in the paraxial volume is investigated.

For 3D azimuth-symmetrical case the structure considered is governed by a scalar equation of Helmholtz type for the azimuthal component of a magnetic field. The longitudinal and radial components of the electric field can be expressed in terms of the magnetic component. The boundary conditions reflect ideal conductivity (current absence) on the metal surface as well as symmetry and periodicity of the structure.

The problem is solved using the discrete source method. The solution is sought in the form of a linear combination of Green's functions of the operator in an enveloping rectangular domain. The boundary conditions are assumed to be satisfied exactly at individual points of the boundary, which yields a set of homogeneous linear algebraic equations (SHLAE) in the coefficients determining the strength of the sources. The numerical algorithm for obtaining the boundary profile parameters for which the matrix of the SHLAE has zero eigenvalue is based on the procedure of singular decomposition of matrices [2].

To obtain stable fields of the skew-symmetric type required, various configurations of the paraxial volume are considered. In addition to the traditional elliptic profiles, configurations of a resonant type [2] that form an electric field with the longitudinal component synchronous with injected electrons are considered. A comparative analysis of fields in the accelerating structure of an electron-positron collider in cases of the paraxial volume boundaries of the two types, including with an elliptic and resonant shape, shows the better properties of the latter system with respect to forming of an electric field pattern synchronous to injected electrons as well as their higher stability with respect to small perturbations of the boundary profile.

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On the spectrum of two-dimensional periodic operator with a localized perturbation

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We consider a two-dimensional periodic operator on the plane with a localized perturbation. The perturbation is described by an abstract operator acting from $L_2(\mathbb{R}^2)$ in a weighted Sobolev space. We study the structure and the asymptotic behavior of the spectrum of such operator.

Let $x=(x_1,x_2)$ and $\square$ be Cartesian coordinates and the unit cube in $\mathbb{R}^2$. By $C^\beta_{per}(\overline{\square})$ we denote the space of $\square$-periodic functions belonging to Hölder space $C^\beta(\overline{\square})$. In $L_2(\mathbb{R}^2)$ we introduce the operator

$$\mathcal{H}_0 := -\sum_{i,j=1}^{2}\frac{\partial}{\partial x_i}a_{ij}\frac{\partial}{\partial x_j}+\mathrm{i}\sum_{i=1}^{2}\left(a_i\frac{\partial}{\partial x_i}+\frac{\partial}{\partial x_i}a_i\right)+a_0,$$

where $a_{ij}=a_{ij}(x)\in C^{1+\beta}_{per}(\overline{\square})$, $a_i=a_i(x)\in C^{1+\beta}_{per}(\overline{\square})$, $a_0=a_0(x)\in C^{\beta}_{per}(\overline{\square})$ for some $\beta\in(0,1)$, $a_{ij}=a_{ji}$,

$$\nu_1|\xi|^2\leqslant\sum_{i,j=1}^{2}a_{ij}(x)\xi_i\overline{\xi_j}\leqslant\nu_2|\xi|^2,\qquad (x,\xi)\in\mathbb{R}^2\times\mathbb{C}^2,\quad \nu_i>0.$$

As the domain of $\mathcal{H}_0$ we choose $W_2^2(\mathbb{R}^2)$. Let χ_1, χ_2 be non-negative real functions such that

$$(1+|x|^2)\chi_1^2\in L_2(\mathbb{R}^2)\cap L_\infty(\mathbb{R}^2),\qquad (1+|x|^2)\chi_2\in L_2(\mathbb{R}^2),$$

$$\lim_{|x|\to\infty}\chi_2=\lim_{|x|\to\infty}\frac{\partial\chi_2}{\partial x_i}=\lim_{|x|\to\infty}\frac{\partial^2\chi_2}{\partial x_i\partial x_j}=0.$$

We introduce a weighted Sobolev space $W_2^2(\mathbb{R}^2,\chi_1^2\mathrm{d}\,x)$ with the norm

$$\|u\|^2_{W_2^2(\mathbb{R}^2,\chi_1^2\mathrm{d}\,x)}:=\sum_{i,j=1}^{2}\|\chi_1u_{x_ix_j}\|^2_{L_2(\mathbb{R}^2)}+\sum_{i=1}^{2}\|\chi_1u_{x_i}\|^2_{L_2(\mathbb{R}^2)}+\|\chi_1u\|^2_{L_2(\mathbb{R}^2)}.$$

By ε we indicate a small positive parameter, and $\mathcal{L}_\varepsilon^{(0)}$ is an arbitrary operator from $W_2^2(\mathbb{R}^2,\chi_1^2\mathrm{d}\,x)$ in $L_2(\mathbb{R}^2)$ bounded uniformly in ε. We let $\mathcal{L}_\varepsilon:=\chi_2\mathcal{L}_\varepsilon^{(0)}$, and $\mathcal{H}_\varepsilon:=\mathcal{H}_0-\varepsilon\mathcal{L}_\varepsilon$ is an operator in $L_2(\mathbb{R}^2)$ with the domain $W_2^2(\mathbb{R}^2)$. The operator $\mathcal{H}_\varepsilon$ but not necessary symmetric since we do not assume this property for $\mathcal{L}_\varepsilon$.

The main aim is to study the behavior of the spectrum of $\mathcal{H}_\varepsilon$ as $\varepsilon\to+0$. We prove that the essential spectrum is stable and independent of $\mathcal{L}_\varepsilon$, while the residual one is empty. The part of the point spectrum separated from the essential spectrum is countable and consists of the eigenvalues of finite multiplicities. We prove the convergence theorems for these eigenvalues. We show that under the considered perturbation there can be isolated eigenvalues emerging from the edges of the gaps in the essential spectrum. We prove necessary and sufficient conditions for such eigenvalues to exist and to be absent. In the case of existence we obtain the first terms of the asymptotic expansions of these eigenvalues.

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Comparison of vectorial laser beams radiation pressure on two-level and (1+3)-level neutral atoms

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The action of laser beams on neutral atoms, ions and molecules is of great interests during last decades [1, 2]. The mean radiation force is due to interaction between electric field of laser beam and an atom. If the interaction scheme is two-level then spatial distributions of laser beam intensity and phase plays main role in force distribution. If magnetic sublevels of ground and exited states are taken into consideration spatial distribution of laser beam polarization also important.

In [3–5], an approach to designing electromagnetic fields, based on the use of plane-wave superpositions, differentiable manifolds, and the group of rotation, is presented. The approach provides a broad spectrum of tools to design laser beams with built-in symmetry properties of electric and magnetic fields and allows to govern the distribution of beams energy densities, phases and polarizations.

In this work we use density-matrix method [1, 6] to calculate radiation force of vectorial laser beam designed by above approach. We discuss the difference between radiation force of the same laser beam acting on two-level and (1+3)-level neutral atoms.

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Radiation pressure of vectorial laser beams on (1+3)-level atoms

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Laser beams are known to exert a radiation pressure on neutral atoms, ions and molecules thereby affect their translational motion [1, 2]. Subject to interaction scheme between an atom and electrical field of laser beam different characteristics of latter play the main role. Thus two-level interaction scheme implies the key role of intensity and phase spatial distribution of laser beam. When magnetic sublevels of ground and exited states are taken into consideration laser beam polarization distribution also important.

In present work the translational motion of (1+3)-level atoms in vectorial laser beams is considered. Beam electric field is taken in the form [3, 4, 5]

$$\mathbf{E(r,t)} = \mathbf{E_0} \int_{\mathcal{F}} \mathbf{Bu(b)} \left[\mathrm{e}^{\mathbf{i(k(b)\cdot(r-r_p(b))}-\omega \mathbf{t})} \mathbf{E(b)} + \text{c.c.} \right] \mathrm{d}\mathcal{F}\mathrm{B}, \tag{1}$$

where functions $u(b)$, $\mathbf{k(b)}$, $\mathbf{E(b)}$, $\mathbf{r_p(b)}$ are defined on manifold $\mathcal{F}B$ and describe the distributions of intensity, wave vector, polarization and initial phase of partial plane wave respectively, ω is radiation frequency.

Radiation force on an atom is characterized by the atom state, that can be described by atomic density-matrix $\rho_{ab}(\mathbf{r})$. Based on the density-matrix approach [1, 6] one can derive that density-matrix of (1+3)-level atom in field (1) obeys the following system of equations

$$\begin{aligned} \mathbf{v}\frac{\partial}{\partial \mathbf{r}} \rho_{\mathbf{e}_{\sigma_1}\mathbf{e}_{\sigma_2}} &= iA^{\sigma_1}(\mathbf{r})\chi_{\mathbf{g_0}\mathbf{e}_{\sigma_2}} - \mathbf{i}\mathbf{B}^{\sigma_2}(\mathbf{r})\chi_{\mathbf{e}_{\sigma_1}\mathbf{g_0}} - \mathbf{2}\gamma\rho_{\mathbf{e}_{\sigma_1}\mathbf{e}_{\sigma_2}} \\ \mathbf{v}\frac{\partial}{\partial \mathbf{r}} \chi_{\mathbf{e}_{\sigma_1}\mathbf{g_0}} &= iA^{\sigma_1}(\mathbf{r})\rho_{\mathbf{g_0}\mathbf{g_0}} - \sum_{\sigma_2} \mathbf{i}\mathbf{A}^{\sigma_2}(\mathbf{r})\rho_{\mathbf{e}_{\sigma_1}\mathbf{e}_{\sigma_2}} - (\gamma - \mathbf{i}\boldsymbol{\Delta})\chi_{\mathbf{e}_{\sigma_1}\mathbf{g_0}} \end{aligned} \tag{2}$$

$$\sum_{\sigma} \rho_{e_\sigma e_\sigma} + \rho_{g_0 g_0} = 1, \quad \chi_{e_{\sigma_1} g_0} = e^{-i\Delta t} \rho_{e_{\sigma_1} g_0}, \quad \rho_{\alpha\beta} = \rho^*_{\beta\alpha},$$

$$A^\sigma(\mathbf{r}, \mathbf{t}) = (\mathbf{B}^\sigma(\mathbf{r}, \mathbf{t}))^* = \boldsymbol{\Omega} \int_{\mathcal{F}} \mathbf{B}\mathbf{E}^\sigma(\mathbf{b})\mathbf{u}(\mathbf{b})\mathbf{e}^{\mathbf{i}(\mathbf{k}(\mathbf{b})\cdot(\mathbf{r}-\mathbf{r_P}(\mathbf{b}))}\mathrm{d}\mathcal{F}\mathrm{B},$$

where $\sigma = 0, \pm 1$, subscripts g_0 and e_σ denote ground and exited levels respectively, $E^\sigma(b)$ are circular components of vector $\mathbf{E}(\mathbf{b})$, Δ is laser detuning, Ω is Rabi frequency, 2γ is spontaneous emission rate.

The investigation of stationary solution of system (2) in the case of light field periodical in propagation direction is carried out. Influence of spatial distribution of intensity, phase and polarization on radiation force and therefore translational motion of atom is studied.

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Composite model for generalized Chebyshev oscillator

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Representations of the Chebyshev algebra, i.e. generalized oscillator algebra, generated by Jacobi matrix for "non-standard" Chebyshev polynomials are considered. Unlike the Jacobi matrix for standard Chebyshev polynomials, in a considered case on the main diagonal of such matrix there stands a periodically repeating sequence of N complex numbers. In the report on example N=3 we consider connection of representations of this algebra with representations of N algebras for standard Chebyshev oscillators. In our talk the polynomials generated by considered Jacobi matrix will be constructed together with the solution of the related moments problem. The carrier of the constructed measure is located on the system of the rays going through the beginning of a complex plane and symmetric under $\frac{2\pi}{3}$−rotations.

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Dynamic equations for an orthotropic plate

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There are a lot of dynamic plate equations derived in the literature, both for isotropic and anisotropic materials. Here a recently developed approach is used to derive such equations for an orthotropic plate in a very systematic way. The equations can be given to any order in the thickness and they are believed to be asymptotically correct to the given order.

Consider a plate of thickness $2h$ with traction-free faces. Introduce a coordinate system with the origin in the middle of the plate and the z axis normal to the plate. The density of the plate is ρ and the stiffness constants of the orthotropic material are c_{IJ} in standard abbreviated indices. Consider the antisymmetric (bending) motion of the plate. Then the displacement components can be expanded as

$$\begin{aligned} u_1(x,y,z,t) &= zu_1(x,y,t) + z^3u_3(x,y,t) + \ldots, \\ u_2(x,y,z,t) &= zv_1(x,y,t) + z^3v_3(x,y,t) + \ldots, \\ u_3(x,y,z,t) &= w_0(x,y,t) + z^2w_2(x,y,t) + \ldots. \end{aligned}$$

Insertion of these expansions into the 3D equations of motion, the x component leads to the following recursion relation

$$u_{n+2} = \frac{1}{(n+1)(n+2)c_{55}}[\rho\partial_t^2 u_n - c_{11}\partial_x^2 u_n - c_{66}\partial_y^2 u_n - c_{126}\partial_x\partial_y v_n - (n+1)c_{135}\partial_x w_{n+1}],$$

for $n = 1, 3, \ldots$, and similarly for the other two components. Here the stiffness constants $c_{126} = c_{12}+c_{66}$ and $c_{135} = c_{12} + c_{55}$ enter. Used recursively these equations can be used to eliminate all expansion functions except w_0, u_1 and v_1. This is really the crucial point of the method, and it is noted that no approximations or truncations are performed so far.

The boundary conditions on the faces of the plate remain. Inserting the displacement expansions, eliminating all but the three lowest order expansion functions and truncating to order h^3, the x component of the boundary condition gives the first plate equation

$$\begin{aligned} &c_{55}(u_1 + \partial_x w_0) + \frac{h^2}{2}\big[\rho\partial_t^2 u_1 + (\beta_{133}c_{135} - c_{11})\partial_x^2 u_1 - c_{66}\partial_y^2 u_1 \\ &+ (\beta_{133}c_{234} - c_{126})\partial_x\partial_y v_1 - \beta_{133}\rho\partial_t^2\partial_x w_0 + \beta_{133}c_{55}\partial_x^3 w_0 + \beta_{133}c_{44}\partial_x\partial_y^2 w_0\big] = 0. \end{aligned}$$

Here $c_{234} = c_{23} + c_{44}$ and $\beta_{133} = c_{13}/c_{33}$. The y component gives a similar equation with the changes $u_1 \Leftrightarrow v_1$, indices $1 \Leftrightarrow 2$ and indices $4 \Leftrightarrow 5$. The z component gives

$$\begin{aligned} &h[\rho\partial_t^2 w_0 - c_{55}(\partial_x u_1 + \partial_x^2 w_0) - c_{44}(\partial_y v_1 + \partial_y^2 w_0)] \\ &+ \frac{h^3}{6c_{33}}\big[\rho^2\partial_t^4 w_0 - c_{13}c_{55}\partial_x^4 w_0 - c_{23}c_{44}\partial_y^4 w_0 - (c_{55} - c_{13})\rho\partial_t^2\partial_x^2 w_0 \\ &- (c_{44} - c_{23})\rho\partial_t^2\partial_y^2 w_0 - (c_{13}c_{44} + c_{23}c_{55})\partial_x^2\partial_y^2 w_0 - (c_{33} + c_{135})\rho\partial_t^2\partial_x u_1 \\ &- (c_{33} + c_{234})\rho\partial_t^2\partial_y v_1 + (c_{11}c_{33} - c_{13}c_{135})\partial_x^3 u_1 + (c_{22}c_{33} - c_{23}c_{234})\partial_y^3 v_1 \\ &+ [c_{33}(c_{126} + c_{66}) - c_{23}c_{135}]\partial_x\partial_y^2 u_1 + [c_{33}(c_{126} + c_{66}) - c_{13}c_{234}]\partial_x^2\partial_y v_1\big] = 0. \end{aligned}$$

The three plate equations resemble the classical Mindlin equations. However, the Mindlin equations omit the last three w_0 terms in the first equation and all the h^3 terms in the last equation. Furthermore, the Mindlin equations have smaller changes in some of the coefficients and also include the shear correction factor.

By a variational approach the method also gives the boundary conditions that are to be satisfied along edges of the plate. To illustrate the accuracy of the method the dispersion relation and displacement and stress components are compared with other methods and exact 3D calculations.

Wave dynamics of non-harmonic internal gravity wave in stratified ocean

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We consider the problem of reconstructing non-harmonic internal gravity wave packets generated by a source moving in a stratified ocean. The solution is proposed in terms of modes, propagating independently at the adiabatic approximation, and described as a non-integral degree series of a small parameter characterizing the stratified medium. A specific form of the wave packets, which can be parameterized in terms of model functions, e.g. Airy functions or Fresnel functions, depends on a local behavior of the dispersion curves of individual wave mode. We modified the space-time ray method, which belongs to the class of geometrical optics methods. The key point of the proposed technique is the possibility to derive the asymptotic representation of the solution in terms of a non-integral degree series of the some small parameter ε. Specifically, we are looking for a solution W as the sum of modes propagating independently (the adiabatic approximation), namely: $W = A(\varepsilon x, \varepsilon y, z, \varepsilon t)R_0(\sigma) + \varepsilon^\alpha B(\varepsilon x, \varepsilon y, z, \varepsilon t)R_1(\sigma) + \ldots$, $R'_{i+1}(\sigma) = R(\sigma)$, $\sigma \equiv (S(\varepsilon x, \varepsilon y, \varepsilon t)/a\varepsilon)^\alpha$, where σ is on the order of one, and the functions S, A are to be found. The function $R_0(\sigma)$ is expressed in terms of Airy functions (shelf zone) or the Fresnel integrals (deep ocean).

The explicit form of the asymptotic solution was determined based on the principles of locality and asymptotic behavior of the solution in case of a stationary and horizontally homogeneous medium. The wave packet phase is calculated from the corresponding eikonal equations that are numerically solved along the characteristic curves. Specifically, the eikonal equation is defined as: $\frac{\partial^2 A}{\partial z^2} + |k|^2(\frac{N^2(z,x,y)}{\omega^2} - 1)A = 0$, $k(\omega, x, y) = -\nabla S$, $\omega = \frac{\partial S}{\partial t}$, $|\nabla S|^2 = |k|^2$, $N^2(z, x, y)$ — Brent-Vaisala frequency. The wave packet amplitudes A are determined from the energy conservation laws along the characteristic curves: $\frac{d}{dt}\ln(DA^2\frac{\partial K}{\partial \omega}K^{-1}) = 0$, where $K(\omega, x, y) = |k|^2$ and D is the Jacobian determinant to define transformation from the ray coordinates into the Cartesian ones.

Our modification of the geometrical optics method allows us to describe the wave field structure both far from and at the vicinity of the wave front. This work solved the problem of describing the evolution of the non-harmonic packets of the internal gravity waves in a layer of an arbitrary stratified medium of varying depth with a non-stationary, horizontally non-uniform density. We show that it is possible to observe some peculiarities in the wave field structure, depending on the shape of ocean floor, water stratification and the trajectory of a moving source. Numerical analyses that are performed using typical ocean parameters reveal that actual dynamics of the internal gravity waves are strongly influenced by nonstationarity and horizontal inhomogeneity.

Radio coverage simulation for three-dimensional urban environment using physical optics, physical theory of diffraction and the near-to-far-field transformation method

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The objective of this paper is the presentation of a three-dimensional (3-D) model which simulates the electromagnetic propagation in outdoor urban areas for GSM frequencies (900 - 1800 MHz), through the combination of separate propagation mechanisms. The simulation results of each propagation mechanism are analyzed in order to derive its particular contribution to the total received field.

The mechanisms considered for evaluation at each receiver point of the three-dimensional space are the Line-of-Sight (LOS), scattered and diffracted fields. The scattered field is calculated using the Physical Optics method, while the diffracted field is calculated using the Physical Theory of diffraction (PTD). In particular, for the calculation of the diffracted field the method of Mitzner's Incremental Length Diffraction Coefficients is applied, which is an extension of Ufimtsev's Physical Theory of Diffraction. An additional simulation is performed, in order to define the contribution of ground reflection to the total received field, which is based on Image Theory.

In order to compute the scattered field from a particular building wall, we use the 'Near to Far Field Transformation' method. Specifically, we perform segmentation of the scattering surface into an appropriate number of small rectangles (cells), when the receiving antenna is located in the near or Fresnel zone of the scatterers. By such a subdivision of the electrically large scatterer, an observation point which is originally located in the near or Fresnel zone of the scatterer, is then transferred to the far region of the smaller cells. The same method is also applied for the calculation of the diffracted field from the buildings' wedges which are not in the far field area. This allows us to apply the far-field equations for small segments of the wedge and integrate the results to calculate the diffracted field from the wedge.

The simulation is implemented by creating a software tool for the modeling of the 3-D space, which is created in Matlab environment. We create a 3-D model including buildings of rectangular shape and subsequently we apply a specially designed shadowing algorithm to define the illuminated areas of the outdoor urban space according to the given position of the transmitter. These illuminated areas are categorized in three distinct groups. The first group includes illuminated facets of the existing buildings. These facets are considered as the input for the algorithm that applies the Image Theory to calculate the scattered field. The second group includes illuminated wedges of existing buildings. For each of these wedges we apply the formulae for diffraction which yields the diffracted field according to the Physical Theory of Diffraction. The third group includes all the illuminated points which do not belong to the first two groups, and takes into account only the LOS propagation mechanism.

The results of these simulations indicate the importance of scattering and diffraction as mechanisms of electromagnetic propagation in mobile telecommunications coverage in dense urban environments. In addition they provide a satisfactory radio-coverage prediction for three-dimensional space taking into account the difference in height between the transmitter and the receiver. They are also found to provide accurate radio coverage diagrams in different model configurations, also revealing the particular contribution of each propagation mechanism for several geometries of the urban scene.

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On violation of Rayleigh law of scattering in case of subsurface deterministic inhomogeneity

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The problem of Rayleigh wave scattering by near-surface three-dimensional deterministic inhomogeneity of isotropic solid is solved in Born (Rayleigh-Born) approximation of perturbation theory. Inhomogeneity is described by arbitrary function factorized in three coordinates. Expressions for displacement field and intensity in scattered Rayleigh wave, asymptotic expressions for intensity in Rayleigh limit $\lambda \gg L$ (λ is wavelength, L is characteristic size of inhomogeneity) are derived. It is shown, that inhomogeneity structure (its form-factor) strongly influences on frequency dependence of scattered Rayleigh wave intensity in Rayleigh limit, giving rise to violation of Rayleigh law of scattering (law about proportionality of scattered Rayleigh wave intensity to the fifth power of frequency $I \sim \omega^5$). Connection between topological characteristics of inhomogeneity and frequency dependence and value of scattered Rayleigh wave intensity in Rayleigh limit is established. It is obtained, that frequency dependence of intensity can have form $I \sim \omega^{5+2n}$, where $n = 0, 1, 2, 3\ldots$ It is shown, that variation of dimension of inhomogeneity symmetry determined by zeroing of topological characteristics of inhomogeneity results in additional variation of scattered Rayleigh wave intensity frequency dependence in Rayleigh limit. It is obtained, that imposition of cylindrical symmetry on coordinate dependence of inhomogeneity structure gives rise to new effect in connection between frequency dependence of intensity and topological characteristic of inhomogeneity in plane of cylindrical symmetry: topological characteristics only of even order influence on intensity and its frequency dependence. It is found, that inhomogeneity structure strongly influences on the scattering angular distribution form in Rayleigh limit. Zeroing of topological characteristics of certain order results in violation of the angular isotropy of Rayleigh scattering indicatrix and in appearance of Rayleigh scattering indicatrix zeroes in angular of scattering. Increasing of dimension of inhomogeneity symmetry defined by zeroing of the inhomogeneity topological characteristics gives rise to increasing of scattering indicatrix zeroes number. At variation of wavelength indicatrix zeroes defined by the inhomogeneity structure in direction perpendicular to the surface move in scattering angular. Indicatrix zeroes defined by the inhomogeneity structure in the plane parallel to the surface do not move. Position of arised Rayleigh scattering indicatrix zeroes depends on inhomogeneity form and on its location with respect to the incident Rayleigh wave direction of propagation. Only forbidden forward scattering direction is fixed.

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Oscillations of scattering in Rayleigh limit

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The problem of Rayleigh wave scattering is solved in Born (Rayleigh-Born) approximation of perturbation theory (in roughness amplitude) for three-dimensional deterministic (not statistical) roughness $x_3 = f(x_1, x_2) = \delta_0 f_0(x_{\shortparallel})$, $x_{\shortparallel} = \sqrt{x_1^2 + x_2^2}$ of isotropic solid occupying half-space $x_3 \geqslant 0$, where

$$f_0(x_{\shortparallel}) = \sum_{i=i_0}^{i_m} a_i^{(p)} \Phi_{n^{(i)}+1}(x_{\shortparallel}); \quad \Phi_{n^{(i)}+1}(x_{\shortparallel}) = \sum_{m=1}^{m_{n^{(i)}+1}} f_1(m)\psi_0^{(i)}(m) f_\theta(\frac{x_{\shortparallel}}{d}, \frac{(m-1)}{m_{n^{(i)}+1}}, \frac{m}{m_{n^{(i)}+1}}); \quad (1)$$

$m_{n^{(i)}+1} = 2^{n^{(i)}}$; $n^{(i)} = 0, 1, 2, 3, \dots$; $f_\theta(x, a, b) = 1$ for $a \leqslant x \leqslant b$ and 0 for $x < a, x > b$; $f_1(m) = (-1)^{k_m}$, where $\delta_{i,k}$ - Kronecker symbol; k_m is found from equation $m - 1 = \sum_{k=1}^{k_m} 2^{l_k}$, $k_1 = 0$, $m > 1$, integers l_k, $k = 1, 2, 3, \dots, k_m$ are so, that $0 \leqslant l_1 < l_2 < l_3 < \dots < l_{k_m}$, i.e. k_m is sum of digits in the binary representation of the number $m - 1$; $\psi_0^{(i)}(m) = q_1^{(i)} m_{n^{(i)}+1}^{p_1^{(i)}+p_2^{(i)}} (x_{\shortparallel}/d - (m - 1)/m_{n^{(i)}+1})^{p_1^{(i)}} (m/m_{n^{(i)}+1} - x_{\shortparallel}/d)^{p_2^{(i)}}$; $q_1^{(i)} = (p_1^{(i)} + p_2^{(i)})^{p_1^{(i)}+p_2^{(i)}} / (p_1^{(i)p_1^{(i)}} p_2^{(i)p_2^{(i)}})$; $p_1^{(i)}, p_2^{(i)} = 0, 1, 2, \dots$; $n^{(i)}$ is order of the i - th partial roughness spatial statistical symmetry.

It is found, that in all range of wave-lengths, i.e. from $d/\bar{\lambda} \ll 1$ up to $d/\bar{\lambda} \gg 1$ ($\bar{\lambda} = \lambda/(2\pi)$) intensity of scattered Rayleigh wave vertical component $I_3 = I_3^{(0)}/x_{\shortparallel}$ at big distances from the roughness is defined by the formula $I_3^{(0)} = (A^{(0)}\delta_0)^2 I_3^{(R)}/d$, where indicatrix of scattering $I_3^{(R)}$ is

$$I_3^{(R)} = \left(\Phi_z^{(P)}(z)\right)^2 \frac{(k_R d)^5}{8\pi R_2^2} \beta^2 \frac{c_R^4}{c_t^4} A_2^2(x_3)(1 - \cos\varphi_s)^2(\gamma + \cos\varphi_s)^2;$$

$$\Phi_z^{(P)} = \sum_{i=i_0}^{i_m} a_i^{(p)} \Phi_{n^{(i)}+1}^{(z)}; \; z = k_R d\sqrt{2(1 - \cos\varphi_s)}; \quad (2)$$

$$\Phi_{n^{(i)}+1}^{(z)}(z) = (-1)^{p_1^{(i)}} \frac{2\pi q_1^{(i)}}{z} \left(\sum_{i_1=0}^{p_1^{(i)}+p_2^{(i)}} \frac{1}{z^{i_1}} \sum_{m=1}^{m_{n^{(i)}+1}} D_{mi_1}^{(2;i)} J_{i_1+1}(z_m^{(n^{(i)}+1)}) - \right.$$

$$\left. - \sum_{i_2=0}^{\left[(p_1^{(i)}+p_2^{(i)}-1)/2\right]} \frac{D_{i_2}^{(1)}}{z^{2i_2+1}} \sum_{m=1}^{m_{n^{(i)}+1}} C_{m(2i_2+1)}^{(5;i)} \left(J_{i_2+1}(z_m^{(n^{(i)}+1)}) H_{i_2}(z_m^{(n^{(i)}+1)}) - H_{i_2+1}(z_m^{(n^{(i)}+1)}) J_{i_2}(z_m^{(n^{(i)}+1)}) \right) \right), \quad (3)$$

where $A^{(0)}$ is amplitude of incident Rayleigh wave x_1-component; k_R, c_R-it's wave-vector and velocity; $c_{l,t}$-velocities of bulk longitudinal and transverse waves; φ_s-angle of scattering; $\{\alpha, \beta\} = \sqrt{1 - c_R^2/c_{l,t}^2}$ respectively, $\gamma = 1 - c_R^2/(2c_t^2)$; $\alpha\beta = \gamma^2$ - Rayleigh wave dispersion equation; $R_2 = (\alpha^2 + \beta^2 + 2\gamma^4 - 4\gamma^3)/\gamma^2$; $A_2(x_3) = \alpha e^{-\alpha k_R x_3} - (\gamma/\beta) e^{-\beta k_R x_3}$; $J_m(x), H_m(x)$ - Bessel and Struve functions of order m respectively; $[\dots]$ - integer part of a number;

$$a_i^{(p)} = a_i^{(p0)} / \left(C_i^{(P)}(n^{(i)})\right)^{1/2}; \; C_i^{(P)}(n^{(i)}) = C_{n^{(i)}} \frac{\pi\beta^2}{2R_2^2} \frac{c_R^4}{c_t^4} A_2^2(x_3) \left(P^{(i)}\right)^2 (\gamma + \cos\varphi_s)^2 (1 - \cos\varphi_s)^{2\left[\frac{n^{(i)}}{2}\right]+2};$$

$$C_{p_1}^{i_1} = \frac{p_1!}{i_1!(p_1 - i_1)!}; \; z_m^{(n^{(i)})} = mz/m_{n^{(i)}}; \; C_{n^{(i)}} = \frac{\left(\left(2\left[\frac{n^{(i)}}{2}\right] + 1\right)!\right)^2}{2^{2\left[\frac{n^{(i)}}{2}\right]} \left(\left[\frac{n^{(i)}}{2}\right]!\right)^4}; \; q_{pk}^{(i)} = \frac{q_1^{(i)}}{n^{(i)}! m_{n^{(i)}+1}^{(n^{(i)}+k)}};$$

$$p_{qk}^{(i)} = p_1^{(i)} + p_2^{(i)} + k;\ M_j^{(i)} = \sum_{m=1}^{m_{n^{(i)}+1}} (-1)^{k_m} m^{n^{(i)}+j};$$

$$P^{(i)} = \begin{cases} q_{p1}^{(i)} \frac{p_1^{(i)}!p_2^{(i)}!}{p_{q1}^{(i)}!} M_0^{(i)}, & n^{(i)} - \text{odd} \\ q_{p2}^{(i)} \frac{p_1^{(i)}!p_2^{(i)}!}{p_{q1}^{(i)}!} \left(\frac{1}{(n^{(i)}+1)} M_1^{(i)} - \frac{(p_2^{(i)}+1)}{p_{q2}^{(i)}} M_0^{(i)} \right), & n^{(i)} - \text{even} \end{cases};\ D_{mi_1}^{(2;i)} = \sum_{j=2i_1}^{p_1^{(i)}+p_2^{(i)}} D_{mji_1}^{(0;i)};$$

$$C_{mj}^{(5;i)} = \frac{m}{m_{n^{(i)}+1}} C_{mj}^{(3;i)};\ C_{mj}^{(1;i)} = C_{mj}^{(i)} m_{n^{(i)}+1}^{j} / m^j;\ D_{n_1}^{(1)} = (-1)^{n_1+1} \frac{\pi}{2} ((2n_1+1)!!)^2;$$

$$D_{mji_1}^{(0;i)} = (-1)^{j+i_1} \frac{j!!}{(j-2i_1)!!} \left(\frac{m}{m_{n^{(i)}+1}} \right)^{j+1-i_1} C_{mj}^{(3;i)};\ C_{mj}^{(i)} = \sum_{i_1 = max\{0; j-p_2^{(i)}\}}^{min\{j; p_1^{(i)}\}} C_{p_1^{(i)}}^{i_1} C_{p_2^{(i)}}^{j-i_1} m^{i_1+p_2^{(i)}} (m-1)^{p_1^{(i)}-i_1}.$$

$$C_{mj}^{(3;i)} = (-1)^{k_m} C_{mj}^{(1;i)} - (1 - \delta_{m,m_{n^{(i)}+1}})(-1)^{k_{m+1}} C_{(m+1)j}^{(1;i)};$$

If $(p_1^{(i)} + p_2^{(i)}) = 0$, then the second term in (3) is absent. (1)-(3) give oscillations of $I_3^{(R)}$ in Rayleigh limit $d/\bar{\lambda} \ll 1$ (Fig.1-6).

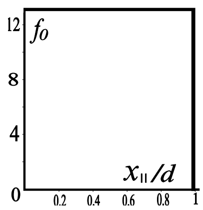

Fig. 1

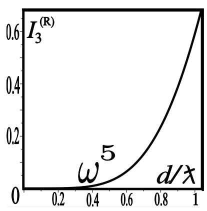

Fig. 2

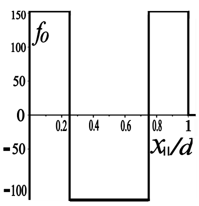

Fig. 3

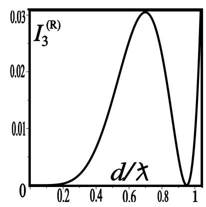

Fig. 4

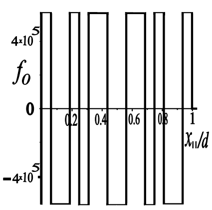

Fig. 5

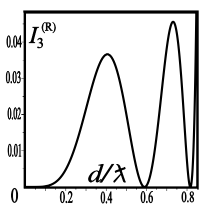

Fig. 6

$\{p_1^{(i)} = 0;\ p_2^{(i)} = 0;\ n^{(i)} = 2i\}$ for Fig. 1–6. Fig. 1, 2: $\{i_0 = 0;\ i_m = 0;\ a_i^{(p0)} = 1\}$. Fig. 3, 4: $\{i_0 = 0;\ i_m = 1;\ a_i^{(p0)} = 1\}$. Fig. 5, 6: $\{i_0 = 0;\ i_m = 2;\ a_0^{(p0)} = 4.7;\ a_i^{(p0)} = 20$ for $i = 1,2\}$; $x_3 = 0, \varphi_s = \pi/2$, Poisson coefficient $\sigma = 0.25$ everywhere.

These oscillations (Fig. 4, 6) are violation of Rayleigh law (Fig. 2) of scattering.

Abstract approach and explicit asymptotic solutions of 2-D wave equation with variable velocity and localized right-hand side

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We consider the Cauchy problem for the inhomogeneous wave equation wave equation with a variable velocity and perturbation in a form of a right hand side localized in space (near the origin) and in time. In particular, this problem is connected with the question about the creation of the tsunami and Rayleigh waves. Using the abstract operator theory we show that the solution is separated into two parts: the transient one which localized in the neighborhood of the origin and decreases in time and the propagating part one, which propagates in the space like the wave created by the momentary "equivalent source". We present several examples covering wide range of perturbation resulting in quite explicit formulas expressing solutions in terms of the error function of the complex argument.

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Asymptotics of the solution to the mixed boundary elliptic problem

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The following two-dimension problem is considered: equation $\Delta u = f(x)$ in some domain $G \in \mathbb{R}^2$ with piecewise smooth boundary. The boundary condition is following: the derivative on a normal is equal zero everywhere, except a small segment γ, where function $u(x)$ is given. The length of the segment equal to a small parameter ε. The problem is to find the asymptotics of the solution $u(x, \varepsilon)$ as $\varepsilon \to 0$. The full asymptotic expansion was constructed and proved.

Generalized solution to the light scattering problem for axisymmetric particles

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We consider scattering of a plane wave by an axisymmetric particle. Our approach is based on electromagnetic field expansions in terms of spherical/spheroidal wave functions and involves ideas of the separation of variables (SVM), extended boundary condition (EBCM) and point-matching (PMM) methods [1-3]. These methods were never considered together because of essentially different light scattering problem formulations applied. However, as they use the same field expansions expressions of all scatterer characteristics in the SVM, EBCM, and PMM are similar. The methods differ in the way of determination of unknown field expansion coefficients, which leads to different systems of linear algebraic equations relative to these coefficients. We study the relation of the methods when spherical/spheroidal basis is utilized. From a practical point of view, the approach suggested allows one to calculate optical properties of different shape scatterers, including strongly flattened/elongated layered ones with surface ripples.

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The nonstationary problem of membrane vibrations, partially submerged into the layer of liquid

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The problem of oscillations of elastic constructions partially submerged into the water is one of the actual problems of modern techniques. Ships, oil platforms, sea airports are the examples of such bodies. However the exact calculation of such bodies vibrations is rather complicated. So it is useful to explore the possible oscillations in these objects taking as an example more simple mechanical systems.

The aim of this work is to analyze oscillations problem of the rather simple mechanical model of this class - the membrane partially submerged into liquid, build the solution for forced vibrations and analyze the streams of energy in the system membrane-liquid. The problem of forced oscillations of the membrane partially submerged into the layer of liquid is considered in the rigorous mathematical statement.

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Absolute continuity of the spectrum of the periodic Schrödinger operator in a layer and in a smooth multidimensional cylinder

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We study the Schrödinger operator $H = -\Delta + V$ in a d-dimensional cylinder

$$\Xi = U \times \mathbb{R}^m \subset \mathbb{R}^d, \quad d = k + m, \quad m \geq 1, \quad k \geq 2,$$

where $U \subset \mathbb{R}^k$ is a bounded domain, $\partial U \in C^\infty$. The potential is assumed to be periodic with respect to a lattice Γ in $\mathbb{R}^m$:

$$V(x, y + l) = V(x, y), \quad l \in \Gamma, \quad (x, y) \in \Xi.$$

We establish sufficient conditions on the potential V for the spectrum of H to be absolutely continuous. The results are expressed in terms of $V \in L_p(U \times \Omega)$, where Ω is an elementary cell of Γ. Various boundary conditions on $\partial U \times \mathbf{R}^m$ are studied.

In the case of a plane-parallel layer ($k = 1$) the established sufficient condition is $p > d/2$ for $d \geqslant 3$. In the case of a cylinder ($k \geqslant 2$) and $d = 3, 4$ the condition is also $p > d/2$. Finally, for $k \geqslant 2$ and $d \geqslant 5$ we assume $p > d - 2$.

The proofs of both cases are based on the Thomas scheme [4], the operator is decomposed into a Floquet-Bloch-Gelfand direct integral. In the case of a layer we then use Sogge [2] spectral cluster L_p-estimates of an elliptic operator on a compact manifold without boundary, having modified them for the case of a product $M \times [0, a]$. In the case of a cylinder we use an analogous result for manifolds with boundary, derived by Smith and Sogge [1] using Strichartz estimates for the wave equation.

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On laplacian in domain perforated along the boundary

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We consider a boundary value problem for Laplace operator in a domain perforated along the boundary. On the internal boundary we impose homogenous Neumann boundary condition, while on the boundary of the cavities we impose Dirichlet one. We construct the asymptotic expansions for the eigenvalues of this problem converging to simple eigenvalue of the limiting (homogenized) problem. We show that an eigenvalue of the original problem is strictly less that the corresponding eigenvalue of the limiting problem. This is a joint work with G. Chechkin and Yu. Koroleva.

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Smith-Purcell radiation resonant regimes in open type waveguide on tori sequence in relativistic diffraction generator

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In the present work the numerically studies of the multiwave mechanisms of interaction between tubular electron beam and fields of super-dimensional axisymmetric periodical slow-wave structures (SWS) on the sequence of tori in output section of relativistic diffraction generator are discussed. The main phenomenon that determines the action of this type of generator of long-pulsed high power coherent microwave radiation in the centimeter and millimeter wavelengths range is the Smith-Purcell radiation.

The problem of Smith-Purcell radiation detection consists in definition of a resultant field being the sum of proper field of moving charges (grazing field) and field scattered on the periodic obstacles (in this work in form of tori sequence). The assumption of axial symmetry of system permits us to consider only axisymmetric modes of E_0n- type. This approach to the problem of diffraction on slowing axially symmetric periodic structure is a numeric realisation of rigorous solution of a diffraction problem of proper radiation of relativistic tubular electron beam modulated on frequency ω on periodic obstacles of axisymmetric SWS. It evolves the construction of a source function (Green function) for free space. Maxwell equations with boundary conditions for perfectly conducting surface are reduced to surface Fredholm integral equation of second kind in H_ϕ considering the axisymmetric mode. Its fundamental solution is represented as the azimuthal integral depending on the distance between the point of observation and the integrating point and then solved numerically using conjugate operator formalism [1].

Since the periodic form of a surface of axisymmetric slowing down structure is typical for devices of relativistic diffraction electronics it permitted us to consider by reduction of surface integral equation to system of linear equations its matrix as a block Toeplitz matrix and thus to use an efficient algorithm for computational solution. The surface currents derived this way as a result of solving of Toeplitz matrix equation are used for pointing out the fields inside the whole bulk of periodic structure as the second step of the problem of Smith-Parcel radiation detection solution. This integral equation method can be used for investigations of radiation generation resonance regimes on smooth and even noncontinuous surfaces.

In the report the flux energy directions as a correspondence of rings number and the correlation between different geometrical parameters of tori sequence are presented and proved. Some common detailed formulae, in particular for continuous sinusoidally corrugated surface, can be found in [2].

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Efficient surface integral algorithms for three dimensional electromagnetic scattering

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Many important physical processes involve scattering of electromagnetic waves by ensembles of deterministic and stochastic particles.

In particular, for applications such as light scattering in (i) the atmospheric sciences, with configurations consisting of computer models of atmospheric ice crystals and dust particles with rough non-convex surfaces having unique stochastic description; (ii) medical diagnostics involving, for example, several red blood cells; (iii) electromagnetic scattering by surfaces with conical singularities (for example well known benchmark targets: cone-sphere, NASA Almond, ogive) and in several other classes of wave propagation problems, it is efficient to develop algorithms that directly incorporate local mapping properties of each obstacle in the configuration and use such mappings to reduce the computational complexity.

In this work, we discuss high-order spectral-Galerkin surface integral algorithms with specific focus on simulating the scattering of electromagnetic waves by a collection particles arising from specific applications discussed above.

Calculation of synthetic seismograms by summation of gaussian beams of a given width

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The function $G_\varphi(s,n)$, describing behavior of Gaussian beams [1], [2] in 2D-case, can be written in a following form

$$G_\varphi(s,n) \sim \exp\left(-\frac{n^2}{L^2(s)}\right)\exp\left(i\omega\tau(s)\left(1+\frac{1}{2\tau(s)}K(s)n^2\right)\right),$$

$$K(s) = Re\left(\frac{P(s)}{Q(s)}\right),\ L(s) = \left(\frac{\omega}{2}Im\left(\frac{P(s)}{Q(s)}\right)\right)^{-1/2},$$

$$Q(s) = z_1 q_1(s) + z_2 q_2(s), \; P(s) = z_1 p_1(s) + z_2 p_2(s), \; Z = (z_1, z_2),$$

where (s, n) – curvilinear coordinates of a receiver M, $\tau(s)$ – eikonal, ω – wave frequency and

$$W(s) = \begin{pmatrix} q_1(s) & q_2(s) \\ p_1(s) & p_2(s) \end{pmatrix}$$

– fundamental solutions of the elastodynamic equations. It is well known [1] that the function $G_\varphi(s, n)$ is rather sensitive to a choice of the initial parameters Z, which determine a form of Gaussian beams. To get a correct solution it is very important to choose an adequate Z. This can be done on basis of the following conditions. Firstly, it is obviously necessary to specify Z in such a way that the width L to be narrow enough as that allows limiting only to rays sufficiently closed to point M where the solution is sought. This reduces the time of calculations. Secondly, it is natural to construct the Gaussian beams so that the function $G_\varphi(s, n)$ would have a minimum of oscillations as it leads to increase of accuracy of sum of the Gaussian beams. In this study a special procedure for choice of the initial parameters Z is suggested. This procedure, satisfying both of the above requirements, allows us to construct Gaussian beams with any beforehand given width L_0 and simultaneously with the least number of oscillations of $G_\varphi(s, n)$ on basis of the following equations

$$\begin{cases} q_1(s) \neq 0 \\ e = -\frac{q_2(s)}{q_1(s)} - i\frac{\omega \det W(0)}{2q_1^2(s)} L_0^2 \\ Z = (e, 1) \end{cases} \; or \; \begin{cases} q_2(s) \neq 0 \\ e = -\frac{q_1(s)}{q_2(s)} + i\frac{\omega \det W(0)}{2q_2^2(s)} L_0^2 \\ Z = (1, e) \end{cases} \Longrightarrow \begin{cases} k = 1, 2 \\ Q(s) = \mp i\frac{\omega \det W(0)}{2q_k(s)} L_0^2 \\ \frac{P(s)}{Q(s)} = \frac{p_k(s)}{q_k(s)} + i\frac{2}{\omega L_0^2}, \end{cases}$$

As consistent with [2], q_1 and q_2 can not be equal zero simultaneously, therefore initial parameters Z can be always defined from the above system. If the both solutions $q_k (k = 1, 2)$ do not equal zero, we can pick out such pair q_k, p_k that minimizes the factor $\left(1 + \frac{1}{2\tau(s)} \frac{p_k(s)}{q_k(s)} n^2\right)$ that gives the least number of oscillations of the function $G_\varphi(s, n)$ of Gaussian beams for this L_0.

The initial parameters defined in such a way are individual for each ray, even for the system of incident-reflected (transmitted) rays. Moreover, it is shown that using such procedure for choice of these parameters it is possible to construct Gaussian beams with any beforehand given width. From the physical point of view it is quite reasonable to set this width equal to the wave length, however it can be specified in any others ways, for example, as related to the geometrical spreading.

The program for calculating of synthetic seismograms is written on basis of summation of Gaussian beams with fixed width. Numerical modeling shows that synthetic seismograms calculated by summation of the Gaussian beams are regular in the regions where the ray method fails, such as the caustics, vicinity of critical ray, etc. Examples of synthetic seismograms for 2D and 3D structures in inhomogeneous media with smooth interfaces calculated by the suggested approach for construction of the Gaussian beams are presented. Various possible applications of Gaussian beams to seismological problems of practical importance are outlined.

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Diffraction by a dielectric wedge: theory and experiments

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The exact analytical solution to the diffraction of an electromagnetic wave by a perfectly conducting wedge of infinite extent has been found by Mac Donald [1] more than one century ago. A similar solution for the dielectric wedge is not yet available to date, despite several attempts over the last decades, that turned out to be flawed [2][3]. The diffraction on finite-sized objects with complex shapes and made of various materials cannot be solved exactly. Approximate theories have been developed to cope with such real life objects. For example, to account for the multiple interactions of radiowaves with buildings in urban areas, coverage prediction tools often rely on the Uniform Theory of Diffraction (UTD) [4]. The UTD belongs to the family of asymptotic methods and is very accurate in presence of perfectly conducting edges, thanks to the existence of the aforementioned exact canonical solution [1]. In urban environment though, one encounters mostly dielectric structures. The lack of exact canonical solution for the dielectric wedge led to heuristic modifications of the UTD [5]. We review in this paper the strengths and limits of this heuristic theory and compare its predictions with laboratory measurements.

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Trapped-mode, pass- and gap-band effects in waveguides with obstacles

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In our presentation we focus at the resonance phenomena of a time-averaged oscillation $\mathbf{u}e^{-i\omega t}$ featured by the time-harmonic wave energy localization near the obstacles in the form of energy vortices. These phenomena, which are also known as trapped-mode effects, are usually accompanied by a sharp stopping of the wave energy flow along the waveguide and, consequently, in deep and narrow gaps in the frequency plots of transmission coefficients. They are tightly connected with the distribution of natural frequencies (resonance poles) ω_n in the complex frequency plane ω. Specific forms of energy localization are governed by the eigen-solutions $\mathbf{u}_n$ associated with the resonance poles ω_n, which are actually the spectral points of the related boundary-value problems. In ideally-elastic structures certain combinations of obstacles may result in totally real poles.

The study is carried out for elastic layers with obstacles in the form of cracks, voids and inclusions using analytically based computer models relying on wave expressions in terms of path Fourier integrals, Green's matrices for the laminate structures considered, and asymptotics for traveling waves derived from those integrals. To get an inside into the mechanisms of the resonance effects of interest a simplified one-dimensional waveguide model for a spring-supported string with point-wise defects has also been considered.

It was found that although a set of resonance poles ω_n for a group of obstacles cannot be obtained as a simple combination of poles corresponding to the individual obstacles taken alone (due to their mutual wave interaction), the blocking properties of the group as a whole is determined in the main by the stop-bands of its individual members. Therefore, it turned out to be possible to extend a frequency stop range (gap band) by the use of an aperiodic system of a few obstacles (cracks) with individual spectral points lying close to each other and to the real axis instead of the conventional use of large periodic systems.

A further study of elastic waveguides with multiple defects has shown that inside gap bands there might appear pass frequencies. In the transmission coefficient plots such pass modes look as narrow peaks centered at frequencies ω_p located closely to the resonance poles $\omega_n: \ \omega_p \approx \mathrm{Re}\,\omega_n$. The number of such pass frequencies ω_p (and correspondingly of the transmission peaks) is proportional to the number of obstacles N but they all are located in a limited frequency range; therefore, as N increases the peaks fill in tightly this range forming pass band inside a wider gap band. As the number of identical evenly spaced defects increases to infinity, the pass and gap bands become the same as that obtained within the Bloch-Flocket theory. It is also shown that a slight disturbance of the defect's periodicity may considerably change the stop and transmission properties.

Lamb wave excitation, propagation and diffraction in laminate composites with obstacles

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Non-destructive inspection of plate-like structures based on ultrasonic Lamb waves requires theoretical and experimental investigations into elastic wave diffraction at defects of different kind (surface cracks, notches, holes and so on) in isotropic and composite plates. Another important problem is to choose suitable devices for the elastic wave actuation and registration. In recent years piezoceramics, which could be applied both as input and output devices, became widespread since they are cheap, easy to use and can also become an integrated part of a monitored structure.

In the course of research work a series of experimental measurements has been carried out accompanied by theoretical computer simulations which aimed at the investigation of piezo- electrically induced Lamb wave propagation and diffraction at different surface obstacles. In the context of general linear elasticity the transient displacements of a layered waveguide with surface obstacles are expressed through their time-harmonic spectra that are obtained from the boundary value problems (BVPs) for the full system of elastodynamic equations. The inci- dent and diffracted wave fields are modeled using the integral and asymptotic representations in terms of Green's matrix of the structure under consideration. The distribution of unknown contact stresses under obstacles is obtained from the Wiener-Hopf type integral equations using expansion in terms of specially constructed axially symmetric delta-like functions. The experimental investigations have been performed with an isotropic aluminium plate of 1 mm thickness. Circular piezoactuators are used as a wave source while the data are recorded using the laser-vibrometer technique. The following obstacles are used: permanent magnets placed from both sides of the plate, pieces of steel glued to the aluminium plate and drops of molten solder placed on the surface.

The comparison of theoretical and experimental results for the incident pulse propagation in the plate without defects and for the Lamb waves diffracted by surface obstacles has shown a very good coincidence. In view of further experimental investigations, the expansion of the theoretical model for the case of anisotropic laminates is proposed and illustrated by numerical examples.

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Spring boundary conditions and modeling of 2D wave propagation in composites with imperfect interfaces

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Composite materials, owing to their intrinsic heterogeneity in properties, are often accompanied by a damage occurrence such as void growth, micro-cracking and debonding between different phases. In contrast to "pure crack" implying stress-free faces of the crack it might be called "delamination", the latter is just a partially debonded part of the interface. Wave diffraction by a delamination in an elastic waveguide might be of a notably different properties compare to a crack, examples are included. A damage ("delamination") can be modeled as a set of cracks, by replacement of the thin layer with a damage on a visco-elastic layer or using other approaches.

Baik and Thompson [1] used quasistatic approximation to simulate a real imperfect interface by a distributed over the interface spring with mass. Distributed spring stiffnesses have been derived from the equality of the transmission coefficients for transverse and longitudinal waves, accuracy of the derived spring model at low frequencies have been also shown in [1], the same trick is used in the present work to define value of spring stiffness. Spring boundary conditions have been also utilized by Boström and Wickham to model partially closed crack in [2]. Some ideas from [2] concerns stiffness estimation using ensemble average technique have been also utilized in [3] and in the present work.

Thus as a natural continuation of the work carried out in [3] on modeling of imperfect contact between materials for SH problem is a model of an imperfect contact for in-plane problem which is presented here. A delamination is replaced by distributed springs (horizontal and vertical displacement), where springs' stiffnesses are defined by the properties of contacting materials and crack density. The derivation of the stiffness is made in similar way, but here not only normally incident plane waves are considered. Lamb wave diffraction on a crack and on a delamination in a layered waveguide is considered, corresponding integral equation is derived and solved using integral approach with Galerkine scheme, appropriate transmission coefficients and resonance phenomena are compared [4].

The work is supported by the Ministry of Education and Science of Russian Federation (Project No. 2.1.1/1231) which is gratefully acknowledged.

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Analysis of 2D photonic crystal slabs of any rod shape and conductivity using a very fast conical integral equation method

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The boundary-integral-equation-based method [1,2] has been used to calculate the sensitive optical response of 2D photonic crystal slabs (PCS), including dielectric, absorbing, and high-conductive rods of various boundary shapes. It turned out that a small number of collocation points per boundary combined with a high convergence rate can provide adequate description of the dependence on diffracted energy of multilayered band gaps illuminated at arbitrary incident and polarization angles. The numerical results presented demonstrate the significant impact of rod shape on diffraction in various PCS supporting polariton-plasmon excitation and other types of anomalies (i.e. waveguiding anomalies, cavity modes, Fabry-Perot resonances, Rayleigh orders), particularly in the vicinity of resonances and at high filling ratios. The diffracted energy response calculated vs. array cell geometry parameters was found to vary from a few percent up to a few hundred percent. Thus, the simple effective medium theory cannot be applied to design and analysis of such PCS.

A comparison of dispersion curves of metallic subwavelength PCS performed in the visible and near IR photon ranges revealed a very strong effect of nanowire form-factor and arrangement, both on the position and amplitude of the energy peaks inside the plasmon resonances. The rectangular profile of the rods appears to be most sensitive out of the shapes considered, because of its low symmetry and strong dependence on absorption. The code developed and tested for different types of PCS is found to be very accurate and fast and applicable to studies of complex periodic structures, including almost perfectly-conductive rods, inclusions with edges, and multilayer gratings with any boundary profiles operating with arbitrary incident radiation.

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Diffraction of high intensive acoustic wave in the stratified atmosphere

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The nonlinear wave equation and modified Khokhlov-Zabolotskaya type equation for high intensive acoustics waves propagating in stratified atmosphere with inhomogeneous of sound speed is set up. The geometrical acoustics approximation and modified Raley integral for this problem is suggested. The profile distortion of broadband waves and waves with discontinuities is investigated. Time profiles of the single impulse, dependencies of its peak amplitude and duration are obtained.

Error estimates for Filon-Clenshaw-Curtis rules for highly-oscillatory integrals

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The Filon-Clenshaw-Curtis rule approximates integrals of the form:

$$I_k(f) = \int_{-1}^{1} f(x)\exp(ikx)dx,$$

where the parameter k is often large, by replacing the slowly oscillatory function $f(x)$ with its polynomial interpolant $P_N(x)$ of degree N at the Clenshaw-Curtis nodes $x = \cos(j\pi/N)$, for $j = 0, \dots, N$. Thus

$$I_k(f) = \int_{-1}^{1} P_N(x)\exp(ikx)dx.$$

It can be implemented in $O(NlogN)$ time with FFT. In this talk, we present the error estimates for the Filon-Clenshaw-Curtis rule that show explicitly the error dependance on the parameters k and N and on the regularity of f. We also present a piecewise version of this rule which is useful when the function f suffers from certain types of algebraic singularities, with the mesh refined locally near the singularities, and for these we give optimal error estimates which are explicit in k, N and the number of mesh subintervals. We also discuss the stable implementation of the rule when k is large with respect to N and when N is large with respect to k and give numerical illustrations. The rules are very useful for implementing hybrid asymptotic-numerical methods in high frequency scattering.

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The singular boundary problem for elliptic equation

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The singular boundary value for elliptic second order equation in the domain with some small holes is considered. The most complicated case arises if dimension of domain is equal two: the rational functions of logarithms of ε appear in asymptotic expansion. The uniform asymptotic expansion is constructed up to any power of ε. In addition we studied the case when the domain contains many small holes while the distance between them tends to zero.

Seismic Migration in Terms of Locally Supported Wavelets

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Due to the increasing interest in renewable energy productions, geothermal energy is one of its main representatives because of its independence of external factors like the climatic behavior. However, the modeling of geothermal reservoirs is a difficult challenge for many scientific disciplines.

The first step in the chain of the deep geothermal energy exploration is the high resolution determination of the fault patterns and its accompanying fractures in the deeper Earth's underground. Due to the huge success of wavelets in signal processing, noise reduction, etc and its possibility to break complicate functions into many simple pieces at different scales and positions, that makes detection and interpretation of local events relevant in geothermal energy projects significantly easier, new methods for exploration and modeling of deeper geothermal reservoirs based on a wavelet approach are becoming available.

The construction of wavelet techniques for solving boundary value problem involving the Helmholtz equation corresponding to regular surfaces dates back to Freeden and Mayer (2003). However, strategies, applying for modeling the seismic wave propagation, concern usually to regular surfaces possessing edges and corners.

For this reason, in this paper, the classical limit- and jump-relations for Helmholtz potential operators, defined on regular surfaces by Freeden et al. (2003), is extended to regions with non-smooth boundaries. According to the proposed limit- and jump-relations, locally supported wavelet functions are constructed, that approximate the fundamental solution of the Helmholtz equation. In addition, the wavelet functions regularize the singular integral representation of the Helmholtz type boundary value problems, and approximate recursively the solution in regions with edges and corners. In order to handle velocity models with strong lateral and vertical variations, the Born approximation is applied.

Furthermore, it should be noted, that the contruction of wavelets allows the efficient and economical implementation in form of a tree algorithm for the fast numerical computation.

Finally, several examples of seismic migration applied to synthetic data sets in frequency-space as well as in time-space domain are presented, which define reflectors in the interior by data recorded on the surface and by approximating velocity values given in the volume.

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On the extension of the wave based method

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In the end of 90ies of the twentieth century, a new numerical method has been developed for steady–state acoustic analysis in bounded domains. This novel deterministic numerical technique is based on the indirect Trefftz approach, cf. [11], and has been designed especially for mid–frequency range cases. The reason is simple: neither Finite Element Methods (FEM), cf. [6], nor Statistical Energy Analysis (SEA), cf. [5], cannot be applied exactly in this frequency range, cf. [1], [10], [9], [8], [13], [12], [3], [4]. Some similar ideas have been used in 80ies by Prof. Dr. Willi Freeden et al to solve exterior Dirichlet problems for the homogeneous Helmholtz equations. The main idea was to use an interpolation method using metaharmonic splines to find solution of above mentioned problem in 3D case from given discrete data, [2], [7].

The Finite Element Method usually is applicable in low–frequency cases, where the frequency limit of the FEM is related to the growing number of finite elements required to describe the short wavelength behavior at increasing frequencies. The SEA is a prediction method designed for the high frequency range providing the averaged results which are based on the power balance relations. The classical Wave Based Method (WBM) utilizes complex valued wave functions which have been used to expand the dynamic pressure function and which *a priori* satisfy the homogeneous Helmholtz equation, [1], [10]. Hence, no discretization of the domain is required and the sizes of appropriate governed matrices are rather small. This, obviously, gives the possibility to touch the mid–frequency range. Moreover, by using the natural basis functions, namely wave functions, we reduce certain "numerical stress" of the numerical approximation of the problem. This was exactly the ideology of Trefftz, who applied basis functions which *a priori* exactly solve one or another differential equation, [11].

We present a little different wave functions, which allow to build, in general, real valued matrices. Moreover, in classical wave based approach one assumes that the Helmholtz equation is either homogeneous or has point source as the right hand side function. In this paper, we will consider also non–homogeneous elliptic differential equations what allow to apply the wave based technique in a much broad way, namely, we use non-uniform rational B-splines (NURBS) to treat the inhomogeneous part. We would like to emphasize that the same Trefftz ideas can be applied to the Helmholtz, Poisson, Laplace or even to more general elliptic problems, where the so-called *maximum principle* can be valid. Generally, in the classical steady–state acoustics maximum principle is not valid, however, this does not play a big role here. We use this property here to separate so–called "bad" elliptic problems, where so far WBM was used, from "good" ones. We present and discus the possibility to extend and apply the ideas of the Wave Based Method in non–acoustics areas such as steady–state temperature propagation, calculation of the velocity potential function of a liquid flux, calculation of the light irradience in a liver tissue/tumor, etc.

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Diffraction of monochromatic electromagnetic waves on 3D-dielectric bodies of arbitrary shapes

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The work is devoted to the problem of diffraction of monochromatic electromagnetic waves on heterogeneous dielectric inclusions of arbitrary shapes. For the numerical solution of the problem, an integral representation for the electromagnetic field in heterogeneous media is used. In result, the problem is reduced to a volume integral equation for the electric field in the region occupied by the inclusion. Existence and uniqueness of the solution of this equation was studied in [1]. Discretization of this equation is carried out by Gaussian approximating functions. The theory of approximation by the Gaussian and similar functions was developed in [2]. For these functions, the elements of the matrix of the discretized problem are calculated in explicit analytical forms. For a regular grid of approximating nodes, the matrix of the discretized problem proves to have the Toeplitz structure, and the matrix-vector product with such matrices can be carried out by the Fast Fourier Transform technique. The latter strongly accelerates the process of iterative solution of the disretized problem. Numerical calculations for a medium with a spherical inclusion are compared with the exact (Mie) solution for various wave lengths of the incident field and the contrasts in the properties of the medium and the inclusion. The results of the calculations of the electric fields inside a cylindrical inclusion are presented for various wave lengths and directions of the wave vector for the incident field. The inside fields are used for the construction of the differential cross-sections of the considered inclusions.

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Elastic shell impact on a thin layer of water

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The paper is concerned with a problem of elastic shell impact on a thin layer of an ideal incompressible liquid. Cases of a cylindrical shell (2D problem) and a spherical shell (antisymmetrical problem) are considered. The shell initially touches the liquid free surface at a single point and then penetrates the liquid layer at a constant vertical velocity (Fig. 1). The problem is coupled because the liquid flow, the shape of the elastic shell and the geometry of the contact region between the body and the liquid must be determined simultaneously. The liquid flow is analyze using Korobkin's approach [1], via the method of matched asymptotic expansions. In the framework of this approach the flow region is subdivided into four complementary regions that exhibit different properties: the region beneath the entering body surface, the jet root, the spray jet, and the outer region (Fig. 2). A complete solution is obtained by matching the solutions within these four subdomains. The structural analysis is based on the normal-mode method. The fluid-structure coupled analysis leads to a system of nonlinear differential equations for the evolution of principal coordinates of the shell shape and the fluid hydrodynamic pressure. The solution of this evolution system provides both the deformation and stresses for the shell, the contact line between the shell and the liquid, and the liquid flow.

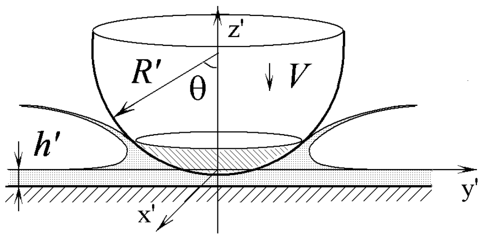

Fig. 1.

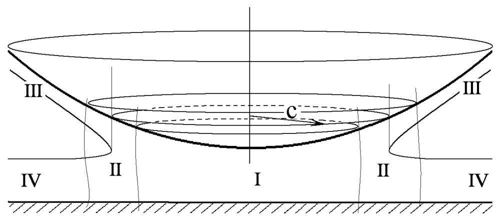

Fig. 2.

A main result of the analysis is that shallow-water impact is more dangerous than deep-water impact because stresses and the deformation of the shell increase as the thickness of the water layer decreases. But starting from some relatively large depth of the water, maximal values of the stresses and deflections remain constant with increasing water depth.

In the case of a flexible shell several distinct regimes of the impact process were found (Fig. 3). For a high impact velocity the lower part of the shell flattens and the shell does not enter the water (a), but it is possible also, that "thin tongue" of the shell touch the bottom with high velocity (b). For a moderate impact velocity the shell reaches the bottom and an effect of "fluid capture" may occur (c). For a low impact velocity the shell penetrates the liquid, but the size of the contact region decreases before the shell reaches the bottom (d). This behavior corresponds to exit or "reflection" of the shell from the water layer. These intricate behaviour of the shell cannot be found if only the first few modes of the vibratory motions of the shell are considered.

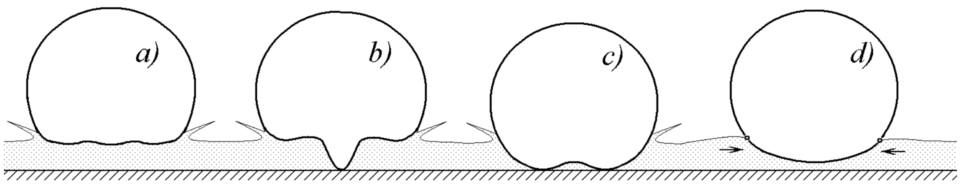

Fig. 3.

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Non-plane surface and interface elastic waves with arbitrary waveforms

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Non-stationary non-plane waves in homogeneous elastic half-space and at a contact of two half-spaces are discussed in a uniform manner. Rayleigh, Stoneley and Shölte-Gogoladze waves having arbitrary time-dependencies are considered. The basic tools are integral representations of the wavefields employed earlier for time-harmonic regimes in [1] and "membrane equations" introduced first by Achenbach [2]. The talk is based on [3].

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Exact Gaussian localized waves via paraxial solutions

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We describe a general simple procedure providing Bateman-type relatively undistorted exact solutions of the wave equation $u_{xx} + u_{yy} + u_{zz} - c^{-2}u_{tt} = 0$. These are of the form $u = gf(\theta)$, where the amplitude $g = g(x, y, z, t)$ and the phase $\theta = \theta(x, y, z, t)$ are fixed functions, while the waveform f is an arbitrary function of one variable, see, e. g., [1,2]. Proper choices of f allow particular solutions, which show highly localized beam-like and particlelike behavior. Here, we present solutions with general Bateman-type phases θ depending on 6 complex parameters. Technically, the approach is based on solutions of approximate paraxial 'parabolic equation'.

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Analysis of integro-differential operator of equation for eddy currents in thin conductor

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Many engineering problems require the simulation of stationary and quasi-stationary magnetic fields in the presence of the conductive layers (cases, plates). Such layers are used expensivly as protective shields, frames and load-bearing elements of electro technical and electrical survey devices. One of the way for solving of these problems is the Integral Equation Method. Using this method the boundary value problem can be reduced to the integro-differential equation for a second sources density τ, which can be chosen, so that it is non zero only in the conductive layer. The integro-differential equation will have features in each of all considering cases (transient state, steady state). But the operator noted by T will be common for all these problems. This operator has the following generalized form

$$(T\tau,\xi)_{\overset{o}{W}{}_2^1} = -\frac{\mu_0}{4\pi}\int\limits_{\Gamma}\int\limits_{\Gamma}\frac{[\mathbf{n}_M,\ [\mathbf{n}_Q,\ \mathrm{grad}_s\tau\,(Q)]]}{r_{QM}}d\Gamma_Q\,\mathrm{grad}_s\xi\,(M)\,d\Gamma_M \quad \text{for} \quad \forall\tau,\xi\in\overset{o}{W}{}_2^1(\Gamma).$$

Here Γ is median surface of considering thin conductor, $\overset{o}{W}{}_2^1(\Gamma)$ is a Hilbert space of complex-valued functions with zero mean value on Γ. In this work we established that the operator T is linear, self-adjoint, positive and completely continuose one in $\overset{o}{W}{}_2^1(\Gamma)$. Taking into account these properties we can solve the assigned problems by the most optimal way. For example, in transient state the solution can be presented in analitical form using eigenfunctions of the operator T.

Singular nonlinear problems for self–similar solutions to the steady–state boundary layer equations with zero pressure gradient

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We present an outline of papers [1,2] where results on singular Cauchy problems, smooth stable manifolds of solutions and exponential parametric Lyapunov series are applied to correctly state and analyze a singular "initial–boundary" value problem (IBVP) for a third–order nonlinear ordinary differential equation defined on the entire real axis. The problem arises in incomperssible viscous fluid mechanics and describes self–similar solutions of boundary layer equation for a stream function with zero pressure gradient (the case of a plane–parallel flow in a mixing layer). In [3,4] this problem depending on self–similarity parameter $m > 0$ is given in the form

$$\Phi''' + \Phi\Phi'' - [(m-1)/m](\Phi')^2 = 0, \qquad -\infty < \tau < \infty, \tag{1}$$

$$\lim_{\tau\to-\infty}\Phi'(\tau) = 0, \tag{2}$$

$$\Phi(0) = 0, \tag{3}$$

$$\lim_{\tau\to\infty}\Phi(\tau)/\tau^m = b, \qquad 0 < b \text{ is fixed}; \tag{4}$$

it is erroneously considered as a three–point boundary value problem (BVP), since condition (2) is not equivalent to one condition in a finite point. Condition (2) should be replaced by more accurate limit condition

$$\lim_{\tau\to-\infty} \exp(-\varepsilon\tau)\{\Phi(\tau)+a, \Phi'(\tau), \Phi''(\tau)\} = \{0,0,0\} \quad \forall\varepsilon : 0<\varepsilon<a, \tag{5}$$

which corresponds to a solution's tending to stationary point $(-a, 0, 0)$. For any fixed $a > 0$, it is a pseudohyperbolic equilibrium point with one–dimensional stable separatrix in the phase space of Eq.(1). Condition (5) for the solutions of (1) is equivalent to two nonlinear relations in a finite point which specify a stable saddle separatrix. Thus, provided $-\infty < \tau \le 0$, two–point BVP (1),(5),(3) with parameter $a > 0$ is defined (as is shown in [1,2], for any fixed $m \ge 1/3$ and $a > 0$ it is uniquely solvable). The parameter $a = a(b)$ is found from condition (4) when it is valid.

The approach of [1,2], different from rather complicated methods of [3,4], allowed us not only to state the singular nonlinear IBVP more accurately, but also to analyze it in a more precise and thorough way and suggest a simple numerical method to solve it. Constraints on parameter m required for univalent solvability of IBVP (1),(5),(3),(4) for any fixed $b > 0$ where $a = a(b)$ are given, namely $1/2 < m < \infty$; two–sided estimates of the solution are deduced; its properties and the properties of other solutions to Eq.(1) for different values of $m > 0$, due to their independent physical meaning, are investigated; the results of numerical computations are given.

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A class of localized solutions of the linear and nonlinear wave equations

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There is one main problem in the diffraction theory namely whether it is possible to build such 3D+1 diffraction (and dispersion) model that corresponds to the following experimental results: a) at one diffraction length the spot of any spectrally limited laser pulse satisfies the Fresnel diffraction. b) at several diffraction lengths one-two cycle optical pulses diffract semi-spherically. The linear Diffraction - Dispersion Equation (DDE) governing the propagation in approximation up to second order of dispersion is [1]:

$$-2ik_0\left(\frac{\partial A}{\partial z}+\frac{1}{v_{gr}}\frac{\partial A}{\partial t}\right) = \Delta A - \frac{1+\beta}{v_{gr}^2}\frac{\partial^2 A}{\partial t^2}, \tag{1}$$

where $\beta = k" k_0 v^2$ is a number counting the influence of the second order of dispersion. In vacuum and dispesionless media is obtained also the following Diffraction Equation (DE) ($v \sim c$):

$$-2ik_0\left(\frac{\partial V}{\partial z} + \frac{1}{c}\frac{\partial V}{\partial t}\right) = \Delta V - \frac{1}{c^2}\frac{\partial^2 V}{\partial t^2}. \tag{2}$$

We solve DDE (1) and DE (2), applying spatial Fourier transformation to the amplitude functions $\vec{x}A$ and $\vec{x}V$. The fundamental solutions of the Fourier images $\hat{A}$ and $\hat{V}$ in (k_x, k_y, k_z, t) space are correspondingly:

$$\hat{A} = \hat{A}(k_x, k_y, k_z, t = 0)\exp\left\{i\frac{v}{\beta+1}\left(k_0 \pm \sqrt{k_0^2 + (\beta+1)\left({k_x}^2 + {k_y}^2 + ({k_z}^2 - 2k_0k_z)\right)}\right)t\right\}, \tag{3}$$

$$\hat{V} = \hat{V}(k_x, k_y, k_z, t = 0)\exp\left\{ic\left(k_0 \pm \sqrt{{k_x}^2 + {k_y}^2 + (k_z - k_0)^2}\right)t\right\}. \tag{4}$$

In air $\beta \simeq 2.1 \times 10^{-5}$, DDE (1) is equal to DE (2), and at hundred diffraction lengths appear only diffraction problems. We solve analytically the convolution problem (4) for initial Gaussian light bullet of kind of $V(x, y, z, t = 0) = \exp\left(-(x^2 + y^2 + z^2)/2r_0^2\right)$. The solution is:

$$\begin{aligned} V(x,y,z,t) &= \frac{i}{2\hat{r}}\exp\left[-\frac{k_0^2 r_0^2}{2} + ik_0(vt - z)\right] \times \\ &\left\{ i(vt+\hat{r})\exp\left[-\frac{1}{2r_0^2}(vt+\hat{r})^2\right] erfc\left[\frac{i}{\sqrt{2}r_0}(vt+\hat{r})\right] \right. \\ &\left. -i(vt-\hat{r})\exp\left[-\frac{1}{2r_0^2}(vt-\hat{r})^2\right] erfc\left[\frac{i}{\sqrt{2}r_0}(vt-\hat{r})\right] \right\} \end{aligned} \tag{5}$$

where $\hat{r} = \sqrt{x^2 + y^2 + (z - ir_0^2k_0)^2}$. The numerical solutions of the DDE (1) and the DE (2), as well as the analytical solution (5) of DE (2), satisfy exactly the conditions a) and b) obtained in the experiments with fs and attosecond pulses. Multiplying the solution (5) with the main phase, we obtain also an exact solution of the wave equation $E(x, y, z, t) = V(x, y, z, t)\exp(i(k_0 z - \omega_0 t))$, where ω_0 and k_0 are carrier frequency and carrier wave number in the wave packet:

$$\Delta E = \frac{1}{v^2}\frac{\partial^2 E}{\partial t^2}, \tag{6}$$

$$\begin{aligned} E(x,y,z,t) &= \frac{i}{2\hat{r}}\exp\left(-\frac{k_0^2 r_0^2}{2}\right) \times \\ &\left\{ i(vt+\hat{r})\exp\left[-\frac{1}{2r_0^2}(vt+\hat{r})^2\right] erfc\left[\frac{i}{\sqrt{2}r_0}(vt+\hat{r})\right] \right. \\ &\left. -i(vt-\hat{r})\exp\left[-\frac{1}{2r_0^2}(vt-\hat{r})^2\right] erfc\left[\frac{i}{\sqrt{2}r_0}(vt-\hat{r})\right] \right\}. \end{aligned} \tag{7}$$

One systematic study on different kinds of exact solutions and methods for solving the wave equation (6) was performed recently in [2]. Here and also in [3] we suggest one new method: In the beginning, we use the ansatz $E(x, y, z, t) = V(x, y, z, t)\exp(i(k_0 z - \omega_0 t))$ to separate the main phase and reduce the wave equation to 3D+1 parabolic type one (2). Thus, one initial value problem can be solved and exact (5) (or numerical) solutions of the corresponding amplitude equation (2) can be obtained . The solution (5), multiplied with the main phase, gives an exact solution (7) of the wave equation (6).

In nonlinear regime, when the offset frequency is included, the nonlinear amplitude equation for nanosecond pulses in air can be transformed to:

$$\Delta B - \frac{\partial^2 B}{\partial t^2} + \gamma |B|^2 B = 0, \tag{8}$$

where $\gamma = B_0^2 n_2 k_0^2$ is the nonlinear coefficient. After neglecting two small perturbation terms, the corresponding amplitude equation for short femtosecond pulses can be reduced to:

$$\Delta C - \frac{\partial^2 C}{\partial t^2} + \gamma C^3 = 0. \tag{9}$$

The different kind of the nonlinear part in Eq. (8) in respect to Eq. (9) is important. While Eq. (8), governing ns and ps pulse dynamics, admits exact analytical solution with components propagating in forward and backward direction, the Eq. (9) admits soliton solution propagating in forward direction only. A partial soliton solution of Eq. (8) is $B = sech(ln(\hat{r}))/\hat{r}$, where $\gamma = 2$, and we use a transformation of kind $\hat{r} = \sqrt{x^2 + y^2 + (z+a)^2 - v^2(t + a/v)^2}$. The solution separates the initial pulse to two maximums propagating in forward and backward direction with group velocity. The analytical solution of Eq. (9) is of the same kind: $C = 1/(1+\tilde{r}^2)$, but with a new complex pseudo-spherical radial vector $\tilde{r} = \sqrt{x^2 + y^2 + (z+ia)^2 - v^2(t + ia/v)^2}$. Thus, the fs solitons propagates in forward direction only [4].

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Radial atomic functions in digital signal processing

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The class of radial basis functions with compact support on the basis of the theory of atomic functions (AF) [1-9] is consider. As is known [2-9] AF are the solutions of functional differential equations with shifted argument with compact support. The important thing is their smoothness and high speed of convergence of bases on their basis.

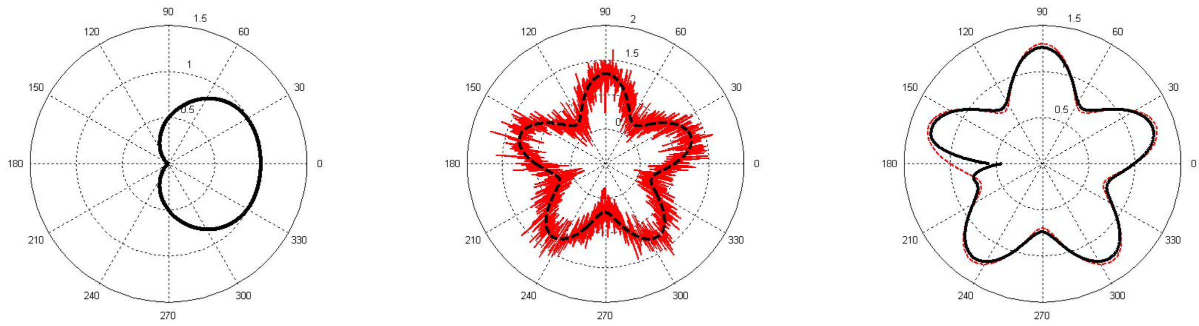
Radial spherical atomic functions. Consider the following differential equation with shifted arguments on the unit sphere: $\frac{df(\phi)}{d\phi} = 2f(2\phi + \pi) - 2f(2\phi - \pi)$. In [2] showed that its solution is the function

$$\widetilde{up}(\phi) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ju\phi} \prod_{k=1}^{\infty} \frac{\sin\left(\pi u 2^{-k}\right)}{\pi u 2^{-k}} du.$$

Evidently that $\widetilde{up}(0) = 1$ and $\sup p\left(\widetilde{up}(\phi)\right) = [-\pi, \pi]$. The basic properties [2] of $\widetilde{up}(\phi)$ functions are following:

$$\widetilde{up}\,(\phi) = \widetilde{up}\,(-\phi),\ \max\left(\widetilde{up}\,(\phi)\right) = 1,\ \int\limits_{-\pi}^{\pi} \widetilde{up}\,(\phi)\,d\phi = \pi.$$

The spectrum of functions is even real function, decreases rapidly. Function $\widetilde{up}\,(\phi)$ are shown on Figure *a*. Example of filtering of noised radial signal on interval $[-\pi, \pi]$ are presented on Figure *b,c*.



a) *b*) *c*)

Figure : (*a*): function $\widetilde{up}\,(\phi)$, (*b*): radial signal without (dotted line) with noise (solid line), (*c*): filtered signal (solid line).

They have a compact support and infinitely differentiable. This allows to apply them in many physical applications, improving existing and obtain new efficient algorithms for processing. They can be used in solving boundary value problems of mathematical physics, antennas theory and the nonlinear Schrödinger equation. On basis of these functions analytical and orthogonal radial wavelets [1, 3-9] are constructed.

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New analytical WA-systems of Kravchenko functions

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In this report the new class of analytical WA-systems of Kravchenko functions on the basis of the theory of atomic functions (AF) [1-9] are constructed and proof. The all wavelet conditions [5-7] are satisfied exact. The expression for the wavelet function has the following form: $\psi(t) = w(t)\{\exp(i\eta t) - A(\eta)\}$, $\psi_{a,b}(t) = \frac{1}{\sqrt{|\alpha|}}\psi\left(\frac{t-b}{\alpha}\right)$, where $w(t)$ is AF [1-4], $A(\eta) = \frac{\hat{w}(\eta)}{\hat{w}(0)}$, α is the dilatation variable and b is represents time shift (α, b are real). For example, for the AF $h_a(t)$ we have $A(\eta) = \prod_{k=1}^{\infty} \operatorname{sinc}\left(\frac{\eta}{a^k}\right)$, and $\psi_h(t) = h_a(t)\left\{\exp(i\eta t) - \prod_{k=1}^{\infty} \operatorname{sinc}\left(\frac{\eta}{a^k}\right)\right\}$. Behavior of this function and its spectra for $a = 4$, $\eta = 2.5\pi$ are presented on Figure 1 *a*,*b*.

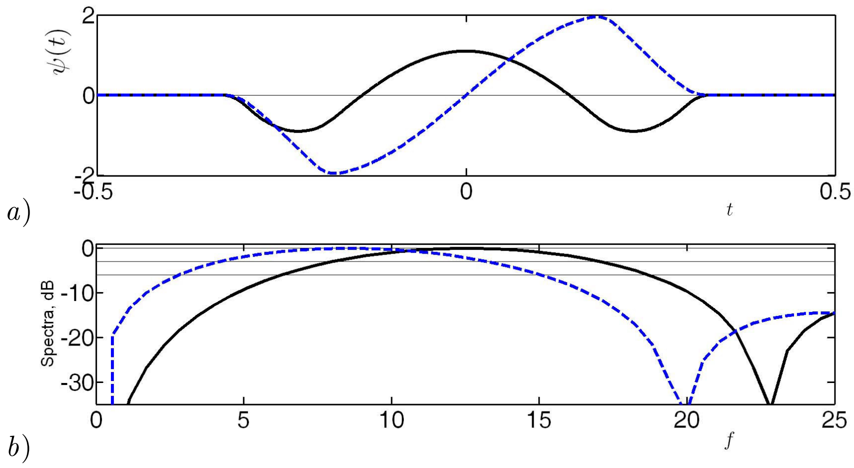

Figure : (a): function $\psi_h(t)$, (b): its spectra for $a = 4$, $\eta = 2.5\pi$(solid line is real part, dashed line is imaginary one).

The constructed wavelets have a compact support and they are infinitely differentiable. On their basis the 2D and orthogonal wavelets are constructed.

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Atomic and R-functions in p-adic analysis theory

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The theory of atomic functions (AF) [1-4] which are the solutions with compact support of special type of p-adic [5] functional differential equations (FDE) is considered. Uniqueness theorems are formulated in which the uniqueness of their solutions and their application to boundary value problems of mathematical physics. The solution of p-adic boundary value problems as shifts of corresponding atomic functions is discussed. The theory of p-adic R-functions consisting in the following: on the first stage basis system of real-valued R-functions of p-adic argument is introduced and on the second stage reasonable total system of R-functions are described. Adelic formulas [5] linking p-adic with real-valued R-functions are considered. Logical and differential properties of defined R-functions are investigated.

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Atomic functions and spectral operators theory in quantum scattering problems

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A new method of solution direct and inverse problems of quantum scattering theory [1-4] is considered on the basis of atomic functions (AF) and spectral operators theory [5-8]. Central idea a new approach is the following:
• the finite solutions of functional-differential equations of n-variables with delay type are found by means Heaviside operator method,
• the solutions of nonlinear Schrodinger equation is constructed of their shifts,
• the completeness and uniqueness theorems of such representation are proved.

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Hyperbolic systems with characteristics of variable multiplicity

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Consider hyperbolic systems of the first order y let some characteristics roots of the system coincide at a some set M in the extended phase space. When the wave front of the Cauchy problem permeate this set M of multiplicity it generates the waves propagating by the all characteristics roots multiple at the set M. Hence the waves propagate along ramified characteristics. Such ramification can provide cycles in the case of multiplicity not less than three, and make the well-posedness of the hyperbolic system dependent on low order terms. As an example the system of magneto hydrodynamics is considered.

Excitation of electromagnetic waves by a pulsed ring current in a magnetoplasma

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Excitation of monochromatic waves by electromagnetic sources immersed in homogeneous and inhomogeneous magnetized plasmas has received much careful study and there are many accounts of it (see, e.g., [1, 2] and references therein). Over the past decade, there has been shown a substantial degree of interest in the excitation and propagation of nonmonochromatic signals in a magnetoplasma [3, 4]. This interest has been motivated by the importance of transient wave phenomena for propagation of whistler-mode waves through the magnetosphere and the ionosphere, as well as plasma diagnostics using pulsed signals launched from antennas on spacecraft. Much previous theoretical work on the subject applies to calculation of the fields due to various idealized pulsed current sources in a magnetoplasma. However, there exists a very little theory of the energy characteristics of such sources. It is the purpose of the present paper to discuss the energy radiation characteristics of an electromagnetic source in the form of a pulsed ring current immersed in a cold homogeneous magnetoplasma.

We consider a ring source whose axis of symmetry is aligned with an external static magnetic field. The current of the source is either a pulse containing a few half-periods of a monochromatic oscillation, or a single pulse without modulation. At first, we find the total field excited by such a source. To describe the temporal behavior of the field, the Laplace transform technique is employed. The spatial structure of the source-excited field is represented in the form of expansion over the eigenwaves of a homogeneous magnetoplasma [2]. Then, using the field representation obtained, we derive a general expression for the radiated energy and analyze its distribution over the spatial and frequency spectra of the excited waves as a function of the source-current parameters. The emphasis has been placed on the practically important case where the frequency spectrum of the source current is concentrated in the whistler frequency range. It is shown that in this case, almost all of the energy emitted from the source goes to the resonant part of that range, in which the whistler-mode refractive index surface has unbounded branches corresponding to quasi-electrostatic waves. One of the most interesting results obtained is that the radiated energy of a pulsed current that contains only a few half-periods of a monochromatic oscillation with the frequency lying in the resonant part of the whistler range is very close to the product of the current duration by the time-averaged radiated power of the corresponding monochromatic source. It is shown that such behavior of the radiated energy is related to the features of excitation of whistler-mode waves by the ring current in a magnetoplasma. In addition, conditions have been determined under which the radiation characteristics of the ring current with one half-period of a monochromatic oscillation and those of the same source with a realistic-shape single current pulse of comparable duration turn out to be almost identical. Detailed numerical results will be reported for the above-mentioned cases. The results obtained are shown to be useful for explanations of the data of experiments on the excitation of whistler-mode waves by pulsed sources in magnetized plasmas.

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Eigenvalues and eigenfunctions for Steklov-Dirichlet problems in half-plane

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The two-dimensional *sloshing problem* in in an infinite container $\Omega = \mathbf{R} \times (-\infty, 0)$ covered by a rigid dock $B \subseteq \mathbf{R}$ is the Steklov-Neumann spectral problem:

$$\begin{cases} \nabla^2 u(x,y) = 0, & x \in \mathbf{R},\ y < 0; \\ \partial_y u(x,0) = \mu u(x,0), & x \in \mathbf{R} \setminus B; \\ \partial_y u(x,0) = 0, & x \in B, \end{cases}$$

where $\mu \geq 0$ is the spectral parameter. For semi-infinite dock $B = (-\infty, 0]$, an explicit formula for the solution $u(x,y)$ was given by Friedrichs and Lewy in 1947. For an infinite dock with a single aperture

$B = \mathbf{R} \setminus (-1, 1)$, it is relatively easy to show that the eigenvalues μ_n are simple, and an asymptotic expansion of the form $\mu_n = \frac{n\pi}{2} + \frac{\pi}{8} + ... + o(\frac{1}{n^2})$ was given by A.M.J. Davis in 1970.

A similar Steklov-Dirichlet problem:

$$\begin{cases} \nabla^2 v(x,y) = 0, & x \in \mathbf{R},\ y < 0; \\ \partial_y v(x,0) = \nu v(x,0), & x \in \mathbf{R} \setminus B; \\ v(x,0) = 0, & x \in B, \end{cases}$$

has recently attracted much attention for its applications in probability theory. I will discuss this connection and the results of my joint paper *Spectral properties of the Cauchy process on half-line and interval* with T. Kulczycki, J. Małecki and A. Stós (arXiv:0906.3113). We derived an explicit formula for $v(x, y)$ when $B = (-\infty, 0]$ by solving a Riemann-Hilbert problem, a method applied by Friedrichs and Lewy. For $B = \mathbf{R} \setminus (-1, 1)$, we proved simplicity of eigenvalues and an asymptotic formula $\nu_n = \frac{n\pi}{2} - \frac{\pi}{8} + O(\frac{1}{n})$. This required different methods than those used in the case of the sloshing problem. We also applied standard numerical methods for estimation of eigenvalues to effectively find numerical lower and upper bounds for eigenvalues.

Comparison of the T-matrix and the pattern equations methods

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T-matrix method (TMM), proposed by Watermen more than forty years ago [1], is currently commonly used for solving wave diffraction problems arising in optics, radiophysics, radio-astronomy, etc. [2], [3]. T-matrix interrelates incident and scattered wave spherical basis expansion coefficients. As such, T-matrix depends only on physics and geometric characteristics of scatterer and is absolutely independent on propagation and polarization directions of the incident and scattered fields [2], [3]. Pattern equation method (PEM), for the first time proposed in paper [4], also allows obtaining the solution of the diffraction problem in the form similar to TMM, but it is applicable at significantly less stringent restrictions on scatterer geometry. So, it is of interest to compare these two methods. In paper [5] it is shown, that TMM is correct only if the scatterer geometry belongs to the class of Rayleigh bodies, i.e. such bodies that all wave field analytic continuation singularities are located inside of the sphere inscribed in scatterer. Such class of geometries is particularly narrow. PEM allows to obtain the rigorous diffraction problem solution (i.e. theoretically with any given accuracy) for so called weakly non-convex bodies [4]. All convex bodies are part of this class. As an example, let's consider the diffraction problem for the plane wave with incident angle $\varphi_0 = 0$ on Rayleigh ellipse with semiaxis $ka = 8, kc = 11$. We calculate the scattering pattern as: $g(\varphi) = \sum_{n=-\infty}^{\infty} c_n i^n e^{in\varphi}$. Let's denote $g_N(\varphi)$ - the scattering pattern, obtained by solving the truncated TMM or PEM algebraic system (when its size is equal to $2N+1 \times 2N+1$). We calculate the difference between the patterns at different N as $\Delta g^{\max}_{N_1,N_2} = \max |g_{N_1}(\varphi) - g_{N_2}(\varphi)|$. If $\Delta g^{\max}_{N_1,N_2} < 10^{-6}$, i.e. at least 7 significant digits are agreeing in the patterns, we consider that adequate accuracy is achieved and there is no point to increase N any more. Additionally, we assess the graphic overlap of patterns. The calculated values of $\Delta g^{\max}_{N_1,N_2}$ at different N_1, N_2 for PEM and TMM are given in the table.

	PEM	TMM
$\Delta g^{\max}_{10,15}$	$4.7334279 \cdot 10^{-1}$	$4.7133309 \cdot 10^{-1}$
$\Delta g^{\max}_{15,20}$	$6.0765886 \cdot 10^{-4}$	$1.1055532 \cdot 10^{-2}$
$\Delta g^{\max}_{20,25}$	$3.3979730 \cdot 10^{-7}$	$1.7063574 \cdot 10^{-4}$
$\Delta g^{\max}_{25,30}$	$2.4759473 \cdot 10^{-11}$	$1.6644684 \cdot 10^{-6}$
$\Delta g^{\max}_{30,35}$	$8.3348103 \cdot 10^{-14}$	$3.2701861 \cdot 10^{-6}$

As it shows, the PEM has much higher convergence rate and allows obtaining twice as much accuracy than TMM. In PEM we have reached the desired accuracy of 10^{-6} already at $N = 20$, whereas TMM did not obtain the desired accuracy at all. The highest possible accuracy, which PEM provides for a given scatterer is $8.3348103 \cdot 10^{-14}$, but TMM achieves only $1.6644684 \cdot 10^{-6}$. As it can be seen, at $N > 35$ for PEM and at $N > 25$ for TMM the accuracy begins to decrease. This is caused by the increase of special function calculation error, which eventually leads to the failure of the algorithm (see [5]). At $N = 25$ the computation time of PEM is 10.779 seconds and TMM is 9.224 seconds. At $N = 60$ the computation time of PEM is 50.136 seconds and TMM is 21.502 seconds. The comparison of PEM and TMM demonstrate, that PEM is unconditionally superior than TMM in terms of accuracy and applicability. The price for this is some increase of computation time. The averaging of scattering characteristics by orientation of the particle is similarly simple in both PEM and TMM.
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High-frequency upper and lower bounds for the total cross section in scattering by obstacles

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We consider the scattering of plane waves by a bounded obstacle $\mathcal{O} \in \mathbb{R}^3$ with a smooth boundary $\partial\mathcal{O} \in C^2$ and impedance boundary conditions. The scattered field $u = u(r)$, $r = (x, y, z)$ satisfies the Helmholtz equation in $\Omega = \mathbb{R}^3 \backslash \mathcal{O}$ and radiation conditions:

$$\begin{cases} \Delta u(r) + k^2 u(r) = 0, \quad r \in \Omega, \quad k > 0, \\ \int_{|r|=R} \left| \frac{\partial u(r)}{\partial |r|} - iku(r) \right| dS = o(1), \quad R \to \infty. \end{cases} \tag{1}$$

The Robin boundary condition holds at the boundary:

$$\frac{\partial u}{\partial n} + k\gamma u = -\left(\frac{\partial e^{ik(r\cdot\alpha)}}{\partial n} + k\gamma e^{ik(r\cdot\alpha)} \right), \quad r \in \partial\Omega, \tag{2}$$

where $\alpha \in S^2$ is the direction of the incident plane wave, n is the exterior normal for $\mathcal{O}$ (directed into Ω).

In [3] one can find the theorem on the existence of the solution to the problem (1)-(2) with $\Im\gamma \geq 0$.

Let us recall that any solution of the problem (1) has the following asymptotic behavior at infinity:

$$u(r) = u_\infty(\theta)\,\frac{e^{ik|r|}}{|r|} + o\left(\frac{1}{|r|}\right), \ \theta = \frac{r}{|r|}, \quad |r| \to \infty. \tag{3}$$

Function $u_\infty \in L_2(S^2)$ is called the scattering amplitude (it depends also on k and α), and the square of its norm

$$\sigma_k(u) = \int_{S^2} |u_\infty(\theta)|^2 dS, \tag{4}$$

is called the total cross-section.

Transport Cross Section equals to the full momentum transmitted to the obstacle:

$$\sigma_k^T(u) = \int_{S^2} (1 - \theta\cdot\alpha)|u_\infty(\theta)|^2 dS, \tag{5}$$

is called the total cross-section.

We present several lower and upper bounds for σ_k that are uniform for large $k >> 1$ both in the case of $\Im\gamma > 0$ and in the case when $\gamma \in \mathbb{R}$.

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Elastic wave scattering and inverse scattering in anisotropic solid materials

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Linear Elastic waves in linear solid materials are solutions of the elastodynamic governing equations that are similar to Maxwells equations for electromagnetic waves relating first time derivatives of field quantities to first spatial derivatives. Fundamental solutions come as plane waves for the homogeneous equations (no given sources) and Green functions for the inhomogenenous equations. Plane waves already tell a lot about the particular existence of wave modes, in the case of elastic waves in homogeneous anisotropic materials one quasi-pressure and two quasi-shear modes being orthogonal to each other and exhibiting a phase velocity depending on the direction of the phase vector. Interesting enough the direction of energy flow as given by the elastodynamic Poynting-vector is different from the phase velocity as well as in magnitude and in direction. Analytic expressions for both can be given for linear time-invariant homogeneous anisotropic instantaneously and locally reacting materials.

For scattering and inverse scattering purposes appropriate Green functions are required, unfortunately no analytic expressions exist for neither kind of materials. Yet, applying discretization methods to the governing equations a most interesting feature is observed: Band-limited time domain Huygens elementary wavelets emanating from point sources exhibit the same spatial structure as the energy velocity diagrams for plane waves. That way, standard time domain backpropagation imaging algorithms as developed for isotropic materials and non-destructive purposes can be, at least approximately, extended to anisotropic materials. The same is true for Born-type scattering in the time domain.

Alternatively, analytic Green function expressions in the frequency wave number domain can be exploited to formulate diffraction tomographic inverse scattering algorithms as counterpart of time domain backpropagation.

Results and examples will be given for a particular problem in non-destructive testing, i.e. the integrity assessment of tendon ducts in concrete.

Non-stationary reflection of a nonlinear electromagnetic wave from smoothly non-uniform isotropic plasmas

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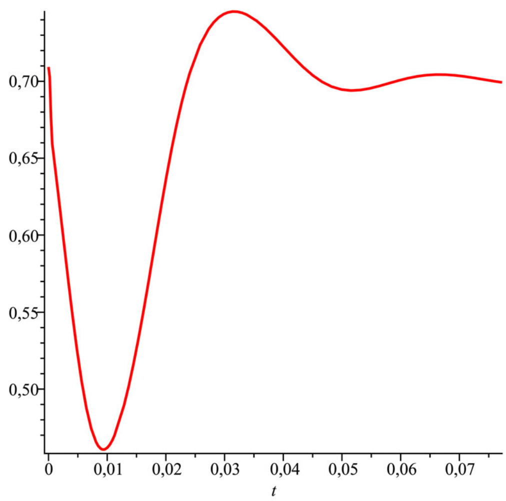
The problem is considered about nonlinear interaction of the falling and reflected electromagnetic waves with raised by them ion - sound waves in a non-uniform layer of isotropic plasmas. At the initial moment of time plasma is free of disturbances and fills half -space with linear dependence of concentration with height. The electromagnetic wave falls on the plasma- vacuum border in a direction of a gradient of concentration. Such problem statement corresponds to ionosphere experiments condition and is directed on explanation of the physical model of electromagnetic radiation propagating from plasma. Previously we already investigated analytically some aspects of this interaction connected to scattering of a probe pulse after short-term influence of a powerful wave on plasma (Lapin V.G. Proceedings of "Days on Diffraction' 06"). However character of evolution of a powerful wave at long time interaction has not been investigated. In the present message some results of research of these phenomena with application of numerical methods are discussed.

Solutions of the equations for a field of an electromagnetic wave and plasma concentration disturbances by forced ion sound wave were investigated at $z > -L$:

$$\begin{aligned}
\frac{\partial^2 E}{\partial z^2} - \frac{2i\hat{k}}{V_g}\cdot\frac{\partial E}{\partial t} + \hat{k}^2 E &= \frac{4\pi e^2}{m_e c^2} nE;\\
\frac{\partial^2 n}{\partial t^2} - 2\gamma\cdot\frac{\partial n}{\partial t} - V_g^2\frac{\partial^2 n}{\partial z^2} &= \frac{\omega_p^2}{16\pi\omega^2 M_i}\frac{\partial^2}{\partial z^2}|E|^2;\\
\hat{k}^2 = \frac{\omega^2}{c^2}\varepsilon_0; \qquad \varepsilon_0 = 1 &- \frac{\omega^2}{\omega_p^2}\left(1+\frac{i\nu}{\omega}\right) = -\frac{z}{L}\left(1+\frac{i\nu}{\omega}\right).
\end{aligned}$$



Boundary conditions corresponded to falling of a plane monochromatic wave on plasma (at $z = -L$) and to attenuation of a wave field in over critical region (at $z > 0$).

At the initial moment of time ion sound wave was absent $n(z, t = 0) = 0$, and the field of an electromagnetic wave is expressed by Airy function (if collision frequency $\nu = 0$):

$$E(z, t = 0) = 2E_0\sqrt{\pi}(kL)^{1/6}\cdot Ai((kL)^{2/3}z/L); \quad k = \omega/c.$$

Calculation parameters correspond to the experiments which were earlier carried out on ionosphere experimental facility "Sura" (Radiophysical research institute, Nyzhny Novgorod), and correspond to plasma of

F- layer of ionosphere: $\omega/2\pi \approx 6 \cdot 10^6\ Hertz$, $L = 50\,km$, $\nu/\omega \approx 10^{-3}$.

On the figure the initial stage is represented of evolution of amplitude of the electromagnetic wave reflected by plasma. We see, that development of ion sound disturbances is accompanied by reduction of reflection from a layer of plasma, and also to occurrence of impulses in the reflected field. Qualitatively it is possible to explain these features of a field, involving results of our former researches on multiple scattering of waves in the media with periodic structure.

Lieb-Thirring inequality for Schrödinger operator with δ-potential on a loop

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In the talk Lieb-Thirring type inequalities for Schrödinger operators with surface potentials will be discussed. We consider two-dimensional Schrödinger operator with δ-potential supported by a closed C^2 curve Γ

$$H_{\alpha,\Gamma} = -\Delta - \alpha\delta_\Gamma(\cdot).$$

with $\alpha > 0$. The operator $H_{\alpha,\Gamma}$ has discrete negative spectrum. We compose Lieb-Thirring sums of negative eigenvalues of $H_{\alpha,\Gamma}$

$$\sum_{j\geq 1} |\lambda_j|^\gamma, \quad \gamma > \frac{1}{2}, \tag{1}$$

and give an upper bound for such sums in terms of α, the geometry of Γ and the parameter γ itself.

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Angular dependence and field distribution in pressed wave

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It was shown earlier [1], that in conditions of surface electromagnetic wave (SEW) excitation by input grating coupler the fields of generated wave are differed significantly from those of SEW. This wave was named pressed wave (PW). PW field amplitudes are given by Fourier-integrals on x coordinate (the wave propagates x-axis direction). In this work we examine the dependence of PW intensity as a function of laser beam incidence angle for number of values of grating width and complex dielectric constant of metal. It is shown that if the width of grating relief Fourier-spectrum is much more (the grating width is small enough) than the width of angular dependence curve for infinite grating the angular dependence of PW intensity coincides with Fourier-spectrum of grating relief. Otherwise the angular dependence of PW intensity is given by angular dependence curve for infinite grating (note that infinite grating angular dependence is determined by complex dielectric permittivity). In the intermediate case both factors contribute in angular dependence curve. The calculated results are compared with experimental ones.

We also have calculated the distributions of field amplitudes versus z-coordinate (z axis is perpendicular to the metal surface). It is obtained that PW field amplitudes first decrease exponentially (as for SEW) and then increases again, reach a maximum and then gradually decrease to zero. This amplitude field behavior is consistent with the intensity distribution of near surface bulk radiation observed in the experiment [2, 3]. The field distributions are also calculated as a function of the grating width and complex dielectric permittivity of metal and the obtained results are discussed.

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Correlation functions of integrable spin chains with boundaries

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Various integrable systems, which belong to $U_q(sl_2)$-symmetry, such as the sine (sinh)-Gordon model and the XXZ model, have been studied. Calculation of correlation functions of them is one of the most interesting topics in study of integrable systems.

The biggest problem in computation of correlation functions via the Bethe ansatz is how to deal with sums which arise as a result of commutation relations among the monodromy matrix elements. The induction method in respect to the total spin of a system and change of basis are well-known methods to resolve this problem.

We derived multi-integral expressions of correlation functions for higher spin integrable systems. Furthermore, it was showed that the number of terms in sums appeared in correlation functions of higher spin integrable systems are reduced to that of spin 1/2 integrable systems by considering actions of the $U_q(sl_2)$ algebra on irreducible subspaces in multiple tensor products.

Shear viscosity like a consequence of angular momentum relaxation at hydrodynamical description

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The generalized variational principle (GVP) was derived in the previous papers of the author [1-3]. GVP combines Hamilton's variational principle for dissipationless mechanics with Onsager's variational principle for dissipative thermodynamical systems. It was shown that the motion equations of dissipative hydrodynamics can be derived on the basis of GVP. The shear and bulk viscosities can be introduced into equations of disspative hydrodynamics by using of the Mandelshtam-Leontovich theory of internal parameters [1,3]. This approach generalizes Navier-Stokes equation taking into account viscosity relaxation phenomenon. Nevertheless there is a question about physical interpretation of the used internal parameter.

It is shown in the report that the internal parameter responsible for shear viscosity can be interpreted as a consequence of relaxation of angular momentum of material points constituting mechanical continuum. The rotational degree of freedom as independent variable appears additionally to the mean mass displacement field. For the dissipationless case this approach leads to the well-known Cosserat continuum. When dissipation prevails over inertion this approach describes local relaxation of angular momentum and corresponds to the sense of internal parameter. Frequency dependencies of wave number of eigen modes propagating in the dissipative Cosserat continuum are considered in the report.

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On the phase shift in the Kuzmak-Whitham method for nonlinear problems

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We discuss asymptotic solutions in the form of Kuzmak-Whitham anzats for nonlinear oscillator, wave equation, KdV equation etc. It is well known that in this case the leading term of asymptotic solution could be presented in the form X(S(t,x)/h+f(x,t)),E(x,t),x,t)+O(h), where h¡¡1, the phase S and the "slow varying" parameter E are found from the system of the "averaged" Whitham equations. In nonlinear case the equation for the so called phase shift f(x,t) is obtained during an investigation of the second correction to the leading term and the corresponding procedure is not uniform with respect to a passage to a linear and a weak linear case. Our main observation is that combining the phase shift f(x,t) with the phase S(t,x) and correcting E(x,t) one obtains the same averaged Whitham equations, the difference is in a change of corresponding initial data.

This work was done together with S.Yu. Dobrokhotov and supported by RFBR grant 08-01-00726.

Sound generated by impact on thin ice

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At the times of the late of autumn or the early winter time when the thin layer of ice covers the pond the beautiful and vivid acoustical effect can be observed. If one threw the small pebble or sprig far from the pond's bank the melodious quasitonal sound is generated. The farther the exciter is thrown the longer the sound lasts. The thicker is the ice the lower is the frequency of tone. The snow layer decreases, but does not suppress the effect. In the entry suggested the natural sounds will be demonstrated, that were registered by dictaphone on the frozen forest pond near Moscow (with the nice photo of the pond). The spectra and the spectrograms of the sounds will be given. The appearance of the narrow frequency band sound, generated by the wideband source (-shaped stroke) and the lasting will be qualitatively and quantitatively explained. The analytical solution in the form of double Fourier transformation will be analyzed by means of asymptotic decomposition.

Investigation of Rayleigh waves on free curvilinear boundaries of elastic media

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Propagation of Rayleigh waves on free curvilinear boundary of homogeneous isotropic elastic medium is considered. We consider these waves propagating along element and directrix of the cylinder surfaces and along meridian on the sphere. In these cases we construct the precise solutions of the equations of the theory elasticity and use asymptotics of Hankel and Legendre functions. On the basis of comparison of results, we make assumption about dependence of velocity of the Rayleigh wave on the small curvature of route and on the small curvature in perpendicular direction.

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Surface water waves trapped near submerged cylindrical bodies

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In the work we study the linear problem of surface wave theory which describes interaction (radiation or diffraction phenomena) between an ideal unbounded fluid, having a free surface, and a system of totally submerged cylindrical bodies of arbitrary cross-section. Namely, infinitely long horizontal cylinders in oblique waves or cylinders spanning vertical walls of a channel are considered. It is also assumed that the fluid motion is small-amplitude, irrotational, and harmonic in time. These assumptions lead to a two-dimensional boundary-value problem for a velocity potential u in a cross-section of the fluid domain orthogonal to the generators of the cylinders. The potential u satisfies $\nabla^2 u - k^2 u = 0$

in the cross-section of the fluid domain, the condition $\partial_y u - \nu u = 0$ on the averaged free surface $\{y = 0\}$ (y decreases with depth), Neumann condition on the surface of cylinders and a radiation condition at infinity (see e.g. [1]).

Of interest here is the existence of so-called trapped modes, i.e. solutions to the homogeneous problem ($\partial_n u = 0$ on the surface of bodies), corresponding to unforced oscillations of the fluid, localized near the obstacles. The trapping can occur at isolated values of the spectral parameter ν, which are treated as eigenvalues belonging to the discrete spectrum when $\nu < k$ and as point eigenvalues embedded into the continuous spectrum when $\nu > k$, while $\nu = k$ is the threshold frequency. A good review on the subject is given in [1]; recent papers [2, 4] can also be mentioned. The existence of trapped modes below the threshold is known for some geometries since 1951 (see [5]). In [2] the existence of trapped modes for $\nu < k$ is proved for any number of totally submerged cylinders. Trapped modes above the threshold for such geometries are only known for $k = 0$ (see [3] and references therein).

In this work we extend to the case $k \neq 0$ the methods of [3], which are based on boundary integral equations of potential theory, introduction of two compact self-adjoint operators and investigation of some functionals on their eigenfunctions. Following [3], for $k \neq 0$ a criterion of unique solvability of the boundary-value problem is proved and applied for development of algorithms for detecting non-uniqueness (existence of trapped modes) both below and above the threshold. The algorithms allow us to seek trapped modes for given bodies of arbitrary shape (but above the threshold we demand that geometry is symmetric with respect to a vertical line). Numerical realization and validity of numerical results are discussed. A number of examples of trapped modes are found numerically: in particular, we give first examples of non-uniqueness for totally submerged bodies when $\nu > k \neq 0$.

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Direct use of the far-field patterns in the multi-wave enclosure method

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The usual multi-wave enclosure method for the inverse scattering problem is done by transforming it to the inverse boundary value problem. In this talk we will propose a method which directly use the far-fields patterns. The key is to analyze the behavior of the reflected solutions of the complex geometric solutions.

Asymptotic solution of 2-D wave equation with vanishing variable velocity and localized initial data

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In linear approximation we study amplitudes and profiles of waves near the beach generated by the source localized in the neighborhood of the point $q = (q_1, q_2)$ placed far from the beach. As a model we use the special Cauchy problem for 2-D wave equation on the plane (x_1, x_2)

$$U_{tt} = \operatorname{div}\left(c^2(x)\operatorname{grad}U\right) = 0; \quad U|_{t=0} = V((x-q)/\mu) = 0), \quad U_t|_{t=0} = 0, \quad \mu \ll 1,$$

with the velocity $c(x)$ vanishing on a certain smooth open line L on the plane $(x_1.x_2)$ and the function $V(y_1, y_2)$ decaying at infinity. The asymptotic solutions are constructed by means of the Maslov canonical operator modified for the localized asymptotics and singular Lagrangian manifolds.

This work was done together with S.Yu.Dobrokhotov and B.Tirozzi and supported by RFBR grant 08-01-00726.

On the existence of the fundamental modes of the wedge guide

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The existence of waves propagating along the edge of the elastic wedge was established by many authors (see, e.g.,[2,3,4]) at the physically rigorous level on the basis of numerical computations. The mathematically rigorous proof for wedge with angles less than $\pi/2$ was presented by I.Kamotskii in [1].

We amplify the I. Kamotskii's result and prove the existence of the fundamental modes for some range of angles greater than $\pi/2$.

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Trapped modes in cranked and branched waveguides

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For a wide class of cracked and branched waveguides, the existence of trapped modes with frequencies below the continuous spectrum is readily demonstrated by means of the variational method. A tool to calculate the total multiplicity of the discrete spectrum is shown as well. A method to derive asymptotic formulas for eigenvalues is explained. Several open questions are formulated. All these resulted are mainly related to the case of the Dirichlet boundary condition, i.e. quantum waveguides and acoustic waveguides with soft walls.

New results are presented about the asymptotics of eigenvalues embedded into the continuous spectrum, that is for the Neumann boundary conditions which occur for acoustic waveguides with hard walls and for water-wave problems in channels with vertical walls and straight bottom. The approach is based on the notion of the augmented scattering matrix. Many open questions are formulated, too.

Analytical expansion of highly focused optical beams into vector spherical harmonics

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During the past decade numerous experimental and theoretical works have been undertaken to characterize, elaborate and manipulate single nanoscaled objects. The metal nanoparticles are among the hot topics of active research in nanoscience and related branches. The classical theory of linearly polarized plane wave scattering on a sphere [1] was extended after the invention of the laser and deviations from the classical theory were revealed. The first works have clearly demonstrated that the optical response is strongly dependent on the particle location relative to the beam focus and differs noticeably from that of plane wave irradiated.

The recently renowned interest in highly focused optical beams is mainly concerned with the polarization state of the beam, which strongly influences the size of the focal spot [2]. In particular, the role of azimuthal and radial polarization has been investigated [3]. The scalar complex source beam (CSB) model leads to exact solutions of the wave equation [4]. It can be extended towards vector diffraction theory to accurately describe linearly, radially and azimuthally polarized light [5].

The analytical expansion of linearly, azimuthally and radially polarized vigorous beam-type solutions of the Maxwell equations into vector spherical harmonics is presented in this work. We report on the dominance of the high order multipoles in the highly focused radially and azimuthally polarized beams compared under similar conditions with their linearly polarized counterparts, see Fig .1. Expansions of the vector complex source beams and beams obtained in the common high numerical aperture systems are compared. The generalized Mie theory as one of the possible applications of expansions is used to investigate a scattering of studied beams on a spherical golden nanoparticle. We found that the optical response of the particle is lower for the radially polarized light than for the linearly, when the energies of both beams are the same.

We also introduce expressions of the so-called "real" vector beams obtained in a variety of focusing systems and the conditions under which the spatial extent of the analytical and "real" beams is the same. To this end we compare the scattering of the analytical and "real" beams with the same spatial extent on a golden spherical nanoparticle. The differences in the scattering of the analytical solutions and high NA beams are discussed. We report on a good approximation of the high NA beams by their vector complex source counterparts.

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Reconstruction of individual electric field components of the highly focused optical beam by the Mie scattering scans

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The growing interest in extremely focused optical beams requires a proper treatment of the polarization state of the beam, which strongly influences the size of the focal spot [1]. In particular, the role of azimuthal and radial polarization has been investigated [2]: as the beam is focused sharper, by high numerical aperture (NA) objectives, the symmetry of the focal spot is broken for a linearly polarized beam and strong longitudinal components appear. Therefore, a precise characterization of tightly focused laser beams is not just a challenge but is essential for further applications of such beams.

Many beam characterization methods for are well known and established: a) a knife-edge method, b) a point scan method, c) a slit method etc. Though quite different in their nature all those share the common thing: their background is the scalar diffraction theory, so the precise characterization of highly focused vector beams by the classical evaluation schemes is not a trivial and often a hard task. For an example, the classical knife-edge method without optimization of material and knife parameters can be quite polarization sensitive [3], thus introducing additional difficulties in the reconstruction of the beam's electric field.

The aim of our report is a detailed study on the implementation of the scattering of highly focused various polarizations optical beams on a small metal sphere for the reconstruction of the individual electric field components. We do investigate the beam profiling situations for two different detectors numerical apertures either in the transmitted light or in the reflected. Our theoretical model is based on the classical extended Mie scattering theory. We develop an algorithm, which enables us to reconstruct the longitudinal and transverse components of the incident electric field, see Fig. 1. The optimization questions are also discussed: we do investigate best NA choices in order to eliminate the quadrupole response and develop the first order corrections to the algorithm. As a result, a good reconstruction of the electric field is possible.

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Hodge-Helmholtz decompositions of weighted Sobolev spaces

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We study Hodge-Helmholtz decompositions in nonsmooth exterior domains $\Omega \subset \mathbb{R}^N$ filled with inhomogeneous and anisotropic media. We show decompositions of alternating differential forms of rank q belonging to the weighted L^2-space $L^{2,q}_s(\Omega)$, $s \in \mathbb{R}$, into irrotational and solenoidal q-forms. These decompositions are essential tools, for example, in electro-magnetic theory for exterior domains, in particular, to describe the low frequency asymptotic of time-harmonic electro-magnetic fields properly.

Explicit formulae for higher modes of a nonplanar cavity with odd number of mirrors

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For optical ring-type cavity with a nonplanar contour and alone focusing element (a curved mirror or a lens), stability conditions were investigated and explicit formulae for the fundamental mode were obtained in [1] using technique outlined in [2]–[5]. Unlike the traditional approach [6]–[8], these formulae do not contain eigenvectors of a cavity's monodromy matrix (i.e. round-trip ray matrix). In this paper, on the contrary, the solution in question is used to find analytical expressions for eigenvectors mentioned, to construct ladder operators and to obtain cavity's higher modes in the case of odd number of mirrors.

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Transient current source in two-layer medium: time-domain version of Sommerfeld integral

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Electromagnetic (EM) radiation from a dipole placed at the interface of two half-spaces with different dielectric permittivities was studied in a number of classical works focused at radio propagation along the earth surface, e.g. [1-2]. Monochromatic radiation pattern of a line source lying at a plane interface has been calculated in [3]. The problem of pulsed EM radiation is much less studied. An analytical approach based on the Smirnov-Sobolev concept of functional-invariant solutions was developed in [4] in relation with seismic prospecting. In our paper, motivated by ground penetrating radar research, we construct an elementary solution to a key 2D radiation problem: a line transient current source lying at the interface between two homogeneous, non-dispersive dielectric media. The simplest solution, corresponding to a special current pulse (Heavyside function), is a homogeneous function of $r = \sqrt{x^2 + z^2}$ and $s = ct$ of order -1: $E(r, \vartheta, s) = \frac{1}{r} V(\tau, \vartheta)$.where $\tau = s/r$, $\vartheta = \arctan x/z$. Our reasoning is close to that of [4] but the method of constructing the solution is different. We start from the transient Green function $G(r, s) = (s^2 - n^2 r^2)_+^{-1/2} \equiv \frac{1}{r} V_0(\tau)$ represented as a superposition of pulsed plane waves of the same order of homogeneity:

$$V_0(\tau) = (\tau^2 - n^2)_+^{-1/2} = \frac{1}{2\pi} \oint\limits_{\Gamma_0} \frac{d\beta}{\tau - n\cos(\beta - \vartheta)} \tag{1}$$

where the integration path circumvents the complex pole $\beta_0 = \vartheta + i\mu$, $\cosh\mu = \frac{\tau}{n} > 1$. By generalization we look for a solution to our problem in the form

$$V(\tau, \vartheta) = \frac{1}{2\pi} \operatorname{Re} \begin{cases} \int\limits_{\Gamma_\beta} \frac{B(\beta)}{\tau - n\cos(\beta - \vartheta)} d\beta, & |\vartheta| < \frac{\pi}{2} \\ \int\limits_{\Gamma_\alpha} \frac{A(\alpha)}{\tau + \cos(\alpha + \vartheta)} d\alpha, & \frac{\pi}{2} < \vartheta < \frac{3\pi}{2} \end{cases} \tag{2}$$

(time-domain version of the Sommerfeld-Malyuzhinets integral [5-6]). It is assumed that the integration paths $\Gamma_{\alpha,\beta}$ contain the semi-strips $\left|\operatorname{Re} \frac{\alpha}{\beta}\right| < \frac{\pi}{2} + \delta_1$, $\left|\operatorname{Im} \frac{\alpha}{\beta}\right| > \delta_2$ while the functions $A(\alpha)$, $B(\alpha)$ are analytical and regular inside $\Gamma_{\alpha,\beta}$. By satisfying matching conditions for $V(\tau, \vartheta)$ and $\frac{\partial V}{\partial \vartheta}(\tau, \vartheta)$ at the interface $|\vartheta| = \pi/2$ and ensuring the right solution behavior at $\tau \to \infty$ we come to the Snell law $n \sin\beta = \sin\alpha$ and obtain two pairs of functional equations for $A(\alpha)$ and $B(\alpha)$. As a result, we get explicit formulae

$$A(\alpha) = K \frac{\cos\alpha}{\cos\alpha + n\cos\beta(\alpha)}, \qquad B(\beta) = K \frac{n\cos\beta}{\cos\alpha(\beta) + n\cos\beta} \tag{3}$$

and the integrals (2) are evaluated by residues. Analytic continuation describes all the wave field singularities at the geometrical wavefronts.

The developed analytical method, being a time-domain analog of the Sommerfeld integral, can be applied to more complicated problems: arbitrary pulse waveform, lossy media, wedge-shaped domains, etc.

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Model of point-like opening for Maxwell operator

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A model of interaction through the point orifice for Maxwell equations is discussed. Self-adjoint Maxwell operator for domains with smooth boundary was suggested by M.Sh. Birman [1]:

$$M = \begin{pmatrix} 0 & \varepsilon^{-1}\epsilon\mathbf{p}\mu^{-\mathbf{1}} \\ -\epsilon\mathbf{p} & 0 \end{pmatrix}.$$

Here $\varepsilon(\mathbf{x})$, $\mu(\mathbf{x})$ are smooth, strictly positive, bounded functions of $\mathbf{x} \in R^{\mathbf{3}}$, $\mathbf{p} = -i\partial_{\mathbf{x}}$ is a momentum operator and ϵ is Levi-Chivita tensor. For domains with ideal conducting boundary in the absence of free charges and currents the following conditions are satisfied:

$$\partial_{\mathbf{x}}(\varepsilon\mathbf{E}) = 0, \quad \partial_{\mathbf{x}}\mathbf{B} = 0, \quad \gamma_\tau\mathbf{E} = 0, \quad \gamma_\nu\mathbf{B} = 0.$$

Here $\mathbf{E}$ and $\mathbf{B}$ are electrical and magnetic fields, $\gamma_\tau\omega$ and $\gamma_\nu\omega$ are tangential and normal components of the field ω at the boundary of the domain.

In [2] a problem of scattering of electromagnetic waves by a system of small spheres was considered. A model of generalized point interaction (GPI) [3] for Maxwell equations was suggested. In this case the restriction of the operator on to the set of smooth functions vanishing at a point is essentially self-adjoint. Due to this fact the authors in [2] use Pontrjagin spaces with indefinite metric. We suggest a model of point-like opening in a screen for this operator. The model is based on the theory of operator extensions in Potrjagin space. Particularly a problem of point-like window in a plain is considered.

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True amplitude depth migration by Gaussian beam summation method

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True amplitude depth migration remains an actual topic in geophysics because it enhances migrated seismic image of a subsurface model. It occurs to be important in tomography for the velocity reconstruction. The problem which underlies the true amplitude migration consists in the following. All migration methods are based on computations of wave fields generated by sources or by recorded seismograms. As it is well known, the magnitude of wave fields decreases with increasing distance between the source and observation point and this phenomenon worsens the migration image in a deep part of the subsurface domain. If high-frequency asymptotic methods are used for the wave field description, this phenomenon stems from the geometrical spreading and therefore it has to be eliminated, or at least suppressed, to implement the true amplitude concept of migration.

Recently (see. [1] and [2]) we have suggested and tested on widely used benchmark models an approach to seismic depth migration based on Gaussian beam summation method which also belongs to high-frequency asymptotics of wave fields. Visually this approach can be explained as follows. Suppose on a seismogram we have a reflected wave. Then, for given smooth velocity model, we propagate this wave back in time together with the direct wave field generated by a source, located on the seismic surface, and fix the wave in such a depth position in the migration domain where both fields coincide in phase, i.e. are coherent. The coherence between the direct $U^{(d)}(M,t)$ and back propagated $U^{(0)}(M,t)$ wave fields is estimated by computations of correlation function $W(M) = \int dt\, U^{(d)}(M,t)\, U^{(0)}(M,t)$. Since coincidence in phase of the direct and backward wave fields on an interface is only a necessary and not sufficient condition, we perform stacking of $W(M)$ over sources to avoid areas with casual coherence. Thus, positions of interfaces are fixed in migration domain by the extremes of correlation function after stacking. Propagation of the both direct and backwards wave fields is performed by Gaussian beam summation methods.

In the report we demonstrate capabilities and advantages of our depth migration method:
1. By modification of the imaging condition we are able to compute the angle-depended reflection coefficient on the interface and therefore to evaluate velocity contrast V_1/V_2 on this interface.
2. By using weight-functions in Gaussian beam integral for the wave fields we are able to suppress the geometrical spreading in the wave field computations and to construct therefore the true amplitude migration image.

The above mentioned capabilities are verified by application of our method to several widely used in geophysics benchmark models.

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Application of the Rayleigh wave model to a moving load problem

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The paper is concerned with the application of an asymptotic model for the Rayleigh surface wave [1] to the plane strain problem for an elastic half-space subject to a line force moving with uniform speed along the surface. Both steady-state and transient formulations are investigated. The motion of the half-space is governed by the simplified equations of the asymptotic model recently suggested for the Rayleigh wave, containing quasi-static elliptic equations for the elastic Lamé potentials over the interior along with the hyperbolic equation for one of the potentials at the boundary and also a differential relation for the other potential. In case of stationary formulation the asymptotic expressions [2] for the displacements and stresses turn out to be the leading order Taylor series expansion of the known exact solution [3]. Application of the model to transient moving load problem allows a simple explicit solution for the displacements [4] along with the associated rigid body motion components. In this case, the vertical component of rigid body motion has a logarithmic growth in time. We also immediately establish the asymptotic behavior of the solution at the Rayleigh wave speed, demonstrating a linear growth in time. This behavior is similar to that predicted in [5] for a Heaviside-like moving load using a rather sophisticated procedure applied to the related exact solution. In addition, we discover that formulae, originally derived for non-resonant regimes, are valid for a load moving with the Rayleigh wave speed as well. Numerical comparison with the exact integral solution [6] of the original problem establishes the validity range of the approximate model.

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High frequency Diffraction of an electromagnetic plane wave by an imperfectly conducting rectangular cylinder

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We shall consider the the problem of determining the scattered far wave field produced when a plane E-polarized wave is incident on an imperfectly conducting rectangular cylinder. By using the the uniform asymptotic solution for the problem of the diffraction of a plane wave by a right-angled impedance wedge, in conjunction with Keller's method, the a high frequency far field solution to the problem is given.

On the new model for protein concentration dynamics in bounded domain

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We propose the refinement of modelling of protein concentration dynamics in space (long range diffusion), leading to the 3d order partial differential equation, together with the refinement of them in time (via the time delay or relaxation). The combination of these improvements is based on general concepts of the Extended Irreversible Thermodynamics (EIT). It allows us to derive the new reaction-diffusion-mobility (RDM) model. We show that both additions to the classic approach are necessary, and the RDM leads to a new approach in mathematical modelling of protein concentration dynamics, free of some inconsistencies typical for parabolic or hyperbolic reaction-diffusion equations. In the autoregulation problem the new statement provides an exact solution for a piecewise linear source term. The examples of the new description of gene expression dynamics in early development of Drosophila embryo are considered.

Asymptotic methods for some hydrodynamics problems with rapidly oscillating data

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Let ε be a small positive parameter and (u,p) be a Hopf's solution of the initial-boundary value problem for unsteady Navier-Stokes equations

$$u'_t - \nu\,\Delta\,u + u\cdot\nabla u + \nabla p = F_\varepsilon \quad \text{in} \quad \Omega\times(0,T),$$
$$\operatorname{div} u = 0 \quad \text{in} \quad \Omega\times(0,T), \tag{1}$$
$$u\big|_{t=0} = 0 \quad \text{in} \quad \Omega, \qquad u = 0 \quad \text{on} \quad \partial\Omega\times(0,T),$$

where $F_\varepsilon = F(t,x,x/\varepsilon)$, $F(t,x,y) \in L^2(0,T;L^2(\Omega;L^\infty_{per}(Y)/\mathrm{R})^n)$, $\Omega\subset \mathrm{R}^n$ is a bounded domain with a smooth boundary, T is a positive number, and $2\le n\le 4$. Here, a subscript *per* means 1-periodicity with respect to $y\in\mathrm{R}^n$ and $Y=[0,1]^n$ is a periodicity cell. Therefore, by definition $F(t,x,y)$ is 1-periodic in y, $\int_Y F(t,x,y)\,dy = 0$ for a. e. $(t,x)\in(0,T)\times\Omega$, and the restriction of $F(t,x,y)$ to Y is an element of $L^2(0,T;L^2(\Omega;L^\infty(Y))^n)$. Thus, F_ε is a rapidly oscillating vector function.

Theorem. *Let $\nabla_x F \in L^1(0,T;L^2(\Omega;L^\infty_{per}(Y)/\mathrm{R})^{n\times n})$ and (u,p) is a solution of problem (1). Then, there are positive ε_0 and ν_0 such that*

$$\|\,u\,\|^2_{L^\infty(\,0,T\,;\,L^2(\Omega)^n)} + \nu\,\|\,\nabla u\,\|^2_{L^2(\,0,T\,;\,L^2(\Omega)^{n\times n})} \le C\,(\,\varepsilon^2 + \varepsilon^2\,\nu^{-1}\,),$$

and

$$\|\,p\,\|_{W^{-1,\infty}(0,T;L^2(\Omega)/\mathrm{R})} \le C\,(\,\varepsilon + \varepsilon^2\,\nu^{-1-n/4}\,),$$

where C is a constant independent of ε and ν whenever $0<\varepsilon\le\varepsilon_0$ and $0<\nu\le\nu_0$.

Asymptotic and homogenization methods are used to prove of the theorem (see [1],[2]). Similar theorems for equations (1) and the linearized equations will be discussed also, for example, when $\int_Y F(t,x,y)\,dy \ne 0$.

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Integral methods for conical diffraction by multi-profile gratings

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In the talk we discuss two approaches for studying the diffraction of plane waves by 1D multi-profile gratings and photonic multi-grid structures. In many cases these problems can be reduced to a sequence of diffraction problems on relief or rod gratings with one profile, which can be modeled by a 2×2 system of singular integral equations. We give some existence and uniqueness results for solutions of that system and present an efficient collocation method for its numerical solution. In combination with the marching procedures the presented integral method allows to solve conical diffraction for rather complicated grating geometries.

Nonstationary diffraction of a single pulse for a generator of encoded pulse sequence

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Using the pulse approach developed in [1], diffraction from the thin ring lens as a component of the Fresnel lens was studied. The analysis demonstrated transformation of the initial plane δ-wave into a combination of
- passing through component of the initial wave conversion into a section of spherical wave, and
- two edge toroidal waves possessing all specific features described in [2].

While propagation towards the focal point, the internal self-crossing point of the tore surface runs down the passing wave and reaches it just in focus. In the focal point, the combination of those pulses forms the first derivative of δ-signal. The analytically obtained conclusion is supported with numerical calculations using Gaussian-modulated cosine signal.

By replacing the Fresnel lens with similar reflection system, one can realize the generator of pulses sequence corresponding to different numerical codes consisting of (-1, +1), (0, +1) or (- 1, 0,+1) depending on reflection properties of ring mirror components.

Fourier transform of pulse response provides the response of the ring or lens to the monochrome wave. It permits to explain the main properties of the Fresnel lens as traditional optical instrument.

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Traveling water waves: a global variational approach

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We consider the classical problem of periodic traveling water waves without vorticity, in presence of gravity and surface tension. We give a global variational formulation of this free-boundary problem. We find weak solutions of arbitrary momentum as minimizers of an energy functional. When the momentum is not too large, we prove that our weak solutions are classical solutions.

Weinstein's problem with double set of screens: Matrix Wiener-Hopf approach and ODE approach

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A classical Weinstein's problem is the problem of scattering of a plane high frequency wave by a periodic set of ideally absorbing parallel half-planes (in fact, it is a reformulation of the problem of the problem of radiation from an open end of a plane waveguide). This problem has been solved in the late 40's by using the Wiener-Hopf method. Here we consider the closest generalization of the problem, namely now the period of the scatterer consists of two half-planes instead of one. This problem can be reduced to a matrix Wiener-Hopf problem, for which no solution is known.

An alternative approach is proposed based on the new techniques developed recently, namely the embedding formula and the spectral equation. As a result, we get the ODE with unknown coefficient, but with known boundary data.

Use of eigenfunctions of integral operator with weakly singular kernel for a magnetostatic problem solving

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Consider a thin magnetic layer in the stationary magnetic field of intensity $\mathbf{H}^0$. Let h be the layer's thickness, S its median surface, and μ the relative permeability. Suppose $\mathbf{H}^0$ is given.

By $\mathbf{H}^*$ denote the intensity of microcurrents induced in the layer. It can be described by the scalar potential φ^* that is in fact a simple layer potential. The density σ of the potential φ^* is a solution of the following equation

$$\int_S K\sigma\,\xi dS + \int_S \mu h\,\mathrm{grad}_s K\sigma\,\mathrm{grad}_s K\xi dS = \int_S \mu h\,\mathbf{H}^0_s\,\mathrm{grad}_s K\xi dS.$$

The operator K has the following form

$$K\xi\left(M\right) = \frac{1}{4\pi}\int_S \frac{\xi\left(Q\right)}{r_{QM}} dS_Q, \quad M \in S.$$

One way to solve the equation is reducing its to a system of linear algebraic equations. It is proved that eigenfunctions of operator K can be used as basis functions for the system forming. Thus one can simplify the process of the system forming.

The software package for the problem solving has been made. Numerical results have been tested by comparison with analytical ones. Eigenfunctions of operator K have been calculated by the way described in [1]. Influence of number of eigenfunctions on the solution precision has been analysed.

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Multidimensional zero-pressure gas dynamics with the energy conservation law

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We study *delta-shock wave type solutions* in zero-pressure gas dynamics

$$\rho_t + \nabla \cdot (\rho U) = 0,$$

$$\begin{aligned}(\rho U)_t + \nabla \cdot (\rho U \otimes U) &= 0,\\ \Big(\frac{\rho|U|^2}{2} + H\Big)_t + \nabla \cdot \Big(\Big(\frac{\rho|U|^2}{2} + H\Big)U\Big) &= 0,\end{aligned}$$

where $\rho(x,t) \geq 0$ is the density, $U(x,t) = (u_1(x,t), \dots, u_n(x,t)) \in \mathbf{R}^n$ is the velocity, $H(x,t) \geq 0$ is the specific enthalpy, $x \in \mathbf{R}^n$, $\otimes$ is the usual tensor product of vectors.

Delta-shocks are discontinuities which are different from classical ones in the sense that *carry mass, impulse and energy.*

By generalizations of our results [1], [2], we introduce integral identities to define delta-shocks for the above system. Using this definition, the Rankine–Hugoniot conditions for delta-shock is derived.

We show that delta-shocks are connected with *transport and concentration processes* and derive the balance laws describing mass, momentum, and energy transport between the volume outside of the δ-shock wave front and the δ-shock moving wave front. We prove that these processes are going on in such a way that the mass and energy of the delta-shock wave front is an increasing quantity, while the kinetic energy of the volume (outside of the delta-shock wave front) and the total kinetic energy are nonincreasing quantities.

These results can be used in modeling of mediums which can be treated as a *pressureless continuum* (dusty gases, two-phase flows with solid particles or droplets, granular gases).

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The numerical method for 2D Helmholtz equation in complicated regions

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The new method for numerical solving difference boundary value problem for Helmholtz equation in complicated regions is supposed and comparison with existing methods is discussed. The method is based on the application of Fast Fourier Transformation in the rectangle containing the considered region. The procedure of the solving of initial problem is reduced to calculation of some "low dimensional" boundary operator and the solving of corresponding boundary equation. It's shown that the method supposed demonstrates high efficiency and accuracy compared with most of known algorithms of solving of the problem of such kind.

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Temporal and spectral evolution of electric field and complex envelope of few-cycle light pulses experiencing paraxial self-focusing in transparent media

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There are two common mathematical formalisms for the description of nonlinear evolution of ultrashort pulses in transparent nonmagnetic media. One opportunity is to deal directly with the real electric field or its spectrum, another variant is to use complex envelopes (or amplitudes) in derivations. A choice of a correct and convenient formalism for nonlinear optics of femtosecond pulses with ultrabroad spectra is a point for discussions since 1990s. The envelope approach was fundamental for quasi-monochromatic light pulses with many field cycles and narrow spectra [1]. But it was originally introduced to analyze slow variation of pulse profile as a whole, rather than fast field oscillations. The characteristic time scale for distinguishing "slow" and "fast" processes was a period of a field cycle. Later, the envelope equations were generalized to the case of femtosecond pulses with broad or even continuum spectra with spectral width of the order of the central frequency [2]. However the utility and validity of such extension for continuum spectra and few-cycle pulses with 10 or less field oscillations was questionable. Particularly, it was unclear how to treat and calculate the envelope of ultrafast fields with characteristic features in the time scale of one cycle or even shorter.

The envelope consideration is not necessary for few-cycle pulses, so many researchers constructed their models directly for the optical field in 1990s. Kozlov and Sazonov derived equation for the electric field of femtosecond pulses with ultrabroad temporal and narrow spatial spectra propagating in transparent media with dispersion and cubic electronic nonlinearity [3]. The equation is a reduction of Maxwell equations to the case of non-resonant medium response and unidirectional propagation without generation of self-reflected waves. The equation was used to qualify scenarios of paraxial self-focusing of few-cycle light pulses in media with normal and anomalous group dispersion [4]. It was shown that the non-stationary self-focusing leads to development of complicated spatiotemporal field structures like light "dumbbells" at moderate intensities or light "bubbles" at higher intensities.

Here we compare field and envelope formalisms. Using a consistent definition of complex envelope of light filed with arbitrary temporal profile [5] we present the generalized envelope equation which accounts for third harmonic generation (THG) and exact medium dispersion relation without Taylor expansion. We develop similarly arranged numerical models of field and envelope equations with equivalent approximation accuracy and show that the computation results for self-focusing of axisymmetric few-cycle wave packets are identical if THG is considered in envelope equation. However the computed temporal and spectral structures can differ substantially when THG is ignored as is done conventionally. The field in nonlinear focus is overpredicted without THG because the higher harmonics lag behind due to group dispersion and carry the optical energy out of the focusing region. The results illustrate unusual features of the envelope of few-cycle pulses. The trailing edge of the envelope is self-steepening and shortening down to duration of a half of a period on laser frequency. The spectral width of generated continuum is twofold to the central frequency. The amplitude and phase have oscillations considerably faster than the field cycle of input pulse.

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Discrete spectrum of periodic Schrodinger operator with non-constant metric in the case of non-negative perturbations

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The talk is dedicated to the joint work with M.Sh. Birman.

Let A be an elliptic periodic self-adjoint operator of the second order in $L_2(\mathbb{R}^d)$, $d \geq 1$, and let V be the multiplication by the function $V(x) \geq 0$ which tends (in an appropriate sense) to zero as $|x| \to +\infty$. Let (α, β) be an *inner* gap in the spectrum of A and $\lambda \in [\alpha, \beta]$ be a fixed number. The spectrum of the operator $B(t) := A + tV$, $t > 0$ inside the gap (α, β) is discrete. Denote by $N(\lambda, \tau)$ the number of eigenvalues of the operator $B(t)$ that have passed the point λ as t increased from 0 to τ. In the paper the asymptotics of $N(\lambda, \tau)$ is obtained for $\tau \to +\infty$ in the case when the perturbation $V(x)$ has power-like asymptotics at infinity, $V(x) \sim \omega(x/|x|)|x|^{-\rho}$, $|x| \to +\infty$, $\rho \in (0, d)$.

The main result can be represented in the following form: $N(\lambda, \tau) \sim \Gamma_\rho(\lambda)\tau^{d/\rho}$, $\tau \to +\infty$. Here the coefficient $\Gamma_\rho(\lambda)$ is computed in terms of *the zone functions* of the operator A. Under certain conditions, this asymptotics holds on the left edge of the gap as well, $\lambda = \alpha$. We impose no additional restrictions on the smoothness of the coefficients of the operator A. The condition $\rho < d$ is a technical one and can be dropped if the coefficients of the operator A are smooth enough.

The verification of the main result is based on analysis of the asymptotics of singular numbers of certain integral operators. Down this route, we employ different generalizations of the Cwikel estimate. The derived asymptotics is non-local with respect to energy, its order is different from the "standard" $\tau^{d/2}$. The Weyl nature of the asymptotics reveals itself if the roles of coordinates and quasi-impulses are switched.

On a method of metrological self assurance in a problem of control of orbital complexes

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In a problem of control of an orbital complex of space apparatus such as GPS, GLONASS, GALILEO appear tasks that require a full solution of the eigenvalue problem. The matter is that to such cases correspond systems of large dimensions. For solution of such systems iterative methods are usually used. However, for stiff and super stiff systems provided by these methods metrological level becomes insufficient. In the paper a modified method of metrological self assurance for solving systems of this type is proposed.

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Active protection from noise propagation in cylindrical waveguide

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We consider a cylindrical waveguide with acoustic noise propagating from one side. To reduce the noise we suggest to install in the walls of the waveguide a set of microphones, which will register amplitudes of waves. This data is processed and a "protective" wave is generated with the help of sound speakers also installed in the walls of the waveguide. Results of numerical simulation of such a device are to be presented.

Coercivity of boundary integral operators in high frequency scattering

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Much research effort in recent years has been focused on designing effective numerical methods for high frequency acoustic scattering. The main difficulty is that, as the frequency increases, the solution becomes more oscillatory, leading to a rapid increase of degrees of freedom in conventional methods to maintain accuracy. Once these methods have been designed, an interesting and challenging question is whether rigorous error bounds can be established which are explicit in the frequency.

One strategy for proving rigorous error bounds for boundary integral methods for these high frequency problems is to seek to prove that the integral operator has a property known as coercivity. This is a strong definiteness property of the operator which in particular implies boundedness of an inverse. For these high frequency problems one ideally wants to establish coercivity independent of (or at least explicit in) the frequency.

Coercivity has so far been established only for the case of the circle (in 2d) and sphere (3d) using Fourier analysis. This talk will outline why coercivity is important, what is known about it already, and then discuss some new results on proving coercivity using PDE techniques involving certain identities, and on numerically computing the so-called coercivity constants (which appear in the error bounds).

This is joint work with Timo Betcke and Simon Chandler-Wilde (University of Reading), and Ivan Graham and Valery Smyshlyaev (University of Bath).

Simulation of a laminar flow in a porous medium

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In this presentation we analyzed and compared the software packages Eclipse 2004A, NGT BOS v.2.1 and ANSYS CFX v11.0 for the task of the oil and gas flows simulation in a porous medium. We identified the common features, examined the differences and explored recommended fields of application.

PO/GTD method for 3D modeling of the aperture antenna with a radome

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The generalised image theory and exact mathematical model of an aperture antenna are employed in order to develop exact and PO integral representations of the fields radiated by 3-dimensional radome-enclosed aperture antenna. The desired problem is reduced to finding fields of a plane wave diffracted on the "symmetrized" radome. Radiation patterns for antennas with a specified ampliphase distribution enclosed in the radomes of different geometries are analysed.

Correction of bore-sight errors induced by a radome

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We consider an advantage of using dielectric materials with different permittivity in the radome design to reduce far side field and to improve directivity properties of a two-dimensional antenna array in a radome. For calculations, the method based on solving the volume integral equation relatively the total field in the radome is used.

A novel Fisher information criterion to study electromagnetic resonances in lamellar gratings

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A novel information criterion based on the principle of minimum Fisher information[1] is presented in order to locate resonant wavelengths at which field enhancement [2] occurs in the interaction of electromagnetic beams with finite lamellar gratings. A comparison with the results obtained using Maxwell equations is done. Nevertheless both theories agree well, we show the former method is numerically more efficient and reliable.

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Near field spectrum in the neighborhood of a subwavelength metallic slit at resonant wavelengths

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At wavelengths where enhanced transmission occurs we show that the near field spectrum in the neighborhood of a subwavelength metallic slit presents a topology which favors the energy flow (characterized by the Poynting vector) through the slit no matter how deep it is. However a little shift (about 5completely different topology which turns the energy flow away from the aperture entrance (kind of turbulence). A simulated movie of this phenomenon is shown.

Homogenization of nonstationary periodic equations

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In $L_2(\mathbf{R}^d;\mathbf{C}^n)$, we consider a second order differential operator $\mathcal{A}_\varepsilon = b(D)^* g(\varepsilon^{-1}x) b(D)$, $\varepsilon > 0$. Here $g(x)$ is an $(m \times m)$-matrix-valued function in $\mathbf{R}^d$ such that $g, g^{-1} \in L_\infty$, $g(x) > 0$, and $g(x)$ is periodic with respect to some lattice. Next, $b(D)$ is a first order differential operator; its symbol $b(\xi)$ is an $(m \times n)$-matrix-valued linear homogeneous function of $\xi \in \mathbf{R}^d$ such that $\operatorname{rank} b(\xi) = n$, $\xi \neq 0$. We assume that $m \geq n$. We study the following Cauchy problem for the Schrödinger type equation for a function $u_\varepsilon(x,\tau)$, $x \in \mathbf{R}^d$, $\tau \in \mathbf{R}$:

$$i\partial_\tau u_\varepsilon(x,\tau) = \mathcal{A}_\varepsilon u_\varepsilon(x,\tau), \quad u_\varepsilon(x,0) = \phi(x).$$

We also study the Cauchy problem for the hyperbolic equation for a function $v_\varepsilon(x,\tau)$, $x \in \mathbf{R}^d$, $\tau \in \mathbf{R}$:

$$\partial_\tau^2 v_\varepsilon(x,\tau) = -\mathcal{A}_\varepsilon v_\varepsilon(x,\tau), \quad v_\varepsilon(x,0) = \varphi(x),\ \partial_\tau v_\varepsilon(x,0) = \psi(x).$$

The corresponding "homogenized" problems look as follows:

$$i\partial_\tau u_0(x,\tau) = \mathcal{A}^0 u_0(x,\tau), \quad u_0(x,0) = \phi(x);$$

$$\partial_\tau^2 v_0(x,\tau) = -\mathcal{A}^0 v_0(x,\tau), \quad v_0(x,0) = \varphi(x), \ \partial_\tau v_0(x,0) = \psi(x).$$

Here $\mathcal{A}^0 = b(D)^* g^0 b(D)$ is the effective operator.

Theorem 1. *If $\phi \in L_2(\mathbf{R}^d;\mathbf{C}^n)$, then u_ε tends to u_0 in $L_2(\mathbf{R}^d;\mathbf{C}^n)$ for a fixed $\tau \in \mathbf{R}$, as $\varepsilon \to 0$. If $\phi \in H^s(\mathbf{R}^d;\mathbf{C}^n)$, $0 < s \le 3$, then*

$$\|u_\varepsilon(\cdot,\tau) - u_0(\cdot,\tau)\|_{L_2} \le \varepsilon^{s/3} C_s(\tau) \|\phi\|_{H^s}.$$

Here $C_s(\tau) = O(|\tau|^{s/3})$ for large values of $|\tau|$.

Theorem 2. *If $\varphi, \psi \in L_2(\mathbf{R}^d;\mathbf{C}^n)$, then v_ε tends to v_0 in $L_2(\mathbf{R}^d;\mathbf{C}^n)$ for a fixed $\tau \in \mathbf{R}$, as $\varepsilon \to 0$. If $\varphi, \psi \in H^s(\mathbf{R}^d;\mathbf{C}^n)$, $0 < s \le 2$, then*

$$\|v_\varepsilon(\cdot,\tau) - v_0(\cdot,\tau)\|_{L_2} \le \varepsilon^{s/2} \left(C_s^{(1)}(\tau)\|\varphi\|_{H^s} + C_s^{(2)}(\tau)\|\psi\|_{H^s} \right).$$

Here $C_s^{(1)}(\tau) = O(|\tau|^{s/2})$, $C_s^{(2)}(\tau) = O(|\tau|^{1+s/2})$ for large values of $|\tau|$.

We also prove analogs of Theorems 1 and 2 for more general class of operators [1].

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Properties of quasi-Rayleigh waves near cylindrical cavity subject to surface impedance load

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Theory and applications of surface waves present significant practical interest [1, 2], with particular attention directed at Rayleigh waves. For example, axially symmetric quasi-Rayleigh and Stoneley waves near cylindrical cavity were considered in paper [3]. Properties of quasi-Rayleigh waves near the plane boundary of isotropic half-space under two component impedance load were studied in article [4]. This work investigates the effect of surface impedance load on the characteristics of axially symmetric quasi-Rayleigh waves propagating along the boundary of cylindrical cavity in the isotropic elastic medium. Such waves are of importance for many practical applications.

Dispersion equation for the waves under consideration is derived under assumption that the impedance load can be described by Hermitian impedance matrix $\mathbf{Z}(L)$. Such a load causes additional normal (n) and tangential (τ) stresses on the surface. As a result full stress $\overline{\sigma}^{(f)} = (\sigma_n^{(f)}, \sigma_\tau^{(f)})^T$ can be expressed through the displacement vector $\overline{u}^{(f)} = (u_n^{(f)}, u_\tau^{(f)})^T$ as follows:

$$\overline{\sigma}^{(f)} = -i\omega \left(\mathbf{Z}^{(0)} + \mathbf{Z}^{(L)} \right) \overline{u}. \tag{1}$$

Here $\mathbf{Z}^{(0)}$ stands for the impedance matrix of the medium (its derivation is illustrated in the present work) and ω is the angular frequency. Condition of absence of full stress at the cavity surface immediately leads to the dispersion equation, describing the behavior of quasi-Rayleigh waves:

$$\det \left(\mathbf{Z}^{(0)} + \mathbf{Z}^{(L)} \right) = 0. \tag{2}$$

In limiting cases (zero load, plane boundary case, etc.), it was shown to coincide with results [3,4].

It was found that for certain loads quasi-Rayleigh waves in question cannot exist in cylindrical cavity. Such loads can be determined from condition of cutoff frequency ω_{cr} turning to infinity $\omega_{cr} \to \infty$. This condition follows from equation (2) if one sets wavenumber k equal to that of shear waves k_t and tends frequency ω to infinity.

The properties of the waves under consideration were investigated in detail for the case of diagonal impedance load matrix $\mathbf{Z}^{(L)} = \mathrm{diag}(Z_{rr}, Z_{zz})$. It was demonstrated that depending on the load three situations are possible: quasi-Rayleigh waves do not exist, there is one such wave, and two of them exist simultaneously. To prove it one should consider a point in space (k, ω) and find loads such that the dispersion curve of the quasi-Rayleigh wave passes through this point. They are described by equation (2), with this point fixed, and form a hyperbole in space (Z_{rr}, Z_{zz}). Considering such hyperboles for all possible positions of this point one can note that impedance space can be separated in three regions. No hyperboles lie in the first of them; the points of the second region belong to just one hyperbole; the points of the third one pertain to two hyperbolae. It justifies above mentioned classification. Noteworthy, if one considers loads from the third region and knows dispersion properties of one of two quasi-Rayleigh waves then it is possible to determine restrictions for those of the other wave. These restrictions are discussed in the work as well.

Obtained results were exemplified with two models of surface impedance load. As the first model the cavity filled with the non-viscous fluid was considered. This fluid was treated as impedance load with only one non-zero element Z_{rr} of the matrix $\mathbf{Z}^{(L)}$. Its substitution into equation (2) leads to classical dispersion equation of Stoneley wave in fluid-filled cavity. For the second model comb-like layer was chosen (with the elements of height l, which is small as compared to the shear wave length). The impedance matrix for this layer was derived before [4]. It was shown that such a load always causes two quasi-Rayleigh waves to appear. The cutoff frequencies for each of them diminish with the increase of ratio l/a, where a is the radius of the cavity.

Theoretical results described above can be used for the development and construction of the devices, which will allow controlling characteristics of quasi-Rayleigh waves in cylindrical geometry.

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"Complex source" in two-dimensional real space

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We consider the complexified Green function for the 2D Helmholtz operator, $g_* = -\frac{i}{4}\mathcal{H}_0^{(1)}(kr_*)$, $r_* = \sqrt{x^2 + (z - ia)^2}$, $a > 0$, which is analogous to the 3D construction by Izmest'ev and Deschamps [1, 2]. The function g_* is interesting as an exact solution of the Helmholtz equation showing a Gaussian beam behavior near z-axis and is widely discussed in literature (see e.g. [3, 4]). The square root in r_* is not defined uniquely but branches at $z = 0$, $x = \pm a$, and thus g_* jumps on a certain curve S, depending on the choice of the branch cut. Thus, g_* satisfies the inhomogeneous Helmholtz equation $(\triangle + k^2)g_* = F$ in the real space $\mathbb{R}^2$ with a source function F localized on S. We calculate F for several choices of branch cut.

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Surface acoustic waves in a rotating solid

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Surface acoustic waves (SAW) properties are dependent on the rotation speed of the medium. This effect is happened because the Coriolis force $m \cdot 2[\Omega 3.0 \times v]$ is applied on the particle of mass m oscillating with velocity v when the body is rotating with angular velocity Ω. Similar effect in optics is called the Faraday effect [1] when that the speed and the polarization of light in the material depends on the applied magnetic field. This effect arises because the Lorentz force $q \cdot [v 3.0 \times B]$ acts on electrons with the charge q and velocity v in a magnetic field B.

Two methods of analysis have been considered. The first is the numerical solution of equations for SAW propagation in the rotating half infinite anisotropic piezoelectric substrate. The second one is qualitative description of the Coriolis force action on the moving particles while SAW propagation.

In [1-2] the rotation investigation was restricted only by the SAW velocity changing and this shift is very small for mainly all practical applications. However it is important to take into account other effects (for example, appearance of new components of displacement vector) which can be measured and further on can be used in the devices. According to [1-2] the maximum shift in SAW velocity is achieved by the body rotation around the normal to the sagittal plane, that kind of rotation usually considered as preferable. In this case, the Coriolis force will act along the radius of curvature of the particle locus. This will lead in material stiffness changing and hence the wave velocity shift. On the other hand, according to [2], the minimal shift in the SAW velocity is achieved by rotation around the direction of SAW propagation. In this case, the Coriolis force is perpendicular to the sagittal plane. Nevertheless, this kind of rotation will result in the declination of the plane of particle oscillation from the sagittal plane. As a result, the additional component of displacement will appear and also the power flow angle of SAW will be different. These effects have been recently confirmed by the corresponding numerical calculations [3].

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Super short Bessel beam Ffrmation by axicon

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Peculiarities of femtosecond Bessel light beams formation depending on axicon shape are investigated. It is shown that conical lens blunt tip causes significant differences of structure of light fields formed after it in comparison with one obtained by means of ideal axicon. A new method is proposed to eliminate influence of real axicon blunt tip: dielectric mirror (photonic crystal) is carried on the place of cut blunt tip.

Introduction. Generation of Bessel light beams by means of an axicon attracts increasing attention of scientists due to law light loss as compared with the other methods of such light fields obtaining (for example, using circular diaphragm [1]). Structure of this type light fields is strongly influenced by a shape of an axicon though. In works [2], [3] it is shown that taking into consideration peculiarities of axicon shape (blunt point on axicon apex) leads to significant differences in intensity distribution. Lately femtosecond laser pulses characterizing high light energy localization are widely adopted in electric discharge control, clustered plasma formation, media miniaturization. However, dispersing blurring of such light fields is a significant problem. Therefore formation of dispersion-free, diffraction-free pulsed Bessel beams by means of an axicon is of great interest. Detailed investigation of super short Bessel light beams formation by means of axicon of various shape is carried out.

Description of femtosecond Bessel light beams formation. Let the pulse of Gaussian envelope and amplitude depending on radial coordinate r $E(r,t) = E_0 \cdot \exp(-r^2/2w^2) \cdot \exp(-t^2/t_0^2) \cdot \exp(i\omega_0 t)$, falls on an axicon along longitudinal coordinate z. Transformation of a light field frequency component made by the conical lens can be presented as the Kirhgof integral $E(r,z,\omega) = -(ik/z)\cdot\exp(ikr^2/2z)\int_0^R T_{id/nonid}(r',\omega)\cdot\exp(-r'^2/2w^2)\cdot\exp(ikr'^2/2z)\cdot J_0(kr'r/z)\cdot r'dr'$, where $T_{id/nonid}$ - transmission function of ideal or nonideal axicon with blunt tip ($T_{id}(r,\omega) = \exp[-ik\gamma(n(\omega)-1)r]$, $T_{nonid}(r,\omega) = \exp[-ik\gamma(n(\omega)-1)R_h \cdot \tan^2(\gamma)\sqrt{1+r^2/R_h^2 \cdot \tan^2(\gamma)}]$) for $r \le r_h$, $T_{id}(r,\omega) = \exp[-ik\gamma(n(\omega)-1)[(R-r)\cdot\tan(\gamma)]]$ for $r > r_h$). Expression of pulsed beam electrical field looks like: $E(r,z,t) = (1/2\sqrt{\pi})\int_{-\infty}^{+\infty} t_0 \cdot \exp(-\omega^2 t_0^2/4) \cdot E(r,z,\omega) \cdot \exp[i(k_z(\omega)z - \omega t))]d\omega$, where $k_z(\omega) = k_{z,0} + k'_{z,0}(\omega-\omega_0) + (1/2)k''_{z,0}(\omega-\omega_0)^2 + (1/6)k'''_{z,0}(\omega-\omega_0)^3 + \ldots$.

Results. Calculation conducted according to the obtained formula has shown significant differences in paraxial intensity profiles of super short light beam formed by axicon with blunt tip and ideal axicon. Intensity distribution after the conical lens with blunt tip distinguishes by numerous oscillations and additional maximum outside diffraction-free zone in comparison with one formed by ideal axicon. The oscillations and the additional maximum are caused by interference of super short Bessel beam formed by peripheral part of nonideal axicon and pulsed Gaussian beam produced by blunt tip of axicon central part (acting as a lens).

We propose a new method to eliminate the influence of conical lens imperfection. It lies in the following: dielectric mirror consisting of periodically alternating dielectric layers of different refraction indexes (photonic crystal) is carried on cut axicon blunt point. This method is based on the property of photonic crystal not to transmit radiation of wavelength from so-called photonic band gap. It possesses a number of advantages as compared with the other methodic (with an addition of a beam stop [2]) such as disappearance of unwanted oscillations and light losses. In Fig. there are illustrated paraxial intensity profiles of super short Bessel light beams formed by means of ideal, nonideal conical lenses and with photonic crystal on the cut apex of axicon. It is obvious structure of the light field formed after the axicon with photonic crystal follows one formed by ideal conical lens.

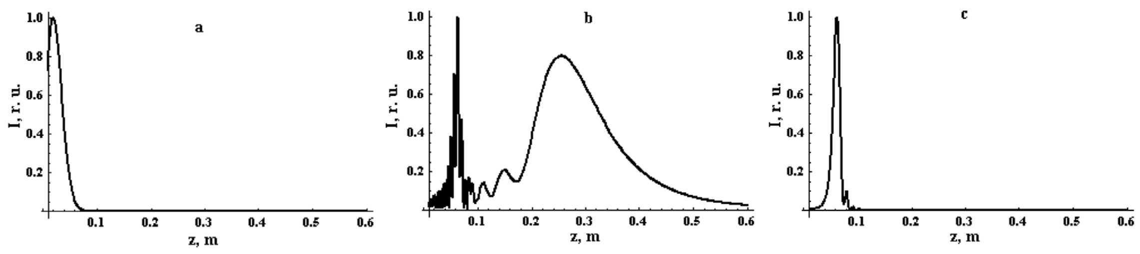

Figure : Normalized on its maxima paraxial intensity profiles of super short Bessel light beams formed by means of ideal (a), nonideal (b) conical lenses and axicon with photonic crystal on the cut apex (c).

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Exact radiation from an antenna on an oblate metallic spheroid coated with layers of isorefractive and anti-isorefractive materials

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The geometry analyzed in this work consists of an oblate metallic spheroid that is coated with confocal oblate spheroidal layers made of lossless materials whose refractive indexes are either equal (isorefractive) or opposite in sign (anti-isorefractive) to the refractive index of the infinite medium (e.g., air) surrounding the structure. The intrinsic impedances of the materials of the coating layers are real and positive, but may take any value. The primary source is an electric dipole antenna mounted outside the structure, on the axis of symmetry, and axially oriented. The analysis is performed in phasor domain.

The boundary-value problem is amenable to exact solution by separation of variables. The fields in the various regions of space are written as infinite series of products of radial and angular oblate spheroidal functions, in the notation of Flammer (1957). The modal expansion coefficients are determined by imposing the boundary conditions at the spheroidal interfaces and the radiation condition at infinity. The explicit analytical determination of the coefficients is possible because the angular oblate spheroidal functions are independent of the sign of the refractive index. Thus, a new canonical solution is obtained, that is of interest not only in itself, but also because it may be used to validate computer codes developed for penetrable structures. The particular cases in which the metallic spheroid becomes either a sphere or a circular disk are examined in detail.

The radiated field is discussed as a function of the geometry (thickness of each layer, number of layers, eccentricity of the metal spheroid) and of the intrinsic impedances of the layers, and is compared to the field radiated in the case of an uncoated spheroid.

Modal representation of transient waves constrained by an elliptical cylinder

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The generation of transient waves by physically admissible (space-time limited) sources is discussed. We assume that both the wavefunction and the source term equals zero on the surface of an elliptical cylinder (which, for example, corresponds to an elliptic-waveguide boundary condition).

The investigation is carried out in the elliptic-cylindrical coordinate system u, v, z , related to the Cartesian coordinates x, y, z with the same axis z by

$$x = h \cosh u \cos v, \qquad y = h \sinh u \sin v$$

where h is a positive real parameter with dimensions of length. The wave equation takes the form

$$\left[\frac{\partial^2}{\partial\tau^2} - \frac{1}{h^2\left(\cosh^2 u - \cos^2 v\right)}\left(\frac{\partial^2}{\partial u^2} + \frac{\partial^2}{\partial v^2}\right) - \frac{\partial^2}{\partial z^2}\right]\Psi\left(u, v, z, \tau\right) = S\left(u, v, z, \tau\right),$$

where Ψ is the wavefunction, S is the source term and $\tau = ct$ is the time t in units of length (c is the wavefront velocity). Both the wavefunction and the source term are supposed to be confined by an elliptical cylinder $u = u_0$, viz.

$$\Psi\left(u, v, z, \tau\right)\big|_{u=u_0} = 0, \qquad S\left(u, v, z, \tau\right)\big|_{u=u_0} = 0.$$

Then, according to the result obtained in [1], p. 297, Ψ and S admit expansion in terms of the Mathieu (elliptic) sine (Se, se) and cosine (Ce, ce) functions

$$\begin{aligned}\begin{pmatrix}\Psi\left(u, v, z, \tau\right)\\ S\left(u, v, z, \tau\right)\end{pmatrix} &= \sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\begin{pmatrix}\psi_{mn}^{(c)}\left(z, \tau\right)\\ s_{mn}^{(c)}\left(z, \tau\right)\end{pmatrix}\mathrm{Ce}_m\left(u, q_{mn}^{(c)}\right)\mathrm{ce}_m\left(v, q_{mn}^{(c)}\right)\\ &+ \sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\begin{pmatrix}\psi_{mn}^{(s)}\left(z, \tau\right)\\ s_{mn}^{(s)}\left(z, \tau\right)\end{pmatrix}\mathrm{Se}_m\left(u, q_{mn}^{(s)}\right)\mathrm{se}_m\left(v, q_{mn}^{(s)}\right)\end{aligned} \tag{1}$$

where $q_{mn}^{(c)}$ and $q_{mn}^{(s)}$ are the roots of equations

$$\mathrm{Ce}_m\left(u_0, q^{(c)}\right) = 0, \qquad \mathrm{Se}_m\left(u_0, q^{(s)}\right) = 0$$

Substitution of (1) into the wave equation yields the PDE

$$\left[\frac{\partial^2}{\partial\tau^2} - \frac{\partial^2}{\partial z^2} + \left(\frac{2q_{mn}^{(c,s)}}{h}\right)^2\right]\psi_{mn}^{(c,s)}\left(z, \tau\right) = s_{mn}^{(c,s)}\left(z, \tau\right)$$

of known Riemann function

$$R\left(z', \tau'; z, \tau\right) = J_0\left(\frac{2q_{mn}^{(c,s)}}{h}\sqrt{\left(\tau - \tau'\right)^2 - \left(z - z'\right)^2}\right)$$

Thus, the desired longitudinal-temporal components $\psi_{mn}^{(c,s)}\left(z, \tau\right)$ of the modal representation (1) can be obtained using the Riemann method.

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Derivation of modified Smyshlyaev's formulae using integral transform of Kontorovich-Lebedev type

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The aim of this work is to fill the gap between the embedding formulae for cones and the modified Smyshlyaev's formulae. Embedding formulae for cones represent the directivity of the scattered field as multiple integrals over spatial variables. Modified Smyshlyaev's formulae represent the same directivity as a single contour integral over parameter ν. This situation resemble the convolution theorem for Fourier transform: multiple convolutions can be represented as a single integral over frequency.

Originally, Modified Smyshlyaev's formulae have been "guessed" and then proved by study of the poles of the integrands instead of being regularly derived. Extension of the analogy with Fourier transform allows to obtain a regular method for deriving the modified Smyshlyaev's formulae.

The most straightforward way to extend the described analogy to the conical case is to use the Kontorovich-Lebedev transform. However, we cannot use it directly due to convergence problems. Namely, for the classical Kontorovich-Lebedev procedure it is necessary for the parameter k_0 of the Helmholtz equation to be purely imaginary, which is hardly interesting from the practical point of view.

That is why we develop a slightly different approach. Instead of the Kontorovich-Lebedev transform we use the "Kontorovich-Lebedev-Smyshlyaev representation" that differs by the choice of the cylindrical function (Bessel instead of Hankel), and, more important, by the contour of integration. As the result, the functions participating in the representation stop being orthogonal. However, for our needs the orthogonality (and even the uniqueness and invertibility of the representation) is not important, we need only the analogs of well known for Fourier transform Plancherel formula and convolution formula. That is why we prove only these important formulae without using orthogonality and demonstrate the possibility to derive the modified Smyshlyaev's formulae.

The numerical calculation of eigen modes of rectangular electrodynamic waveguide with metal partition

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We present the numerical method for calculating eigen modes of rectangular electrodynamic waveguide with metal partition. Such a devices appear in particular in the high frequency radio technic systems as polarizers. Due to the quite complicated geometry the calculating of eigen modes of such devices needs the numerical approach. We suppose method based on decomposition through the basis of eigenfunctions of rectangle without partition. This method reduce the problem to the solution of dispersion equation, which can be solved essentially more efficiently in compare with the direct diagonalization of the matrix of transversal Helmholtz operator. We discuss also some problems connected with calculation electromagnetic field through the modes computed.

Existence of edge waves along periodic structures

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We address the problem of interaction of linear water waves with three-dimensional periodic structures, totally or partially submerged within a homogeneous, inviscid and incompressible liquid. Considering, as usual, the problem as a spectral boundary-value problem, we derive a condition ensuring the existence of trapped modes by introducing a trace operator in a suitable functional setting and investigating its spectrum. The trapped modes correspond to waves propagating along the periodic structures but vanishing at large distances from them.

The sufficient condition is a simple inequality comparing a weighted volume integral, taken over the submerged part of an element in the infinite array of identical obstacles, to the area of the free surface pierced by the obstacle.

Various examples are given and the results are extended to edge waves along periodic coastlines and over periodically varying ocean floor.

This is a joint work with Sergey Nazarov from the Institute of Mechanical Engineering Problems at the Russian Academy of Sciences at St. Petersburg.

Instability of electromagnetic surface waves guided by the hiral column

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The study is made of the linear stage of the parametric instability of electromagnetic surface waves guided by the hiral infinitely extended cylinder immersed in a uniform unbounded dielectric space. Note that the parametric instability of whistler surface waves guided by an axially magnetized plasma column surrounded by a dielectric space has been considered in [1]. Here we consider the bi-anisotropic column whose permittivity and permeability are described by tensors $\hat{\varepsilon}$ and $\hat{\mu}$ with nonzero off-diagonal elements. The axis of considered hiral cylinder is assumed to coincide with the gyrotropic axis which is parallel to z axe. For a monochromatic signal tensors $\hat{\varepsilon}$ and $\hat{\mu}$ can be written as follows

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_1 & i\varepsilon_2 & 0 \\ -i\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix}, \qquad \hat{\mu} = \begin{pmatrix} \mu_1 & i\mu_2 & 0 \\ -i\mu_2 & \mu_1 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix},$$

here $\varepsilon_2 = \dfrac{\chi}{\eta}$, $\mu_2 = -\chi\eta$, $\eta = \sqrt{\dfrac{\mu_1}{\varepsilon_1}}$, χ is the chiral parameter [2,3].

Our investigation has shown that the possibility of existence of proper electromagnetic surface waves is determined by the sign of components of permittivity and permeability tensors. We obtained the conditions under which the surface waves can be supported by the column. The dispersion characteristics of these waves are analyzed. As it is known, the intense external electromagnetic field may effect on the medium properties and as a result of such action, tensors diagonal elements are the functions of the amplitude of the external field. A three wave interaction can occur if the space-time conditions between the external electromagnetic field and the guided surface waves take place. The expressions of the instability increment of guided waves are obtained. For the some practically interesting cases numerical analyzes have been performed and the results of computations will be reported.

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Resonance scattering and generation of the third harmonic by the diffraction of a plane wave on cubically polarisable dielectric layered structure

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Results of the analysis of process of resonance scattering and generation of the third harmonic in the case of the diffraction of a plane electromagnetic wave through an isotropic, cubically polarisable, non-magnetic, linearly polarised (E polarisation) medium with a non-linear, layered dielectric structure are considered. The electromagnetic waves in a non-linear medium with a cubic polarisability are described by an infinite system of non-linear equations [1-3]. In the study of particular non-linear effects it proves to be possible to restrict the examination to a finite number of equations, and also to leave certain terms in the representation of the polarisation coefficients, which characterize the physical problem under investigation. Thus, in the analysis of the non-linear effects of the processes of resonance scattering of the field (at the frequency of excitation) and generation of the third harmonic (at the triple frequency) it is possible to restrict the investigation to a system of two equations, where only the non-trivial terms in the expansion of the polarisation coefficients are taken into account [4]. This leads to the strong formulation of a boundary-value problem, which in turn can be reduced to a system of one-dimensional non-linear integral equations (defined along the height of the structure) with respect to the complex Fourier amplitudes of the scattered fields in the non-linear layer at the basic and triple frequencies. The system of non-linear integral equations and also the boundary-value problem are reduced to a system of non-linear boundary-value problems of Sturm-Liouville type, which indicates the equivalence of both problems. The one of the possible iterative schemes of the solution of the system of non-linear integral equations (based on the application of a quadrature rule to each of the non-linear integral equations) is considered. The analysis of the scattering problem and the generation of the third harmonic by excitation by a plane wave passing a non-linear three-layered structure is carried out. Results of the numerical investigation of both the values of the non-linear dielectric constants corresponding to a given amplitude of the incident field and of the scattered and generated fields are presented [4]. The dependence characterizing the portions of generated energy in the third harmonic on the value of the amplitude of the excitation field of the non-linear structure is investigated. Within the framework of the conservative system under consideration it is shown that the imaginary part of the dielectric constant, determined by the value of the non-linear part of the polarization at the excitation frequency, characterises the loss of energy in the non-linear medium (at the frequency of the incident field) due to the generation of the electromagnetic field of the third harmonic (at the triple frequency).

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Resonance properties of wave propagation in the heterogeneous composites with nematic coatings

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The peculiarities of the wave propagation in the elasto-nematic heterogeneous composites are investigated. The so-called liquid crystalline rubber-like nematic elastomers (or simply nematics) are relatively new non traditional class of materials combining the properties of classical viscoelastic solids, liquid crystals and Cosserat media. These combinations attract the interest of research community in context of medical and bio- applications, as well as in nano mechanics (as a specific matrix). In the latter the additional problem of quality control is arisen while facing the appearance of nano tube clusters. In what follows we focus the attention on the acoustical properties of composites which possess the potential of applications in many areas. The recently introduced effective low frequency model of nematic medium is used.

The phase speeds and attenuations of the surface and guided waves is the subject of this study. The quasi Rayleigh and quasi Love waves propagating in the coated half-space are considered first under different orientation of the nematic anisotropy. The similar situations for the fundamental S0, A0 and SH0 modes in the coated plates are also studied.

The solution to the respective Christoffel equation is obtained analytically, than the impedance matrices for each configuration of composite are obtained. The solution to the dispersion equations are sought numerically based on their long wave asymptotic behavior.

The obtained results are classified and parametrically analyzed. Two kinds of resonance effects are revealed and their physical meanings are discussed.

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High order asymptotics of the near field, radiated by a normal or angled beam fluid couple ultrasonic transducer, into an elasic plate or a half-space

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The reasonable formalization of the radiation problem for the case of normal or angled beam fluid coupled ultrasonic transducer is discussed in context of NDT needs. The inspection solid is assumed to be a plate or, as a limit case, an elastic half-space. The Green tensor is introduced first in the frequency domain. In the plate it is represented in the modal form. The radiated field is represented by a convolution integral using effective contact loading. The field is analyzed in the first Fresnel zone where the characteristic wavelength is comparable with the zone size. As known, this problem is usually most time consuming.

The high frequency asymptotics of the convolution integral of the highly oscillating functions over the contact spot with smooth contour are studied using the stationary phase method. For the case of rectangular transducer the leading asymptotic terms are obtained in the special geometrical zones. On the boundaries of these zones the transition areas are investigated.

The numerical examples in the frequency and time domains show that the satisfactory accuracy is achieved at the distance of a half-wavelength from the transducer lobes with simultaneous decreasing the calculation time with four decimal orders.

In the limit case of large (infinite) thickness of solid the field underneath the rectangular transducer is also discussed.

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Semiclassical analysis of conductance fluctuations in open electronic resonators

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Quantum transport of electrons through semiconductor microjunctions has been observed in many experiments in the middle of nineties. Less than micron-size 2D junctions are made with such purity that both quantum coherence length and the mean free path for elastic collisions of electrons with defects are large compared to the size of junction. Thus, electrons can be described as 2D ideal Fermi gas of non interactive particles. In this case electron would bounce ballistically through the resonator cavity, and its wave function scatters elastically from the walls of the junction. The conductance of such junctions was measured, and was found to oscillate strongly as the Fermi energy of electron or the strength of applied magnetic field is varied. Statistical properties of these fluctuations have been studied, and compared with predictions from random matrix theory. It was found common to all generic microscopic conductors in electronic transport in mesoscopic systems [1], [2], [3] that the high-frequency part of the power spectrum of the oscillations had structure of power-law decay for regular systems versus exponential decay for chaotic ones.

On the theoretical side, physicists were trying to develop simple algorithms that predict the fluctuations of conductance with respect to electron Fermi energy or magnetic field. Conductance of electronic waveguide through resonator is determined by total transmission coefficient (Landauer formula for the zero temperature conductance of a structure) $T = \sum_{n,m} |t_{n,m}|^2$, where $t_{n,m}$ are the transmission amplitudes of transverse modes propagating in waveguide. The quantities $t_{n,m}$ may be computed semiclassically. Toward this end a number of groups ([4], [5], [6], [7], and others) performed calculations for some specific shapes of resonators such as rectangular or circular, and found the same qualitative behaviour of the rapid oscillations of conductance. However, most of the papers on semiclassical analysis of conductance oscillations dealt with acoustic model for quantum electron behaviour and seems to have not been able to present quite accurate results for magnitude of the conductance.

In this talk basic details of a developed semiclassical analysis, based on the uniform GTD ([8]), that predicts some of the large scale structure of conductance fluctuations of waveguide-resonator junction are presented. Using effective hamiltonian approximation for the junction of waveguide and rectangular resonator, made of metal or semiconducting materials, this problem can be reduced to 2D problem for the Schrödinger operator with magnetic field and parabolic confinement potential.

In the paper ([9]) a semiclassical analysis of the high-energy eigen-states of an electron inside a closed resonator was described. On the basis of numerical analysis carried out by finite element method (FEMLAB), it was established that these high energy eigen-states are excited in the intersection of waveguide and resonator by a waveguide traveling mode. In the case of resonance excitation of a high-energy eigen-state (Fermi energy of the incident mode coincides with the closed resonator eigen energy), the transmission of waveguide through resonator is blocked. Firstly, this effect was discover on the basis of FEMLAB for single-mode waveguide propagation ([9]). The mathematical model generated in FEMLAB studying a waveguide propagation through resonator is able to takle only single-mode propagation. It encounters computational difficulties for large values of Fermi energy of travelling electron. The presented semiclassical analysis of the junction conductance works for a multi-mode waveguide propagation, and it becomes more accurate the larger electron Fermi energy is taken. In this talk full agreement is shown between the FEMLAB computations and the results obtained by semiclassical analysis of the junction conductance for the case of one mode waveguide propagation. It is near the border between the resonance and high energies bands. For larger electron Fermi energy and two modes waveguide propagation, we demonstrate that the blockage of waveguide propagation by excited resonator takes place again. It is due to excitation of a high-energy eigen-states based on stable periodic orbits. Such type of mesoscopic device may be considered as an ideal filter for electronic transport through semiconductors.

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Stationary phase based asymptotic analysis of inter-pulse interference from a frequency comb source in dispersive media

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In this work, we investigate the formation of cross-correlation patterns generated by a frequency comb laser in dispersive media. The source consists of equidistant pulses which contain modes only at specific frequencies. Each pulse has a fixed phase relation with every other pulse yielding a discrete spectrum with regularly spaced frequencies $\omega_m = m\omega_r + \omega_0$ where ω_0 is the common offset frequency, m is a non-negative integer and ω_r is the repetition frequency f_r expressed in angular notation $\omega_r = 2\pi f_r = 2\pi/T_r$ and $T_r = 1/f_r$. Here T_r is the time distance between the pulses. The field of the pulse emitted by the laser, propagating in the direction of positive x, at $x = 0$ can be written as

$$E(0,t) = \sum_{m=0}^{\infty} A_m \cos\left[(m\omega_r + \omega_0)\, t + \phi_m\right] \tag{1}$$

where A_m is a real amplitude and ϕ_m is a phase. We consider the problem of the pulse propagation in a dispersive unbalanced interferometer. It can be shown that the discrete spectrum of the laser yields discrete cross-correlations that can be described by a series as

$$\Gamma(X) = \sum_{m=0}^{\infty} |a_m|^2 \cos\left[(m\omega_r + \omega_0)\, n\,(m\omega_r + \omega_0)\,\frac{X}{c}\right] \tag{2}$$

where $a_m = A_m \exp(i\phi_m)$, $X = x_2 - x_1$ is the delay distance, and $n()$ is a non-linear function denoting the dispersion properties of the medium. When the pulses emitted from the frequency comb laser propagate in a non-linear dispersive medium, the resulting cross-correlation patterns are distorted. This distortion at a short length scale is non-linear and relaxes to linear broadening for longer length scale. We observe that the cross-correlation distortion and the interference fringe formation cannot be simply understood by using the discrete cross-correlation model (2). Therefore, we used the Poisson summation formula to extend the formalism to an integral-based continuous model [1]. The continuous model is found to be more helpful in understanding the dispersion effects on cross-correlation functions. Furthermore, to study the asymptotic behaviour of the cross-correlations formed in the unbalanced interferometer for very long distances, we used the method of stationary phase [2]. From this analysis it is seen that the contributing stationary frequency remains constant in the evolution of a particular optical fringe in the correlations found periodically at increasing delay distances.

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Flexural-gravity wave scattering by heterogeneities in an elastic plate floating on water

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In this paper we consider the scattering of a flexural-gravity wave propagating on a thin elastic plate by an arbitrary number of infinitely-long parallel straight line heterogeneities. The plate is partially submerged into an ideal incompressible fluid of finite constant depth and covers its entire surface. The plate's heterogeneities are specified via boundary-contact conditions. We consider three following conditions: the plate is clamped, hinged or cracked along a line. The problem is designed to model a pontoon-type very large floating structure [1-7] which is either supported by a number of fixed columns to the sea bed, or tethered by extensible mooring lines. Cracks also may occur between adjacent plates making up the structure. Exact mathematical formulation of the problem is given. Exact expressions are obtained for the wave field in the fluid and the flexural field in the plate. The transmission coefficient of the incident flexural-gravity wave from infinity and its reflection coefficient are determined. The internal forces which arise in the supports are found. The existence of the resonant phenomena which follows to the perfect transmission of the wave is shown. The problem is also solved for two approximate models of water depth: infinite and shallow. Numerical results obtained are compared to determine the validity ranges of these models.

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SPECIAL SECTION: METAMATERIALS

The paradox of zero forward-scattering in relation with the optical theorem

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Metamaterials and exotic material properties have raised a strong interest in recent years for cloaking and transparency, as well as other anomalous scattering phenomena. In this regard, one of the recognized venues to metamaterial cloaking, the plasmonic cloaking technique, put forward in 2005 [1], has been often associated with the anomalous scattering from small nanoparticles coated by "complementary" permittivity materials, which were shown few decades earlier by Kerker [2] to provide identically zero scattering in the static limit.

We will talk here about a different and less known anomalous scattering property of magnetodielectric nanoparticles in the static limit, also put forward by Kerker [3], which consists of the interesting theoretical possibility of conceiving objects that may provide identically zero scattering in the forward direction, despite significantly larger scattering in any other direction. Recent experimental and theoretical papers on the topic have further discussed this possibility in more realistic scenarios [4-8]. Inspecting some of their analyses, it seems indeed possible to conceive a significant scattering pattern presenting a sharp minimum (but not identically zero) in the forward direction and a much stronger scattering in all other directions. However, from a theoretical standpoint, it is well known that the total scattered power from any object has to be proportional to a portion of the scattered field in the forward direction, implying that zero (or near-zero) forward scattering should be synonymous to zero (or even closer to zero) total scattering, regardless of the nature of the object and of its design! Using analytical theory and an accurate scattering formulation, in this talk we will discuss the nature of this apparent optical paradox and the limitations of this phenomenon in practical situations. In this way, we show that the optical theorem is indeed satisfied and we will shed some light on theoretical and experimental papers on the topic, showing relevant missteps in some of their physical interpretation, and discussing the general possibility of verifying these effects in practice. Moreover, we will relate this discussion to the recent interest in cloaking applications using exotic artificial materials, and in particular on the possibility to achieve minimum-scattering devices.

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A simplified analytical model for receiving wire antennas consistent with power conservation

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The interest in artificial materials and metamaterials composed of short loaded wires has led in recent years to a revived interest for simplified, but accurate models for the scattering and receiving properties of dipole antennas [1-2]. Currently available models that calculate the scattering and receiving properties of these antennas are often oversimplified and they are easily shown not to satisfy basic constraints on power conservation. This interest has developed independently, but in parallel with a revived interest for circuit equivalent models of receiving antennas, for which recent discussions have pointed out how power relations may not always hold in their circuit representation [3-5].

In this talk, we present an improved and self-consistent analytical model for the approximate current distribution induced on an arbitrarily loaded wire antenna of moderate length operating in its receiving operation. We derive interesting novel closed-form conditions on the values of input impedance parameters of an arbitrarily loaded wire, and we relate these quantities with its approximate current distribution models and with a novel accurate expression for its polarizability.

The results of our analysis not only shed some new light on a debated issue regarding the circuit model of receiving antennas, and improve currently available simplified analytical models for receiving dipoles, but are also shown to be of great interest to the proper homogenization of composite materials and surfaces formed by loaded short wires, and to minimum-scattering receivers and absorbers.

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Microwave heat of copper powder with varying particle size

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In recent years, using of microwaves for heating has extended to compacted metal powders. Microwave heating is more preferred than conventional due to its various advantages such as: time and energy saving, rapid heating rates, considerably reduced processing cycle time and temperature, fine microstructure and improved mechanical properties, better product performance, etc. There are a lot of experimental and theoretical publications in microwave heating of metal powders. But mechanisms of microwave heating of metal powders have not yet been explained clearly. In the study [1] was obtained that as particle size increases the heating rate decreases and the heating rate increases as the porosity increases.

This work is a theoretical verification of experimental paper [1] where microwave heating curves of copper powder with varying particle size and porosity were obtained.

Approximate theoretical model for calculation of microwave heating of copper powder is suggested in this work. Copper particles are covered with thin oxide shells. We use effective medium approximation to take into account the impact of this shell on the effective permittivity and permeability. We use Mi theory to take into account the impact of skin-effect on the permeability of the copper core [2]. Electric and magnetic fields penetrated into copper powder are calculated using transition matrix method.

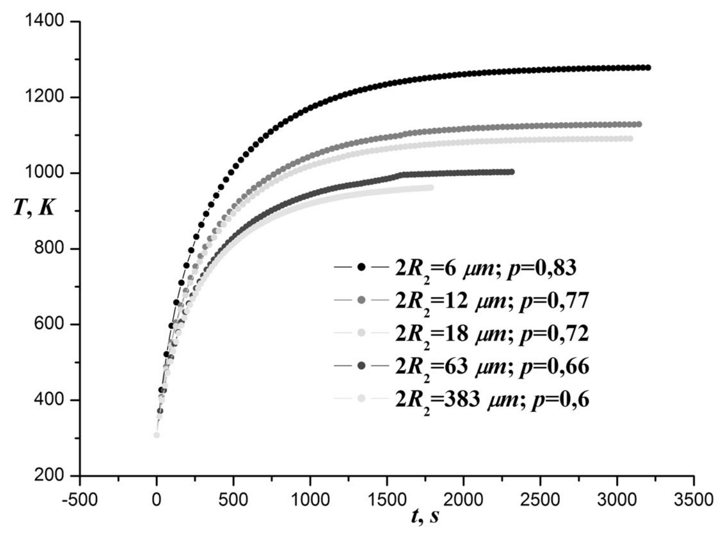

Fig.: Calculated heating curves of copper powder as a function of particle size.

Theoretical heating curves of copper powder are obtained in the present study (fig. 1). And these results are in a good qualitative correspondence with the experiment results [1]. Presence the oxide shell onto the copper cores provides the penetration of the electromagnetic waves into the volume of the copper powder. And heating of the powder is provided by complex effective permeability of the copper core appeared due to skin effect and magnetization of the particle.

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FDTD Mmdelling of transformation electromagnetic based devices

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Transformation electromagnetics [1] enables the design of exotic devices for the manipulation of electromagnetic waves in ways that are not occurring naturally. The most prominent application so far has been the cloak of invisibility [1], a structure that can be constructed using dispersive metamaterials [2]. The cloak of invisibility and other interesting transformation electromagnetic based devices, such as the field rotator, concentrator and the optical clack hole, will be studied in this presentation. So far, however, such devices have been mostly studied under single frequency plane wave illumination [3], which effectively ignores their inherently dispersive nature. For example, the investigation of the cloaking bandwidth has been very limited in the literature to mostly analytical treatments [4].

In this presentation, we will examine the steady-state and transient responses of transformation-based devices. The goal is to demonstrate their bandwidth performance and better understand the physics involved in their frequency response. This is achieved using the robust and efficient dispersive radially-dependent FDTD numerical technique [5], which will be thoroughly explained during the presentation. This numerical modelling method is advantageous compared to the Finite Element Method (FEM) used in previous works [6], since the transient response and the operational bandwidth of a device can be easily computed. Dispersive FDTD also self-consistently accounts for the frequency-dependent effects of the electric and magnetic components that arise in resonance-based metamaterial structures.

The FDTD simulations of the lossless and lossy ($\tan\delta = 0.1$) cylindrical cloaks, after the steady state has been reached, can be seen in Fig. (a), (b), respectively. More results will be presented at the conference.

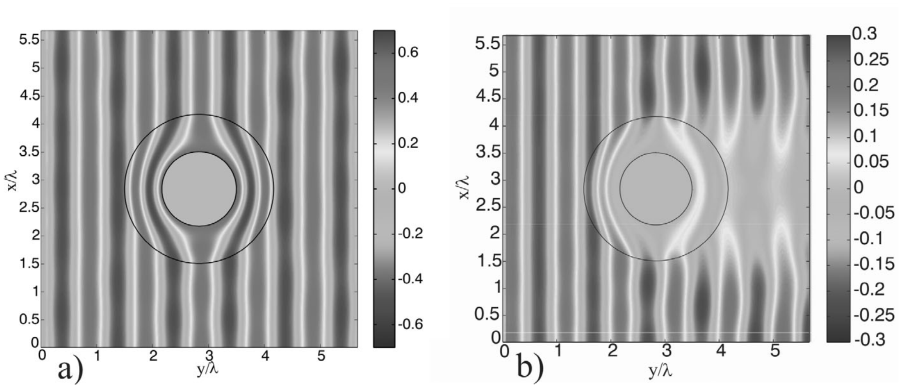

Fig. Magnetic field (Hz) amplitude distribution of the lossless (a) and a lossy (b) ideal cylindrical cloak.

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Optimal parameters of metallic nanorods arrays for subwavelength imaging

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Arrays of metallic nanorods are capable of transmitting images with subwavelength resolution [1,2]. There are two typical geometries of such arrays, suggested by A. Ono, J. Kato, S. Kawata [1] and M.Silveirinha, P.Belov, C.Simovski [2] for different frequency ranges of optical spectrum. In this work we examine the geometries in order to identify optimal parameters of the structures which enable best imaging performance. In the case of A.Ono et al the length of rods can be tuned so that imaging of cophasal sources is possible. This tuning provides much better range of applications of the nanorods arrays by providing possibility of imaging arbitrarily shaped sources. These improvements may dramatically change operation of cascaded geometries suggested in [3] for colour subwavelength imaging. In the case of M. Silveirinha et al the tuning of the rods diameter provides wider operation bandwidth. The optimum diameter is approximately equal to half of the square lattice period. Astonishingly, the arrays of thicker rods are capable of operating with sources located at significant distances away from the front interface. The arrays in such regime operate very similarly to perfect lens [4], but the distance through which the sources can be detected is limited by the periodicity of the structure rather than the thickness of the slab as in the case of the perfect lens.

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Nonreciprocal transmission of surface microwaves along "ferrite-grating of resonant elements" metasandwiches

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First unique effect of giant nonreciprocal transmission (up to 35 dB) has been discovered in "transversely magnetized ferrite plate" grating of resonant double split rings metasandwiches, arranged at distance of a quarter of the waveguide width from the narrow side wall in a rectangular waveguide [1]. In [2] nonreciprocity was observed in situations when nonreciprocity is absent without a resonant grating. So, the effect was observed when the metasandwiche was placed in free space or along the waveguide axis. These effects take place with both chiral elements and electric-dipole elements (polygonal nonconvex loops and zigzag dipoles) [2, 3]. Therefore theoretical investigations of wave processes around gratings of resonant elements are of interest.

The paper under presentation is devoted to theoretical study of surface microwaves formed by a bianisotropic layer (BL) as well by a metasandwich "ferrite plate – bianisotropic layer". Bianisotropic layers simulate gratings of resonant elements. When resonant elements are identically oriented double split rings or other chiral elements it is supposed that the BLs matter possesses permittivity and permeability with nonzero and different from each other diagonal components , $\varepsilon_{jj} = \varepsilon_j, \mu_{jj} = \mu_j (j = x, y, z)$ and the nonzero components $\kappa_{yz} = \kappa_{zy}^T = \kappa$ of chiral tensors. When resonant elements are electric-dipoles the BLs permeability is unit tensor, and chirality is absent. We suppose that a ferrite permittivity ε is isotropic, and tensor permeability possesses elements μ and $\pm i\mu_a$, as usually [4]. To allow for dissipation of electromagnetic energy by a ferrite, μ and μ_a are assumed to be complex numbers.

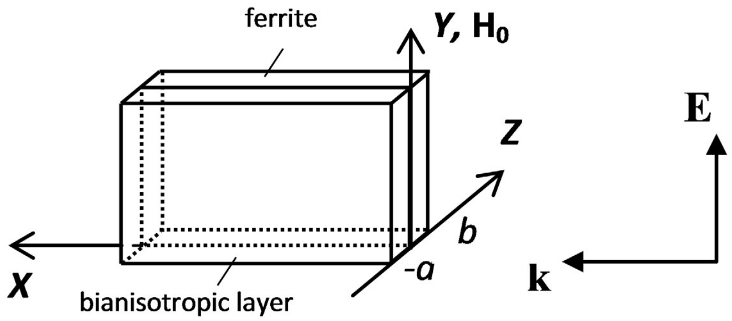

Figure : A bianisotropic-ferrite metasandwich in free space; **k** and **E** are the wave vector and electric field of an incident wave, $\mathbf{H_0}$ is a magnetostatic field.

The dispersion equation has been got. The distribution of energy fluxes and the polarization of the magnetic field of surface waves of BL have been studied. Dispersive characteristics of the metasandwich have been found. Some basic transmission laws for the waves in the metasandwiches ferrite plate bianisotropic layer have been revealed. These laws include the nonreciprocity of microwaves transmission along the metasandwich placed in free space, the manifestation of the transmission nonreciprocity near the resonance in the grating elements, the change in the nonreciprocity sign by transference of the ferrite plate to opposite side the BL, dependence of the nonreciprocity sign on the relative positions of the ferromagnetic resonance and the resonance in the BL elements, and (qualitative) independence of the observed effects on the design of the grating elements (they are chiral or electric - dipole). All these peculiarities were experimentally observed in [1-3, 5].

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Nonlocal homogenization theory of multilayered metal-dielectric nanostructured metamaterials

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In our presentation we consider an optical metamaterial consisting of spatially periodically repeated nanolayers of metal and dielectric. Such the metamaterial possess plenty of unique electromagnetic properties. For instance, it can transmit evanescent waves from one of its surfaces to another one. This property could be used to create superlenses, which is capable of transforming evanescent waves to propagating ones. Also, the metamaterial under consideration could be applied in subwavelength microscopy, as image magnification possibility recently was demonstrated in a number of works [1-5]. Another useful applications include nanolithography [6], as well as cloaking [7].

In order to describe such metamaterials method of local homogenization is in widely use. However, we will show that in the multilayered metal-dielectric nanostructure there are effects of strong spatial dispersion, which cannot be avoid. As expected, a local model does not provide satisfactory description of the metamaterial's electromagnetic behaviour because it does not depend on the wave vector. The latter is necessary for description of spatially dispersive effects. Thus we need nonlocal homogenization theory.

A method of nonlocal homogenization has been proposed by Mario Silveirinha in 2007 [8]. The main idea consists in excitation of system by an external electric current with certain field distribution. Solving Maxwell's equations with such a continuous source, it is possible to calculate average electric fields and then find the effective nonlocal dielectric permittivity $\varepsilon_{\text{eff}}(\omega, \mathbf{k})$ depending on a wave vector $\mathbf{k}$ by means of the following expression:

$$D_{av} = \varepsilon_0 E_{av} + P_g = \bar{\bar{\varepsilon}}_{\text{eff}}(\omega, \mathbf{k}) E_{av},$$

where D_{av}, E_{av} - average electric displacement and electric field, and P_g polarization.

We apply the given method to the special case of a multilayered metamaterial. Solutions of Maxwell's equations are written in the form of the sum of three components: direct and backward waves (eigenmodes of the structure), and the induced component caused by an external current, which is the solution of the equations for infinite space. Amplitudes of direct and backward waves are found by solving the system of the linear equations. Then from expressions for these amplitudes average electric fields are calculated. Finally we obtain analytical expressions for components of the nonlocal dielectric permittivity tensor $\bar{\bar{\varepsilon}}_{\text{eff}}$ of the multilayered metamaterial in directions along the layers and normal to the layers. Expressions obtained are fully spatial dispersive as they depend on the wave vector $\mathbf{k}$.

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Metallic nanorods dimer: from optical nano-antennas to planar chiral metamaterials

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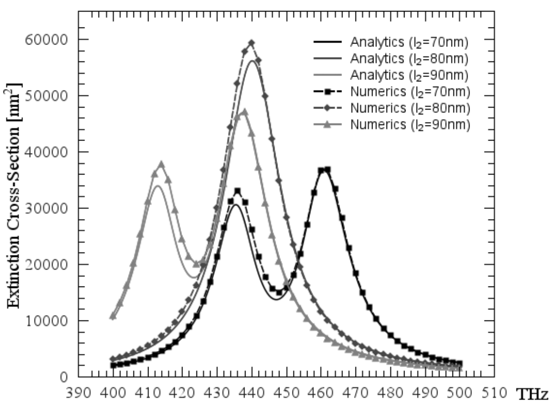
Recently, plasmon-polariton resonances of metallic nanoparticles have received much attention due to their exotic electromagnetic properties. By choosing the shape of the particles, it becomes possible to engineer both scattering and near-field properties, promising in, e.g., near-field microscopy, biosensing, optical antennas, solar cells, etc. A pair of closely placed metallic nanorods forms a strongly coupled system, whose properties are strongly influenced by the dimer geometry. Such a nanorod dimer can act as an optical nano-antenna or as a resonant unit cell of optical metamaterial. Metamaterials are known to possess unusual physical properties rare or absent in naturally occurring media. Two notable examples are (i) giant optical activity in materials made of spiral-like or otherwise twisted elements ("meta-atoms") and (ii) planar chirality if the meta-atoms possess 2D rather than 3D enantiomeric asymmetry. Such planar chiral metamaterials (PCMs) have polarization eigenstates that are elliptical and co-rotating, unlike 3D chiral or Faraday media. This leads to exotic polarization properties such as asymmetry in transmission for left-handed vs. right-handed circularly polarized incident wave.

In this presentation we introduce a simple but powerful model for the metallic nano-dimer within dipole-dipole approximation. Resonances of an isolated nanorod are governed by the coupled electromagnetic and surface plasmon excitations of the particle and determined by the nanorod material and size. For particles much smaller in size than the wavelength of the incident wave the dominant contribution to the fundamental resonance is seen to be dipole in nature. If a particle is sufficiently elongated in one dimension, only the longitudinal plasmon-polariton excitation contributes to the fundamental resonance. The proposed model employs this to derive explicit analytical expressions for effective dipole, quadrupole, and magnetic dipole polarizabilites of a dimer.

These calculated polarizabilities can be used to further determine the extinction and scattering cross-sections of an individual dimer meta-atom, as well as the effective material tensors of a PCM composed of such meta-atoms. The results in Figure 1 show a reasonable agreement with direct numerical simulations. Analytical nature of the model facilitates an efficient and systematic study of dimer-based nano-antennas and metamaterials. By varying the length of the rods and their placement in the dimer, it is possible to arrive at the geometries that maximize the scattering efficiency of an antenna or that provide the desired planar chiral properties. The model lends itself to a straightforward generalization to describe both PCMs and 3D chiral metamatetials within a single framework.



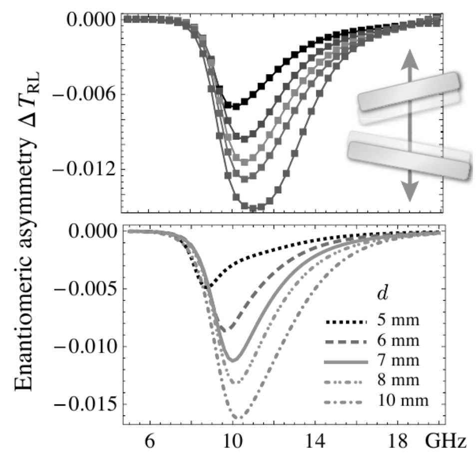


Figure: (Left) Gold nanorods dimer. Extinction cross-section of a pair of parallel gold nanorods of different length. One rod has fixed length $l_1 = 80$ nm, distance between rods is $d = 100$ nm. Comparison of the dipole-dipole model and direct numerical simulations is shown. (Right) Planar chiral metamaterial. Numerical (top) and analytical (bottom) dependence of the transmittance difference for left-handed vs. right-handed circularly polarized waves for different inter-rod distance. Unit cell consists of a pair of copper mm-scale rods rotated 45 degree with respect to each other. Rods lengths are $l_1 = 13$ mm and $l_1 = 10$ mm.

Acoustic metamaterials

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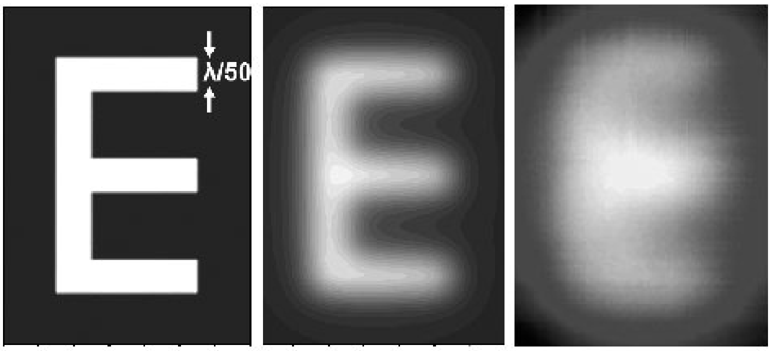
In this lecture I will start out with a brief review on sonic metamaterials that are man-made structures enabling the design of exotic properties, such as negative mass densities, subdiffraction limited focusing and resonant blockage of sound [1,2]. Metamaterials gain their material properties from geometrical parameters rather than their chemical composition. In this context we demonstrate how to obtain quasi-perfect lensing (see Fig. 1) by means of a perforated steel plate, which is able to serve as an imaging device when Fabry-Perot resonances are excited within the holes [3,4].



Source (Simulation) Image (Simulation) Image (Experiment)

The mass conservation equation constitutes the bulk modulus (spring constant or compressibility for air) and governs linear acoustics. With a so-called acoustic double fishnet structure we show how the effective bulk modulus can be tuned to negative values, giving rise to a complete suppression of sound transmission. This negativity implies that a fluid element in the structure on resonance is expanding upon compression, hence exhibiting a negative group velocity [5].

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Lossy wave propagation through graded interfaces between RHM and LHM media

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In the last decade, a new class of artificial composite materials called electromagnetic metamaterials or left-handed materials (LHM) has emerged. LHM are produced using "particles" such as split-ring resonators and nanowires as their structural units, and they exhibit unusual electromagnetic properties like the simultaneously negative permittivity and permeability. In the theoretical work of Veselago, it was shown that left-handed materials have a negative index of refraction (and, hence, negative phase velocity), inverse Doppler effect, and radiation tension instead of pressure. These properties are related to the fact that the Poynting vector in these materials is antiparallel to the wavevector, i.e., the electric field, the magnetic field and the wavevector of a plane electromagnetic wave form a left-handed system of reference.

We investigate the transmission and reflection properties of lossy structures involving left-handed materials with graded permittivity and permeability. An exact analytical solution to Helmholtz' equation for a lossy case, with the graded real parts of permittivity and permeability profile changing according to a hyperbolic tangent function along the direction of propagation, is presented. We obtain the exact analytic expressions in the closed form as well as the graphical results for the field intensity along the graded structure. The model straightforwardly allows for arbitrary spectral dispersion function.

Thus we consider the transmission and reflection properties of lossy structures including left-handed materials with graded permittivity and permeability. Such structures, with neglected losses, were studied in the framework of metamaterial gradient index lenses by a few authors, who have shown that this provides an additional degree of freedom that can be used to reduce geometrical aberrations. A gradient metamaterial lens was also demonstrated experimentally by Smith. Theoretical investigations of structures including left-handed materials with graded permittivity and permeability has been done only very recently.

Here we present an exact analytical solution of Helmholtz' equation for the propagation of electromagnetic waves through a lossy graded metamaterial structure. We choose a graded profile for which the real parts of both the permittivity and the permeability vary according to a hyperbolic tangent function, such that there is a perfect impedance match between the RHM and LHM Media.

Investigation of electrodynamic properties of multilayer structures from biisotropic materials by means of nonlocal bilateral boundary conditions

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Multilayer structures find wide application in various fields of optics and the microwave techniques (radio absorbing coats, frequency-selective and polarizable filters, radio transparent antenna radoms, one-dimensional photon crystals, antireflection films). In this connection, investigation of electromagnetic performances of the stratified structures consisting from biisotropic materials is of interest.

The rigorous solution of the following diffraction problem was obtained. In the free space R^3 with the permittivity and permeability ε_0, μ_0 stratified structure $D(0 < z < \Delta$, consisting from n layers $\Omega_s(z_s < z < z_{s+1}), s = \overline{1,n}, z_1 = 0, z_{n+1} = \Delta$ is allocated. Layers consist from biisotropic materials with electromagnetic parameters $\varepsilon_s, \mu_s, Z_s, G_s$. From a half-space $D_1(z<0)$ on the screen D the plane electromagnetic wave $\vec{E}_0, \vec{H}_0$ with arbitrary direction of propagation and polarization was incident.

Let's designate: $\vec{E}'_1, \vec{H}'_1$ - scattering field in D_1; $\vec{E}_1 = \vec{E}_0 + \vec{E}'_1 \vec{H}_1 = \vec{H}_0 + \vec{H}'_1$ - total field in D_1; $\vec{E}_2, \vec{H}_2$ - the field which has transited in region $D_2(z>\Delta)$; $\vec{E}^{(s)}, \vec{H}^{(s)}$ - field in layers Ω_s.

Boundary-value problem statement. Field equations

$$rot\vec{E}^{(s)} = i\omega(\mu_s\vec{H}^{(s)} + Z_s\vec{E}^{(s)}),\ rot\vec{H}^{(s)} = -i\omega(\varepsilon_s\vec{E}^{(s)} + G_s\vec{H}^{(s)})\ in\ \Omega_s, \quad (1)$$

$$rot\vec{E}_j = i\omega\mu_0\vec{H}_j,\ rot\vec{H}_j = -i\omega\varepsilon_0\vec{E}_j\ in\ D_j, \quad (2)$$

where $\varepsilon_s, \mu_s, Z_s, G_s$ - the arbitrary complex quantities.

Boundary conditions on planes $\Gamma_s(z=z_s)$ $(s=1,2,...,n+1)$

$$(\vec{E}_\tau^{s-1} - \vec{E}_\tau^{s})|_{\Gamma_s} = 0,\ (\vec{H}_\tau^{s-1} - \vec{H}_\tau^{s})|_{\Gamma_s} = 0,\ (\vec{E}^{(-1)} = \vec{E}_1, \vec{H}^{(-1)} = \vec{H}_1, \vec{E}^{(n+1)} = \vec{E}_2, \vec{H}^{(n+1)} = \vec{H}_2) \quad (3)$$

Boundary conditions of infinity.

For the problem solution (1) (3) the nonlocal bilateral boundary conditions linking fields on either side of the screen are used

$$\vec{E}_{2\tau}|_{\Gamma_{n+1}} = (B_{11}\vec{E}_{1\tau} + B_{12}\vec{H}_{1\tau})|_{\Gamma_1}, \quad (4)$$

which are equivalent to the system of boundary conditions (3) in case of monochromatic fields [1,2].

The technique of the solution of diffraction problems on stratified structures from biisotropic materials is used for investigations of electromagnetic properties of the one-dimensional photon crystals, the electromagnetic screens, radio absorbing coats. Influence of biisotropic material parameters on electrodynamics performances of stratified structures is presented.

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Homogenization of arrays of nanorods

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A metamaterial made of nanorods arranged periodically in two directions of space is considered in the low frequency regime. The materials constituting the nanorods can be a dielectric or a metal. The effective permittivity and permeability tensors are derived. It is shown that a magnetic activity is possible for dielectric rods while the effective medium is non-local for metallic rods.

A lot of efforts have been made to describe the effective properties of metamaterials [1, 2, 3]. However, the complexity of these structures is generally an obstacle to a rigorous theory. In the present work, we consider a structure that is a bidimensional periodic array of finite length nanorods. Using a multiple scale approach, we derive rigorously the effective behavior of the metamaterial. The domain of validity of the results is precised by numerical experiments.

Homogenization. The behavior of dielectric rods is the same as the one obtain for infinitely long rods [3]. Let us simply describe here the behavior of ohmic nanorods. In that case, the medium has a strong spatial dispersion, the displacement field $\mathbf{D}$ being given by [4]: $\mathbf{D} = \varepsilon_0(\mathbf{E} + iPe_3)$, where P is a polarization field satisfying a propagation equation, with the vertical component of the electric field as a source term:

$$\frac{\partial^2 P}{\partial x_3^2} + (k^2 + \frac{2i\pi\gamma}{\kappa})P = 2i\pi\gamma E_3, \tag{1}$$

here $\kappa\varepsilon_0\omega$ is the total conductivity of the wires over the entire medium and $1/\gamma = d^2 \log(\frac{d}{2\pi a})$. To these relations one should add the boundary condition: $\frac{\partial J}{\partial x_3} = 0$ on the top and bottom of the structure. Clearly, in that case it is not possible to define an effective local permittivity.

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Light transport in disordered metamaterials made of nanorods

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Light transport in two-dimensional disordered metamaterials made of high-permittivity rods is studied theoretically. Different regimes of transport are observed and explained in terms of coupled electric and magnetic dipolar resonances. Light propagation at frequencies close to the magnetic dipole resonance is shown to rely on hybrid, necklace-like, states.

All-dielectric metamaterials have attracted much attention recently because of their potential ability to manipulate light without loss at optical frequencies [1]. Dielectric nano-rods in s-polarization have been shown to support overlapping electric and magnetic dipole resonances, yielding a left-handed behavior in periodic arrays of them [2, 3]. Interestingly, experiments by Peng and coworkers [2] revealed that this behavior was not particularly sensitive to structural disorder. Previous studies had also shown that photonic band gaps could resist a relatively high degree of disorder [4]. Actually, disorder is of critical importance for nanophotonics applications and is known to result in complex optical phenomena [5].

Light Transport. In this work, we study light transport in disordered arrays of high-permittivity nanorods by means of the scattering matrix method [6]. Such arrays are shown to exhibit three

distinct regimes of transport, described by positive and/or negative effective permittivity ε_{eff} and permeability μ_{eff} (in s-polarisation). We find that disorder has only a weak effect on the "dielectric" optical features of the structure, i.e. right-handed bands ($\varepsilon_{\text{eff}} > 0, \mu_{\text{eff}} > 0$) and photonic band gaps ($\varepsilon_{\text{eff}} < 0, \mu_{\text{eff}} > 0$). The existence of an artificial magnetic activity at frequencies close to the magnetic dipolar resonance is evidenced by calculating the total magnetic moment. The structural disorder is found to play a critical role on light propagation in the double-negative ($\varepsilon_{\text{eff}} < 0, \mu_{\text{eff}} < 0$) frequency range. Microscopically, light transport is supported by hybrid modes assimilable to necklace states [7].
Acknowledgements. This work was realized in the framework of the ANR contract POEM PNANO 06-0030. Support from the Institut Universitaire de France is gratefully acknowledged.

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Meeting the phase requirement for an EBG resonator antenna in two bands using a single-band frequency selective surface

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In recent years, metamaterials have been considered for enhancing the performance of antennas or for obtaining characteristics that are not easily attainable otherwise. Among them are the EBG structures with special properties that are required to form EBG resonator antennas. Such antennas have the advantages of low planar profile, high directivity and low cost of production. The main resonator in such an antenna is an air cavity, which is bounded by two surfaces. One of the surfaces is either a fully reflecting electric conductor or a fully reflecting artificial magnetic conductor (AMC). The other surface is usually formed by a strongly but partially reflecting EBG structure. 1-D, 2-D and 3-D EBG structures have been considered for this purpose. In the 2-D case, it is essentially a specially designed frequency selective surface (FSS) or a partially reflective surface (PRS). Both the reflection coefficient magnitude and the phase of this surface are crucial to the performance of the antenna. The reflection coefficient should be sufficiently large to achieve high gain. Its phase should satisfy the cavity resonance condition at the operating frequency. These requirements have been met by several groups, employing different methods. A wider operating bandwidth can be achieved by designing the surface so that its phase increases with frequency, unlike in a standard PRS where the phase decreases with phase. The authors recently presented two surfaces with such character, for wideband EBG resonator antennas.

Designing an EBG-resonator antenna to operate in two bands is more challenging because the phase condition for cavity resonance has to be satisfied in both bands. Several methods have been developed in the past to design dual-band EBG resonator antennas, including the application of multi-layer EBG surfaces and the combination of a single-layer PRS with an artificial magnetic conductor (AMC) ground. In this paper we explore another approach introducing a steep rise of reflection phase, at a specific frequency, to the phase response of the surface. This can be achieved by making the metamaterial inclusions in the surface to resonate at this frequency. Then, it is possible to meet the phase requirement for cavity resonance at two frequencies, one below the surface resonance frequency and one above the surface resonance frequency. Hence dual-band cavity resonance and dual-band antenna operation can be achieved using a single-band surface.

As an example, we have designed a PRS with a 2-D periodic slot array etched on one side of an FR4 substrate. At its resonant frequency of 11.4 GHz, it exhibits a steep rise in phase. When a resonant cavity is formed using this surface and a conducting ground, the cavity resonates at two frequencies, around 10.5 GHz and 12.3 GHz. The reflection magnitude of the surface is large enough at these frequencies to support cavity resonance. When an EBG resonator antenna is formed using this cavity, the antenna directivity peaks in two bands around these cavity resonance frequencies. The peak directivities, predicted by full-wave electromagnetic simulations, at the two bands are 18.2 dBi and 20.5 dBi, respectively.

It should be noted that the wideband EBG resonator antenna and this dual-band antenna need different reflection phase curves. In the former, the phase should slowly increase with frequency over the operating band. In the latter, it should increase steeply at or around one frequency.

Two-layered waveguide with superconducting film and metamaterial slab: propagation below cutoff

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It is known, that electromagnetic waves can amplify at interaction with some moving system, for example with electron flow in backward-wave tube, when the velocity of electromagnetic wave is equal to velocity of moving system. The electromagnetic waves amplification can take place also in structures with thin superconducting film having electrodynamic parameters in the nonlinearity range of the dynamic mixed state [1, 2]. In this paper the electromagnetic wave propagation in two-layered waveguide with thin superconducting film is considered. One layer is a double negative metamaterial, the other one is an usual dielectric. This layers are divided by thin superconducting film having parameters in the dynamic mixed state. In this case the external magnetic field directed perpendicularly to the plane of the film penetrates into the superconductor in the form of Abrikosov vortex lattice. Under the impact of transport current the flux-line lattice in the superconducting film moves along the interface of the film with the velocity v. The dispersion relation for considered two-layered waveguide structure is examined. The presence of thin superconducting film is described as boundary condition by reason of the small amount of thickness. The propagation of TE modes is studied. It has been shown that the amplification of electromagnetic waves because of the interaction between electromagnetic waves and the moving Abrikosov vortex lattice is possible, when the velocity of electromagnetic waves is equal to the velocity of vortex lattice. The combination of double negative metamaterial and usual dielectric in considered waveguide structure brings to the existence of slow waves which effective interact with flux-line lattice. As result the significant amplification is observed below a cutoff frequency of the two-layered waveguide. The parameters of amplification or attenuation depend on value of external magnetic field and transport current density. The dependency of amplification on external magnetic field can be used by creation new controllable amplifiers and filters.

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Equivalent Surface Impedance of FSSs and Applications

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The extraction of an equivalent surface impedance (admittance) that describes the far field scattering of an FSS can be based on a pole-zero technique. As a consequence the extraction of an equivalent reactance reduces to the extraction of a set of alternating poles and zeros. An extension of the technique that allows extraction of an analytical expression for the surface impedance of dipole FSSs arrays as a function of the dipole lengths will also be described. This is based on the observation that the dependence of the far-field response of FSSs as a function of frequency and as a function of dipole length is similar. This extension allows for synthesis of dipole FSSs. Examples of applications in waveguides for dispersion compensated transmission as well as leaky wave antennas will be demonstrated.

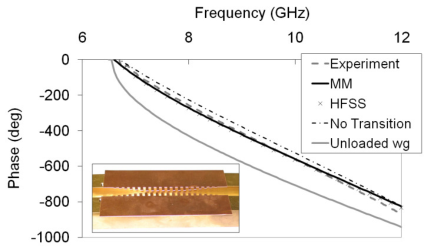

Figure 1: Fabricated prototype of a dispersion compensated waveguide: Measured and simulated unfolded transmission phase of S21 (unfolded phase for transmission along 78 mm of unloaded x-band waveguide shown for comparison).

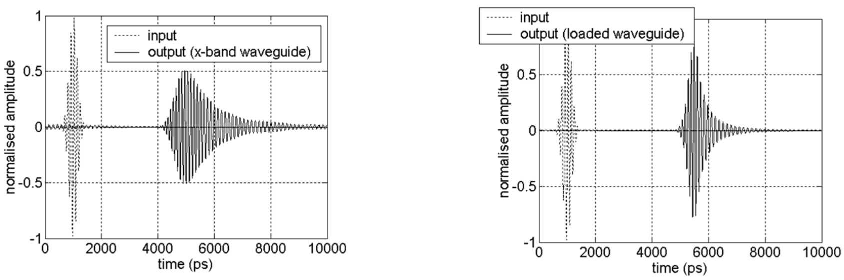

Figure 2: (a) Time domain representation of the pulse at the output of 84.6 cm length of hollow x-band waveguide, (b) and the loaded waveguide of the same length.

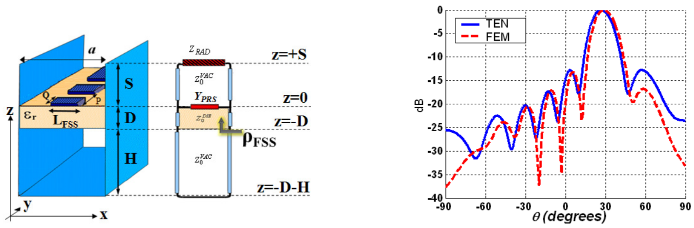

Figure 3: (a) 1D Fabry-Perot leaky-wave antenna formed by a parallel-plate waveguide loaded with a dipole-based FSS acting as a PRS and transverse equivalent network of the structure, (b) Radiation pattern of the designed PRS-LWA (LFSS=8mm, f=15GHz).

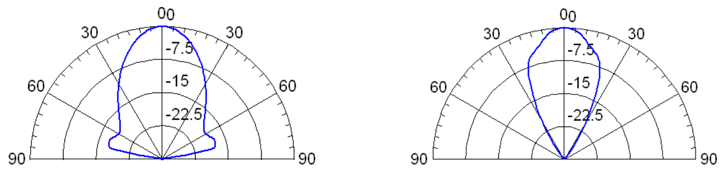

Figure 4: E-plane normalised radiation pattern for 2-layer antenna 6 at a) low-end bandwidth frequency 12.3GHz, b) upper-end bandwidth frequency 14.25GHz.

Modes of a metallic waveguide with the metamaterial insertion

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Currently, the waveguide properties of a single infinite layer of isotropic metamaterial with simultaneously negative permittivities (dielectric ε and magnetic μ, respectively) have been extensively studied and discussed [1-5]. Detailed classification of the layer modes and description of the features of their dispersion characteristics are given in [6, 7]. Influence of anisotropy is investigated in [8].

In this paper, we consider a metallic waveguide with the insertion of an isotropic metamaterial. Such systems can be practically employed for the miniaturization of waveguide paths [9, 10] and the creation of frequency filters and deceleration systems. The presence of metamaterial insertion allows to efficiently manage the dispersion of propagating waves.

A single layer of isotropic metamaterial with negative permittivity and permeability supports only slow the modes. In the presence of the metallic waveguide, both the slow and the fast modes are possible. At the same time, the waveguide itself may be either subcritical or supercritical with respect to the vacuum.

The influence of the waveguide results in the redistribution of energy fluxes in the vacuum and the metamaterials, thus significantly affecting dispersion characteristics. For example, when reducing the size of the waveguide, the forward wave transforms into the backward one. In the case, when initially (without the waveguide) both the forward and backward waves coexist (i.e. the dispersion curve has a bend), the decrease of the size of the waveguide causes the vanishing of the forward wave (the bend vanishes).

By varying the parameters of the waveguide, one can control the position of the bend of the dispersion characteristic and, in particular, localize the bend at the point of the transition from the fast waves to the slow ones. This way, an interesting class of modes that have the fast forward wave and the slow backward one can be implemented.

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Plasmonic extraordinary transmittance

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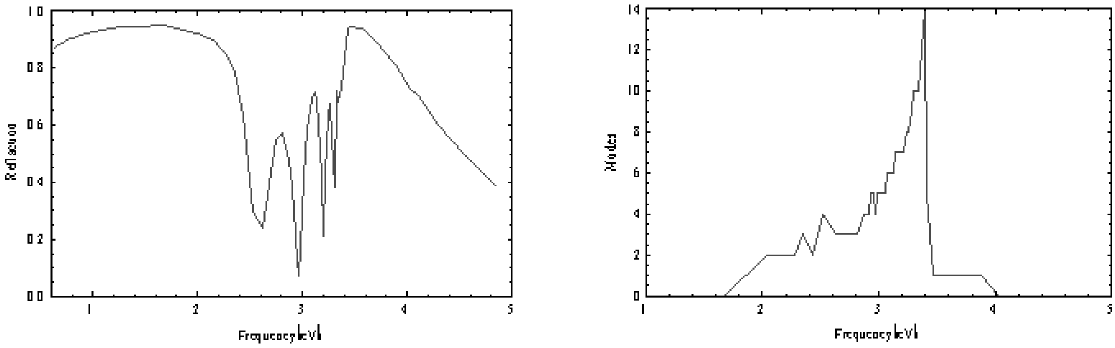
For a few recent years the interest in metal-dielectric metamaterials, which form a large class of nanostructured composite materials, has quickened [1,2]. Nanostructured metal-dielectric composites exhibit fascinating optical properties at visible and near-infrared frequencies due to excitation of surface plasmon modes. Such plasmonic systems, as two-dimensional (2D) arrays of metal nanoparticles embedded in a dielectric medium have been attracting much attention. Surface-enhanced Raman scattering (SERS) signal has a large value (as large as for arrays of silver or gold nano-disks) for such structures. SERS enhancement factor for the nanostructured arrays are strongly dependent on the ratio of composing particle diameter and inter-particle spacing [3,4].

Since the metal permittivity is negative in the optical frequency region and it is inversely proportional to the frequency squared we can model a metal particle as an inductance L. The interaction of a metal particle with electromagnetic (EM) field can be then presented as excitation of R-L-C contour. Inductance L (with small losses described by resistance R) represents the metal particle while capacitance C represents the gap between particles. The resonance in R-L-C contour is analogous to the surface plasmon resonance in a single metal particle [4].

In this work the two-dimensional (2D) model of TM-wave propagation in dielectric medium with array of duplicate silver nanoparticles and the same inter-particle distance was investigated. The calculations of the numerical problem were obtained with COMSOL Multiphysics 3.4. For a few different diameters of nanoparticles (200nm, 100nm, 50nm, 10nm) and the distance between nanoparticles

(20nm, 10nm, 5nm, 1nm) the reflection factor of TM-wave through the system have been calculated. The value of the ratio of particle diameter and inter-particle spacing was constant.



(a) (b)

Figure : (a) Reflection factor at array of silver nanoparticles with diameter D=100nm and distance between nanoparticles d=10nm for different frequencies. (b) Number of created surface plasmon modes in array of silver nanoparticles with diameter D=100nm and distance between nanoparticles d=10nm for different frequencies.

It was shown, that the reflection of TM-wave at array of nanoparticles in the proposed model nonmonotonic depends on frequency (fig.1 (a)) due to collective surface plasmon resonance in metal nanoparticles. In some frequency range nanoparticles exhibit EM modes similar to whispering gallery modes. Number of nodes in the mode depends on frequency (fig.1 (b)).

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Experimental verification of left-handed properties of manganite-perovskite metamateral in microwave band

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The manganite-perovskite composites with left handed media properties seem to be e quite perspective structures for design of electromagnetic materials of GHz and THz bands [1]. In this paper, we present experimental study of transmission properties of manganite-perovskite metamaterials in microwave band.

In our research the specimen of strontium-doped lanthanum manganite La0.775Sr0.225MnO3 (manganite-perovskite) was used [2]. The specific property of this metamaterial is a phase transition from ferromagnetic-metallic phase to the paramagnetic-dielectric one. The temperature of the phase transition is 350 K. Two structures were considered: a) manganite- perovskite slab; b) manganite-perovskite prism (Fig.a,b). The structures were loaded into single-mode waveguide section: a) waveguide; b) H-plane T-junction. Transmission spectra of the manganite- perovskite structures have been

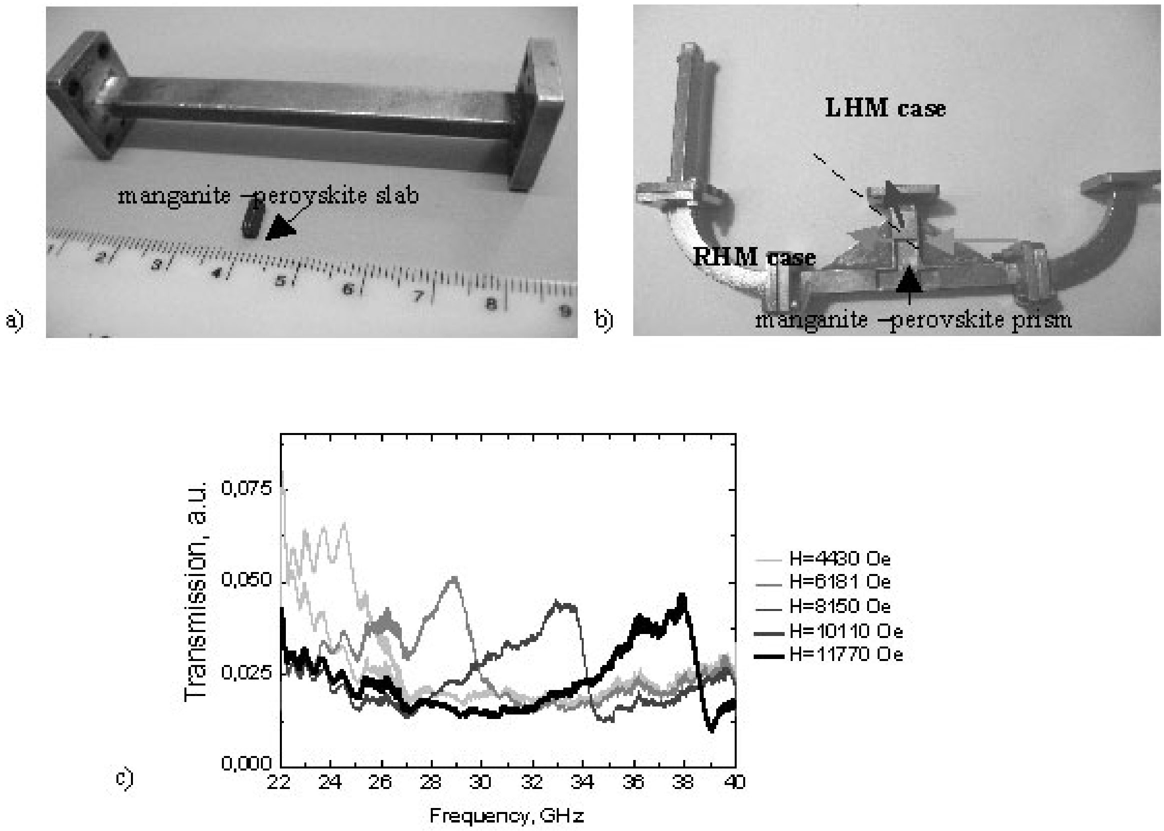

Figure : Photo of of the investigated metamaterial structures: (a) manganite-perovskite slab, (b) manganite-perovskite prism. (c)Transmission through manganite-perovskite metamaterial at various magnetic fields.

experimentally studied for different thicknesses of perovskite slab in the frequency range of 20-40 GHz and magnetic field range of 0-15000 Oe.

We have obtained the following results:

1) The transparency peak (double-negative region [1]) in the transmission spectra has been observed.
2) The influence of the metamaterial thickness on the peak amplitude has been registered.
3) The negative refraction of electromagnetic waves in the manganite-perovskite prism has been shown.
4) The magnetic field tuning of the transparency peak frequency position has been indicated.

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Microwave magnetic response of a cut wire based on interaction with surface plasmons

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It is well known that metamaterials, containing chiral conductive inclusions, possess resonant magnetic properties, which contribute to effective permeability, if microwave magnetic field induces the resonant current in elements in dependence on orientation and size of elements. Permittivity is due to excitation of currents and corresponding resonant effects by microwave electric field. Interest in metamaterials is permanently renovated because of possibility to realize materials with artificial magnetism containing not only chiral elements. This issue is discussed in many works. The emphasis is given to a pair of metal rods or strips which can show a negative response to an electromagnetic plane wave in the case when the electric field is parallel to the axis of rods and magnetic field is oriented perpendicular to the plane of the rods [1]. The magnetic response to radiation is possible because this magnetic field will cause anti-parallel currents in the two rods. The electric field induces parallel currents and provides strong resonant electric response to radiation at similar wavelength. But practically it is very difficult to separate magnetic and electric response and to be certain that magnetic response is created.

Therefore search of another way of looking is urgent. In this presentation a microwave magnetic resonant response of a single cut wire, or of a grating of wires, placed along the propagation direction transversely to the electric field E of a plane electromagnetic wave is experimentally revealed and identified for the first time. This phenomenon is observed when cut wires are placed near a meta-surface SSP, which forms surface plasmons. A strong magnetic resonant response appears in the domain of excitation of plasmons on a low-frequency side of the plasmonic resonance and depends on the distance between a wire and SSP in the case of an asymmetrically located half-wavelength wire. The response is identified as magnetic because a wire shows negative pass-band in a cutoff waveguide and electric resonant response is impossible (a single cut wire is placed transversely to the electric field). Analyzing pass-bands in cutoff waveguide one can testify magnetic or electric excitation of the resonance [2]. The results are confirmed with different SSP, containing spirals, planar double split rings, gratings of cut wires parallel to the E-field. Magnetic response is possible because incoming inhomogeneous magnetic field of plasmons can possess necessary circular polarization to cause the current in a wire and create magnetic response depending on a wire length and location. Tunable magnetic resonance and plasmonic resonance are observed at different frequencies depending on design.

The possibility of exciting separate tunable magnetic and electric microwave resonances, as well as combining and superimposing these resonances, is experimentally demonstrated for the first time. These phenomena are observed in a three-layered meta-sandwich that consists of a wire grating as a meta-surface SSP forming surface plasmons and two layers containing a single wire. The joint magnetic-?electric resonance is excited depending on design and provides simultaneously negative parameters.

Metamaterials, containing such elements, can be easy realized in microwaves and optics.

Interactions of plasmons with media are attractive because new effects have been observed with ferrite [3] and new applications are predicted with active media [4].

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Analytical modeling of artificial impedance surfaces

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This lecture concentrates on analytical modeling of artificial impedance surfaces. We will make an overview to our recent results on this field and concentrate especially on the oblique-incidence excitation. We will represent some simple analytical formulas for the grid impedance of electrically dense arrays of rectangular patches, for the surface impedance of high-impedance surfaces based on such grids over ground planes, with or without metallic vias.

Artificial impedance surfaces are a branch of metamaterials, composed of a capacitive grid over a grounded dielectric slab, whose electromagnetic response can be engineered depending on the application. The interest towards the exotic features of high-impedance surfaces has increased tremendously after the seminal paper [1] by D. Sievenpiper et al. in 1999, in which many promising applications for such surfaces were proposed. In this lecture we will discuss our recent results in analytical modeling of artificial impedance surfaces (see e.g. [2, 3, 4]), including design, optimization, and revise some possible applications in absorbers and antennas. We will concentrate mainly on analytical modeling of artificial impedance surfaces, in which the capacitive grid is composed of metallic patches. Also, the effect of metallic vias in the grounded dielectric slab is discussed.

The main design challenges are the need to provide as uniform operation with respect to the incidence angle (including also evanescent waves in the vicinity of sources or inhomogeneities) and as broad frequency bandwidth as possible. As is usually the case, these requirements conflict with the desire to have as thin and simple structure as possible. Recently, we have made a series of studies with the goal to better understand physical phenomena in artificial magnetic conductors and find better solutions for their design.

The first half of the lecture discusses the modeling of frequency selective surfaces such as the patch arrays. Here we will concentrate on finding an averaged boundary condition for different capacitive grids. We will also look into the possibility to use the extreme material properties of some novel artificial electromagnetic materials favorably in the artificial impedance surface designs. This leads us to a dual-resonance high-impedance surface design in which the plasma resonance of wire medium is used. Finally, we will review some applications in which the discussed artificial impedance surfaces could be used favorably.

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Homogenization of metamaterials on the basis of average scattering matrixes

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For the description of electromagnetic properties of composites and metamaterials in practice effective material parameter (permittivity, permeability, chirality and nonreciprocity) are widely used. For calculation of these parameters various analytical, numerical and numerically-analytical techniques are used. Effective material parameters generally are tensors values, that complicates their use at the solving of electrodynamic problems. Use of effective parameters of the specified type can lead to errors at modeling of the electrodynamic systems which characteristic sizes are comparable with the sizes of structural heterogeneity of a metamaterial (surface layers of macroscopical volumes, thin layers, spikes and edges, objects of the small wave sizes).

The approach to homogenization of composites and metamaterials, free from specified above disadvantages is offered. For the description of average electromagnetic properties of a metamaterial the macro block containing a fragment of a non-uniform material is allocated. Blocks can have the following form: a rectangle or a square - for two-dimensional problems; a rectangular parallelepiped or a cube - for three-dimensional problems. By analogy to a method of the minimum autonomous blocks (MAB) [1] electromagnetic characteristics of the non-uniform block are described by an average scattering matrix in relation to the waves propagating in virtual wave guides, connected to the parties or block sides. On walls of virtual wave guides periodic boundary conditions are set. The order of an average scattering matrix is equal: 4 - for two-dimensional problems; 12 - for three-dimensional problems. The average scattering matrix completely describes electromagnetic properties of a fragment of a composite at any modes of excitation and does not demand preliminary or subsequent definition traditionally used effective constitutive parameters.

Various techniques of calculation of average scattering matrix elements, based on the MAB method and on the finite elements method are offered. Frequency dependences of scattering matrix elements for non-uniform blocks with various internal structure and material structure are investigated. It is shown that average scattering matrixes do not concede to the description of material parameters of composites and metamaterials by tensor values.

The solving of electrodynamic problems with use of average scattering matrixes is possible on the basis of the MAB method with use of recomposition, iterative or hybrid algorithms. Within the limits of this approach non-uniform blocks are a part of the decomposition scheme along with usual homogeneous MABs.

Examples of use of the developed method of homogenization for research of electrodynamic systems which structure includes the metamaterials containing spirals, the opened rings, metal-dielectric strips etc. are presented.

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Channelling Casimir's force: ultra-long range Casimir-Polder interactions in uniaxial nanowire composites

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In 40's of the last century, a Dutch physicist Hendrik B. G. Casimir predicted [1] that two electrically neutral metal plates in vacuum attract when positioned close enough one to another. This attraction is due to the quantum fluctuations of the electromagnetic field in the space that separates the plates. When this space is filled with a high-permittivity liquid and one of the plates is replaced by a low-permittivity dielectric, the sign of the force is flipped and the interaction becomes repulsive [2].

The force between parallel plates decays rather quickly: as $1/a^4$, where a is the plate separation. This behavior of the force is easily predictable with a dimensional analysis: the only combination of the Plank constant $\hbar$, the speed of light c and the plate separation a that has the dimension of the force per unit surface is $\hbar c/a^4$. Usually, at distances of about several hundreds of nanometers Casimir's forces are rather weak (from a macroscopic point of view) and are of the order of micronewtons per square centimeter.

In the parallel plate geometries that had been considered so far the separation a was the only geometrical parameter of the system. What will happen if we add more? Effectively, what will happen if we replace the vacuum in between the plates with a metamaterial that has certain microstructure? In this presentation we study the case when such an intermediate agent is the uniaxial wire medium with nanowires oriented orthogonally to the plates. With a rigorous proof, we show that the additional geometrical parameter in this case is the area b^2 per a single wire (the unit cell area of the wire medium). Respectively, the Casimir's force becomes proportional to $\hbar c/(ba)^2$. Such a force is much stronger at large separations and may be observable even at tens of microns.

The physical reason for this enhancement in the force at large distances is that the quantum fluctuations of the electromagnetic field are effectively guided by the nanowires, in a manner similar to channeling of the near field that happens in wire lenses [3]. In wire lenses, as well as in the system that we consider, each wire plays a role of a separate channel whose characteristic cross-section is b^2. When $a \gg b$ these channels are effectively unidimensional. It is known that in the unidimensional equivalent of the volumetric parallel plate system the Casimir force is proportional to $\hbar c/a^2$. Taking into account that in our problem there are $1/b^2$ effective channels per square meter we immediately obtain the expression for the force mentioned in the previous paragraph.

When compared to the usual Casimir forces, the long-range forces in wire media are stronger by the factor $(a/b)^2$. Such an enhanced attraction results in what might be called "quantum super glue" effect. Not only attractive forces can be enhanced. We have studied several configurations in which exist long-range repulsive forces. Moreover, the effect of channeling of the quantum fluctuations in wire media can be controlled if one can control the conductivity of the nanowires, their separation and (or) orientation. This opens interesting perspectives in micro- and nano-technology where a fine control over the interaction of the exceptionally tiny parts of the micromachines is a necessary requirement.

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Spatial dispersion from a quasi-static model: crossing wires and patches

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In 2003, in a joint publication [1] we demonstrated that the dielectric response of uniaxial wire media was strongly non-local even at very low frequencies. The following wave-vector dependent expression for the longitudinal component of the dielectric permittivity of these media was proposed

$$\frac{\varepsilon_{zz}(\omega, \mathbf{k})}{\varepsilon_0} = 1 - \frac{\omega_{\rm p}^2}{\omega^2 - (k_z c)^2}, \tag{1}$$

where $\omega_{\rm p}$ was the effective plasma frequency of the wire medium. The physical mechanism of this spatial dispersion, however, was not explained in detail in [1]. This was done in [2], based on a simple quasi-static model of wire medium reported in [3]. Later in [4], Demetriadou and Pendry gave a similar explanation, indicating the charge accumulated on the wires and the low effective capacitance of the wires as the main factors resulting in spatial dispersion in wire media. The same authors also mentioned some possibilities how to make the dispersion effects less pronounced.

Recently, in [5] we have extended the model of [3, 2] to include classes of wire media with several subsets of intersecting wires and wires loaded with metallic objects and/or impedance insertions. In short, our model treats the wires as objects of certain effective capacitance and inductance per unit length. These effective parameters are found under a quasi-static approximation. The dielectric permittivity is then obtained by relating the current induced on the wires with the average electric field in the medium. In this presentation we will give an overview of our quasi-static model paying special attention to physical mechanisms that are responsible for the non-local effects in wire media and to the ways how to control them.

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A critical review of extraordinary transmission phenomena

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Unexpected high transmissivitty of light through electrically small holes made in opaque metal screens is called, since the discovery of the phenomenon twelve years ago, as *extraordinary optical transmission* (EOT) [T. W. Ebbesen et al., *Nature*, **391**, pp. 667-669, Feb. 1998]. Transmission efficiencies larger than one were measured in the original experiments. The phenomenon seems to be in apparent contradiction with Bethe's theory for small apertures and even with the optimistic predictions of ray optics. In spite of the word "optical", enhanced transmission has been observed at THz [X.-Y. He et al. *J. of Mod. Opt.*, Sept. 2009] and millimiter-wave frequencies [M. Beruete et al. *IEEE Trans. Antennas & Propagat.*, **53**, no. 6, pp. 1897-1903, June 2005]. Indeed, the phenomenon is closely related with the periodicity of the distribution of holes (or any other perturbation of the metal surface). It is the period of such perturbation what controls the critical frequencies of the phenomenon [A. G. Schuchinsky et al., *J. Opt. A: Pure Appl. Opt.*, **7**, pp. S102-S109, 2005] rather than material properties (which certainly play some role). Nowadays the phenomenon is well understood and its study has deserved three long review papers in the prestigious journal *Reviews of Modern Physics* [F. J. García de Abajo, *Rev. Mod. Phys.*, **79**, pp. 1267-1290, Oct.-Dec. 2007; K. Y. Bliokh et al., *Rev. Mod. Phys.*, **80**, pp. 1201-1213, Oct-Dec. 2008; F. J. García-Vidal et al., *Rev. Mod. Phys.*, **82**, pp. 729-787, Jan.-March 2010]. In spite of this, there remains some confusion about the true meaning of EOT. For instance, many papers have been published reporting on extraordinary transmission through a single hole or a few holes close to each other. In those cases, the size of the holes with significant transmissivitty is far from being subwavelength. The transmission of electromagnetic waves through holes having nearby resonant structures (which are, sometimes, even inside the hole and crossing the hole) has inappropriately been referred to as "extraordinary transmission".

Althouhg in a first stage EOT was linked to periodic structures, it has recently been shown that small diaphragms inside waveguides exhibit exactly the same properties as periodic distributions of holes [N. G. Don et al., *Radiophys. Quantum Electron.*, **51**, no. 2, pp. 101-108, Feb. 2008; Y. Pang et al., *Optics Express*, **17**, no. 6, pp. 4433-4441, 2009; F. Medina et al., *Appl. Phys. Lett.*, vol. 95, pp. 071102-(1-3), Aug. 2009]. A theory reporting on a unified explanation for extraordinary transmission, which is valid for periodic structures and hollow pipes with small diaphragms, has recently been developed [F. Medina et al., *IEEE Trans. on Mic. Theory Tech.*, **56**, no. 12, pp. 1108-1120, Dec, 2008; F. Medina et al., *IEEE Trans. on Mic. Theory Tech.*, **58**, no. 1, pp. 105-115, Jan. 2010]. The basic concept behind this model is impedance matching rather tan surface wave excitation.

Finally, this talk will pay credit to some classic papers focused on the same topic or on closely related phenomena. The reading of those pioneering works might shed some light on the understanding of the extraordinary transmission phenomenon.

Electromagnetic properties of doubly-periodic chiral gratings placed on both surfaces of a dielectric layer

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The normal incidence transmission of circularly polarized waves through a planar doubly-layer structure (DLS) consisting of doubly-periodic strip gratings placed on both surfaces of a dielectric layer was reported. The problem of electromagnetic wave scattering by the DLS is solved using a numerical method. The comparison of the transmission properties of the structures with identical chiral strips and with reflection symmetric of chiral strips in adjacent gratings was presented.

Recently, a few papers dealing with electromagnetic properties of doubly-periodic grating (DPG) of curvilinear strips were published [1, 2]. Such structures are attractive for microwave applications because of their resonance properties in the frequency band of a single-wave regime due to a complex shape of the grating elements. The first experiential observation and theoretical analysis of circular conversion dichroismin of a planar single-layer chiral structure was discussed in [1]. In this paper a peculiarity of circularly polarized waves propagation through the DLS consisting of planar chiral gratings are reported.

The intensities and polarization characteristics of fields were calculated using the method described in [3]. This numerical method involves solving the integral equation for the surface induced current in the metal pattern by the field of the incident wave. This is followed by calculations of scattered fields as a superposition of partial spatial waves.

The perceptible effect of asymmetric transmission of a circularly polarized incident wave was observed for single-layer DPG with chiral strips only if the dielectric substrate was lossy [1]. The DLS with lossless dielectric substrate possess such the feature. We want to draw your attention to the fact that the transmission matrixes $T_{CP} = \begin{pmatrix} T_{++} & T_{-+} \\ T_{+-} & T_{--} \end{pmatrix}$ of the DLS with identical gratings and with reflection symmetric strips in adjacent gratings are differed essentially (see Figure). For DLS with identical gratings of the chiral strips $T_{++} = T_{--}$, $T_{+-} \neq T_{-+}$ (see Figure a), but for DLS with reflection symmetric chiral strips in adjacent gratings (see Figure b). Although the shape parameters of strips are equal, the DLS with identical gratings behave as a plane chiral structure, the DLS with reflection symmetric strips in adjacent gratings as a 3D chiral structure.

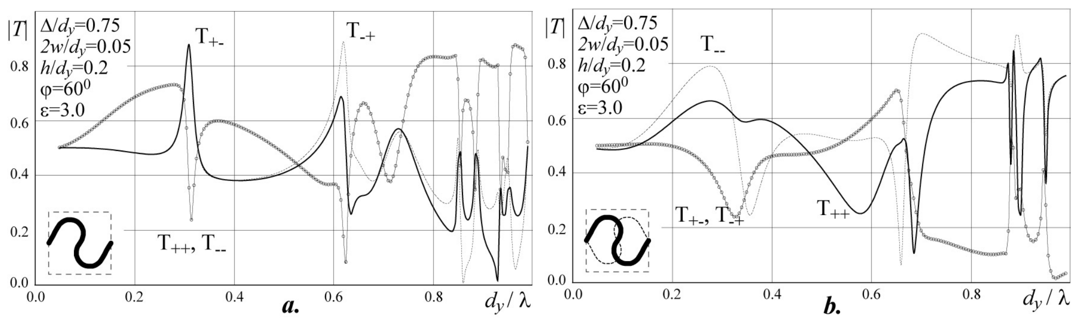

Figure : Normal incidence transmission of the DLS with identical gratings (a) and with reflection symmetric strips in adjacent gratings (b) The strips have the chiral shape.

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Anti-reflection optical coating with silver nanoparticles

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Metal-dielectric composite media exhibit abnormal dispersion characteristics at optical scales and can behave as a medium with unique effective refractive index. From the practical point of view it is interesting that the optical response of such artificial media may be tuned by specific choice of constituents and their concentration or by the detailed morphology of medium. In this work, we consider the possibility of producing of an antireflection structure (a low-index composite layer) in the substrate itself using silver ellipsoidal nanoparticles.

It is theoretically shown that monolayers of disc-like silver nanoparticles embedded in a transparent host (glass) can significantly reduce the reflection by making use of interference effects. One can see from Figure that due to the plasmon resonance of nanoparticles the reflectance in a broad spectral region (> 150 nm) never rising above that of the uncoated glass surface. The lowest reflectance calculated is about 0.04%. Unfortunately, there is no any evident improvement of the transmittance because of the absorption of electromagnetic radiation and the transition of the energy of electromagnetic radiation into the thermal energy of the nanoparticles. Nevertheless, no less than 96% of the light actually enters the glass substrate in a relatively broad spectral region (~ 100 nm) around the wavelength of the lowest reflectance.

This work explores the utility of effective medium representations to simplify the electromagnetic analysis of composite system, and demonstrates the use of this simplification in solving of the boundary problem under consideration. This approach allows us to easily control the parameters of a system and predictably change its optical properties, expressing the necessary conditions in an analytical form. With the help of full-wave finite-element numerical analysis, it is shown that effective-medium approach provides a satisfactory qualitative description of the reflection and transmission spectra in such composite layers and confirms their antireflective behavior.

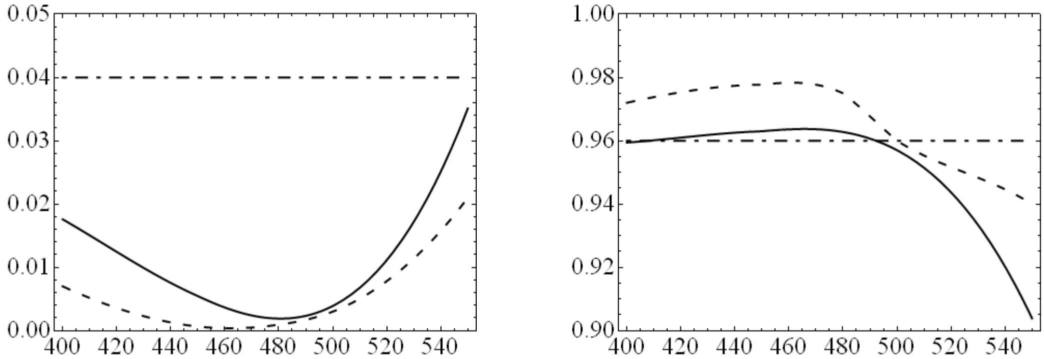

Figure: Reflection and transmittance spectra of composite material on glass substrate computed for normal incidence. Solid line corresponds to the case of three-layer antireflection structure. Dashed line is the case of optimized single-layer antireflection structure. Dot-dashed line corresponds to the case of the uncoated glass surface.

Dynamic extraction of effective material parameters of composites from reflection and transmission coefficient of a single grid

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In recent years, there has been a great deal of attention devoted to composite electromagnetic structures called metamaterials (MTMs). The most convenient way to describe the electromagnetic behaviour of MTMs is the utilization of so-called effective material parameters, obtained by a homogenization procedure. However, in the modern scientific literature there is no well-established method of homogenization and existing solutions often give controversial and ambiguous results.

In this report we apply a dynamic homogenization procedure which allows the extraction of the effective material parameters of MTMs (consisting of small scatterers) from the reflection and transmission coefficient of a single grid of such scatterers. The method is based on the results of our previous works [1, 2, 3, 4] and partially on the results obtained in [5, 6]. To illustrate the applicability of the method we studied the design of MTM, suggested in [7] to obtain negative permeability at optical frequencies, based on effective rings of plasmonic spheres. The results of dynamic homogenization were compared to the same of the quasi-static approach developed in [8].

It was shown that this method gives effective material parameters which satisfy the locality condition, what is the initial indication of their physical adequacy. The comparison with quasi-static approach reveals the fact that the behaviour of the dynamic material parameters is much more complex then is expected by Maxwell Garnett approximation. Particularly, the electric and magnetic resonances are not independent: the electric resonance is supplemented with a weaker magnetic one and vice-versa.

The obtained results clearly show that the dynamic homogenization is a necessary procedure if one wants to characterize MTMs which lattice constant is not negligibly small compared to the wavelength. We hope that the method will have wide application to the design of modern MTMs, especially operating at optical frequencies.

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Tolerable material properties of resonators in all-dielectric bi-spherical metamaterial

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An artificial material with negative refractive index can be developed using a set of closely positioned dielectric spherical particles exhibiting Mie resonance. Interaction between the particles with E111 resonance gives rise to creation of magnetic and electric dipoles in the structure. Simulation shows backward wave existence in the structure. Additionally, a possibility of a design of an invisibility shield for microwave region using resonant dielectric spheres is discussed.

Metamaterial structures consisting of dielectric resonators are widely discussed in number of publications. Most of them are based on dielectric resonators operating on first Mie resonance [1-8]. Dielectric resonators based metamaterial has an advantage of low loss at high frequency and may be easier to fabricate (in comparison with conventional split ring resonators).

In this paper a bi-spherical metamaterial structure consisting of two types of dielectric spherical resonators [2] is observed. This structure has an advantage of isotropy and is promising in construction of tunable metamaterial.

It is well known that effective parameters could be retrieved in the limit of homogenous structure. This is valid in case if the period of the structure is less than the quarter of the wavelength of the electromagnetic wave, propagating in the structure [8, page 4]. This condition limits maximum value of the dimensions of the particles which are used as constituent of the metamaterial. The sum of sizes of two closely placed different resonators could not exceed the period of the structure. At the same time the resonance frequency depends on the dimensions of the resonators. Resonance frequency in turn depends on the permittivity of the resonator. It will be shown that for any chosen operating frequency there is a minimum value of the permittivity of the resonating particle $\varepsilon_{min} = 100$ at which the period of the structure has physical meaning.

Other calculations show that to increase bandwidth of the double negative region (where both effective permittivity and permeability of the structure are negative) the permittivity should be minimal. Besides there is a limit for tolerable loss level in resonators material. In case of high loss level double negative bandwidth tends to zero.

Summing all the conclusions it could be defined optimal properties for the material parameters of resonators used in desired metamaterial structure.

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Spatial dispersion in multilayered metal-dielectric nanostructures

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The multilayered metal-dielectric structures are widely used in optics for manipulation of fields on the subwavelength scale. Their applications include subwavelength imaging [1-5], creation of subwavelength patterns [6] and achieving negative index of refraction [7]. In the most of the cases, the effective medium model is applied for description of the metal-dielectric structures. At the long-wavelength limit, it is assumed that the one-dimensional photonic crystal can be described as a uniaxial dielectric with the permittivity tensor of the following form:

$$\overline{\overline{\varepsilon}} = \begin{pmatrix} \varepsilon_\perp & 0 & 0 \\ 0 & \varepsilon_\parallel & 0 \\ 0 & 0 & \varepsilon_\parallel \end{pmatrix}, \qquad \varepsilon_\parallel = \frac{\varepsilon_1 d_1 + \varepsilon_2 d_2}{d_1 + d_2}, \qquad \varepsilon_\perp = \left[\frac{\varepsilon_1^{-1} d_1 + \varepsilon_2^{-1} d_2}{d_1 + d_2}\right]^{-1},$$

where d_1 and d_2 are thicknesses, ε_1 and ε_2 are permittivities of the constituent layers, respectively.

The effective medium model is widely used for explanation of various phenomena in multilayered metal-dielectric nanostructured optical metamaterials. However, its applicability for description of such structures was not verified yet, up to our knowledge.

We performed detailed full-wave study of dispersion properties of a typical multilayered metal-dielectric structure and compared the obtained results with ones predicted by effective medium model. The comparison of dispersion curves and isofrequency contours revealed very strong spatial dispersion effects in the structure at the frequencies near the resonance of transverse component of permittivity. At these frequencies the local effective medium model is not applicable. For example, the local model predicts existence of the only propagating extraordinary wave, whereas the full-wave results demonstrate two or even more waves at certain directions at some frequencies.

The observed effects are closely related to existence of hybrid plasmon-polariton waves travelling predominantly along interfaces of the layers. These travelling waves causes the spatial dispersion effects in multilayered optical metamaterials in the same manner as magneto-inductive waves causes the spatial dispersion effects in microwave metamaterials consisting of split-ring resonators [8,9].

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Self-organization route to metamaterials

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One possible way to move beyond limitations of now-a-day manufactured metamaterials would be using the self-organization mechanism and chemical methods [1]. The bottom-up such as self-organization and chemical methods approach could be a good alternative to mainstream metamaterial manufacturing techniques, and could result in 3D metamaterials, broadband behaviour and different new functionalities. Recent advances in the manufacturing of engineered self-organized multicomponent structures and their way towards novel electromagnetic functionalities will be presented. The structures are made utilizing directional solidification of eutectics (DSE) [2-3] as well as by nanoparticles doping (NPD) [4]. The product properties of eutectics promise metamaterial-like behaviour of appropriately-designed and engineered materials. The structures with geometries as lammelar, rod-like, spiral, and others are available in eutectics world (Fig. 1). Also structures resembling different metamaterial-like geometries have been shown [3]. And these will be mainly discussed. Also, the ability to make metallodielectric materials will be presented.

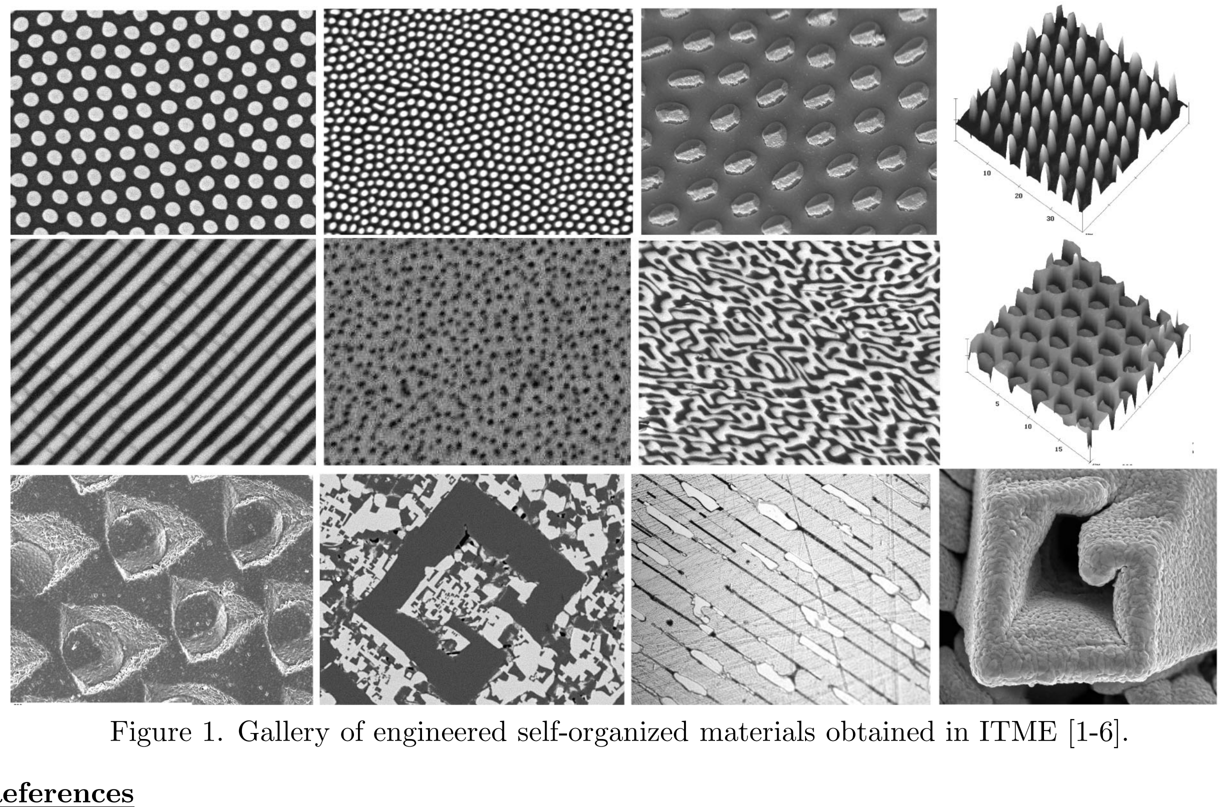

Figure 1. Gallery of engineered self-organized materials obtained in ITME [1-6].

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Inter-element coupling in metamaterials

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Properties of metamaterials assembled from individual resonators are governed by strong interactions between individual 'artificial atoms'. The coupling between individual resonators may lead to propagation of slow waves with the wavelength much shorter than that of the electromagnetic radiation. Due to these slow waves metamaterials may provide the basis for a variety of near-field manipulating devices including miniaturized waveguide components and near field lenses.

This talk discusses recent advances (i) in studies of coupling mechanisms in metamaterials, (ii) properties of slow waves of coupling and (iii) in the developments of novel applications.

(i) *Coupling mechanisms.* It was only recently realized that the properties of a metamaterial assembled from individual resonators are governed by strong interactions between individual artificial atoms. Coupling mechanisms may be quite different depending on the operation frequency. For low-frequency metamaterials like capacitively loaded loops (MHz range) the coupling is purely magnetic and anisotropic, depending on the relative orientation of the elements. For split rings (GHz range) the physical picture is more complicated. Firstly, the coupling coefficient comprises both a magnetic and an electric part; depending on the relative orientations of two such elements and on the distance between them, the coupling may be dominated by either its electric or magnetic part or be a combination of both. Secondly, the coupling constant is a complex quantity due to retardation effects. In the THz range the effects that need to be incorporated are those of kinetic inductance due to the inertia of the electrons (noticeable as dimensions become as small as 100 nm) and of plasmon-polaritons at the metal/dielectric interface (noticeable as the surface plasma frequency is being approached).

(ii) *Magnetoinductive waves.* The coupling between individual resonators may lead to propagation of slow waves with the wavelength much shorter than that of the electromagnetic radiation. These slow waves are eigenmodes of the metamaterial and can be expected to couple to and influence the propagation of electromagnetic waves forming polaritonic modes, similar to plasmon-polaritons in a bulk metal. Magnetoinductive waves, propagating on magnetically coupled resonators, exhibit all the relevant wave phenomena such as refraction, reflection and diffraction and, in a nonlinear variety, are shown to exist in form of solitons and breathers or be suitable for parametric amplification. Experimentally, slow waves of coupling between metamaterial elements have been proven to exist in metamaterials in the entire frequency range from MHz over GHz to hundreds of THz. For any particular realization MI waves can propagate in a limited band but that band may be anywhere in a region from RF frequencies to infrared and possibly beyond.In practice, when a particular device is to be designed it is desirable to have considerable freedom in choosing the dispersion properties of a metamaterial structure. We have explored the flexibility of metamaterial engineering to tailor the dispersion to required specifications. A biatomic structure (with two elements per unit cell) has a dispersion curve split into an upper branch and a lower branch, similar to the acoustic and optical branches of phonons in bi-atomic solids. Possible ways for realizing a biatomic chain are changing parameters of the element (e.g. the resonant frequency) and varying the distance between the elements. (iii) *Applications.* Due to slow waves of coupling, which can be much shorter than the free-space wavelength, magnetic metamaterials may provide the basis for a variety of near-field manipulating devices including miniaturized waveguide components smaller than the wavelength such as power dividers, directional couplers, field concentrators, delay lines, phase shifters, near field lenses, Magnetic Resonance Imaging (MRI) components or optical wave plates. A possible application of biatomic structures is to use them as waveguides providing two distinct pass bands. A particular application may be in MRI when the image is required at two different frequencies far from each other. Another set of potential applications is for nonlinear wave interaction exemplified by parametric amplification.

Transmission through slit diffraction gratings with dielectric slabs: equivalent circuit model

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The issue of the transmission properties of slit diffraction gratings has received a renewed attention in the last few years (see, for instance, [D. C. Skigin et al., *Phy. Rev. Lett.*, **95**, pp. 217402(1-4), 2005; M. Navarro-Cía et al., *App. Phys. Lett.*, **94**, pp. 091107(1-3), 2009] and references therein). Among the unexpected characteristics of these gratings we can emphasize (i) extraordinary transmission peaks associated with the periodicity of the structure and (ii) the presence of total transmission bands with some centered/off-centered transmission dips (provided electrically thick screens and compound gratings with more than one slit per period are considered). In the optics frame, these phenomena were completely accounted for by full dynamic diffraction theories (see references above). Although these theories provide some physical insight into the shape of the transmission spectra, they are complex and it can be said that they mostly provide a "numerical" explanation. A simple and easy physical understanding of the problem would be welcome. Fortunately, all the above mentioned phenomena have also been perfectly explained by the simplified equivalent circuit model reported in [F. Medina et al., *IEEE Trans. Mic. Theory Tech.*, **58**, no. 1, pp. 105-115, 2010]. The equivalent circuit is composed by a simple network of transmission lines and some appropriate capacitances. When the slit thickness is almost negligible, the only "anomalous" phenomenon that appears is the expected total reflection due to the Wood's anomaly, and this is accounted for by the equivalent circuit model. Nevertheless, if the diffraction grating incorporates dielectric slabs in the front and/or back sides of the perforated metal screen, the transmission spectrum becomes much more complex. This is due to the appearance of new phenomenology. In particular, new total reflection and total transmission effects. In this poster we will present some modifications of our original circuit model that are able to account for all the details of the modified transmission characteristics. More specifically, we add a shunt capacitance-loaded transmission line to account for the excitation of possible surface waves of higher-order in the dielectric slab. In this way, our proposal approaches the "standard" explanation of total transmission phenomena as due to the excitation of surface plasmons but providing a quantitative reduced order model whose parameters can be computed from a few full-wave simulations. Astonishing agreement between circuit model results and full-wave mode matching results will be shown.

Towards dynamical metamaterials: electrodynamical relativistic phenomena and invisibility problem

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Most of the currently studied optical metamaterials - artificial media with engineered optical features - are static, i.e. their optical characteristics do not change with time. However, non-stationary media are also of great interest, both from the scientific point of view, and due to a number of promising applications. Non-stationary behavior can be organized, first, by controlled propagation of optical characteristics perturbations in motionless media and, second, by controlled medium motion. In the talk, a review is presented of the both variants, but mainly the first, due to the availability to reach relativistic velocities in this case.

The first variant corresponds to the so-called parametric Doppler effect in motionless media [1]. Here one can consider an optically nonlinear medium whose refractive index depends on the radiation intensity. Irradiation of the medium by a laser pulse or soliton, or by a number of pulses-solitons induces inhomogeneities of medium refractive index moving jointly with pulses, i.e., with the speed of light. Then the medium is linear for additional less intensive radiation, but the medium includes fast moving inhomogeneities of refractive index. Correspondingly, a giant Doppler shift of frequency can occur for reflected radiation. The simplified problem to be solved is a linear wave equation for electric field strength with given non-stationary (moving) inhomogeneities. More detailed description takes into account the effect of additional field on the inhomogeneities important if reflection coefficient is large enough and a significant part of energy of strong pulses forming inhomogeneities is transformed into reflected radiation.

In one-dimensional geometry and under approximation of given localized inhomogeneities propagating with constant speed, the problem of reflection of weak monochromatic radiation can be solved analytically [2, 3]. The speed of the inhomogeneity should be less than the phase velocity of reflected radiation, because otherwise rapidly moving inhomogeneity radiates itself due to the Vavilov-Cherenkov effect. In the regime of homogeneous plane waves (with real wavenumber, the case of purely transparent medium), it occurs that the equation for the reflected radiation frequency has one solution, several solutions (so called complex Doppler effect), or no solutions. In the last case, the reflected radiation is not monochromatic, corresponding to the regime of accumulation of the filed near the front of moving inhomogeneity. For the case of oblique incidence of the weak radiation wave on the moving inhomogeneity, the angle of reflection differs from the angle of incidence. In metamaterials with simultaneously negative dielectric and magnetic permittivity, a new branch of frequencies in the dispersion relation arises. For pulses of weak radiation with narrow frequency spectrum, the reflected radiation has the same shape, but its width is strongly reduced. In the regime of inhomogeneous plane waves forming in media with absorption or under conditions of total internal reflection, it follows from complexity of wavenumber that the frequency of reflected wave occurs to be complex too. This corresponds to exponential temporal variation of the reflected radiation amplitude. Engineering of inhomogeneities, i.e., using of a regular sequence of strong laser pulses forming the inhomogeneity, allows one to increase the reflection coefficient giving the possibility of efficient frequency transformation of electromagnetic radiation.

The second variant with the medium motion is connected with the invisibility problem. Discussed are several approaches for this problem solution: the invisibility of the object transparency, the invisibility due to light rays bending around the object, invisibility with detecting of radiation incident on the object and subsequent generation of radiation restoring the form of the incident radiation - and their limitations are revealed [4]. One of the limitations is due to light partial dragging in moving media (the Fresnel-Fizeau effect). We demonstrate that a hypothetical object invisible when motionless, reflects radiation when it moves due to the Fresnel-Fizeau effect, and it can therefore be detected via reflected radiation.

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An analytical approach for study the spectral properties of a nanosize laser subjected to a random force

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We present a study of a lasing phenomenon which occurs as a result of strong resonant interaction of the dielectric quantum dot with plasmonic oscillations in metallic nanoscale resonator. The whole composite aggregate is considered like a 'nanolaser'. We calculate the spectral characteristics of this nanolaser and discuss the ways for tuning the lasing wavelength. Lasing threshold is defined analytically. The influence of external random force has been studied regarding to functioning the nanolaser.

On electromagnetic characterization of metamaterials

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In this presentation we discuss problems related to electromagnetic characterization of metamaterials in terms of effective material parameters and clarify the difference between characteristic and effective material parameters. Metamaterial is an arrangement of artificial structural elements, designed to achieve advantageous and unusual electromagnetic properties. Not every arrangement possessing such properties is compatible with the concept of material. The concept of material implies homogeneity, i.e. the distance between elements should be small enough. In contrast to photonic

crystals, metamaterials possess such properties due to specific electromagnetic response of their "artificial molecules" and not due to specific wave distances between them. Characterization in material science refers to the use of external techniques to probe into the internal material structure and material properties of a material. The results of characterization of a material should not depend on the sample shape and measuring setup. Respectively, electromagnetic characteristic material parameters are not parameters specific for a given wave process, they do not correspond to a given sample or to a given source. We consider interpretations of retrieved electromagnetic parameters for bulk media and discuss the pitfalls related to inconsistent classification of the material under study. Inconsistent classification may lead to inadequate characterization of the material, when bulk electromagnetic parameters would be meaningless for a structure which they should characterize. Misinterpretation of retrieved material parameters may lead to misleading performance expectations related to the values of these parameters, which are often inconsistent with the definition of characteristic parameters. The reasons of these problems are discussed. The artifact of the so-called antiresonance, the concept of so-called Bloch lattices and the role of transition layers at the interfaces of metamaterial slabs are discussed in details.

Multifrequency local field enhancement by a metamaterial nanopyramid

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We suggest and theoretically study the local field enhancement in a metamaterial sample shaped as a pyramid and formed by alternating metallic and dielectric nanoplates parallel to the pyramid base. Due to very small thickness of metal nanoplates and different transversal sizes of them the structure not only offers the efficient conversion of the light wave field impinging the pyramid base into hot spots near the pyramid apex, but the structure also sustains a large number of plasmonic resonances at which a tremendous field enhancement can be observed in the spatial vicinity to the structure. These resonances cover the whole visible range. Such structure that possesses the ability to provide a broadband local field enhancement may be of use in applications to enhance fluorescence. It can also open new doors for field-enhanced near-field microscopy and spectroscopy of nanoscaled objects. We suggest a nanostructure shaped as a pyramid formed by silver or gold nanoplates that have a varying size along a coordinate axis which is a multi-resonant plasmonic object. The whole pyramid is optically small, and the thickness of silver plates is equal to few nm. Therefore the pyramid is efficiently excited by a wave beam as a whole. The envisioned regime requires an illumination of the pyramid from the base. At a given frequency one of metal nanoplates experiences a plasmon resonance. The spectrum of resonances that is covered can be easily controlled by changing the axis ratio of the nanoplates that form the pyramid. If the light pulse is broadband all nanoplates resonate. It is important that the resonant excitation of a given nanoplate does not suggest that the local field is only concentrated inside the respective nanoplate or only close to its vicinity. The chopped interface of the nanotip supports the surface wave propagation at all frequencies of the visible band. Therefore the resonant excitation of one of nanoplates results in the resonant excitation of surface plasmon polariton propagating towards the pyramid apex. At the apex the local field is enhanced as well as at the surface of the resonating nanoplate. Such operation suggests a possible path towards broadband plasmonic resonators.

Manipulating the light transmission through metamaterial films by applying a magnetic or electric field and by changing of nano-structures shapes

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The light transmission through metallic films with different types of nano-structures was studied both theoretically and experimentally. It is shown that the positions of the surface plasmon resonances [1] depend on nano-structural details. Those can be changed from sample to sample [2] or in given sample by applying an external dc electric [3] or magnetic field [4-6]. The dependence of transmission spectrum on the shape of holes (inclusions) and external fields can be used for manipulation of the light transmission, as well as the polarization [7] of the transmitted light and other optical properties, by external field. Two complementary situations are considered: a metal film with dielectric holes and a dielectric film with metallic islands [2]. A new analytical asymptotic approach for calculation the optical properties of such plasmonic systems is developed [2]. The results of our analytical and numerical studies are in good qualitative agreement with experiment [2,8].

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Nonlinear diffraction and total internal reflection with interaction of optical beams

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We present a review of theoretical investigations and laser experiments on optical beams interaction in media with a nonlinear refractive index, including metemeterials. Effects of total internal reflection and nonlinear diffraction are presented. The equation of a signal trajectory is introduced and analyzed. The expression for a critical angle of total reflection is obtained. Results of numerical simulation of beam interactions in quadratic media, photorefractive crystals and thermal nonlinearity are discussed. Data of experiments on cw helium-neon and argon laser beam interplay in a cell with the tinted spirit are compared with the theoretical results. We also consider the discrete diffraction in the induced grating created by two tilted pump beams.

The phenomenon of total internal reflection from a less dense medium refers to the fundamental phenomena of optics. Total reflection in smoothly inhomogeneous medium, for example, in a ground layer of an atmosphere and gradient a waveguide occurs due to a curvature of a light trajectory. Here Snells law can be used taking into account layered change of a refractive index. New features of total reflection have been opened in nonlinear optics of solitons. Spatial soliton can be described as gradient waveguide which index profile is defined by basic beam intensity in cross-section. In defocusing media dark solitons can propagate. In defocusing media can exist dark solitons, and Gaussian laser beams have additional divergence.

In this lecture we discuss total reflection of optical radiation from the negative inhomogeneity induced by a powerful laser bunch. The mechanism of reflection consists in the following: in defocusing medium the powerful basic wave owing to cross-action creates at the second, signal frequency effective transverse inhomogeneity of a refractive index, and a maximum of pump intensity corresponds to a minimum of the index. When passing through the inhomogeneity is induced by the second beam suffers refraction, there is the mutual repulsion of beams. As a result, the trajectory of the signal beam is curved, and if the angle between the beams is sufficiently small, a nonlinear total internal reflection occurs. The effects of repulsion and reflection have been studied in the quadratic nonlinear media, photorefractive crystals and, finally, in media with thermal nonlinearity. In the latter case we observed total reflection due to the interaction of crossed laser beams of different frequencies in the cell with ethanol.

Theoretical analysis of the dynamics of total reflection and nonlinear diffraction is performed using numerical simulation of wave equations and the method of geometrical optics of inhomogeneous media for the construction of the trajectories of the signal wave. The problem of nonlinear interaction of beams reduces to the analysis of propagation of the signal wave in a inhomogeneous channel formed by the pump beam (compare with wave propagation in a gradient waveguide, the ionosphere, etc.). Since the channel is making a negative heterogeneity, there is a reflected signal from a less dense medium. Numerical simulation showed that under certain conditions, oblique signal beam reflected from the pump beam. If the condition of total reflection not satisfied, then the signal beam passes through the pump beam, slightly curving its trajectory.

It should be noted an important feature of the interaction of beams in a defocusing medium. When the condition of total reflection the beam pump becomes opaque to the signal wave, and thus acts as a convex cylindrical mirror. Therefore narrow beams after reflection are divergent and broad beams round the pump beam, forming a specific pattern of the nonlinear diffraction. The reflection of the signal beam is experiencing a similar effect of pump beam - it deviates in the opposite direction according law of impulse conservation

Total reflection was first observed by us in modeling the incoherent three-wave interaction in media with quadratic nonlinearity, with two-wave mixing in photorefractive crystals and thermal blooming in a weakly absorbing media. In the latter case, we also performed laser experiments in which beams of gas lasers interact with alcohol cell.

In the 3D geometry the phenomenon of nonlinear diffraction gets special features. As a result, reflect a narrow signal beam becomes diverging (the effect of a convex mirror), taking the crescent shape in accordance with the cylindrical shape of the reflecting beam pumping. The wide bunch flows round the induced cylinder, forming behind it extensive area of a shadow which is not washed away by diffraction. As a result the alarm wave is as though cut by a basic pump beam on two parts.

We also consider nonlinear diffraction in a gradient waveguide with defocusing nonlinearity. In this case there is competition between the total reflection effects and focusing on a parabolic profile of refractive index.

The final part deals with the discrete diffraction by a periodic structure, formed by two crossing beams.

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Possibilities of cloaking and invisibility at microwaves

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In this tutorial overview talk we will discuss recent advances in cloaking and invisibility at microwave frequencies. Recently, the topic of making objects "invisible" for electromagnetic radiation has gained much attention, following new ideas of using engineered electromagnetic materials with unusual properties for this purpose. This lecture provides a comparative review of the recent developments in this field and discusses the potentials of utilizing these ideas for various microwave and antenna applications. Recently proposed solutions for cloaking of objects are reviewed and compared, with the emphasis on the fundamental limitations of their performance. This topic is closely linked to the problem of creating of artificial materials with engineered electromagnetic properties. In particular, materials with equal values of relative permittivity and permeability are of interest. The lecture presents our recent developments of such materials based on mixtures of spiral inclusions and their use for cloaking applications. Furthermore, we discuss the use of electrically dense meshes of transmission lines as cloaking devices. It is shown how new cloaking techniques can be used for applications not necessarily related to cloaking of objects, for instance in new microwave lens antennas or in the design of matched absorbing layers.

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Effective permittivity of structure of coated wires

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In present work a new technique of obtaining an effective permittivity of some type of metamaterials is developed. This technique is similar to the averaged boundary conditions method which is effective for some planar periodic structures [1-3]. The technique is applied to a periodic 3D-structure of parallel wires with non-conducting coating.

It is known that the system of wires without coating possesses certain spatial dispersion [4]. This dispersion can be destructive to some prospective application of metamaterials. Therefore taming this effect is an actual problem. One of methods for this consists in using a structure of coated wires [5].

The system under consideration is characterized by two finite periods having the same order. It is assumed that the coating radius is essentially less than these periods which are in turn considerably less than the typical wave length. Thus the method developed is based on two small parameters. As a result, we obtain an expression for longitudinal component of effective permittivity tensor as a function of frequency and wave vector. This function contains two parameters: an effective plasma frequency and a parameter being responsible for spatial dispersion. For the case of non-coated wires, comparison of our result with results of other authors is given.

It is shown that the coating has only small influence on the plasma frequency but it affects essentially spatial dispersion. In principle, it is possible to reach both increase and decrease of spatial dispersion. For taming spatial dispersion we can use both magnetic coating and dielectric one (the variant with magnetic coating had been offered in [5]). Note that coating with high permeability gives bigger decrease of spatial dispersion. However, coating with high permittivity gives essential decrease as well. The latter variant has some preference since the dielectric coating with small losses is simpler in fabrication than the magnetic one.

In the present work we describe some application of the structure under consideration as well. It concerns diagnostics of beams in accelerators. The important problem in this area is measurement of energy of the beam particles. We offered for this a new method which bases on measurement of frequencies of Cherenkov modes in a waveguide containing some material [6,7]. Now we show that the structure of coated wires is a perspective medium for this goal since it allows realization of strong enough dependence of the modes frequencies on the particles energy.

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Optical flip-flop in bistable photonic crystal microlasers

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It has been envisaged for a long time that bistable phenomena in microstructures (e.g., photonic crystals or microcavities) can pave the way towards designing an all-optical memory element (flip-flop) suitable for applications in integrated optics and optical communications. In this respect, multistable microlasers are particularly promising because of their ability to maintain their state indefinitely if they remain above threshold and because they can be flipped by a relatively low-energy pulse in a very short time. However, designing a microlaser that would exhibit multiple-wavelength bistability turned out to be challenging until very recently.

This work employs a systematic theoretical treatment of lasing in a multimode microcavity. Using coupled-mode Maxwell-Bloch equations, it is shown that bistable lasing is unlikely to occur in microcavities unless spatial hole burning is strongly suppressed. Nevertheless, such suppression is demonstrated to occur rather frequently in coupled-cavity geometries where the cavity modes have different symmetry properties but almost identical spatial intensity distribution (see Fig. a). For such cavities, there exists a parameter range where more than one mode can lase single-handedly depending on the initial conditions. So, the lasing mode can be selected by preparing ("injection seeding") the cavity by a signal with the matching phase symmetry pattern. Further, the lasing mode (and hence, the wavelength) can be changed by simply re-seeding the cavity, without any need for an external cavity tuning process. Thus it can be expected that ultrafast operation necessary for an all-optical flip-flop can be achieved in such systems.

In order to evaluate the performance and practical feasibility of such a device, we analyzed the temporal mode dynamics during the flip-flop cycle for the simplest case of two modes. It was confirmed that picosecond-scale wavelength switching is indeed possible by re-seeding the cavity by control pulses with energy slightly exceeding the energy stored in the cavity during steady-state lasing. A tradeoff was identified between switching speed and minimum energy of the control pulse (Fig. b), so the performance of the flip-flop can be geared towards faster or more energy-efficient operation, without

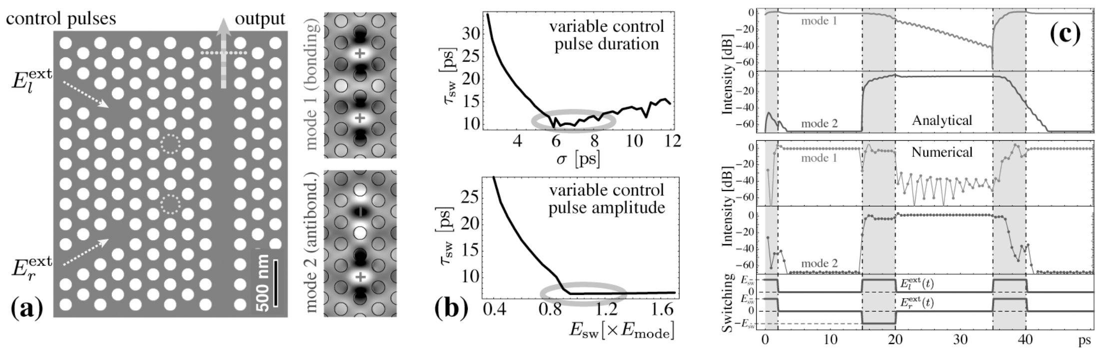

Figure : (a) The twin-defect coupled-cavity microstructure used in the present work, together with typical electric field distributions for the cavity supermodes. Note that the supermodes are different in phase pattern but not in field intensity. (b) The dependencies of the total switching time of the proposed flip-flop cell τ_{sw} on the duration and intensity of the control pulses. Circled are regions of optimum switching performance. (c) Output mode amplitudes during flip-flop cycle for the structure shown in (a), as obtained from the coupled mode theory (top) vs. from direct numerical finite-difference time-domain (FDTD) simulation (bottom).

having to change the cavity parameters.

Theoretical predictions were checked against direct numerical time-domain simulations of a twin-defect cavity in a 2D photonic crystal lattice available for experimental fabrication at state-of-the-art facilities (Fig. a). The results (Fig. c) show that 15-30 fJ top-hat pulses can switch such a cavity in less than 10 ps, with on-off contrast of at least 40 dB. The modes in question are sufficiently (about 3 nm) apart from each other in wavelength. This value can be controlled to a large extent by a simple change in the spacing between the defects in the cavity. Generalization of the proposed operating principle to more than two modes as well as to other geometries such as coupled microdisks, nanopillar waveguides, or multicore photonic crystal fibers is straightforward.

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