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DAYS ON DIFFRACTION 2022

ABSTRACTS

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FOREWORD

"Days on Diffraction" is an annual conference taking place in May–June in St. Petersburg since 1968. The present event is organized by St. Petersburg Department of the Steklov Mathematical Institute, St. Petersburg State University, and the Euler International Mathematical Institute.

The conference is supported by a grant from the Government of the Russian Federation, agreement № 075-15-2022-289, and by Simons Foundation.

The abstracts of 77 talks, presented during 5 days of the conference, form the contents of this booklet. The author index is located on the last pages.

Full-length texts of selected talks will be published in the Conference Proceedings. Format file and instructions can be found at http://www.pdmi.ras.ru/~dd/proceedings.php. The final judgement on accepting the paper for the Proceedings will be made by editorial board after peer reviewing.

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List of talks

Multiresolution quantum field theory in infinite-momentum frame

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We analyze the construction of path integral and causality issues in quantum field theory models written in light-front coordinates $(x^+, x^-, x^\perp)$ and regularized by means of wavelet transform. In contrast to standard instant-form approach, where the Hamiltonian evolution in x^+-time is used to build the Feynman integration, we consider x^+- and x^--translations on equal footing, and speculate on appropriate Poisson brackets (commutation relations, respectively), symmetric with respect to x^+ and x^- coordinates. The causality in our model is governed not only by x^+ and x^- ordering, but first of all by wavelet scale of appropriate operators.

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High-frequency diffraction by an elongated 3-axis ellipsoid

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The problem of high-frequency acoustic wave diffraction by an elongated 3-axis ellipsoid with the semi-axes $a_x < a_y < a_z$ is considered. It is well known that the problem of diffraction by a 3-axis ellipsoid allows variables separation in ellipsoidal coordinates. However, the expression for the field is complicated and is not suitable at high frequencies. It contains solutions of the wave Lame equation belonging to the class of equations with 5 singular points.

We assume that the ellipsoid is so much elongated that the quantity

$$\chi = \frac{k(a_y^2 - a_x^2)}{a_z}$$

remains bounded while $ka_z \to +\infty$. Here a_x, a_y and a_z are the semiaxes of the ellipsoid and $a_x < a_y < a_z$. Under this assumption and considering the incident plane wave propagating along Oz axis we apply the parabolic equation method, which enables the leading order approximation for the field in a boundary layer near the surface to be expressed in the form of the integral containing solutions of confluent Heun equation.

This allows the field on the hard surface and the velocities on the soft surface to be computed. The effects of high-frequency diffraction are to be discussed.

Asymptotic solutions of Helmholtz equation in a strip with localized right-hand side for two-layer medium and application to acoustics

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Consider a domain $\Omega = \{(z,x) : z \in (z_-, z_+), x \in \mathbb{R}^2\} \subset \mathbb{R}^3$. Let $\rho_-(x)$, $\rho_+(x)$, and $D(x)$ be $C^\infty(\mathbb{R}^2)$ functions such that $\rho_\pm(x) > 0$ and $z_- < D(x) < z_+$ for all x. Assume that $k(x,z)$ is piece-wise smooth in z, namely, $k(x,z) = k_-(x) > 0$ for $z \in [z_-, D(x))$, and $k(x,z) = k_+(x) > 0$ for $z \in (D(x), z_+]$, where $k_\pm \in C^\infty(\mathbb{R}^2)$. Finally, let $F(x)$ and $G(z)$ be smooth functions that decay sufficiently fast at infinity. Consider a problem in $\overline{\Omega}$ for Helmholtz equation with localized right-hand side

$$h^2 \Delta_x u + u_{zz} + k^2(x,z)u = F\Big(\frac{x - x^0}{h}\Big) G\Big(\frac{z - z^0}{h}\Big)$$

(where $h \ll 1$ is a small parameter) with boundary conditions as follows:

$$u\Big|_{z=z_-} = u\Big|_{z=z_+} = 0, \quad u\Big|_{z=D(x)-0} = u\Big|_{z=D(x)+0}, \quad \rho_-(x)\frac{\partial u}{\partial z}\Big|_{z=D(x)-0} = \rho_+(x)\frac{\partial u}{\partial z}\Big|_{z=D(x)-0}. \qquad (1)$$

This problem is related to a well known problem in acoustics, namely, the sound field generated by a point source (see e.g. [1,2,3] and references there). Here $z \in [D(x), z_+]$ is a layer of water, and $z = D(x)$ is the bottom. Our model is different in that we have an elastic foundation of finite width, while usually it is taken infinite with conditions as $z \to -\infty$. Also, considering localized functions in the right hand side (as opposed to Dirac's delta in case of a point source) seems to be both appropriate from the physical point of view and allowing to avoid mathematical difficulties.

The use of adiabatic separation of variables enables us to get rid of z-variable, and reduces the problem to an h-pseudo-differential equation with localized right-hand side, which allows to make use of technique developed in [4].

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Semiclassical approximation for bound states in graphene in magnetic field with small trigonal warping correction

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The paper deals with constructive semiclassical approximation of eigenfunctions of the 2-D differential operator describing graphene with trigonal warping effect in a constant magnetic field (small perturbation of the Dirac operator, see [1]):

$$\widehat{\mathcal{L}}_{TW}\psi = E\psi, \quad \widehat{\mathcal{L}}_{TW} = \mathcal{L}_{TW}(\overset{1}{\hat{p}}, \overset{2}{x}),$$
$$\mathcal{L}_{TW}(p,x) = \begin{pmatrix} U(x)+M(x) & \mathbf{p}_1 - i\mathbf{p}_2 \\ \mathbf{p}_1 + i\mathbf{p}_2 & U(x)-M(x) \end{pmatrix} + \mu \begin{pmatrix} 0 & (\mathbf{p}_1 + i\mathbf{p}_2)^2 \\ (\mathbf{p}_1 - i\mathbf{p}_2)^2 & 0 \end{pmatrix}, \tag{1}$$

where $\mathbf{p}_1 = p_1 + \frac{Bx_2}{2}$, $\mathbf{p}_2 = p_2 - \frac{Bx_1}{2}$, $\hat{p}_j = -ih\partial/\partial x_j$, $\mu = \gamma h$, and h is a small parameter. This statement of the problem corresponds to the further physical parameters:

$$E_0 = 6t\mu, \quad l = \frac{\hbar v_F}{E_0 h}, \quad \mathbf{B} = \frac{E_0 B}{e v_F l},$$

where E_0 is typical energy, l is typical length scale, $\mathbf{B}$ is the value of magnetic flux density, $\hbar$ is the Planck constant, $v_F = 0.97 \cdot 10^6$ m/s is Fermi velocity in graphene, and

$$M(x) = m(x)/E_0, \quad U(x) = u(x)/E_0,$$

where $m(x)$ is mass of impurities and $u(x)$ is the electric field potential.

Using standard semiclassical methods, we reduce the problem to a pencil of magnetic Schrödinger operators with a correction term. We assume $u(x)$ to be radially symmetric and $m(x)$ radially symmetric or small ($m(x) = \sqrt{h}\widetilde{m}(x)$), thus the system defined by the principal symbol turns out to be integrable, but the correction term destroys the integrability. Fixing an invariant torus with Diophantine frequencies for the system and solving the transport equation on this torus, we obtain a series of asymptotic eigenfunctions that are in one-to-one correspondence with tori that satisfy the quantization rule and lie in $O(h)$-neighborhood of the Diophantine torus. Constructive semiclassical approximation of the asymptotic eigenfunctions is based on the global representation of the Maslov canonical operator via Airy function and its derivative (see [2]). We present some numerical examples in Wolfram Mathematica that show the efficiency of our formulas.

This work was supported by the Russian Science Foundation under grant № 21-11-00341.

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Stochastic moment equations for ocean waves: Landau damping & rogue waves

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Ocean waves are often studied through envelope equations, such as the Zakharov equation, or the nonlinear Schrödinger equation (NLS). These envelope equations can be considered with stochastic initial data, and lead to closed second order moment equations using a gaussian closure. A distinct advantage of these schemes is that they can incorporate real metocean data, which are overwhelmingly second moment data (power spectra) [1]. In this context, the stability of measured sea states can be quantified. When a sea state is found to be unstable, it supports the generation of localized events with particular intrinsic scalings.

In this talk, I will briefly recall the derivation of the Alber equation [2] from the NLS using a complex Isserlis theorem as a gaussian moment closure. Moreover, using the argument principle, the constructive resolution of the Alber stability condition (which in itself is a system of two nonlinear equations in three unknowns) will be outlined [3]. This allows for a determination whether a homogeneous sea state is stable or unstable, and even then "how stable" or "how unstable" it is. This determination is supported by Monte Carlo simulations, with large extreme events strongly correlated with less stable power spectra [4]. Building on this analysis, a rigorous Landau damping estimate is derived for the stable power spectra [3], while an intrinsic scaling of localised extreme events (i.e. rogue waves) is produced for unstable power spectra.

Finally, it will be demonstrated how this analysis can be extended for the first time to the broadband Crawford–Saffman–Yuen equation (CSY), a stochastic moment closure scheme for the Zakharov equation [5].

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Space-time ray method and quasi-photons for whispering gallery waves

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Space-time ray method (STRM) in the whispering gallery case is constructed. The complex version of the STRM-expansion describing quasiphotons is also considered.

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Transformation of angular momentum of light in a system of anisotropic optical fibers

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The use of optical fibers and current related studies open the tremendous benefits and potential of fiber optics mostly in infocommunication [1–3]. Optical fibers can be considered as a promising medium for data transmission and encoding when OAM (orbital angular momentum) — beams [3] are associated as information carriers, which provides significant advantages in the growth of capacity and data protection.

In order to find the possibilities of the controlling of light beam parameters, transmitting of information encoded in states of photons, quantum computing based on classical fields, various types of optical fibers were investigated [4–6]. In particular, it was shown [5–7] that single twisted anisotropic and multihelical fiber, and also these fibers connected to the system can be used to control parameters of vortex and vector light beams and in addition to constructing fundamental logic gates for OAM-beams.

In continuation of this trend of work, we study the transformation of a paraxial light beam passing through the system of twisted anisotropic and multihelical optical fiber which are in certain resonance regimes. We obtain that incoming circular polarized mode is transformed to the hybrid HE or EH mode in twisted anisotropic fiber. At that, the polarization state of the input field defines the type of output one. Further, passing through the multihelical fiber in the system the hybrid mode transforms to the linearly polarized optical vortex which polarization and topological charge are determined by the parity and type of hybrid mode, respectively. We demonstrate the transformation of the angular momentum of the field in the system. To propose a numerical example we have found the parameters of optical fibers in such a system.

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Electrodynamic characteristics of a multigap loop antenna with phased excitation in a magnetoplasma

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In recent years there has been a great deal of interest in the excitation and propagation of twisted electromagnetic waves in a magnetoplasma. Such waves have a helical phase front which is described by the relation $\omega t - m\phi - hz = \text{const}$, where ϕ and z are the azimuthal and axial cylindrical coordinates, respectively, t is the time, ω is the angular frequency, m is the azimuthal index ($m = 0, \pm 1, \ldots$), and h is the axial wave number. This interest is primarily related to the fact that twisted waves carry orbital angular momentum, which may be used for some promising applications [1]. As is known, phased arrays can be employed for launching twisted waves to a magnetoplasma [1]. Another way for doing this is to excite an appropriately phased current in a single antenna with several feeding gaps.

In this work, we discuss the electrodynamic characteristics of a multigap loop antenna immersed in a homogeneous cold magnetoplasma such as exists in the Earth's ionosphere. The antenna has the form of a perfectly conducting, infinitesimally thin, narrow strip coiled into a ring with its axis parallel to an external static magnetic field $\mathbf{B}_0$, which is aligned with the z direction. The azimuthal current with the surface density $I(\phi, z)$ is excited in the strip conductor of the antenna by an external electric field, which is produced in the feeding gaps of the strip by the voltages $V_k = |V_k| \exp(i\psi_k)$, where $|V_k|$ and ψ_k are the magnitude and phase of the voltage supplied across the kth gap, respectively. To determine the unknown antenna current, we expand its surface density into a Fourier series over the azimuthal coordinate and formulate the integral equations for the expansion coefficients $\mathcal{I}_m(z)$ of such a series. To this end, we derive a representation of the antenna-excited field using the approach of [2] and then satisfy the boundary conditions for the tangential components of the total electric field on the surface of the perfectly conducting strip. Upon finding the solutions of the integral equations for the quantities $\mathcal{I}_m(z)$ by the method described in [3], we calculate the current distribution and the total radiated power of the antenna. It turns out that this power is reduced to the sum of the partial powers P_m. Each quantity P_m describes the power going to twisted waves with the azimuthal index m and is determined by the mth azimuthal harmonic $\mathcal{I}_m(z)$ of the surface current density. We demonstrate that by choosing the quantities $|V_k|$ and ψ_k, it is possible to maximize the partial power with the desired azimuthal index m. Thus, the antenna considered is shown to be capable of selectively exciting twisted electromagnetic waves in a magnetoplasma.

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Recent results on the two-dimensional Electric Impedance Tomography

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Suppose that M is a Riemann surface with boundary ∂M and Λ is its DN-map. The Electric Impedance Tomography problem is to determine M from Λ. An algebraic version of the BC-method is in the use.

In the talk, we provide the following results:

1. For *nonorientable* M, we construct from Λ the conformal copy of M by the use of the algebra of holomorphic functions on the orientable double cover of M [1, 2].

2. Suppose that M has internal holes with grounded or isolated boundaries. We construct the conformal copy of M by the use of the algebra of holomorphic functions on the double cover of M obtained by the gluing two copies of M along the boundaries of internal holes [3].

3. We provide necessary and sufficient conditions for an operator Λ acting on the closed curve Γ to be a DN-map of some Riemann surface with the boundary Γ [4].

4. Suppose that M and M' are two surfaces with common boundary and homeomorphic to each other. We show that, if their DN-maps Λ and Λ' are close, then M and M' are close in the relevant sense [5].

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Artificial damping for hyperbolized nonlinear Schroedinger type equation. The case of unbounded operator

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A lot of problems in physics is governed by a nonlinear Schrödinger type equation with an unbounded operator, the continuous spectrum of which covers the entire real axes. This fact leads to serious computational problems. To avoid these difficulties, a hyperbolization procedure is suggested. Adding of the second derivative of an unknown function in time with a special small parameters leads to a bounded operator. This procedure permits us to obtain explicitly the Green function for the linear part of the operator considered. As a result one can use a split-step procedure, where exact solutions both for a linear part and a nonlinear one exist. Another advantage is the essentially increased time step.

In addition to such a hyperbolization procedure, an artificial damping makes it possible to bring a numerical solution into proximity with the principal part of the solution.

Application of spline collocation iteration method for solving direct and inverse scattering problems

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Direct scattering problems. It is known [1] that various scattering problems are modeling by the Helmholtz equations, which solutions are ill-posed problems. We propose a spline collocation iteration method to solve the Dirichlet and Neumann interior and exterior problems for the Helmholtz equation in a domain with a piecewise smooth boundary.

We apply the method by solving the Dirichlet problem for the Helmholtz equation

$$\frac{1}{4\pi}\int_{\partial D}\frac{e^{ik|x-y|}}{|x-y|}\varphi(y)ds(y) = f(x),\ x\in\partial D, \tag{1}$$

where D is a closed bounded domain.

An approximate solution of (1) is sought as $\varphi_n(y)=\sum\limits_{k=0}^{n}\alpha_k\psi_k(y)$, where $\psi_k(y)$ are basic functions defined on ∂D_n. The coefficients α_k are determined from the system of equations

$$\sum_{j=0}^{n}\alpha_j\frac{1}{4\pi}\int_{\partial D_n}\frac{e^{ik|x_l-y|}}{|x_l-y|}\psi(y)ds(y) = f(x_l),\ l=0,1,\dots,n. \tag{2}$$

Here ∂D_n be a surface obtained by triangulation ∂D, x_l, $l=0,1,\dots,n$ are the collocation points.

Replace integrals in the left-hand side of eq. (2) with a quadrature formula. According to the continuous operator method [2], we associate the following system of differential equations with the equation (2)

$$\frac{d\alpha_l(t)}{dt} = (\beta_l)\left(\sum_{j=0}^{n}\alpha_j(t)\frac{1}{4\pi}\int_{\partial D_n}\frac{e^{ik|x_l-y|}}{|x_l-y|}\psi_j(y)dy - f(x_l)\right),\ l = 0, 1, \ldots, n, \tag{3}$$

where $\beta_l = \pm 1$, $l = 0, 1, \ldots, n$. The signs are chosen so that the logarithmic norm of the right -hand side of (3) is negative. The system of equations (3) can be solved by any numerical method.

Inverse scattering problems. Consider the Helmholtz equation $\Delta u + k^2 u = 0$, in a domain D. Here $k = \|k\| > 0$ is the wave number corresponding to the wave vector k. Let the values of $u(x)$ and $\partial u/\partial \nu$ are known on the boundary ∂D, where ν is the unit outward normal vector. Let at some point $x^* \in D$ the solution $u(x)$ of the Helmholtz equation is known under known boundary conditions. To find the wave number k, we use a continuous method for solving nonlinear operator equations

$$\frac{dk(t)}{dt} = \beta\left(\frac{1}{4\pi}\int_{\partial D}\left[u(y)\frac{\partial}{\partial\nu(y)}\frac{e^{ik(t)|x^*-y|}}{|x^*-y|} - \frac{\partial u}{\partial\nu}(y)\frac{e^{ik(t)|x^*-y|}}{|x^*-y|}\right]ds(y) + u(x^*)\right),$$

where $\beta = \pm 1$ and is chosen so that the logarithmic norm of the Jacobian of the right side of the previous equation was negative.

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Approximation of point interactions by geometric perturbations in two-dimensional domains

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We present a new type of approximation of a second-order elliptic operator in a planar domain with a point interaction. It is of a geometric nature, the approximating family consists of operators with the same symbol and regular coefficients on the domain with a small hole. At the boundary of it, Robin condition is imposed with the coefficient which depends on the linear size of a hole. We show that as the hole shrinks to a point and the parameter in the boundary condition is scaled in a suitable way, nonlinear and singular, the indicated family converges in the norm-resolvent sense to the operator with the point interaction. This resolvent convergence is established with respect to several operator norms and order-sharp estimates of the convergence rates are provided.

This is a joint work with Pavel Exner.

Boundary optimal control of radiative-conductive heat transfer with reflection and refraction effects

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The following steady-state normalized diffusion model (see [1]) describing radiative, conductive, and convective heat transfer in a bounded domain $G \subset \mathbb{R}^3$ is under consideration:

$$-a\Delta\theta + \mathbf{v}\cdot\nabla\theta + b\kappa_a(|\theta|\theta^3 - \varphi) = 0, \quad -\alpha\Delta\varphi + \kappa_a(\varphi - |\theta|\theta^3) = 0, \tag{1}$$

Here, θ is the normalized temperature, φ the normalized radiation intensity averaged over all directions, and $\mathbf{v}$ a given velocity field. The parameters a, b, κ_a, and α describe radiation and thermal properties of the medium.

Assume that the domain G is divided by two subdomains G_1 and G_2 with different refractive indices n_1 and n_2, respectively. Moreover, G_1 is the external subdomain such that $\partial G \subset \partial G_1$ and $\partial G_2 \subset \partial G_1$. Let $\varphi = \varphi_i$, $n = n_i$, and $\alpha = \alpha_i$ if $x \in G_i$, $i = 1, 2$. The equations (1) are supplied by the following conditions at the boundary $\Gamma := \partial G$ and at the interface ∂G_2 :

$$a\partial_n\theta + \gamma(\theta - u) = 0, \quad \alpha\partial_n\varphi + \beta(\varphi - u^4) = 0, \quad x \in \Gamma, \tag{2}$$

$$n_1^2\alpha_1\partial_n\varphi_1(x) = n_2^2\alpha_2\partial_n\varphi_2(x), \quad h(\varphi_2(x) - \varphi_1(x)) = \alpha_1\partial_n\varphi_1(x), \quad x \in \partial G_2. \tag{3}$$

Here, the symbol ∂_n denotes the derivative at the boundary of subdomain G_1 in the outward normal direction $\mathbf{n}$ and the parameter h depends on refractive indices of subdomains G_1 and G_2 (see [1]).

Non-negative function u is considered as a control determined by following form:

$$u = \sum_{j=1}^{m} c_j f_j, \quad u_1 \leq u \leq u_2. \tag{4}$$

Here, u_1, u_2, f_j are given bounded non-negative functions defined on the boundary Γ.

The problem of optimal control consists in the determination of the functions u, θ, φ that satisfy the conditions (1)–(4) and minimize the objective functional J for a given function θ_d:

$$J(\theta, \varphi) := \int_G (\theta - \theta_d)^2 dx \to \inf . \tag{5}$$

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Solution of the problem on the reciprocal lattice nodes of the Ewald circle of reflection without and with the account of the amplitude form-factor of a scattering lattice

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The problem of the surface acoustic Rayleigh wave [1] scattering on isotropic solid surface roughness, having the form of a rectangle band, and containing the periodic lattice of discontinuities, is considered in the frame of a conception of the Ewald circle of reflection [2] for the short-wavelength Laue–Bragg–Wulff limit, when the wavelength is more less than the size of the lattice unit cell. The problem of an arbitrary number, defined beforehand, of the resonances of scattering, i.e. reciprocal lattice nodes, for any angles of scattering, defined beforehand, lying on the Ewald circle of reflection, that is presented in the acoustic Lauegram [3] of a rough solid surface, is first solved both with and without the influence of the amplitude form-factor of a lattice on the Rayleigh wave scattering from the first principles of the dynamical theory of elasticity [1] in the present work. The conception of the Ewald sphere of reflection was not considering and solving this problem earlier since the first works on the Roentgen X-rays scattering in crystals [3]. The analytical formulas for the radius of the Ewald circle of reflection in dependence on the arbitrary numbers, defined beforehand, of resonances, lying on this circle, and on their arbitrary angles of scattering, defined beforehand, are obtained. It is obtained, that increasing of the number of resonances, lying on the Ewald circle of reflection for any arbitrary angles of scattering, is necessarily accompanied by the increasing of the Ewald circle of reflection radius, i.e. of the Rayleigh wave frequency at fixed sizes of a discontinuities lattice. The structure of acoustic Lauegram of the Rayleigh wave scattering as a whole, but not only the resonances of scattering, is investigated from the point of view of the Ewald circle of reflection conception. It is obtained first, that amplitude form-factor of a discontinuities lattice, i.e. dependence of the roughness left and right amplitude difference in a point of discontinuities on a number of this discontinuity in a lattice, strongly influences the structure of the acoustic Lauegram. It is obtained first that arbitrary number of the Rayleigh wave scattering resonances for any arbitrary angles of scattering can be placed on the Ewald circle of reflection without variation of its radius, i.e. of the Rayleigh wave frequency, using the appropriate amplitude form-factor of a discontinuities lattice of a solid roughness. The results of scattering for the different limits in the wavelength as compared to the width of a rough band are obtained. Control of the scattering, i.e. its amplification or suppression, through the conception of the Ewald circle of reflection use is possible. The obtained results can be used in the experimental and theoretical investigations of the wave scattering phenomena, in particular the X-Rays scattering, and for the spectrum of scattering construction [4, 5].

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An estimate of the BMO-norm of a divergence-free vector field in terms of the associated paracommutator

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Let T be a singular integral operator of convolution type that acts as a bounded operator in $L_2(\mathbb{R}^d)$. Under certain additional assumptions on T, its commutator

$$A_b = [b, T] \tag{1}$$

with a pointwise multiplier b is also bounded in $L_2(\mathbb{R}^d)$, provided that $b \in \mathrm{BMO}(\mathbb{R}^d)$ (the fact is evident for $b \in L_\infty(\mathbb{R}^d)$), and the following estimate holds true

$$\|A_b\|_{L_2 \to L_2} \leq C_{d,T} \|b\|_{\mathrm{BMO}}. \tag{2}$$

This well-known fact from harmonic analysis was generalized to a certain class of paradifferential operators A_b (paracommutators), which depend on the coefficient b in a more general way than it is prescribed by (1). In particular, this applies to the operator of the form

$$A_b f = P(b \wedge Pf) \tag{3}$$

acting on vector fields $f \in L_2(\mathbb{R}^3; \mathbb{C}^3)$. Here b is a vector field in $\mathbb{R}^3$, $\wedge$ is the vector product in $\mathbb{C}^3$, and P is the orthogonal projection on the space of vector fields in $L_2(\mathbb{R}^3; \mathbb{C}^3)$ that coincide with gradients of scalar functions.

The present talk concerns the converse of estimate (2) in the case when A_b is given by (3). Namely, our goal is the following inequality

$$\|b\|_{\mathrm{BMO}} \le C\|A_b\|_{L_2\to L_2}. \tag{4}$$

Such estimates are known in the case when a linear mapping $b \mapsto A_b$, which associates a paracommutator A_b to a coefficient b, satisfies a certain nondegeneracy condition. However, the mapping (3) is degenerate in the following sense. It can be shown that if b is a gradient of a scalar function, then $A_b = 0$. We establish estimate (4) under the condition $\operatorname{div} b = 0$. The latter essentially means that b is orthogonal to gradients of scalar functions.

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On asymptotic solutions of the Cauchy problem for the nonlinear system of shallow water equations in a basin with a shallow beach

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Solutions of a small amplitude of a nonlinear system of shallow water equations in a one- or two-dimensional domain are considered. The amplitude is characterized by a small parameter. It is assumed that the function defining the depth of the basin is smooth, and its gradient does not vanish on the set of its zeros (i.e., on the coastline of the basin in the absence of waves). A solution of a system is a triple (region, free surface elevation, velocity) that smoothly depends on the small parameter and is such that the sum of the free surface elevation and depth is positive inside the domain and zero on its boundary, and the functions themselves that specify the free surface elevation and velocity are smooth in this region and satisfy the nonlinear system of shallow water equations everywhere in the region. The asymptotic solution is defined in a similar way, only the system of equations must be satisfied up to some degree of a small parameter. We prove that under the above assumptions on the depth function, the nonlinear system of shallow water equations with small initial data has an asymptotic solution up to an arbitrarily high power of the small parameter, and this asymptotic solution is asymptotically unique. The proof is constructive (and leads to simple explicit formulas for the leading term of the expansion). To construct asymptotic solutions of the Cauchy problem with small smooth initial data for a nonlinear system of shallow water equations, a change of variables (of the type of a simplified Carrier–Greenspan transformation) is used, which depends on the unknown solution itself and transforms the domain in which the latter is defined into an unperturbed domain independent of the solution, and then the resulting nonlinear system is solved by methods of regular perturbation theory. As the zero approximation, a linear hyperbolic system arises with degeneracy at the boundary of the domain. The proof of the existence and uniqueness of a smooth solution of the Cauchy problem with smooth initial data and right-hand sides for a linearized system is based on lifting it to a closed three-dimensional manifold (where the spatial part of the operator of the lifted system turns out to be hypoelliptic).

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Approximation of radially symmetric Gaussian beams by Bessel and Airy functions

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We consider three-dimensional radially symmetric Gaussian beams in a 3-D situation. They are localized in the vicinity of one of the axes and are asymptotic solutions of the Helmholtz equation (or in the paraxial approximation of the Schrodinger equation). The simplest Gaussian beam on the plane of the normal of this axis is described by a Gaussian exponent, beams of a more complex type ("excited states") are determined by the product of the Gaussian exponent and some polynomials of degree m. We show that such "excited" bundles already at not large m are very well uniformly approximated by a pair of special functions which are the Bessel and Airy functions of complex arguments.

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Asymptotics of multiple orthogonal Hermite polynomials $H_{n_1,n_2}(z,\alpha)$ determined by a third-order differential equation

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We consider multiple orthogonal Hermite polynomials $H_{n_1,n_2}(z,\alpha)$ with two indices. These polynomials can be defined as solutions of the third-order differential equation [1]

$$\frac{d^3H}{dz^3} - 4z\frac{d^2H}{dz^2} + \left(4z^2 - 4\alpha^2 + 2(n_1+n_2-1)\right)\frac{dH}{dz} - 4(z(n_1+n_2) - \alpha(n_1-n_2))H = 0. \qquad (1)$$

We discuss an approach to constructing the asymptotics of polynomials at large indices based on the semiclassical approximation, which gives global asymptotics via Airy functions.

The feature of the problem is that the symbol of the corresponding operator is complex-valued. This symbol is associated with the curve defined by a polynomial of the third degree. One can separate the real root of this polynomial and split it into linear and quadratic parts, what allows us to split the original equation into two. Using operational methods ([2]) we can obtain the principal symbols and subsymbols of the corresponding operators. The symbol which corresponds to the quadratic part of the characteristic polynomial is also complex-valued, but we can get rid of the complexity and reduce the equation to the Schrödinger equation with a real potential. The asymptotics of the solution of this equation can be represented in the form of the Airy function Ai and its derivative.

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Spontaneous symmetry breaking in waveguide with periodic complex potential

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Active waveguides with periodically modulated refractive index, also referred as "distributed feedback" lasers (or "DFB" lasers) are of great scientific interest over the past few decades [1]. The index modulation induces rescattering of counterpropagating waves on index lattice and provides optical feedback for the laser. In such systems the working mode can be represented as two resonantly coupled counter propagating waves. At the end of the Brillouin zone, the group velocity becomes equal to zero and thus this mode has the lowest radiative losses. This facilitate the selection of the modes and helps to achieve single-mode generation regime. The same effect can be achieved by gain grating [2]. The combined refractive index and gain gratings are also was studied [3], but nonlinear effects have not been taken into account yet.

In the present work we consider the bifurcations of the stationary states forming in the system of nonlinear active waveguide with periodically changed refractive index and periodically modulated effective gain. The focus point of the study is the spontaneous symmetry breaking bifurcation and the formation of the hybrid stationary states with dominating direction of the energy flow.

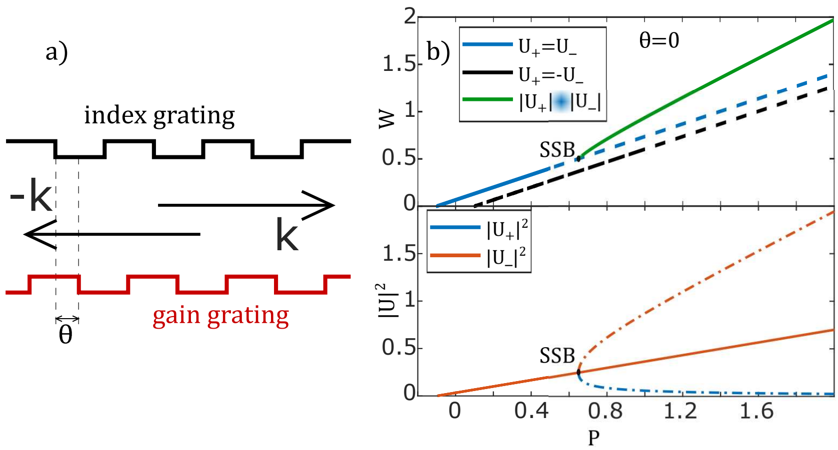

Fig. 1: a) Schematic view of a waveguide with periodical index and gain gratings. b) Bifurcation diagram for spatially uniform states. Upper panel shows dependence of the full field intensity of states on gain $W = |U_b|^2 + |U_d|^2 = |U_+|^2 + |U_-|^2$; dashed lines indicate dynamically unstable states. Lower panels demonstrate intensities of counter-propagating waves; by solid and dash-dotted different branches from corresponding upper panel are shown.

The dynamics of the system schematically shown in Fig. 1(a) is described by two counter-propagating waves approach and can be expressed mathematically by following system of equations:

$$(\partial_t \pm \partial_x)U_\pm = (i\alpha - \gamma)(|U_\pm|^2 + 2|U_\mp|^2)U_\pm + (i\sigma + \Gamma)U_\mp + PU_\pm, \qquad (1)$$

where U_+ and U_- are the slow varying complex amplitudes of the two counter-propagating waves, γ is the nonlinear losses, $\alpha = \pm 1$ is the Kerr-nonlinearity coefficient, σ is the coupling coefficient of the counter-propagating waves caused by index grating, Γ is the gain coupling caused by gain grating and P is gain in the system. It is worth noting here, that σ can be complex $\sigma = |\sigma|e^{i\theta}$, where θ is phase difference between the gratings defined as $\theta = \kappa\Delta x$, κ is the lattice constant and Δx is the shift of the real part of the periodic potential in respect to imaginary one.

In Fig. 1(b) the bifurcation diagram of stationary states is shown. The system under consideration provides three types of homogeneous stationary states: antisymmetric ($U_+ = -U_-$), symmetric

($U_+ = U_-$), and hybrid ($U_+ \neq U_-$) state which appears as a result of spontaneous symmetry breaking.

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Simulation of wave processes in bone phantoms for osteoporosis diagnostics

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Osteoporosis is a progressive systemic skeletal disease associated with a reduction of bone mass and deterioration of bone microarchitectonics, which leads to a decrease in bone quality and an increase in fracture risk. The disease proceeds with few symptoms and is often detected only after a bone fracture. In recent decades, Quantitative Ultrasound (QUS) has become a widespread method for examining human bones to assess their current state and detect developing osteoporosis [1, 2]. Cortical bones are suitable waveguides for ultrasonic guided wave (GW) propagation. The advantage GWs is that they are sensitive to both the mechanical and geometrical properties of the cortical bone. However, understanding the generation and propagation of ultrasonic guided waves in cortical bone structures remains challenging. This requires mathematical and computer models that adequately simulate the wave generation and propagation in the bone structures.

The analytically based computer models [3], previously developed for the ultrasonic inspection of composite materials, have been adapted and applied to the simulation of guided wave excitation and propagation in multilayered phantoms mimicking waveguide properties of tubular bones. The models are based on the explicit GW representation through the inverse Fourier transform path integrals of the waveguide's Green matrix, and the GW extraction using the residue technique. The developed computer model is validated against the finite element simulation (Comsol Multiphysics 5.6). To identify signs predicting osteoporosis, transient GW signals, time-frequency wavelet images, and amplitude-frequency characteristics are analyzed and discussed.

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Exact solution to the light scattering problem for a core-mantle spheroid with non-confocal layer boundaries

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Layered spheroidal scatterers are encountered in a number of applications. The light scattering problem for such particles can be solved by different numerical methods in a range of parameter values. The exact solution to the problem found in [1] in the case of the confocal boundaries of the layers has a much wider applicability range and needs less computational efforts.

So far, a more general case of spheroids with the non-confocal layer boundaries has been rigorously considered in two works [2, 3]. Han, et al. [2] applied the separation of variables method and the relations between the spheroidal and spherical vector functions given in [4]. These authors obtained the solution in the form of the scattered field expansions in terms of the spherical functions. In contrast, Farafonov [3] used the T-matric method and the original relations between the spheroidal and spherical scalar functions later published in [5]. So, he tried to derive the solution in the spheroidal system related to the particle surface. However, he presented only a smaller part of the problem solution and could not provide any numerical results.

In this paper we construct the complete solution to the light scattering problem for core-mantle spheroids with non-confocal layer boundaries by applying the approach of [3]. So, we extend the solution for confocal layer spheroids by including the transitions from the spheroidal system related with one layer boundary to the system related with another layer boundary. Our numerical calculations have demonstrated that such extension does not affect the convergence speed and appliacability range, i.e. both are similar to those in the case of confocal spheroids. The analytics and calculations say that the problem solution with the spherical basis is applicable just in a narrow region of parameter space, and we discuss why the derived solution is found to avoid this limitation.

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2D-topological vectorial solitons in lasers with saturable absorption

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Topological features of polarized optical radiation can be described by a set of three types of isolated or non-isolated singular points: V-points where radiation intensity vanishes, C-points with circular polarization, and L-points with linear polarizations, with corresponding topological indices [1]. Here we present the analysis of localized vectorial structures (solitons) in semiconductor wide-aperture lasers with saturable absorption. We use the mean field approximation with averaging of the field envelopes in the longitudinal direction and the "spin-flip" model [2] with inclusion of additional terms describing the angular selectivity of radiation losses. Our results include a number of new types of stable topological vectorial laser solitons and domains of their stability in the space of the scheme parameters.

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On the spectrum of a quasiperiodic non-self-adjoint operator

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We study the simplest non-trivial (non-self-adjoint) one-dimensional quasiperiodic difference operator. We use the monodromization method, a renormalization approach suggested by Buslaev and Fedotov when trying to extend the Bloch–Floquet theory to difference equations on the real axis with periodic coefficients. We show that there are hidden asymptotic parameters that allow an effective analysis. We describe the geometry of the spectrum of the operator, compute the Lyapunov exponent on the spectrum, and describe the conditions under which either the spectrum is purely continuous or a point spectrum appears additionally.

Based on a joint work with D. I. Borisov (Ufa, Russia).

An exact transparent boundary condition for the 2D Helmholtz equation

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In this presentation an exact transparent boundary condition (TBC) for the two-dimensional Helmholtz equation is derived. Its general properties as well as the transition to a Baskakov–Popov type TBC [1] for the 2D parabolic equation are discussed. An unconditionally stable finite-difference implementation of the obtained boundary condition in the framework of Cauchy evolutionary problem for the 2D Helmholtz equation is then considered and a fully discrete analog of the continuous TBC is derived. A number of numerical experiments are carried out, which illustrate how the proposed TBC can be used in practice for the modeling wave propagation in the free space as well as in the inhomogeneous media.

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The influence of porosity and fluid saturation of soils on guided waves and the soils parameter estimation by ultrasound methods

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A deep understanding of soil processes and sub-surface water flow dynamics is need for agriculture and forestry, flood prediction and natural resources conservation, as well as studies on fundamental stability of buildings and other structures, etc [1]. Investigation and estimation of the water saturation, as a consequence soil porosity and soil hydraulic parameters, are important for forecasting and improving of yields. The goals of this investigation is to develop a methodology of soil moisture level, porosity and other effective parameters including hydraulic parameters of the poroelastic fluid-saturated medium estimations based on the measurements of guided waves excited by dynamic surface loads.

At this stage the investigations are only theoretical. For the theoretical study of elastic wave propagation in fluid-saturated media with pores the Biot–Frenkel equations generalizing the Lame equations of the classical linear theory of elasticity to the case of two-phase media are applied. The elements structure-phenomenological approach that uncovered the ration between effective Biot's parameters of poroelastic medium and microstructure contents of soils are presented and discussed. The time-harmonic and transient solutions are obtained in terms of Fourier transforms of Green's matrix and external load, exciting the wave-field [2]. The influence of the porosity and fluid saturation on the phase velocities of surface waves is discussed in details. The solution of inverse problem for the effective parameters of poroelastic medium based on the minimisation of the discrepancy between the experimental measured and simulated phase velocities is discussed too.

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Local recovery of a piecewise constant anisotropic conductivity in EIT on domains with exposed corners

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We study the local recovery of an unknown piecewise constant anisotropic conductivity in EIT (electric impedance tomography) on certain bounded Lipschitz domains Ω in $\mathbb{R}^2$ with corners. The measurement is conducted on a connected open subset of the boundary $\partial\Omega$ of Ω containing corners and is given as a localized Neumann-to-Dirichlet map. The above unknown conductivity is defined via a decomposition of Ω into polygonal cells. Specifically, we consider a parallelogram-based decomposition and a trapezoid-based decomposition. We assume that the decomposition is known, but the conductivity on each cell is unknown. We prove the local recovery near a known piecewise constant anisotropic conductivity γ_0. We do so by proving the injectivity of the Fréchet derivative $F'(\gamma_0)$ of the forward map F, say, at γ_0. The proof presented, here, involves defining different classes of decompositions for γ_0 and a perturbation or contrast H in a proper way so that we can find in the interior of a cell for γ_0 exposed single or double corners of a cell of $\operatorname{supp} H$ for the former decomposition and latter decomposition, respectively. Then, by adapting the usual proof near such corners, we establish the mentioned injectivity.

This is a joint work with Maarten de Hoop (Rice University), Ching-Lung Lin (National Cheng-Kung University), Gen Nakamura (Hokkaido University), Manmohan Vashith (Indian Institute of Technology).

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Asymptotics of reflected and transmitted elastic waves in an anisotropic two-layer half-space with a surface source

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The need to analyze reflected and transmitted elastic waves in compound solids arises in various applications such as geophysics or ultrasonic non-destructive testing (NDT). Although the laws of incident wave reflection and refraction at the interface are well known, the quantitative evaluation of their amplitude-frequency characteristics, taking into account the wave source, remains a challenging problem. The total wavefield can be calculated using a mesh-based numerical solution of the corresponding boundary value problem, for example, by the finite element method (FEM). However, the FEM is usually too computationally expensive for wave simulation, and the total frequency-domain solution cannot directly provide the required (e.g., reflected) waves. Explicit integral representation via Green's matrix of the structure considered provides a quantitative solution, the same as FEM simulation. Moreover, the asymptotics of those path integrals yields closed physically obvious representations for the source-generated waves of various types. In particular, the asymptotics of body waves generated in an isotropic layered half-space by a surface source and transmitted through the interface into the lower half-space is derived as the contribution of stationary point of the oscillating exponents in the integrand [1].

In the present work, these results are extended to the case of anisotropic elastic media. Specifically, we focus on the application to the ultrasonic NDT inspection of metal alloys with cubic anisotropy [2]. The influence of the mutual crystallographic orientation in a compound sample on the wave trajectories, reflection and transmission coefficients, and the manifestation of reflected wave spots on the daylight surface is of prime concern. The asymptotics is derived from the path integrals obtained using the algorithm of Green's matrix numerical calculation for arbitrarily anisotropic multilayered half-spaces [3]. To do this, the parts corresponding to the reflected and transmitted bulk waves are preliminarily selected in the algorithm. Unlike the isotropic case, there are no explicit phase functions here, which also complicates the application of the stationary phase method, in particular, the search for stationary points. The asymptotics obtained are verified against FEM results. Numerical examples illustrating the computer implementation of this approach are presented and discussed.

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Wave propagation in laminates and layered acoustic metamaterials with periodic arrays of cracks

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In recent years, a novel class of composites, the so-called phononic crystals and acoustic metamaterials (AMMs), providing several unusual properties has been receiving special attention in the scientific community. Various AMMs with extraordinary properties have been proposed and developed for the applications in ultrasonics, acoustoelectronics, hydroacoustics, architectural acoustics, sound absorption and others, where AMMs are used to manipulate elastic wave propagation. Layered AMMs with planar cavities or strip-like cracks are considered in this study. Mathematical and computer models for dynamic behaviour simulation of the AMM structures based on the boundary integral equation method and the spectral element method are developed. The results of numerical analysis are presented in this study. The structures under consideration are composites containing elongated interfaces between two materials with different physical and chemical properties. Therefore, manufacturing of such AMMs, where perfect or at least good adhesive contact between the components keeping the planar cavities open at the same time, is discussed and examples are demonstrated.

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Guided waves scattering by impact induced delaminations and sensing by piezoelectric sensors

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Laminates are now widely employed in various areas since they can improve the reliability and durability of elastic structures due to high fatigue resistance or reduction in weight. Ultrasonic guided waves have been widely applied for the non-destructive evaluation and structural health monitoring (SHM) of the composites because of their considerable sensitivity to possible defects such as cracks, pitting corrosion, voids, etc. The numerical methods presented in this work are developed and applied for dynamic behaviour simulation of a multi-layered elastic waveguide with an impact-induced damages in the form of array of bridged cracks, where piezoelectric actuator and sensor are mounted on the surface. The main advantage of the proposed hybrid approach is its efficiency and accuracy for the elastic wave propagation simulation and the determination of the spectral properties of the considered problems including the computation of the eigenfrequencies/eigenmodes and dispersion relations, for unbounded/bounded multi-layered composites with the inhomogeneities such as damages, cracks, electrodes, etc.

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Diffraction on non-plane gratings irradiated by non-planar waves

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The modified boundary integral equation method (MIM) is considered a rigorous theoretical application for the diffraction of cylindrical waves by arbitrary profiled plane gratings, as well as for the diffraction of plane/non-planar waves by concave/convex gratings. This study investigates two-dimensional (2D) diffraction problems of the filiform source electromagnetic field scattered by a plane lamellar grating and of plane waves scattered by a similar cylindrical-shaped grating. Unlike the problem of plane wave diffraction by a plane grating, the field of a localised source does not satisfy the quasi-periodicity requirement. Fourier transform is used to reduce the solution of the problem of localised source diffraction by the grating in the whole region to the solution of the problem of diffraction inside one Floquet channel. By considering the periodicity of the geometry structure, the problem of Floquet terms for the image can be formulated so that it enables the application of the MIM developed for plane wave diffraction problems.

Accounting of the local structure of an incident field enables both the prediction of the corresponding efficiencies and the specification of the bounds within which the approximation of the incident field with plane waves is correct. For 2D diffraction problems of the high-conductive plane grating irradiated by cylindrical waves and the cylindrical high-conductive grating irradiated by plane waves, decompositions in sets of plane waves/sections are investigated [1]. The application of such decomposition, including the dependence on the number of plane waves/sections and radii of the grating and wave front shape, was demonstrated for lamellar, sinusoidal and saw-tooth grating examples in the 0-th and -1-st orders as well as in the transverse electric and transverse magnetic polarisations. The primary effects of plane wave/section partitions of non-planar wave fronts and curved grating shapes on the exact solutions for 2D and three-dimensional (conical) diffraction problems are discussed.

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Simple neural networks and Bayesian methods for diffractions grating efficiency optimization

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A new optimization technique for diffraction grating engineering is proposed. The suggested technique utilizes simple architectures of neural networks (NN) for obtaining parameters of diffractions grating with improved diffraction efficiency. The NN approach is applied to improve the efficiency of diffraction gratings with the oblique-incident (off-plane) radiation scattering from one-periodical gratings (2D structures). The arbitrary conductivity and various functions of the border profile are considered. The efficiency for the given parameters of the grating is obtained through a numerical solution of the Helmholtz equation using the boundary integral equation method ([1], Ch. 12).

Several numerical experiments were performed to investigate the capabilities of the applied method. The objects of study mainly were diffraction grating for the extreme ultraviolet and soft-x-ray wavelength ranges. The optimization experiments were performed for both online (online computation and training of NN) and offline (training on precomputed efficiency values with interpolated values) regimes. Efficiency optimization results and time consumption are compared to the Bayesian-based optimization algorithm introduced earlier [2]. Though, obtained results showed that the Bayesian approach provided faster convergence to the global extremum, the NN method allowed shrinking the search area approximately two times faster and it is also scalable and more convenient for tasks with a large number of parameters (for tasks with more than three parameters).

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Calculation of the field of the high intensive focused ultrasound beam using the modular nonlinearity model

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One of the important problems of nonlinear acoustics is the calculation of the field of high intensive focused beams in the focal region. For this problem, it is not possible to find an exact analytical solution and one has to resort to approximate or numerical calculations. Numerical calculations make it possible to calculate the field amplitude for specific values and parameters of the radiating system. However, in this case, it is necessary to enumerate a large array of realizations to solve the problem of optimizing the field at the focus and obtain the maximum amplitude. A wide class of approximate methods that make it possible to obtain analytical solutions for high-intensity acoustic fields is associated with the approximation of nonlinear geometric acoustics and ray methods. However, these approaches do not allow one to correctly describe the field in the focal region. Asymptotic methods suitable for describing the field at the focus are mainly developed for linear problems and require further development for application in the nonlinear case. One of the possible ways to obtain analytical and qualitative dependences for the field in the focal region of a high intensive focused beam is associated with the use of the modular nonlinearity model. The idea is that the term containing the classical quadratic nonlinearity is replaced by the term containing the modular type nonlinearity. In this case, the qualitative behavior of the nonlinear term is generally preserved. Two significant differences can be identified. First, in a quadratic-nonlinear medium, a discontinuity in the profile is formed smoothly and at some distance from the emitter. In a medium with modular nonlinearity, a discontinuity forms immediately. Secondly, at small amplitudes in a quadratically nonlinear medium, there is a smooth transition to a linear regime, which is absent in a medium with modular nonlinearity. Both of these differences are not very significant when calculating the field at the focus of a high intensive beam, since the main interest is the structure of the field near the beam axis with a large amplitude. In this case, at the initial stage of beam propagation, approximate solutions of equations with quadratic nonlinearity are suitable, which are then used as boundary conditions for solving model equations with modular nonlinearity. The paper proposes model equations with modular nonlinearity suitable for describing the field in the focal region of a high intensive focused beam. It is shown that the temporal profile of the wave is distorted asymmetrically due to diffraction distortions. The peak negative pressure decreases, and the range of negative pressures itself increases. Peak positive pressure, on the contrary, increases, and the interval decreases. According to the model of modular nonlinearity, the wave propagates at different speeds in the intervals of positive and negative polarity. This leads to the formation of areas of ambiguity, in which a discontinuity is drawn according to the rule of equal areas. As a result, a discontinuous profile is formed. The combined effect of diffraction and nonlinear distortions leads to the formation of short pulses of positive polarity with a large amplitude and rather long intervals with a flat negative polarity profile without strongly pronounced peaks of negative polarity. The dynamics of the profile distortion is described by exact analytical expressions, which allow one to proceed to the solution of the problem of optimizing the radiating system in order to obtain the maximum amplitude at the focus.

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On hyperbolicity of close to piecewise constant linear cocycles over irrational rotations

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We study cocycles generated by a skew-product map

$$F_A : (x, v) \mapsto (\sigma_\omega(x), A(x)v), \quad (x, v) \in \mathbb{T}^1 \times \mathbb{R}^2, \tag{1}$$

over irrational rotation $\sigma_\omega(x) = x + \omega \mod 1$, $\omega \in \mathbb{R} \setminus \mathbb{Q}$. The transformation $A : \mathbb{T}^1 \to SL(2, \mathbb{R})$ is supposed to have a special representation

$$A(x) = R(\varphi(x)) \cdot Z(\lambda(x)), \quad R(\varphi) = \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix}, \quad Z(\lambda) = \begin{pmatrix} \lambda & 0 \\ 0 & \lambda^{-1} \end{pmatrix}.$$

where $\lambda(x) = \lambda_0 \gg 1$ is constant and function $\varphi : \mathbb{T}^1 \to \mathbb{T}^1$ is of the form

$$\begin{aligned}\varphi(x; L_1, L_2, r_1, r_2, \varepsilon, t) = {}& \frac{\pi}{2} + \varepsilon^{-1} r_1 x \chi_{[0, \varepsilon r_1^{-1} L_1)} + L_1 \chi_{[\varepsilon r_1^{-1} L_1, t + \varepsilon r_2^{-1} L_1)} \\ & + \varepsilon^{-1} r_2 (x - t) \chi_{[t + \varepsilon r_2^{-1} L_1, t + \varepsilon r_2^{-1} L_2)} + L_2 \chi_{[t + \varepsilon r_2^{-1} L_2, 1 + \varepsilon r_1^{-1} L_2)} + \varepsilon^{-1} r_1 (x - 1) \chi_{[1 + \varepsilon r_1^{-1} L_2, 1)},\end{aligned}$$

where χ_B is the characteristic function of a set B and $L_k, r_k, k = 1, 2, t, \varepsilon$ are real parameters such that

$$L_1 \cdot L_2 < 0, \quad r_1 \cdot r_2 < 0, \quad r_1 \cdot L_1 > 0, \quad t \in [0, 1), \quad 0 < \varepsilon \ll 1.$$

Thus, φ is a continuous function, which is close to a piecewise constant function.

Such cocycles appear in many problems of mathematical physics, e.g., in the theory of almost periodic operators, Hamiltonian systems, diffraction theory.

Using the critical set method ([1, 2]), we obtain conditions on the rotation number ω and parameter t, which guarantee the uniform (resp., non-uniform) hyperbolicity of the cocycle. An application to the Schrödinger cocycle is discussed.

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Modeling of electrodynamic and thermodynamic processes by means of a micropolar continuum

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We develop a linear theory of the Cosserat continuum of a special type. This continuum possesses only rotational degrees of freedom. The constitutive equation for the moment stress tensor is the same as for the elastic continuum. The main feature of our model is that the differential equation relating the angular strain tensor to the angular velocity vector contains a source term [1]. Thanks to a special choice of the constitutive equation for the source term, we obtain a model of continuum that has some properties of a viscoelastic continuum. Considering such a continuum, we associate the main variables characterizing its stress–strain state with quantities characterizing electrodynamic and thermodynamic processes. We identify parameters of our model by comparing the obtained equations with Maxwell's equations and the hyperbolic heat conduction equation. As a result, we arrive at a generalization of Maxwell's equations for conductors. These equations can be reduced to the three-dimensional telegrapher's equation for the electric field vector. This telegrapher's equations account for not only the skin effect described in many literature sources on electrodynamics, but also the so-called static skin effect observed in a number of experiments [2, 3]. In addition, the proposed model describes the conversion of electrical energy into thermal energy due to Joule heat and allows us to obtain the entropy balance equation. In contrast to classical electrodynamics, which contains *two* mutually orthogonal vectors: the electric field vector and the magnetic induction vector, the proposed theory contains *three* mutually orthogonal vectors: the electric field vector, the magnetic induction vector and the temperature gradient. It agrees with experimental facts discovered by Ettingshausen and Nernst (the Ettingshausen effect and the Nernst–Ettingshausen effect).

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Resonance scattering of an extraordinary wave by a smoothed-walled duct with decreased density in a magnetoplasma

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Much previous work on the interaction of electromagnetic waves with density irregularities in a magnetoplasma applies to magnetic-field-aligned plasma channels, commonly known as density ducts [1]. In particular, scattering of the electromagnetic radiation by such plasma structures has attracted considerable interest [2–4]. The resonance scattering of an extraordinary wave by a cylindrical sharp-walled duct with decreased plasma density has recently been studied in [4]. In the present work, it is our purpose to analyze the features of scattering of an extraordinary plane wave by a smooth-walled duct with decreased density in the case where such a wave is incident normally on the duct from the surrounding magnetoplasma. The emphasis is placed on the behavior of scattering and absorption characteristics of the duct at the resonant frequencies of such a plasma structure, which correspond to its plasmon resonances of the surface and volume types [4].

It is found that at the frequencies of the surface plasmon resonances, the maxima of both the field amplitude coefficients and the scattering cross section per unit length of the duct fairly rapidly decrease with smoothing the duct wall. Such behavior is accompanied by a simultaneous increase in the absorption cross section per duct unit length in the frequency interval in which the upper hybrid resonance condition is fulfilled inside the nonuniform wall of the duct. As for the volume plasmon resonances, which are known to exist in the presence of a sharp-walled duct with decreased plasma density [4], they continue to be observed for a smooth-walled duct as well. It turns out that smoothing of the duct wall merely leads to an increase in the frequencies of such resonances and to certain changes in the heights of peaks of the scattering and absorption cross sections at the corresponding frequencies. Analytical and numerical results will be reported for the scattering and absorption characteristics of decreased-density ducts in the upper hybrid frequency range of a magnetoplasma as functions of the density nonuniformity in the duct wall. The results obtained can be helpful in understanding the basic properties of resonance scattering of electromagnetic waves from field-aligned density irregularities in the ionospheric and laboratory plasmas.

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Perturbation of the simple dissipative wave

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We consider the equation, which determines the dynamics of the domain bounds in a weak ferromagnet [1]

$$\frac{\partial^2\phi}{\partial t^2} - c^2\frac{\partial^2\phi}{\partial x^2} + \Omega^2 \sin\phi\cos\phi + \omega^2\ \sin\phi + \alpha\frac{\partial\phi}{\partial t} = 0. \tag{1}$$

The simple (traveling) wave solutions $\phi = \Phi_0(\kappa x - \nu\, t),\ \kappa,\nu = \text{const}$ are determined by the ordinary differential equation

$$\left[\nu^2 - c^2\kappa^2\right]\frac{d^2\Phi}{ds^2} + \Omega^2\sin\phi\cos\Phi + \omega^2\ \sin\Phi - \alpha\,\nu\frac{d\Phi}{ds} = 0. \tag{2}$$

The simple wave under boundary condition

$$\Phi(s)\to 0\ \text{ as } s\to-\infty,\quad \Phi(s)\to\pi\ \text{ as } s\to+\infty. \tag{3}$$

corresponds to the domain bound. Such solution $\Phi_0(\kappa x - \nu\, t)$ exists, if the coefficients of the equation are constant.

We study the equation with slow varying coefficients $c^2, \Omega^2, \omega^2, \alpha$, which depend on the slow time $\tau = \varepsilon t$, [2]. Here $0 < \varepsilon \ll 1$ is a small parameter. For the problem (1), (3) with the initial dates, which correspond to unperturbed simple wave, we construct an asymptotic solution

$$\phi(x,t;\varepsilon) = \Phi(s;\xi,\tau)[1+\mathcal{O}(\varepsilon)],\quad \text{as } \varepsilon\to 0,\ \text{for } t\in[0,\mathcal{O}(\varepsilon^{-1})].$$

The leading order term depends on the fast variable $s = \varepsilon^{-1}S(\xi,\tau)$ and on the slow variables $\xi = \varepsilon\, x,\ \tau = \varepsilon\, t$. The function $\Phi(s;\xi,\tau)$ is a solution of the equations (2), (3) under $\kappa = S_\xi,\ \nu = S_\tau$. The phase function $S(\xi,\tau)$ is formed by two solutions of Hamilton–Jacobi equations

$$\big((S_\tau^\pm)^2 - c^2(S_\xi^\pm)^2\big)\lambda_\pm^2 \mp \alpha S_\tau^\pm\lambda_\pm - \delta^\pm = 0,\ \delta^\pm = \pm\omega^2 - \Omega^2. \tag{4}$$

Here the constants $\lambda_\pm > 0$ are taken from the unperturbed solution in asymptotics at infinity

$$\Phi_0(s) = \exp(\lambda_\pm s)[c_\pm + \mathcal{O}\big(\exp(\lambda_\pm s)\big)],\ s\to\pm\infty,\ c_\pm = \text{const}\neq 0.$$

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Functional model of C^*-algebra associated with a metric graph in the case of simple models

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The eikonal algebra $\mathfrak{E}(\Omega)$ is a C*-algebra associated with the metric graph Ω. This algebra was introduced in [1] and later studied in [2–4]. This paper is part of the application of an algebraic version of the Boundary Control method to metric graphs in order to solve the inverse problem in such a case.

Eikonals (E_γ^T) are bounded self-adjoint operators determined by a dynamical system which describes the propagation of waves from the boundary into the graph with finite velocity. It has been shown that for arbitrary graph $\mathfrak{E}(\Omega)$ has a structure in the following form:

$$\mathfrak{E} \cong \bigoplus_{l=1}^{\mathcal{L}} \dot{C}\big([0,\varepsilon_l];\mathbb{M}^{\kappa_l}\big). \tag{1}$$

The structure of spectrum $\widehat{\mathfrak{E}(\Omega)}$ of the algebra $\mathfrak{E}(\Omega)$ (the set of classes of irreducible representations) defines a functional model for $\mathfrak{E}(\Omega)$ through a relation:

$$\mathfrak{E}(\Omega) \ni E_\gamma^T \to \mathcal{E}_\gamma^T:\ \mathcal{E}_\gamma^T(\pi) := \pi(\mathcal{E}_\gamma^T), \quad \pi \in \hat{\pi} \in \widehat{\mathfrak{E}(\Omega)}. \tag{2}$$

This relation is used to introduce coordinates on the spectrum. They connect the structure of the spectrum to the geometry of the graph. The relation between these two objects is studied for different examples of graphs of simple structure (star, cycle, nonplanar graph).

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New integral representations of the Maslov canonical operator with complex phases

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The canonical Maslov operator with complex phases (the complex germ theory, see e.g. [1]) allows us to construct asymptotic solutions of a wide class of linear partial differential and pseudodifferential equations with a small parameter in the form of oscillating functions localized in the vicinity of surfaces of various dimensions (for example, asymptotics in the form of Gaussian wave packets or Gaussian wave beams). The main geometric object in such problems is a vector bundle over the isotropic manifold in the phase space and with planes in the complexified phase space (a complex germ) as fibers. Asymptotics are represented in an effective complex WKB form in the neighborhood of (regular) points that is diffeomorphically projectable from the isotropic manifold into the configuration space, and in the form of oscillating integrals with a complex phase function in the neighborhood of singular points. Similar to those recently proposed in [2] for the real canonical operator new representations of the canonical operator with complex phases are constructed. New representations allow us to avoid the transition to not very effective in practical applications the momentum-position coordinate system, which is usually necessary to do when using the canonical operator in the standard form. The applied result is to obtain simpler expressions for practical calculations. In some cases an effective representation of asymptotic solutions in the form of special functions is possible.

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On stability of determination of Riemann surface from its DN-map

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Suppose that M is a Riemann surface with boundary ∂M, Λ is its DN-map, and $\mathcal{E}: M \to \mathbb{C}^n$ is a holomorphic immersion. Let M' be a surface diffeomorphic to M and $\partial M = \partial M'$. We provide a canonical way to extend $\mathcal{E}$ to $\mathcal{E}': M' \to \mathbb{C}^n$ and show that the closeness of Λ' to Λ (in the relevant norm) implies the closeness of $\mathcal{E}'(M')$ to $\mathcal{E}(M)$ by the Hausdorff distance in $\mathbb{C}^n$.

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Cerebral oxygen transport model with unknown surface sources

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A two-compartment (blood and tissue) model of oxygen transport is considered. It is assumed that the both compartments occupy the same spatial region $\Omega \subset \mathbb{R}^3$ and have different volume fractions for the blood and tissue compartments, σ and $1-\sigma$, respectively. Following [1], the oxygen transport can be described by the following coupled equations:

$$-\alpha\Delta\varphi + \mathbf{v}\cdot\nabla\varphi = G, \quad -\beta\Delta\theta = -\kappa G - \mu, \quad x \in \Omega. \tag{1}$$

Here, φ and θ are the blood and tissue oxygen concentrations, respectively; μ describes the tissue oxygen consumption; $G = c(\theta - \psi)$ is the intensity of oxygen exchange between the blood and tissue fractions, where ψ is the plasma oxygen concentration; $\kappa = \sigma(1-\sigma)^{-1}$, where σ is the volumetric fraction of vessels; $\mathbf{v}$ is a prescribed continuous velocity field in the entire domain G; α and β are diffusivity parameters of the corresponding phases. There are nonlinear monotonic dependencies of the tissue oxygen metabolic rate μ on the tissue oxygen concentration θ and of the plasma oxygen concentration ψ on the blood oxygen concentration φ.

Equations (1) are supplemented by the following boundary conditions imposed on $\Gamma = \partial\Omega$:

$$\alpha\partial_n\varphi + \gamma(\varphi - \varphi_b)|_\Gamma = 0, \quad \beta\partial_n\theta + \delta(\theta - \psi_b)|_\Gamma = 0. \tag{2}$$

Here, ∂_n denotes the outward normal derivative at points of the domain boundary. Nonnegative functions φ_b, ψ_b, γ, and δ are given.

Suppose that there are a finite number of disjoint subdomains $\Omega_j \subset \Omega$, $j = 1, ..., m$, which are some neighborhoods of the ends of arterioles and venules. The effect of the arterioles and venules on oxygen concentrations can be described by surface functions defined at the boundaries of subdomains $\Gamma_j = \partial\Omega_j$, $j = 1, ..., m$:

$$\alpha\left[\,\partial_n\varphi\,\right] = q_j, \quad x \in \Gamma_j, \quad j = 1, ..., m. \tag{3}$$

Here, $[\,\partial_n\varphi\,]$ denotes the jump of the normal derivative of φ and parameters q_j are constants.

Let $q = (q_1, ..., q_m)$ be unknown. Nevertheless, an additional information with respect to the average blood oxygen concentration at the boundaries of subdomains Ω_j, $j = 1, ..., m$, can be obtained by measurements:

$$\int_{\Gamma_j} \varphi \, d\Gamma = r_j, \quad j = 1, ..., m. \tag{4}$$

As a result, we come to the following inverse problem.

Inverse Problem. *Find a state* $y = (\varphi, \theta)$ *and vector* $q = (q_1, ..., q_m)$ *satisfying* (1)–(3) *such that the overdetermination conditions* (4) *take place.*

The unique solvability of the inverse problem is proven, an algorithm to find solutions is constructed and implemented. The results of numerical experiments are discussed.

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M-functions and metric graphs: hierarchy and inverse problems

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Schrödinger operators on metric graphs, also known as quantum graphs, are determined by the underlying metric graph, the electric and magnetic potentials and the vertex conditions. If the underlying graph is a tree, then the M-function associated with the degree one vertices determines the operator under certain mild assumptions on the vertex conditions. This talk is devoted to the inverse problem for graphs with cycles.

To this end we shall analyse the *hierarchy* of M-functions appeared when graphs are glued together as well as the dependence of M-functions on the *magnetic fluxes* through the cycles. Two approaches leading to unique solution of the inverse problem will be presented:

- *dismantling graphs*: the original graph has sufficiently many contact points that dismantles it into a set of trees;
- *Magnetic Boundary Control*: dependence of the spectral data on the magnetic fluxes through the cycles is used to dissolve vertices and thus reconstruct so-called *infiltration domains.*

Optimal solution of the inverse problem is obtained by combining these two methods.

Sloshing in a vertical-walled cylinder in the presence of a porous layer

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Sloshing in an open vertical-walled cylinder of constant depth and arbitrary cross-section is considered. An inviscid, incompressible, heavy fluid with a free surface occupies the cylinder above a bottom-adjacent porous layer of constant thickness. The model under consideration developed in [1] is based on the nonlinear drag in a porous medium linearized via the Lorentz principle of virtual work. The problem describing free oscillations of the fluid involves two velocity potentials; they satisfy a pair of interface conditions, whereas the Steklov condition containing a spectral parameter is imposed on free surface.

Our approach is similar to that applied in [2], where sloshing in a fluid, consisting of two layers, whose densities are different, was investigated. By separation of variables two sequences (they express eigenvalues and eigensolutions) are obtained. Their elements involve the eigenvalues of the Neumann Laplacian in the two-dimensional domain— the container's horizontal cross-section.

The dependence of eigenvalues on the problem's parameters (namely, porosity, the linear friction factor and the inertial term) is analysed. Also, the obtained eigenvalues are compared with those that describe sloshing in the same container without the porous material. In particular, these results demonstrate that there is a significant distinction between the properties of the spectrum obtained here and studied in [2], despite the obvious similarity of these two problems.

The question of maximizing the time-damping of the free oscillations is studied; it can be achieved by choosing physical parameters of the porous layer. Expressions for angular frequencies ω_n, corresponding to the eigenvalues are obtained. In order to ensure rapid decay in time of the nth eigenmode, the expression for the optimal value of the linear friction factor is found; it guarantees that the maximum of $\operatorname{Im}\omega_n$ is attained.

Inverse sloshing problem is formulated; it consists in finding the characteristics of the porous layer and its thickness from the eigenvalues measured by observations of the free surface. It is demonstrated that for determining two characteristics of the porous layer, one has to measure the lowest sloshing eigenfrequency. Knowledge of two lowest eigenfrequencies allows us to find the depth of the interface as well.

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Analytical finding of eigenfrequencies of an elastic rod with impedance attachment ends

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Reference problems with different boundary conditions for longitudinal oscillations of an elastic homogeneous rod are solved. The eigenfrequencies of the considered mechanical systems are found from the solutions of the Sturm–Liouville problems with the third kind boundary conditions as roots of the transcendental equations. The homogeneous boundary conditions contain parameters whose values are calculated through the parameters of the mechanical system. The number of problem parameters determines the parameterization of the Sturm–Liouville problem. We obtained approximate analytical dependences of the eigenfrequencies on the problem parameter for the considered one-parameter problems. We proposed the method of sequentially using the obtained solutions of one-parametric problems for the solution of multiparametric Sturm–Liouville problems. One-parameter problems can be considered as reference ones in this approach. Eigenfrequencies for the case of the two-parameter Sturm–Liouville problem are found by the proposed method as an example.

Schrödinger operator in a half-plane with singular δ-potential having the support on two half-lines: spectrum and eigenfunctions

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A Schrödinger operator in a half-plane with the Neumann boundary condition is considered. The singular potential of the operator is the Dirac δ-function having its support on two half-lines with the same origin located on the boundary of the half-plane. In the first part of this work we study negative eigenvalues and eigenfunctions of the corresponding self-adjoint operator. We propose an integral representation of the Kontorovich–Lebedev type for the solutions and reduce the problem of description of the spectrum and of the eigenfunctions to a system of functional-difference equations with a characteristic (spectral) parameter. The system is then studied by means of reduction to an integral equation with a selfadjoint integral operator that is interpreted as a perturbation of the so-called Mehler integral operator. We then consider sufficient conditions of existence of the discrete component in the spectrum of the latter perturbation. The corresponding results are applied to description of the eigenfunctions of the Schrödinger operator in hand represented by the the Kontorovich–Lebedev integrals.

Contrary to the first part (spectrum and eigenfunctions) based on the Kontorovich–Lebedev integrals, the second part (asymptotics) of the work deals with an alternative Sommerfeld-type representaion for the eigenfunctions and with the description of their asymptotics at large distances.

On transformation operators and Riesz basis property of root vectors system for $n \times n$ Dirac type operators

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In this talk we investigate spectral properties of selfadjoint and non-selfadjoint boundary value problems (BVP) for the following first order system of ordinary differential equations (ODE)

$$Ly = -iB(x)^{-1}\big(y' + Q(x)y\big) = \lambda y, \quad B(x) = B(x)^*, \quad y = \mathrm{col}(y_1, \dots, y_n), \quad x \in [0, \ell],$$

on a finite interval $[0, \ell]$. Here $Q \in L^1([0, \ell]; \mathbb{C}^{n\times n})$ is a potential matrix and $B \in L^\infty([0, \ell]; \mathbb{R}^{n\times n})$ is an invertible self-adjoint diagonal "weight" matrix. If $n = 2m$ and $B(x) = \mathrm{diag}(-I_m, I_m)$ this equation is equivalent to Dirac equation of order n.

Our first main result is the existence of triangular transformation operators for such equation under certain separation conditions on the entries of $B(x)$. The case of constant $B(x) = B$ is investigated in [1]. Here we discuss applications of this result to the spectral properties of BVP associated with the above equation subject to general BC $U(y) = Cy(0) + Dy(\ell) = 0$, $\mathrm{rank}(CD) = n$.

As a first application of this result, we show that the deviation of the characteristic determinants $\Delta(\lambda) - \Delta_0(\lambda)$ of perturbed and unperturbed (with $Q = 0$) BVPs is a Fourier transform of a certain summable function explicitly expressed via kernels of the transformation operators. In turn, this representation leads to the asymptotic formula $\lambda_m = \lambda_m^0 + o(1)$ as $m \to \infty$, for the eigenvalues $\{\lambda_m\}_{m\in\mathbb{Z}}$ and $\{\lambda_m^0\}_{m\in\mathbb{Z}}$ of perturbed and unperturbed ($Q = 0$) *regular* BVPs, respectively. In the case of $n = 2$ and constant matrix $B(x) = B$ both results are obtained in [2].

Further, we prove that the system of root vectors of the above BVP constitutes a Riesz basis in a certain weighted L^2-space, provided that the boundary conditions are *strictly regular*. Along the way, we also establish completeness, uniform minimality and asymptotic behavior of root vectors. The case of constant matrix $B(x) = B$ was investigated in [2, 3].

The main results are applied to establish asymptotic behavior of eigenvalues and eigenvectors, and the Riesz basis property for the dynamic generator of spatially non-homogenous damped Timoshenko beam model. We also found a new case when eigenvalues have an explicit asymptotic, which to the best of our knowledge is new even in the case of constant parameters of the model.

The talk is based on authors' results published in preprint [4].

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Diabolical points and Rayleigh-wave propagation

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It is well-known that in quantum mechanics many dynamical processes are described by (avoided) level crossings. In accordance with Keck et al. [1], they typically appear in the form of a matrix Hamiltonian, where the matrix elements depend on parameters. These avoided level crossings occur when a single parameter is varied. However true crossings require the variation of more parameters, two in the case of 2×2 matrices, which constitutes the celebrated "diabolical crossing" of the energy levels. Eigenvalue problems with "peculiar properties" were studied in a famous paper by von Neumann and Wigner [2]. In physics, the double cone, also called "diabolo", can be traced back to the work of Hamilton [3] who expounded on an interesting physical effect associated with coincident eigenvalues referred to as *conical refraction*. In modern times, Berry and Wilkinson [4], among others, refer to the double-cone as *diabolo* and the degeneracies themselves as *diabolical points*.

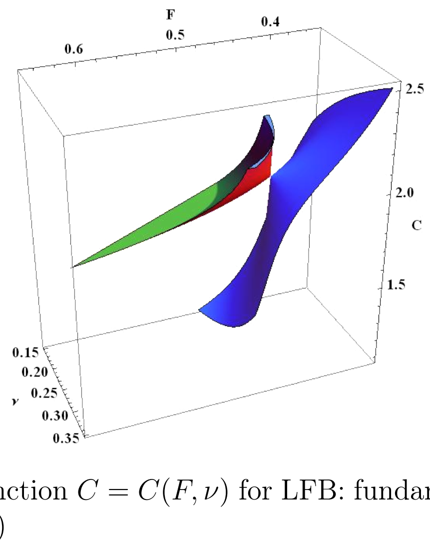

Fig. 1: 3D-picture of the function $C = C(F, \nu)$ for LFB: fundamental mode (blue, right) and higher mode (red, green, left)

A necessary condition for the occurrence of a diabolical point is the degeneration of eigenvalues in that point. It is demonstrated for the first time that such diabolical points may occur at Rayleigh waves propagating in an elastic homogeneous stratum with stress-free surface and fixed bottom (LFB = Layer with Fixed Bottom). Kausel et al. [5] show that this special waveguide LFB has infinitely many degenerated eigenvalues (double roots) for certain rational Poisson ratios such as $\nu = 1/99$, 1/10, 1/4, 13/45, 3/10 ... These ratios are out of the interval $0 < \nu < 1/3$ and follow from special formulas. No double roots exist in the stratum if Poisson's ratio is either an irrational number, or is a rational number that does not follow from the formulas mentioned above. If ν has not the exact value from this list, the phenomenon of avoided crossing occurs which is also called repulsion. A subset of these special Poisson ratios was found in the dissertation of Tran Thanh Tuan [6] in another way. Kausel et al. [5] posit the strong conjecture, that a non-symmetric layered half-space will never exhibit double roots anywhere. In Fig. 1, the dimensionless phase velocity C of the fundamental and first higher mode of Rayleigh waves in LFB is given in dependence on the dimensionless frequency F and Poisson's ratio ν (the necessary 2 parameters!). The formation of the double-cone with the diabolical point is clearly seen for $\nu = 0.25$. These facts are important for the interpretation of dispersion curves of surface waves in seismology and Lamb waves in non-destructive testing.

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Computational techniques for time-fractional modelling of thermal wave propagation in ferroelectrics

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In recent decades, ferroelectrics have become promising materials for numerous engineering applications due to a wide range of significant properties. The constitutive pyroelectric effect in ferroelectrics gives rise to the application of these materials for creating thermal sensors to detect infra-red and microwave radiations. The pyroelectricity is due to changes in internal polarization under temperature exposure. The key characteristic of pyroeffect in ferroelectrics is presented by pyroelectric current, which can be registered as a response to the influence of a heat flux modulated by pulses at a defined frequency.

Ferroelectrics possess self-similar domain structures and exhibit time memory effects during polarization switching. In order to perform modelling of heat conduction in materials characterized by time-memory effect and complex structure, the mathematical apparatus of fractional calculus can be used. Therefore, the current study is devoted to the development of computational techniques for time-fractional modelling of thermal processes induced by modulated heat fluxes in ferroelectrics.

Here we propose a time-fractional diffusion-wave model as a generalization of the classical heat conductivity model to describe space-time temperature distribution in typical ferroelectrics. We derive computational algorithms with a focus on the application of numerical methods designed for time-fractional partial differential equations.

The model is governed by an initial-boundary value problem for a time-fractional diffusion-wave equation for a space-time domain:

$$\rho c(T)\frac{\partial^\alpha T}{\partial t^\alpha} = t^* k_T \Delta T,\ \ 0 < \alpha < 2,\ \ t > t_0,\ \ 0 < x < L, \tag{1}$$

$$T|_{t=t_0} = T_0,\ \ 0 \le x \le L, \tag{2}$$

$$k_T \frac{\partial T}{\partial x}\Big|_{x=0} = -\frac{Q}{2}\left(\operatorname{sign}\left(\sin(\omega t)\right) + 1\right),\ \ t > t_0, \tag{3}$$

$$T|_{x=L} = T_0,\ \ t > t_0, \tag{4}$$

where $T(x,t)$ is the temperature distribution in a sample in K; k_T is the heat conductivity coefficient in $\mathrm{W/(m \cdot K)}$; $c(T)$ is the specific heat capacity in $\mathrm{J/(kg \cdot K)}$; ρ is the density in $\mathrm{kg/m^3}$; L is the sample thickness in m; t_0 is the initial moment of time in s; t^* is the temporal scale parameter in s; where T_0 is the ambient temperature in K; Q is the thermal surface power in $\mathrm{W/m^2}$; $\omega = 2\pi f$ is the radial frequency of the applied field in rad/s and f is the frequency of field oscillations in Hz.

Note that the equation (1) is referred to as the sub-diffusion process if α is from the interval $(0;1)$; hyperdiffusion process when $1 < \alpha < 2$, and the classical wave process if $\alpha = 2$.

To solve the problem (1)–(4) numerically, an implicit computational scheme was derived using finite-difference approximations of the time-fractional Caputo derivative. Constructed numerical schemes were implemented in Matlab. The designed computer program was used to perform simulations of the thermal wave propagation in typical ferroelectrics. This approach significantly expands the possibilities for numerical simulations due to the variation of regimes by means of changes in the fractal dynamical dimension α.

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Analog of de Branges spaces for the Schrödinger operator in a bounded domain

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We introduce spaces which can be considered as a generalization of Hilbert spaces of analytic functions associated with one-dimensional canonical systems introduced by de Branges [1]. Using the approach from [2], we can link with the Schrödinger operator with the Neumann boundary conditions in the bounded domain the family of Hilbert spaces parametrized by $T > 0$. The properties of these spaces are related with the controllability properties (on the interval $(0, T)$) of a dynamical system given by an initial-boundary value problem for the wave equation related to Schrödinger operator.

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Asymptotics of the 1D shallow water equations in the form of running waves in a basin with variable bottom with vertical and gentle walls

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The Cauchy problem for the one-dimensional shallow water equations with variable bottom $D(x)$ and localized initial data is considered [1]. The domain under consideration is confined by a vertical wall on the right, where the Neumann conditions are set, and a movable border on the left. An asymptotics of the Carrier–Greenspan transform is used to get equations with fixed boundaries and small nonlinear terms, which allows constructing (formal) asymptotics to the initial problem [2]. Wave profile changes and its relation to the Maslov index [3] are of interest.

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Classical unique continuation property for multi-term time-fractional evolution equations

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In this talk we consider the classical unique continuation property (UCP) of solutions in $(0, T) \times \Omega$ with a domain $\Omega \subset \mathbb{R}^n$, $n \in \mathbb{N}$ for a multi-terms time fractional evolution equation, which has been awaited for a long time. Here the second order strongly elliptic operator for this evolution equation can depend on time and the orders of its time-fractional derivatives are in $(0, 2)$. We will report that the classical UCP holds for solutions $u \in H^{\alpha,2}((0, T))$ of this evolution equation, where $\alpha \in (0, 2)$ is the largest order of time fractional derivative of the equation. The proof of this result is based on using the usual Holmgren transformation, a Holmgren type transformation and Treve's argument.

Interaction of distant spectral perturbations of the Neumann conditions in application to ice fishing

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One of the problem under provided asymptotic analysis is the Steklov–Neumann problem

$$-\Delta_x u^\varepsilon(x) = 0,\ x \in \Omega, \quad \partial_z u^\varepsilon(x) = \lambda^\varepsilon u^\varepsilon(x),\ x \in \omega^\varepsilon = \omega_1^\varepsilon \cup \cdots \cup \omega_J^\varepsilon,$$

$$\partial_n u^\varepsilon(x) = 0,\ x \in \partial\Omega \setminus \big(\Upsilon \cup \overline{\omega^\varepsilon}\big).$$

Here, $\Omega \subset \mathbb{R}^3$ is a domain bounded by the planar surface $\Gamma \subset \mathbb{R}^3_0 = \{x = (y, z) \in \mathbb{R}^3 : z = 0\}$ and a smooth curved surface Σ in the lower half-space $\mathbb{R}^3_- = \{(y, z) : z < 0\}$ which meet each other at a smooth contour $\Upsilon \subset \mathbb{R}^3_0$. Furthermore, $\omega^\varepsilon_j = \{y \in \mathbb{R}^2 : \eta^j = \varepsilon^{-1}(y - y^j) \in \omega_j\}$, ω_j is a domain in the plane and $P^1 = (y^1, 0), \dots, P^J = (y^J, 0)$ are pairwise different points in Γ. Finally, $\varepsilon > 0$ is a small parameter and ∂_n is the outward normal derivative, $\partial_n = \partial_z$ on Γ while u^ε is the velocity potential and λ^ε the spectral parameter. This problem describes water-waves in a lake covered with ice where several small ice holes are made for winter fishing. The eigenvalue sequence of the problem

$$\lambda^\varepsilon_0 < \lambda^\varepsilon_1 \le \lambda^\varepsilon_2 \le \dots \le \lambda^\varepsilon_m \le \dots \to +\infty$$

starts with $\lambda^\varepsilon_0 = 0$ and other positive eigenvalues get the following asymptotic form as $\varepsilon \to +0$:

$$\lambda^\varepsilon_m = \varepsilon^{-1}\mu_m + O(1), \quad m \in \{1, 2, 3, \dots\}.$$

Entries of the positive unbounded monotone sequence

$$0 < \mu_1 \le \mu_2 \le \mu_3 \le \dots \le \mu_m \le \dots \to +\infty$$

are eigenvalues of the combined family ($j = 1, \dots, J$) of the spectral problems in the half-spaces

$$-\Delta_{\xi^j} w^j(\xi^j) = 0,\ \xi^j \in \mathbb{R}^3_-, \quad \frac{\partial w^j}{\partial \zeta^j}(\eta^j, 0) = 0,\ \eta^j \in \mathbb{R}^2 \setminus \overline{\omega_j},$$

$$\frac{\partial w^j}{\partial \zeta^j}(\eta^j, 0) = \mu\left(w^j(\eta^j, 0) - \left(\sum_{p=1}^J |\omega_p|\right)^{-1} \sum_{k=1}^J \int_{\omega_k} w^k(\eta^k, 0) d\eta^k\right),\ \eta^j \in \omega_j,$$

where $\zeta^j = \varepsilon^{-1} z$ and $|\omega_p|$ stands for area of ω_p. The integral terms that perturb the Steklov boundary conditions on the subdomains $\omega_1, \dots, \omega_J$ in the limit problem, demonstrate the interaction of the Steklov conditions on the small sets $\omega^\varepsilon_1, \dots, \omega^\varepsilon_J$ in the original problem.

Similar effects of interaction of small singular spectral perturbations are found in various elasticity problems and the bi-harmonic equation describing deflection of the Kirchhoff plate suspended by small springs.

Some of the presented results are obtained in cooperation with Valeria Chado Piat.

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Array scattering resonance in the context of Foldy's approximation

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This talk considers the special case of resonance in the problem of diffraction by a wedge consisting of two semi-infinite periodic arrays of point scatterers; see [1–4]. In a recent paper [5], the solution was obtained in terms of two coupled systems, each of which is solved using the discrete Wiener–Hopf technique. An effective and accurate iterative numerical procedure was then developed to solve the diffraction problem, which allows us to compute the interaction of thousands of scatterers forming the wedge. Following a brief overview on resonance for infinite and semi-infinite arrays, the talk will turn to resonance in the point scatterer wedge. Specifically, the aim is to understand the coupling extent in the conditions for resonance and in the resonance waves themselves.

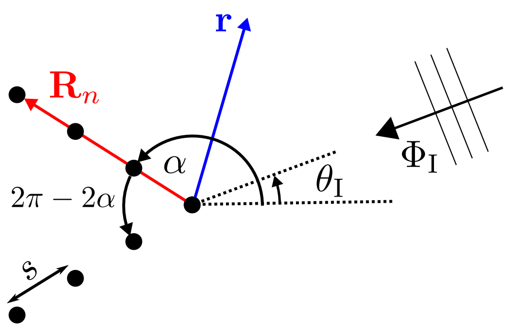

Fig. 1: Diagram of the point scatterer wedge with scatterers located at $\mathbf{R}_n$, the position vector $\mathbf{r}$ and the incident wave Φ_{I}.

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Reconstruction from the Fourier transform on the ball

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We give formulas for finding a compactly supported function v on $\mathbb{R}^d$, $d \geq 1$, from its Fourier transform Fv given within the ball B_r. For the one-dimensional case, these formulas are based on the theory of prolate spheroidal wave functions. In multidimensions, well-known results of the Radon transform theory reduce the problem to the one-dimensional case. We also present a numerical implementation of these results. In particular, the results obtained give super-resolution reconstruction, that is, they allow recovering details beyond the diffraction limit, that is, details of size less than π/r, where r is the radius of the ball mentioned above.

This talk is based on the joint works with M. Isaev and G. Sabinin [1, 2].

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Plasmon resonances of spherical semiconductor-metal core-shell nanostructure

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As is known, the interaction of nanoparticles with external optical radiation can lead to the excitation of collective electronic oscillations (plasmons) in them. The effects of a resonant increase in the near field and in the absorption and scattering cross sections upon excitation of plasmons underlie a large number of promising practical applications. In particular, core-shell nanostructures allow the existence of plasmon resonances in the transparency region of biological tissues (located in the near-IR region of the spectrum), which makes them suitable for a number of biomedical applications (see, for example [1, 2]). In such nanostructures, the shift of the resonant peak down in frequency (from the visible in the IR region of the spectrum), as is known, can be carried out by placing a core made of a material with a large value of the permittivity inside a metal shell. In this case, at the interface between the materials between the shells, a boundary transition layer appears with an inhomogeneous distribution of the electron density [3]. The presence in this layer of a transition through the plasma resonance region, as is known, leads to the appearance of an additional energy loss mechanism associated with the excitation of a longitudinal (plasma) wave in this region and its damping in regions with a low electron density. However, in fact, to date, this loss mechanism has not been studied in relation to semiconductor-metal core-shell nanostructures, which is the purpose of this work.

In this paper, based on the hydrodynamic approach [4], equations are formulated that determine, in the quasi-static approximation, the field in a semiconductor core, a metal shell, and the boundary layer between them. The field in the boundary layer was calculated both on the basis of a numerical solution of the obtained equations and analytically in the case when the thickness of the layer is much smaller than its radius, but significantly exceeds the characteristic scale of polarizability nonlocality (Thomas–Fermi radius). The absorption and scattering cross sections of the core-shell nanostructure interacting with an external field were calculated, and the frequencies of its plasmon modes and the widths of resonance maxima determined by this loss mechanism were obtained. It is shown that resonant absorption in the boundary layer can lead to a noticeable decrease in the quality factor of resonances compared to the case when it is determined only by bulk dielectric losses in the nanostructure material (when the presence of the boundary layer is not taken into account).

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Asymptotic analysis of tunneling through a potential barrier in graphene in the presence of a magnetic field

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The possibilities of controlling electron transport in graphene by using external electromagnetic field are studied in several physical papers for model situations, where fields are constant and the effect of magnetic field is estimated by using classical considerations, see for example, [1]. We assume the smooth and slow dependence of the fields on one spatial coordinate x and study solutions of the Dirac equations asymptotically in the semiclassical approximation. In the absence of magnetic field, the problem is the problem of the reflection of electrons on a smooth electrostatic barrier, this problem was studied asymptotically in [2, 3]. The phenomenon of the Klein tunneling was confirmed in this case: there is no reflection for normal incidence on a potential barrier exceeding the energy of an electron $\mathcal{E}$, and the electrons completely transmit through the barrier. However, in the case of abnormal incidence, reflection takes place, see detailed study in [2, 3].

Here we take into account the external magnetic field $\vec{B}$ and study its influence on the Klein tunneling. We assume the following gauge for the vector potential $\vec{A} = (0, A_y(x), 0)$, $B_z = A'_y \equiv dA_y/dx$. We assume that the electrostatic potential is monotone. If the magnetic field is absent, the single turning point $\kappa_\mathcal{E}$ exists, i.e., $U(\kappa_\mathcal{E}) = \mathcal{E}$, where $U(x)$ denotes an electrostatic potential. We show that the nonzero magnetic field yields the splitting of the turning point $\kappa_\mathcal{E}$ into two simple turning points with the classically forbidden zone between them. We construct the adiabatic solutions outside the turning points in the hole and electron regions, find the field near two turning points, and determine the reflection and transmittance coefficients with account of their phases applying the technique developed in [4].

The magnetic field B is assumed to satisfy the condition $v_f^{-1}|U'(\kappa_\mathcal{E})| \geq eA'_y(\kappa_\mathcal{E})$, where v_f is the Fermi velocity, e is an electron charge, $U'(x) = dU(x)/dx$. Let consider the wave function of electron depending on y through the factor $exp(ip_y y/\hbar)$, where $\hbar$ is the reduced Planck constant. We find that the parameter governing the reflection process reads

$$\nu = -i\frac{(p_y + eA_y(\kappa_\mathcal{E}))^2 (v_f^{-1}U'(\kappa_\mathcal{E}))^2}{2\hbar\left((v_f^{-2}U'(\kappa_\mathcal{E}))^2 - e^2(A'_y(\kappa_\mathcal{E}))^2\right)^{3/2}}. \tag{1}$$

The study was supported by the Russian Foundation for Basic Research (RFBR) under grant № 20-02-00490.

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The role of Airy beam parameters in optical manipulation

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The Airy beams are widely used in optical manipulation because of their diffraction-free and self-healing propagation along a curved trajectory [1]. The scattering force and angular momentum acting on micro- and nanoparticles depend on the orbital momentum, spin momentum and the angular momentum density of the electromagnetic field, respectively. In optical manipulation with counter beams the transverse components of the orbital momentum, spin momentum and the angular momentum density are the most significant. They appear due to diffraction [2].

In this paper, the dependence of the orbital momentum, spin momentum and the angular momentum density on such parameters of the Airy beam as its polarization, initial launch angles, and exponential truncation factor are found in the paraxial approximation.

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Excitation of whispering gallery waves in sea area with bowl-like bottom by an external source

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In recent paper [1] a possibility of acoustic energy localization near curvilinear isobath in a shallow sea area with bowl-like bottom was discussed. It was shown that under certain conditions acoustic field in the horizontal plane exhibits mode structure, and the mode functions are localized near isobaths family. These modes are similar in their properties to whispering gallery modes formed near the wall of a cylindrical building. This effect can be conveniently described using modal representation of acoustic field [2] and the concept of horizontal rays associated with given vertical mode [1].

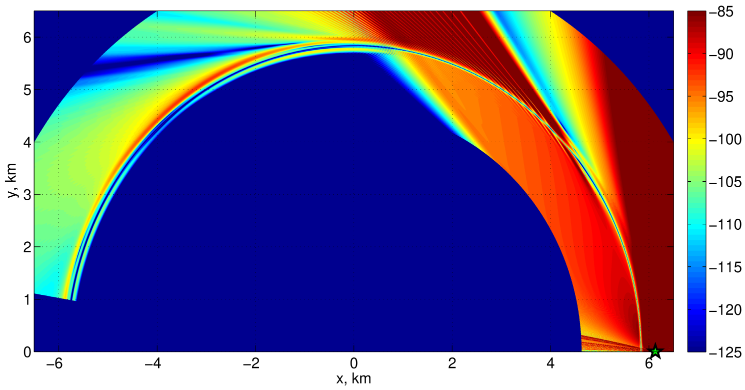

Fig. 1: Contour plot of the magnitude of acoustic field as a function of horizontal coordinates x, y (in dB re 1 m) at the depth $z = 10$ m due to a point source in a shallow sea. Water depth decreases between the isobaths $r = 5.8$ km and $r = 6$ km from 26 to 24 m. Source position is marked by a star.

In this study we investigate the possibility of excitation of such modes by a point source located away from the isobaths family where the modes are localized (see figure). This phenomenon is similar to quantum tunneling and cannot be described within the framework of horizontal rays theory. A qualitative and quantitative description of such external excitation is given in terms of WKBJ theory, and the modeling of this effect is performed numerically using pseudo-differential mode parabolic equations theory.

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A numerical method for estimating anthropogenic acoustic noise levels using wide-angle Mode parabolic equations

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Sound exposure level (SEL) is a characteristic used to estimate human impact arising from seismic survey in a given sea area. The characteristic is defined for a frequency range $[f_1, f_2]$ as follows [1]:

$$\mathrm{SEL}\left(f_1, f_2, x, y, z\right) = 10\log_{10}\left(\frac{\Delta t\int_{f_1}^{f_2}\hat{P}\left(\omega, x, y, z\right)^2 d\omega}{p_0^2\Delta t_0}\right), \tag{1}$$

where $\hat{P}\left(\omega, x, y, z\right)$ is an acoustic field in frequency domain, $p_0 = 1$ µPa, $\Delta t_0 = 1$ s, Δt is a step size over time. Therefore the main challenge is to compute acoustic field in 3D domain stretching for tens of kilometers in both horizontal directions. Mode parabolic equations provide a good compromise between accuracy and efficiency [2]. Acoustic field is represented in the form of a modal expansion

$$\hat{P}\left(\omega, x, y, z\right) = \hat{S}\left(\omega\right)\sum_{j=1}^{J} A_j\left(x, y\right)\varphi_j\left(z, x, y\right), \tag{2}$$

$$\frac{\partial A_j}{\partial x} = ik_{j,0}\left(\sqrt{1+L_j} - 1\right)A_j\,, \quad k_{j,0}^2 L_j = \frac{\partial^2}{\partial y^2} + k_j^2 - k_{j,0}^2\,, \tag{3}$$

where $\hat{S}\left(\omega\right)$ is the source spectrum, $\varphi_j\left(z, x, y\right)$ are normal modes with corresponding eigenvalues $k_j = k_j(x, y)$, $k_{j,0}$ is reference wavenumber. A_j are obtained using split-step Padé method resulting in a wide-angle MPE solution. SEL computation is implemented as a part of AMPLE project that aims to provide a general and efficient way to compute acoustic fields for various acoustical problems.

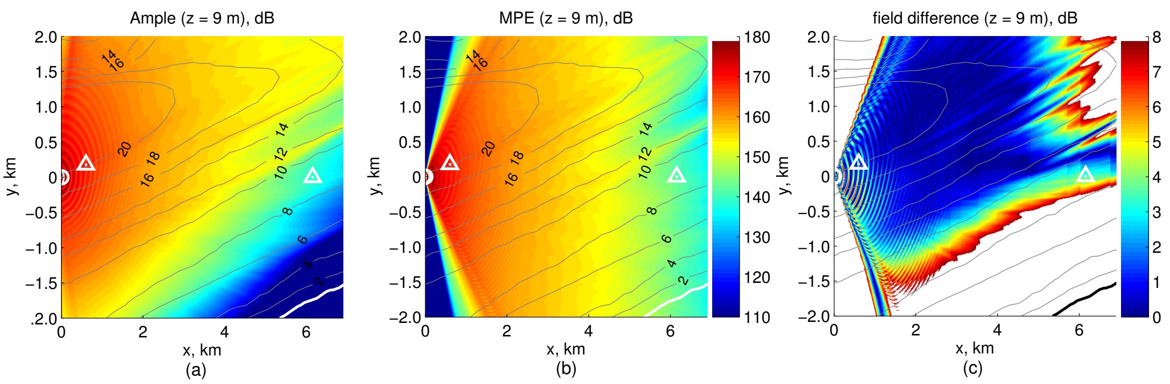

Fig. 1: SEL distribution computed using wide-angle (a) and narrow-angle (b) mode parabolic equations.

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Transparent boundary conditions for subsurface sounding problems

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The transfer, or "dragging" of the radiation condition from infinity is used in applied mathematics in order to reduce the infinite computational domain when solving diffraction and wave propagation problems [1, 2]. The use of transparent boundary conditions in calculation of unidirectional wave propagation by parabolic equation method has gained considerable popularity [3–5]. At the same time, in some problems, such as remote sensing or subsurface EM sounding (ground penetrating radar, GPR), the solution of the full wave equations in an "infinite" open region is required, to take into account the most important backscatter signal. In this paper, on the example of 2D Helmholtz equation, an explicit form of transparency conditions at the boundaries of the selected strip is derived and used to model practical problems of GPR probing.

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Radiation patterns of borehole GPR antennas

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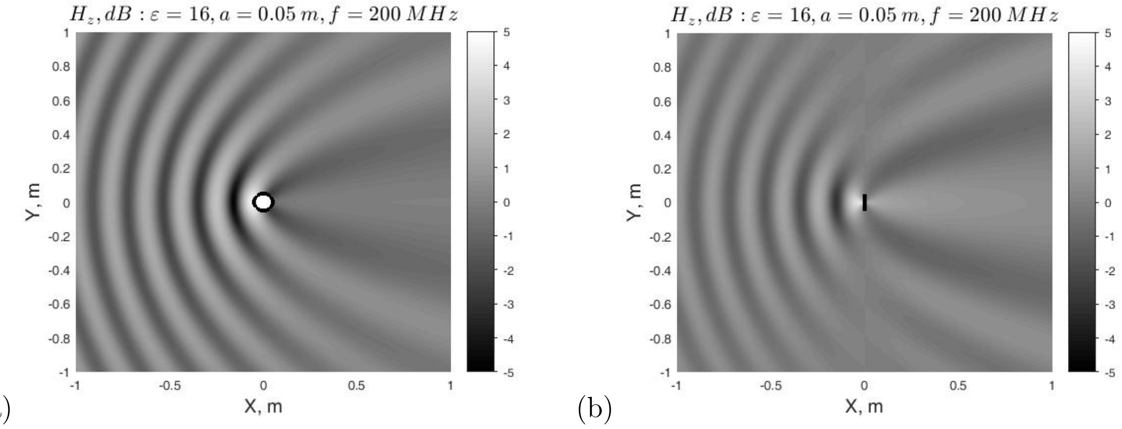
Borehole ground penetrating radar is a branch of widespread GPR technique using EM waves for investigating hidden underground structures. One of the important problems arising in borehole GPR logging is the design of antennas with azimuthal directivity [1]. In our investigation work, we consider several types of directional GPR transmitter antennas, namely: cylindrical slot antenna, planar slot antenna and shielded dipole with grounded arms. All antennas were fed by a pulsed current formed by the discharge of a capacitor with large capacity. For a rough estimate of the borehole GPR directivity, we analytically compare model problems of plane wave diffraction by two scattering objects: conductive cylinder and plane strip immersed in a uniform dielectric medium. We estimate directivity of the antenna by comparison of the wave field amplitude at the front and rear sides of the antenna. In the first case, angular dependence of the scattered wave is achieved due to creeping wave attenuation in the shadow region, see Fig. 1a, in the latter case — by constructive interference of two edge waves, see Fig. 1b. Numerical comparison makes it possible to estimate the directional efficiency of the selected antennas. In this work, along with these simple analytical models, we have done computer simulation using FDTD numerical technique, and a series of measurements of these antennas radiation pattern on a simple experimental stand. Our modeling and experiments confirm feasibility of the proposed solution.

This work has been supported by the Russian Science Foundation grant № 22-12-00083.



(a) (b)

Fig. 1: Scattered field amplitude distribution for two diffracting objects: a) cylinder, b) plane strip.

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Formally determined inverse problems for hyperbolic PDEs

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We describe two stability results for formally determined inverse problems for hyperbolic PDEs. Our first result is Lipschitz stability for the fixed angle scattering problem for the operator $\Box + q(x)$ where the data consists of the medium response to incoming plane waves from two opposite directions. This is joint work with Mikko Salo. Our second result is Lipschitz stability for the fixed angle scattering problem for the operator $\Box + q(x, t)$ where the data consists of the medium response to plane waves coming from the same fixed direction but with different time delays. This is joint work with Venky Krishnan and Soumen Senapati. For both these problems, there are also results for the operator $(\partial_t - a)^2 - (\nabla - b)^2 + c$, by Salo et al for the time independent case and by Krishnan, Rakesh and Senapati for the time independent case, and also with point sources instead of plane wave sources.

Estimates of total bandwidth for magnetic Laplacians on periodic graphs

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We consider Laplace operators with periodic magnetic potentials on periodic discrete graphs. The spectrum of the magnetic Laplacian consists of an absolutely continuous part (a union of a finite number of non-degenerate bands) and a finite number of eigenvalues of infinite multiplicity. We obtain estimates of the total bandwidth for the magnetic Laplacians in terms of geometric parameters of the graph and magnetic fluxes. The proof is based on the decomposition of the magnetic Laplacian into a constant fiber direct integral and trace formulas for fiber operators. The fiber operator depends on the so-called quasimomentum and acts on the finite quotient graph of the periodic one. The traces of the fiber operator are expressed as finite Fourier series of the quasimomentum with coefficients depending on magnetic fluxes through cycles of the quotient graph from some specific cycle sets. This is a joint work with Korotyaev E. L. from St. Petersburg State University.

Convolution maximizers in L_p: recent results and open questions

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The motivation of the main result presented here, which pertains to functional analysis, came from a concrete computational question: to determine the best constant in Hardy's inequality for the norm of the Laplace transform operator $f(x) \mapsto Lf(s) = \int_0^\infty f(x)e^{-xs}\,dx$ in Lebesgue spaces.

For any $p \in [1, +\infty)$, the operator L acts boundedly from $L_p(\mathbb{R}_+)$ to $L_{p'}(\mathbb{R}_+)$, where $p'^{-1} = 1 - p^{-1}$ (it is not possible to replace p' with any other exponent). Let N_p be its operator norm. The classical Hardy inequality says: $N_p \le (2\pi/p')^{1/p'}$. The proof goes by reduction to Young's convolution norm inequality. Let $x = e^y$, $s = e^u$, $F(y) = e^{y/p}f(e^y)$, $G(u) = e^{u/p'}(Lf)(e^u)$; then $G = h_p * F$, where the convolution kernel is $h_p(u) = \exp(u/p' - e^u)$. Then Young's inequality $\|G\|_{p'} \le \|h_p\|_{(2-2/p)^{-1}}\|F\|_p$ and calculations yield Hardy's estimate.

For the Fourier transform, which can be thought of as a Laplace transform evaluated along the imaginary s-axis, the exact $L_p \to L_{p'}$ norm is known due to K. I. Babenko and W. Beckner. This is not the case for the Laplace transform except when $p = 2$. For $p = 2$, Hardy's constant is sharp, which can be demonstrated by the explicit spectral analysis of the operator L^*L in L_2. Its spectrum has no atoms; in particular, there is no function $f \in L_2$ (a maximizer) such that $\|Lf\|_2 = N_2\|f\|_2$. The operator $F \mapsto F * h_2$ acting on L_2 with $h_2(u) = \exp(u/2 - e^u) \in L_1$ does not have a maximizer.

The situation is opposite for convolution operators $f \mapsto h * f$ with $h \in L_q$ acting from L_p to L_r, where $p^{-1} + q^{-1} = 1 + r^{-1}$ and neither of p, q, r equals 1 or ∞. The boundedness of any such operator follows from Young's inequality. In [1] we proved that the maximizer of convolution in this situation always exists. As a consequence, constructing approximations converging to a maximizer becomes a feasible approach for a numerical calculation of the convolution norm. (The norms N_p for the Laplace transform have been thus computed.)

The maximizer existence theorem is not completely new. It was proved by M. Pearson in 1999 in the case of a symmetric convolution kernel (and with weak additional assumptions), but his proof does not generalize. Our method is similar to the Concentration Compactness principle of P. L. Lions.

Some related questions, resolved and open, are to be discussed. To give examples, define $\mathcal{K}_p$ to be the class of kernels $0 \ne h \in L_1$ for which the operator $f \mapsto h * f$, $L_p \to L_p$, possesses a maximizer. In [2] we were able to characterize $\mathcal{K}_1$. (By a standard exercise, any $h \ge 0$ belongs to $\mathcal{K}_1$, but anything beyond this is nontrivial.) An argument based on the Fourier-tranform and Plancherel's theorem, shows that $\mathcal{K}_2$ is nonempty. Whether $\mathcal{K}_p$ is nonempty for some $p \in (1,2) \cup (2,\infty)$ is an open question.

Major advances bordering the present topic have been reported by M. Christ and his school [3]. Employing modern tools of additive combinatorics, they study the behaviour of near-extremizers of Beckner's (= sharp Young) inequality $\|h * f\|_r \le c_{p,q} \|h\|_q \, \|f\|_p$; here both h and f are free to vary.

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An equivariant local index formula for the metaplectic group

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Consider the algebra A of bounded operators on the Hilbert space $L^2(\mathbb{R}^n)$ generated by quantizations of isometric affine canonical transformations. We define a spectral triple (A, H, D) for this algebra, where $H = L^2(\mathbb{R}^n, \Lambda(\mathbb{R}^n))$ is a module over A, while D is the Euler operator (a first-order operator of index one).

We show that this spectral triple has a simple dimension spectrum: namely, for each operator B in the algebra generated by pseudodifferential operators of Shubin type and elements in A, the zeta-function $\zeta_B(z) = \mathrm{Tr}(B|D|^{-2z})$ admits a meromorphic continuation to $\mathbb{C}$ with at most simple poles. Our main result is an explicit algebraic expression for the Connes–Moscovici cocycle of this spectral triple. As a corollary, we obtain local index formulas for noncommutative tori and toric orbifolds.

The results are a joint work with E. Schrohe (Hannover).

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Asymptotic solution of the Cauchy problem for the wave propagation with time dispersion

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The dispersion phenomenon for the wave propagation is well known. In many such problems, the so-called time dispersion effects appear (for example, Maxwell equations), which means that the dependence on frequency can be non-polynomial.

We consider the following Cauchy problem

$$g^2\big(\hat{\omega}\big)u = \langle \hat{p},\, c^2(x)\hat{p}\rangle u, \quad x \in \mathbb{R}^2,$$
$$u|_{t=0} = V\Big(\frac{x}{h}\Big), \quad \frac{\partial}{\partial t}u\Big|_{t=0} = 0.$$

Here $h \ll 1$, $\hat{\omega} = -ih\frac{\partial}{\partial t}$, $\hat{p} = -ih\nabla$. Function $c(x)$ is smooth and bounded: $c_M \geq c(x) \geq c_m > 0$. Initial function $V(y)$ is smooth and fast decaying while $|y| \to +\infty$ with all derivations.

As for the function $g(\omega)$, we assume that it is smooth, odd and monotonically increasing not faster than some polynomial, and $g(\omega) \to \infty$ while $\omega \to \infty$. With small ω, we have

$$g(\omega) = \omega + \frac{g_3}{6}\omega^3 + O(\omega^5),$$

where $g_3 > 0$. Also we suggest that $g''(\omega)$ is not zero for $\omega \neq 0$. Also we assume $g'(\omega) > 1$ with $\omega > 0$.

These conditions on the function $g(\omega)$ allow us to define the pseudo-differential operator $\tilde{g}(\omega^2) \equiv g^2(\hat{\omega})$ as function on operator $\hat{\omega}^2$. Then we can inverse this pseudo-differential operator and obtain the second-order Cauchy problem.

The task is the construction the asymptotic formulae for the solution of posted problem. The case is that in the approach described above it is necessary to evaluate the inverse function $g^{-1}(p)$. In order to obtain the constructive asymptotic formula for the solution we have to avoid evaluation of the inverse function. It can be done it we consider the extended phase space including time and frequency with the space variables and corresponding momenta.

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On convergence of the ray generating function for a multilayered waveguide field

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A planar multilayered acoustical waveguide is considered (Fig. 1). Each layer is homogeneous and characterized by its value of sound velocity c_i and density ρ_i, $i = 1, \ldots, N$. The waveguide is excited by a point source (point "s" in the figure). The receiver is shown in the figure by "r".

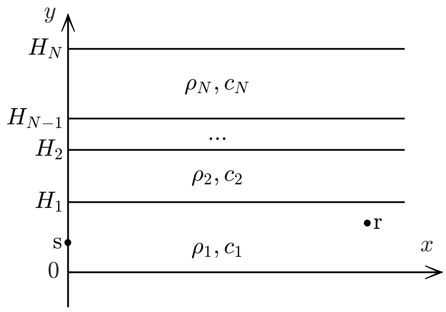

Fig. 1: Multilayered waveguide.

It is well known (for example, [1, 2]) that the acoustical potential $\varphi(x, y, t)$ in such a system can be represented through the reverberaion matrix $\mathbf{R}$ as follows:

$$\varphi(x, y, t) = \int_{-\infty+i\varepsilon}^{\infty+i\varepsilon} \int_{-\infty}^{\infty} \left[\mathrm{Y}_r (\mathbf{I} - \mathbf{R})^{-1} \mathrm{Y}_s + p \right] e^{-i\omega t + ikx} dk\, d\omega.$$

Here $\mathbf{I}$ is the identity matrix, Y_r is a row vector describing the receiver position, Y_s is a column vector describing the source position, p is an additional term, which is non-zero only if the source and the receiver are placed in the same waveguide layer, ε is small positive value. Reverberation matrix is determined by the reflection and refraction coefficients. In case $(\mathbf{I}-\mathbf{R})^{-1}$ can be represented as a Neumann series:

$$(\mathbf{I}-\mathbf{R})^{-1}=\sum_{m=0}^{\infty}\mathbf{R}^m,$$

the wavefield is represented as a sum of integrals. This sum is a generating function for rays in the waveguide, i.e. a term number m describes all rays in the waveguide, which reflect or refract m times.

In the talk we study the convergence of the series. We show that there are some real values k and ω such that the Neumann series cannot be used. For these values of k and ω, we propose a reduced reverberation matrix: a layer causing the series divergence is changed into a thin layer, where rays cannot be distinguished.

The work is supported by the RFBR grant 19-29-06048.

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Experimental study of ultrasound diffraction by a thin cone using maximum length sequence method

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A problem of diffraction of ultrasound acoustic waves by a hard thin cone is studied, the wavelength is small compared to the size of the cone. A diffraction experiment is conducted to measure the total field on the surface and nearby the surface of the cone. The cone is made of duraluminium, the length of the cone is 30 cm, the aperture angle of the cone is approximately 26 degrees. The continuous 40 kHz sine wave modulated by the maximum length sequence of order 9 and discretization frequency of 1000 Hz is emitted by piezoelectric source. The field is measured by 1/8-inch microphone. Only the case of axial incidence is studied. An impulse response of the system is calculated and a frequency response is picked out. Then the frequency response is compared to analytical solution based on the parabolic approximation [1]. Theory and experiment are in good agreement with each other (see Fig. 1).

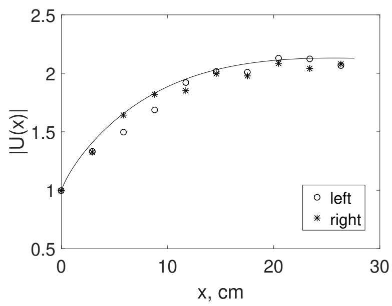

Fig. 1: Comparison between theoretical value of total field on the cone surface calculated from [1] and field measured in the diffraction experiment.

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Phase retrieval problem with background information

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We consider the phase retrieval problem, that is the problem of reconstruction of a function v from phaseless Fourier transform. We use a priori background information. In particular, in dimension $d \geq 1$, we show that the phaseless Fourier transform $|F(v+w)|^2$ and background function w uniquely determine unknown function v, under the condition that $\operatorname{supp} v$ and $\operatorname{supp} w$ are sufficiently disjoint. If this condition is relaxed, then we give similar formulas for finding v from $|Fv|^2$, $|F(v+w)|^2$. We also illustrate these results by numerical examples in the framework of phaseless inverse scattering in the Born appriximation.

This talk is based on joint work with R. G. Novikov and T. Hohage; see, in particular, [1].

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Analysis of the oscillatory process inside an acoustic interference antenna

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The study analyzes the sound field inside an acoustic interference antenna when a sound wave is exposed to it. An interference antenna is a narrow tube of constant cross-section with perforations (holes) on the side surface along its entire length (Fig. 1). Such antennas are used in narrow directional interference-type microphones. One end of the antenna is attached to the microphone capsule (directional or non-directional), and the other end remains open. The side openings are tightened with a cloth (usually silk) or closed with nets. Acutely directional interference microphones (the so-called microphone cannon) are used for sound recording mainly in the field of television, and are also rarely used for acoustic measurements in the field of architectural acoustics.

a)
b)

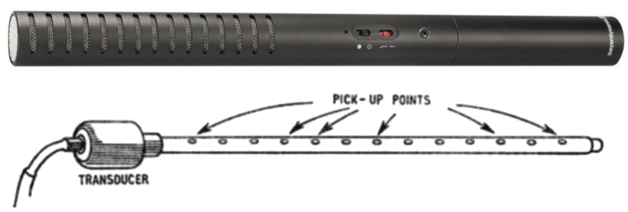

Fig. 1: View of the modern interference microphone Beyerdynamic MCE85 (a) and schematic representation of the microphone (b).

The mathematical model of such microphones is known, but not sufficiently developed. In particular, it does not take into account the wave effects inside the interference antenna and the characteristics of the material that closes the holes.

The analysis of the sound field was carried out by two methods – the method of electroacoustic analogies (Lumped Modeling) and the method of reverberation matrix.

In the method of electroacoustic analogies, the antenna was represented as an equivalent electrical circuit of RLC-elements and Z-blocks, which replaced the acoustic elements of the antenna (air in the tube, holes, grids). The electric current in the branch of the circuit is equivalent to the volumetric oscillatory velocity, and the voltage is equivalent to the sound pressure.

In the reverberation matrix method, the sound field is described by a matrix equation that allows determining the sum of all elementary straight lines and the sum of all elementary inverse waves of any given diffraction order in each segment of the interference antenna.

The calculation results in the formulated mathematical models coincide well with the experimental results (Fig. 2). In the experiment, a mock-up of an interference microphone with side holes not covered with fabric or mesh was studied. It can be seen that open holes do not provide the microphone with narrow directional properties, which cannot be determined from the classical model of an interference microphone. The reason for the absence of acute directivity is the dispersion inside the interference antenna.

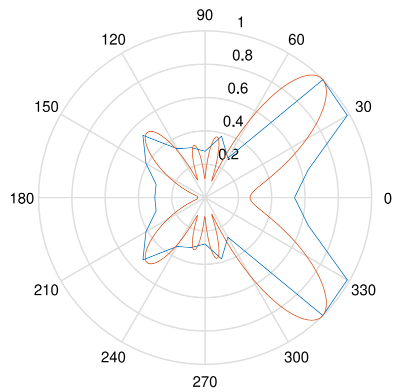

Fig. 2: Directivity characteristic at a frequency of 5 kHz. Comparison of calculation results (red) and experiment results (blue).

Deep learning and successive approximations for the BCM-based image enhancement

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Let $\Omega \subset \mathbb{R}^d (d = 2, 3)$ be a bounded and simply connected domain with the smooth boundary $\partial\Omega$, and $T > 0$ be the final time.

Consider a dynamical system

$$\rho(x)u_{tt} - \Delta u = 0 \text{ in } \Omega \times (0, T], \tag{1}$$
$$u(x, 0) = u_t(x, 0) = 0 \text{ in } \overline{\Omega} \times \{0\}, \tag{2}$$
$$\nabla u \cdot \nu = f \text{ on } \partial\Omega \times [0, T], \tag{3}$$

where ν is the unit normal vector on $\partial\Omega$, and the variable coefficient $\rho(x)$ is supposed to be sufficiently smooth and such that $0 < \underline{\rho} \le \rho(x) \le \overline{\rho}$, $\underline{\rho} = const$, $\overline{\rho} = const$. All functions are supposed to be real valued. If ρ represents the mass density then the quantity $c = \rho^{-1/2}$ is the sound speed, which is supposed to satisfy the non-trapping condition.

A subject of our consideration is the following problem: *Given the set* $\left\{\left(f, u^f_{|\partial\Omega\times(0,2T)}\right) : f \in L^2(\partial\Omega \times (0, 2T))\right\}$, *determine numerically an approximate mass density in* Ω.

To solve numerically this problem, we regularize the Pestov's version of the Boundary Control Method (BCM), which is based on the approximate H^1-controllability. Enhancement of a BCM-based image is motivated by the fact that it is often not appropriately sharp for a number of applications due to undersampling artefacts and oversmoothing penalties in the Tikhonov's functional. To overcome these constraints, two approaches are proposed and implemented numerically. These are deep learning and successive approximations.

Deep learning is a state-of-the-art technique for image processing. One of convolutional neural networks, a slightly modified U-net, has been implemented on the Lambda TensorBook with the 32 GB RAM, GPU NVIDIA RTX 2080 Max-Q with 8 GB memory and 6-core Intel i7 running under Ubuntu 18.04 to enhance the spatial and contrast resolutions of a regularized BCM-based image. The software Keras-Python running on TensorFlow platform is used to implement the U-net.

The other approach is based on the Klibanov's implementation (for a scalar time-domain wave equation) of the Lavrentiev's discovery of transforming a nonlinear inverse problem to a linear integral equation of the first kind. This reduction is utilized to construct the successive approximations and to update iteratively the regularized BCM-based image together with the wave field inside a region of interest. It should, however, be emphasized that instead of solving the aforementioned integral equation, an explicit form of the scattering potential obtained after applying the Laplace operator to this equation is exploited by the successive approximations.

It is demonstrated in the numerical experiments with deep learning that despite the encouraging results, there is the pronounced sensitivity of enhanced images to selecting and configuring a neural network, particularly the U-net, as well as to selecting a regularization technique used to provide stability. Also, it is numerically shown that using the proposed successive approximations, it is possible to obtain the high resolution BCM-based images.

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Solution of the two-dimensional massless Dirac equation with a linear potential and localized right hand side

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We consider the two-dimensional massless Dirac equation with a linear potential and localized right hand side:

$$x_1\sigma_0\psi + \sigma_1(-ih\psi_{x_1}) + \sigma_2(-ih\psi_{x_2}) = \psi^0\left(\frac{x-x^0}{h}\right),$$

where $x^0 = (-a, 0)$, $a > 0$, $\psi^0(x)$ is smooth, fast-decaying function, $h \ll 1$, σ_j are Pauli matrices. The solution must satisfy the absorption limit principle. The talk will be devoted to the construction of an asymptotic solution as $h \to 0$. Using the method of [1], we can construct an asymptotic solution outside a neighborhood of a singular line $x_2 = 0$. Earlier in the paper [2], the asymptotics of the fundamental solution for singular ray $x_2 = 0, x_1 > 0$ was obtained.

This is joint work with S. Yu. Dobrokhotov and I. A. Bogaevsky. Supported by Russian Science Foundation under grant № 16-11-10282.

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Mathematical aspects of monochromatic electromagnetic wave propagation in a plane waveguide filled with inhomogeneous nonlinear medium

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The report focuses on the problem of propagation of a monochromatic wave $(\mathbf{E},\mathbf{H})e^{-i\omega t}$ in the waveguide $\Sigma := \{(x,y,z) : 0 \leqslant x \leqslant h, (y,z) \in \mathbb{R}^2\}$, where

$$\mathbf{E} = (\mathrm{E}_x(x), 0, \mathrm{E}_z(x))\, e^{i\gamma z}, \quad \mathbf{H} = (0, \mathrm{H}_y(x), 0)\, e^{i\gamma z} \tag{1}$$

quantity ω is the circular frequency, γ is an unknown (real) parameter (propagation constant of the waveguide). There are perfectly conducted walls at $x = 0$ and $x = h$.

Inside Σ, the permittivity is described by the formula $\epsilon = \epsilon_l + a|\mathbf{E}|^2$, where $\epsilon_l \equiv \epsilon_l(x) \in C^1[0,h]$ is monotonically increasing positive function and $a > 0$ is a real constant. Everywhere $\mu = \mu_0$, where μ_0 is the magnetic permeability of free space [1].

The fields $\mathbf{E},\mathbf{H}$, introduced in (1), satisfy Maxwell's equations

$$\mathrm{rot}\,\mathbf{H} = -i\omega\varepsilon\mathbf{E}, \quad \mathrm{rot}\,\mathbf{E} = i\omega\mu\mathbf{H} \tag{2}$$

It is well known that tangential components of the electric field $\mathbf{E}$ vanish on the perfectly conductive walls; thus, solution E_z to system (2) must satisfy to the conditions $\mathrm{E}_z|_{x=0} = \mathrm{E}_z|_{x=h} = 0$. We also impose an additional local condition on E_x at the point $x = 0$, see [2, 3] for details.

For the considered problem, a rigorous analytical approach is suggested for the first time. It is proved that even for small values of the nonlinearity coefficient a, the problem has infinitely many nonperturbative solutions (propagation constants and eigenmodes), whereas the corresponding linear problem always has a finite number of solutions. Similar results for shielded plane waveguide filled with homogeneous isotropic/anisotropic cubic nonlinearity are represented in [2, 3]. We also present numerical simulations and draw comparison between similar problems.

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Influence of the imperfect interlaminar contact on the edge waves in laminate structures with thin soft interlayers

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In this work, elastic waves guided by an edge of a plate (edge waves) are investigated in the case of a plate, composed of two isotropic and homogeneous elastic layers and a thin film between them. The investigation is based on the three layer model with spring-type boundary conditions employed for imperfect contact simulation. In the case when thickness of the internal soft layer is sufficiently small compared to the wavelength, the film can be replaced by certain effective boundary conditions (EBCs) coupling two external laminae and tuned to address interlayer mechanical properties and the contact quality. In [1], higher order EBCs are derived via asymptotic expansion technique and used for analysis of the peculiar properties of elastic guided waves in considered laminates. With the use of these boundary conditions, the family of edge wave is investigated for the case of a perfect contact between the layers.

The goals of the present work is to determine the limits of applicability of EBCs in describing of edge waves, and to investigate the influence of the weakened adhesion or damaged interfaces on dispersion and attenuation of these waves. The semi-analytical method based on modal expansion [2] is employed for numerical investigation.

The most interesting property of EWs in a laminate under consideration is that the high order edge waves $\mathrm{EA}_{0.5}$, $\mathrm{ES}_{0.5}$, ES_1 are close to fundamental waves in their main features: their SCs are in general lay close together, and their cut-off frequencies and attenuation is small. These modes are most likely to be observed in the experiments, and can provide additional information for experimental evaluation of laminate material constants and interlaminar bond stiffness. As to the limits of applicability ob EBCs, it is revealed that in the case of a soft interlayer they are restricted because of the presence of the film-related edge wave.

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Dynamics and radiation effects for charges propagating in ultraintense laser fields of Gaussian beams

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We discuss dynamics and radiation effects for charges propagating in ultraintense laser fields described by Gaussian beams. This problem of classical radiation has been studied by generations of physicists since its first formulation in 1892 by the famous physicist Lorentz. In the fields of high intensities, a charge is accelerated so strongly that its own radiation emission may significantly affect its motion. In our analysis the incident electromagnetic field is based on an asymptotic solution of the space-time Gaussian beam (see [1] and the corresponding references). It was generalized for the case of vector electrodynamics wave propagation. The space-time Gaussian beam solution may include the Gaussian–Hermit and Gaussian–Laguerre modes. The dynamics of the charge motion is evaluated numerically on the basis of the corresponding canonical Hamiltonian system with well-known relativistic Hamiltonian of a charge in electromagnetic field [2]. Different types of dynamics of the charge motion are analysed and compared for various incident modes of the Gaussian–Hermit and Gaussian–Laguerre pulsed beams. The intensity of the charge radiation excited by the Gaussian pulsed beams is computed with help of the well-known classical formula presented in [2]. A particular attention is paid to the comparative analysis of the corresponding spectra of the intensity of the charge radiation. The details of the dynamics of the charge motion and the spectra obtained for the Gaussian beam incident field are compared with the exact solution data for the case of the incident pulse of linearly and circular polarizes plane electromagnetic wave (see [2]). We study and compare results for two canonical cases of video and radio propagating pulses with the Gaussian time-dependence.

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Modelling of ghost imaging reconstruction on the basis of pulsed radiation of Gauss–Laguerre and Gauss–Hermite beams in the terahertz range

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In this paper we explore a possibility of obtaining higher quality ghost imaging (GI) reconstruction of thin film objects in the frequency and time domains based on Gauss–Laguerre and Gauss–Hermite beams in the terahertz (THz) spectrum range. These localized beams have been widely studied in the mathematical theory of wave propagation (see, for example, the aperture in PEC screen diffraction [1–4]). They have also been effective in optical ghost imaging reconstruction. Firstly we study a problem of formation of speckle patterns for pulsed THz radiation and monochromatic radiation. In our paper the speckle patterns are formed by means of transmission of radiation through a transparent homogeneous plate with random rough surface on its back side. This random phase screen provides a phase modulation of the radiation that leads to the appearance of speckle patterns at some distance. The mathematical modelling of propagating Gauss–Laguerre and Gauss–Hermite beams through a random phase screen over a long distances is analyse using Green's formula in the Fresnel paraxial approximation. Generation of a proper speckle patters of Gaussian beams with high contract are very important from point of view of high quality image reconstruction.

The discussed computational GI reconstruction can be considered as a variant of the standard two-detectors method (see, for example, [5]). A spatially incoherent beam is generated by passing a laser beam through a random phase screen. The beam is then split up on a beam-splitter, generating the two spatially correlated beams required for GI. Since for each random realization the controlled phase pattern $\Phi_r(x, y)$ is known, one can evaluate the field right after the screen, $E_r(x, y, z = 0) = E_{in}e^{i\Phi_r(x,y)}$, where E_{in} is the incident field the Gaussian beam mode on the screen. Knowing $E_r(x, y, z = 0)$, the field at any distance z from the screen can be computed using the well-known Fresnel–Huygens propagator:

$$E_r(x, y, z) = \int d\xi\, d\eta\, E_r(x - \xi, y - \eta, z = 0)e^{i\frac{\pi}{\lambda z}\left(\xi^2+\eta^2\right)}$$

where λ is the wavelength of the source. In order to reconstruct the transmission function of an thin film object $T(x, y)$ placed at $z = L$, the computed intensity patterns at the object plane $I_r = |E_r(x, y, z = L)|^2$ are cross-correlated with the intensities measured by the point detector placed behind the object $B_r = \int dx\, dy\, I_r(x, y, L)T(x, y)$. This provides the reconstructed image as a correlation function of I_r and Br in the following form

$$G(x, y) = \frac{1}{N}\sum_{r=1}^{N}\left(B_r - \langle B_r\rangle\right) I_r(x, y)$$

Here $\langle\,\rangle$ denotes an ensemble average over N phase realizations of the random phase screen. Intuitively, one can see that the image is obtained by summing the calculated intensities I_r with the appropriate weights B_r. The larger the overlap between the generated intensity pattern and the transmission object, the higher is the intensity measured by the point detector B_r, and thus the calculated $I_r(x, y)$ is summed with a larger weight. This approach works both in the the frequency and time domains. In the paper we demonstrate some examples of quality GI reconstruction performed with the help of various modes of Gauss–Laguerre and Gauss–Hermite beams in THz spectrum range.

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Modeling of fluid flow in a flexible vessel with elastic walls

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We exploit a two-dimensional model [1–3] describing the elastic behavior of the wall of a flexible blood vessel which takes interaction with surrounding muscle tissue and the 3D fluid flow into account. We study time periodic flows in an infinite cylinder with such intricate boundary conditions. The main result is that solutions of this problem do not depend on the period and they are nothing else but the time independent Poiseuille flow. Similar solutions of the Stokes equations for the rigid wall (the no-slip boundary condition) depend on the period and their profile depends on time.

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Detailed study of Alexey Popov's diffraction problem

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We construct high-frequency asymptotic formulas for the wavefield described by

$$u_{xx} + u_{yy} + k^2 u = 0, \quad \partial_n u|_{\mathcal{C}} = 0, \tag{1}$$

where $k \to \infty$ is a wavenumber and ∂_n denotes the derivative along the normal to a contour $\mathcal{C}$. The latter is composed of the half-line $\mathcal{C}_-$ and the piece of smooth contour $\mathcal{C}_+$, see Fig. 1, with a jump in curvature at the conjugation point $\mathcal{O}$. The incident field is a plane wave $u^{\mathrm{inc}} = e^{ikx}$, and $u = u^{\mathrm{inc}} + u^{\mathrm{out}}$, where u^{out} is the outgoing wave.

The problem has been qualitatively approached in [1], and quantitatively considered in [2] by Alexey Vladimirovich Popov who combined the parabolic-equation method, the Kirchhoff-type heuristics and the Sommerfeld–Malyuzhinets technique to derive an expression for the cylindrical wave arising at the non-smoothness point $\mathcal{O}$.

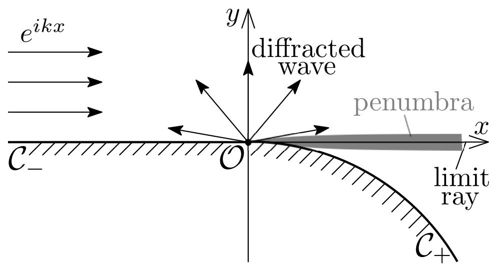

Fig. 1: The geometry of the problem.

Addressing this problem, we continue our work on the systematic application of boundary-layer techniques to diffraction by a jump of curvature and similar problems [3, 4]. We describe the outgoing wavefield u^{out} in a neighborhood of $\mathcal{O}$ via the Leontovich–Fock parabolic-equation method [5, 6]. We seek the outgoing field in the traditional form

$$u^{\mathrm{out}} = e^{iks} U(\sigma, \nu),$$

using stretched coordinates $\sigma = (\varkappa^2 k)^{\frac{1}{3}} s$, $\nu = (\varkappa k^2)^{\frac{1}{3}} n$, where s is the arc length of the contour $\mathcal{C}$ measured from $\mathcal{O}$ and n is the length of normal to $\mathcal{C}$. To the main order, we get the parabolic equation

$$U_{\nu\nu} + iU_\sigma + \theta(\sigma)\nu U = 0$$

with the boundary condition

$$U_\nu|_{\nu=0} = -i\varkappa\theta(\sigma)\sigma e^{-i\sigma^3/3}.$$

Here, $\theta(\sigma) = \{1, \sigma > 0; 0, \sigma \leq 0\}$ is the Heaviside function, $\varkappa$ is the value of curvature of $\mathcal{C}_+$ at the point $\mathcal{O}$.

We have solved this problem explicitly. The solution is, in a sense, similar to that presented in [5, 6] for a smooth contour, but with some classical Airy functions replaced by inhomogeneous Airy functions. We asymptotically analyzed the wavefield in a small vicinity of $\mathcal{O}$ under assumptions that $\sigma > 0$ and $\nu + \sigma \gg 1$, deriving different expressions in boundary layers $\mathcal{D}_2$–$\mathcal{D}_8$ surrounding the limit ray (schematically shown in Fig. 2).

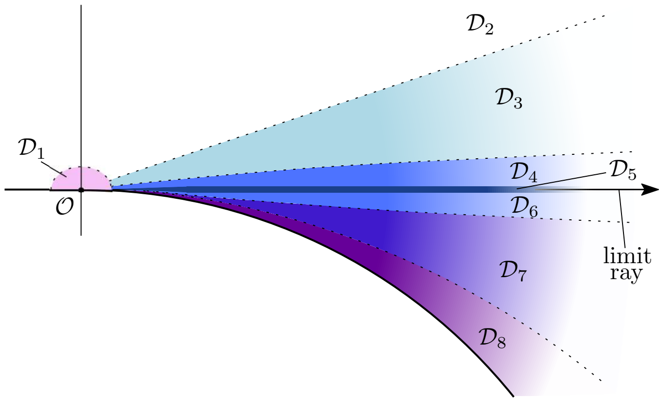

Fig. 2: A sketch of areas under consideration.

An expression for the diffracted wave which we obtained in $\mathcal{D}_2$ agrees with the one found in [2]. As an example, we present a formula for the wavefield in the area $\mathcal{D}_3$ described by $\nu \gg 1$, $\nu^{-\frac{1}{4}} \ll \sqrt{\nu} - \sigma \ll \nu^{\frac{1}{8}}$:

$$u^{\text{out}} \approx \frac{e^{-i\frac{\pi}{4}}}{2\pi} \frac{e^{iks+i\frac{2}{3}\nu^{\frac{3}{2}}}}{\nu^{\frac{1}{4}}} \int\limits_{\gamma} \frac{H'(\xi)}{w_1'(\xi)} e^{i(\sigma-\sqrt{\nu})\xi} d\xi.$$

Here,

$$H(\xi) = \int_0^{\infty} e^{\xi t - \frac{t^3}{3}} dt$$

is the inhomogeneous Airy function, $w_1(\xi)$ is the classical Airy function in Fock's notation [5], and the integration contour γ goes from $-\infty$ to 0 and then to $\infty - i\varepsilon$, $1 \gg \varepsilon > 0$.

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Author index