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DAYS ON DIFFRACTION 2024

ABSTRACTS

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FOREWORD

"Days on Diffraction" is an annual conference taking place in May–June in St. Petersburg since 1968. The present event is organized by St. Petersburg Department of the Steklov Mathematical Institute, St. Petersburg State University, and the Euler International Mathematical Institute.

The conference is supported by the Ministry of Science and Higher Education of the Russian Federation (the grant to the Leonhard Euler International Mathematical Institute in Saint Petersburg, agreement № 075-15-2022-289, the grant to the Steklov International Mathematical Center, agreement № 075-15-2022-265).

The abstracts of 74 talks, presented during 5 days of the conference, form the contents of this booklet. The author index is located on the last pages.

Full-length texts of selected talks will be published in the Conference Proceedings. Format file and instructions can be found at `http://www.pdmi.ras.ru/~dd/proceedings.php`. The final judgement on accepting the paper for the Proceedings will be made by editorial board after peer reviewing.

Organizing Committee

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List of talks

Quasi-classical asymptotics describing the electron-hole interaction and the Klein effect for the (2+1)-Dirac equation in discontinuously varying fields

Allilueva, A.I.[1,2], Shafarevich, A.I.[1,2]
[1]Ishlinsky Institute for Problems in Mechanics of the Russian Acacdemy of Sciences, Prospekt Vernadskogo 101-1, Moscow, 119526, Russia
[2]Moscow Institute of Physics and Technology, 9 Institutskiy per., Dolgoprudny, Moscow Region, 141701, Russia
e-mail: `esina_anna@list.ru`

In this paper, we consider a massless two-dimensional Dirac equation system describing the evolution of wave functions in graphene, which has the form:

$$\begin{aligned} i\varepsilon\frac{\partial u}{\partial t} &= \varepsilon\left(-i\frac{\partial}{\partial x_1}+\frac{\partial}{\partial x_2}\right)v + Fu, \\ i\varepsilon\frac{\partial v}{\partial t} &= \varepsilon\left(-i\frac{\partial}{\partial x_1}-\frac{\partial}{\partial x_2}\right)u + Fv, \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^2$, F is the potential, ε is a small parameter that tends to zero, and characterizes the ratio of the scales of localized inhomogeneity and the general change in the external field. Studying the asymptotics of the solution, we obtain the transmitted and reflected waves from the surface $\Phi(x,t)=0$, where different modes can pass into each other, and also reverse waves appear when the phase and group velocities are directed in different directions.

Two tasks are considered:

1. potential $F = F\bigl(\frac{\Phi(x,t)}{\varepsilon}, x, t\bigr)$ depends on the fast variable $y = \frac{\Phi(x,t)}{\varepsilon}$ and is a smooth function, with $F(x,y,t) \to F^{\pm}(x,t)$ at $y \to \pm\infty$ is faster than any degree of y with all its derivatives. The functions of $F^{\pm}$ are also smooth. This condition reflects the localized nature of the heterogeneity. The parameter $\varepsilon \to 0$, $\Phi(x,t) : \mathbb{R}^3 \to \mathbb{R}$ is a smooth function, and the equation $\Phi(x,t)=0$ defines a smooth regular hypersurface $M \subset \mathbb{R}^3$ (heterogeneity is localized near it).

2. the potential $F(y,x,t)$ has a gap of the 1st kind on the surface of M: $\Phi(x,t)=0$. This means that the limits of $F^{\pm}(x,t)$ are finite and are smooth functions of their arguments. In this case, the solution must be continuously on the surface of the potential gap.

The initial conditions for both problems have the form:

$$\left.\begin{pmatrix} u \\ v \end{pmatrix}\right|_{t=0} = \begin{pmatrix} u^0 \\ v^0 \end{pmatrix} e^{i\frac{S^0}{\varepsilon}}, \tag{2}$$

where $u^0 = u^0(x)$, $u^0 = u^0(x)$ and $S^0(x)$ are smooth functions, u^0 and v^0 are finite, $\nabla S^0|_{\operatorname{supp} u^0} \neq 0$, $\nabla S^0|_{\operatorname{supp} v^0} \neq 0$ and $\operatorname{supp} u^0 \cap M = \varnothing$, $\operatorname{supp} v^0 \cap M = \varnothing$. The initial wave packet is located outside the localized inhomogeneity, the problem is to describe the scattering of such an initial packet by M.

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On Jordan structure of matrices from Lie algebra complex orthogonal group $so(N, C)$

Babich M.V.
PDMI, Fontanka 27, St. Petersburg, Russia
e-mail: mbabich@pdmi.ras.ru

An element of Lie algebra $so(N, C)$ is considered as a linear transformation matrix of some auxiliary Euclidean space. The concept of a Jordan structure is defined for any linear transformation, in particular for elements $so(N, C)$. That is, there is always a basis composed of Jordan vectors. The problem is to find a basis in which the metric tensor of this Euclidean space looks most simple.

The space where the element $A \in so(N, C)$ acts is naturally split into the orthogonal sum of pairs of root spaces $V_{+\lambda_k} \oplus V_{-\lambda_k}$ corresponding to the pairs of eigenvalues $\pm\lambda_k$. We consider the space V_0 corresponding to the zero eigenvalue separately.

For each pair of non-zero $\pm\lambda_k$, the spaces $V_{\pm\lambda_k}$ are isotropic, and the restriction of the scalar product to their sum $V_{+\lambda_k} \oplus V_{-\lambda_k}$ is non-degenerate. The Jordan chains that form the basis of the space $V_{+\lambda_k} \oplus V_{-\lambda_k}$ can be chosen in such a way that the subspaces spanned by pairs of chains are mutually orthogonal. One of the chains of such a pair lies in $V_{-\lambda_k}$, and the other in $V_{+\lambda_k}$.

The chain vectors can be chosen so that, up to sign, they form a hyperbolic basis $V_{+\lambda_k} \oplus V_{-\lambda_k}$, that is, $\langle \vec{e}_i, \vec{e}_j \rangle = (-1)^i \delta_{0,i+j}$. Here, vectors with negative indices form the basis of $V_{-\lambda_k}$, and vectors with positive indices form the basis of $V_{+\lambda_k}$.

The action of the operator reduces the index of the vector by one (Jordan's law), that is, the starting and ending (eigen) vectors of the chains are conjugate to each other. For example, $\vec{e}_{+1} \in V_{+\lambda_k}$ is an eigenvector, that is, $(A - \lambda_k)\vec{e}_{+1} = 0$, and $\vec{e}_{-1} \in V_{+\lambda_k}$ is the starting vector $(A + \lambda_k)\vec{e}_{-1} = \vec{e}_{-2}$, $\langle \vec{e}_{-1}, \vec{e}_{+1} \rangle = -1$.

Now consider the root subspace V_0 corresponding to the zero eigenvalue. The operator A restricted to this subspace is nilpotent. The Jordan basis in V_0 can be chosen so that chains of *even length* (they consist of an even number of vectors) will again be divided into pairs, and the vectors corresponding to each other will be conjugate up to sign. Everything is the same as in the case of non-zero eigenvalues, but the lengths of chains symmetric to each other are necessarily even.

Subspaces spanned by Jordan chains of odd length are orthogonal to each other. The vectors of each of the chains form a hyperbolic (up to sign) basis of the corresponding subspace: $\langle \vec{e}_i, \vec{e}_j \rangle = (-1)^i \delta_{0,i+j}$, $A\vec{e}_{-i_{\max}} = 0$.

Let us describe the generators of the subgroup of orthogonal transformations that preserve this type of Jordan basis. Let us fix some pair of non-zero eigenvalues $\pm\lambda$, it is evident that any orthogonal transformation maps each root subspace into itself, that is, $V_{+\lambda}$ to $V_{+\lambda}$, and $V_{-\lambda}$ to $V_{-\lambda}$.

Let us consider any two pairs of chains $f_{\pm i}$, $1 \leqq i \leqq n$, and $g_{\pm j}$, $1 \leqq j \leqq m$, from $V_{+\lambda} \oplus V_{-\lambda}$. To the starting vector f_{+n} we add $tg_{+n-\delta}, 0 \leqq \delta \leq n, t \in \mathbb{C}$, that is, multiplied by a number, any vector the second chain provided that this vector is no further from the end of its chain than the given starting f_{+n}. To the starting vector g_{-1} we add $(-1)^{\delta+1} t f_{-\delta-1}$. It is easy to check that the vectors obtained by iterations of such starting vectors preserve the pairwise scalar products in question.

For the root subspace corresponding to the zero eigenvalue, there are similar transformations, but this is the subject of subsequent research.

Whistler wave radiation from a circular phased array of loop antennas in a magnetoplasma

Bazhilova E.V.[1], Kudrin A.V.[1], **Zaboronkova T.M.**[1,2], Zaitseva A.S.[1]
[1]University of Nizhny Novgorod, 23 Gagarin Ave., Nizhny Novgorod 603022, Russia
[2]R. E. Alekseev Technical University of Nizhny Novgorod, 24 Minin St., Nizhny Novgorod 603950, Russia
e-mail: `eva-vb@yandex.ru`, `kud@rf.unn.ru`, `t.zaboronkova@rambler.ru`, `zaitseva@rf.unn.ru`

In recent years there has been a considerable interest in the problem of radiation from phased antenna arrays immersed in a magnetoplasma. This interest is stimulated by not only the conventional array applications, such as maximizing the flow of power at a given direction, but also the need to control the radiated-power distribution over the degrees of freedom of electromagnetic waves. In particular, increased attention has been focused on waves that have helical phase fronts and, therefore, carry orbital angular momentum. For antennas capable of launching such waves into a magnetoplasma, the possibility of maximizing the power going to waves with the desired azimuthal index is especially important in the whistler frequency range [1]. To increase the number of selectively excited whistler waves having different azimuthal indices, phased arrays with spaced apart radiating elements should be used [1]. Usually, relatively small magnetic loop antennas are taken as the array elements because of simplicity of their matching with a feeding source. For theoretical considerations in the nonresonant part of the whistler range, i.e., at frequencies below the lower hybrid resonance frequency, where the whistler-mode refractive index is finite for all wave propagation directions, the current loops can be replaced by elementary magnetic dipoles if the loop radii are much smaller than the shortest wavelength [2]. Above the lower hybrid resonance frequency, a cold collisionless magnetoplasma is resonant, and the whistler-mode refractive index surface has unbounded branches. As a result, at such frequencies, there always exist propagating waves whose wavelengths will be shorter than any, even physically small loop radius. In this case, allowance for finite sizes of loop antennas becomes necessary.

In the present work, we extend the theoretical results obtained earlier for a phased array of loop antennas in a nonresonant magnetized plasma medium [2] to the case where such an array is embedded in a resonant magnetoplasma. The loop antennas of finite radius are assumed to be located on a circumference so that their axes are aligned with a static magnetic field superimposed on the plasma. The emphasis is placed on determining the total radiated power of such an array and its partial powers going to different azimuthal field harmonics. For these quantities, rigorous integral representations are obtained using an expansion of the excited field in terms of cylindrical vector eigenfunctions of a magnetized plasma [3]. It is shown that an appropriately phased array is capable of selectively exciting whistler waves with the desired helicity of the phase front and can be useful as a source of such waves in a magnetoplasma. Numerical results will be reported for the radiation characteristics of the considered phased array in cases of interest.

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Triangular factorization of operators, functional models and inverse problems

Belishev M.I.
St. Petersburg Department of Steklov Mathematical Institute of the Russian Academy of Sciences, Fontanka 27, St. Petersburg, Russia
e-mail: belshev@pdmi.ras.ru

Triangular factorization (TF) of an operator C in a Hilbert space $\mathscr{H}$ with respect to a family of subspaces $\{\mathscr{F}^s\}_{s\in[0,T]} \subset \mathscr{H}$ is the representation $C = V^*V$ with an operator V that satisfies $V\mathscr{F}^s \subseteq \mathscr{F}^s$, $0 \leqslant s \leqslant T$. The well-known fact is the deep relations between TF and inverse problems. The TF is the background of one of the variants of the boundary control method, an approach to inverse problems in mathematical physics, using their connections with the control and system theory. The main tool of the factorization is an operator integral, generalizing the classical construction of the triangular truncation integral (M. S. Brodsky, M. G. Krein, I. Ts. Gokhberg). In the talk, we describe the operator integral in detail: its matrix and finite-dimensional versions, leading to the necessary generalizations.

Stability of solutions to the two-dimensional EIT problem

Belishev M.I., **Korikov D.V.**
St. Petersburg Department of V. A. Steklov Institute of Mathematics of the Russian Academy of Sciences, St. Petersburg, Russia
e-mail: belishev@pdmi.ras.ru, thecakeisalie@list.ru

Let (M, g) be a surface (two-dimensional Riemannian manifold) with the boundary Γ and metric tensor g. The Dirichlet-to-Neumann map of (M, g) is given by $\Lambda: \ f \mapsto \partial_\nu u^f|_\Gamma$, where u^f is the harmonic function in (M, g) obeying $u^f|_\Gamma = f$ and ν is the outward normal to Γ. It is well-known that the Λ determines not the surface (M, g) itself but only its conformal class $[(M, g)]$.

In the talk, we discuss the following results on the *stability* of the determination $\Lambda \mapsto [(M, g)]$. In what follows, we consider surfaces with fixed boundary Γ, Euler characteristic χ and orientability o. Let (M, g) and (M', g') be such surfaces and Λ and Λ' be their DN maps, respectively. In [1, 2], we prove that the closeness of Λ' to Λ in the operator norm implies the existence of of the near-conformal diffeomorphism β between (M, g) and (M', g') which does not move the points of Γ. Thereby we establish the continuity of the determination $\Lambda \mapsto [(M, g)]$, where $[(M, g)]$ is the conformal class of (M, g) and the set of such conformal classes is endowed with the Teichmüller metric d_T. We also provide [2] quantitative estimates of $d_T([(M, g)], [(M', g')])$ via the operator norm of the difference $\Lambda' - \Lambda$. These estimates are optimal in the orientable case.

It is worth noting that the topological condition in the above result cannot be weakened since surfaces with the different topologies may have arbitrarily close DN maps [2, 3].

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Inverse dynamic problem for a one-dimensional dissipative system

Belishev M.I., Mikhaylov A.S., **Mikhaylov V.S.**
St. Petersburg Department of V. A. Steklov Institute of Mathematics of the Russian Academy of Sciences, 7 Fontanka, St. Petersburg 191023, Russia
e-mail: `belishev@pdmi.ras.ru, mikhaylov@pdmi.ras.ru, vsmikhaylov@pdmi.ras.ru`

We consider the inverse dynamic problem for a one-dimensional dissipative system governed by the equation $u_{tt}(x,t)+\sigma(x)u_t(x,t)-u_{xx}(x,t)=0$, $x>0$. We show the linear procedure of recovering the damping term σ from a response operator (dynamic Dirichlet-to-Neumann operator). We also answer a question on the characterization of dynamic inverse data.

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A functional model for a class of symmetric semi-bounded operators

M.I. Belishev, **S.A. Simonov**
St. Petersburg Department of the Steklov Mathematical Institute
e-mail: `sergey.a.simonov@gmail.com`

Let $\{\mathscr{H},\mathscr{K};L_0;\Gamma_1,\Gamma_2\}$ be the boundary triple (by M. I. Vishik) for a positive definite symmetric operator L_0, so that

$$(L_0^*u,v)_{\mathscr{H}}-(u,L_0^*v)_{\mathscr{H}}=(\Gamma_1u,\Gamma_2v)_{\mathscr{K}}-(\Gamma_2u,\Gamma_1v)_{\mathscr{K}},\qquad u,v\in\operatorname{Dom}L_0^*,$$

where

$$\mathscr{K}:=\operatorname{Ker}L_0^*,\quad \Gamma_1:=\mathbb{I}-L^{-1}L_0^*,\quad \Gamma_2=PL_0^*,$$

L is the Friedrichs extension of L_0, P is the projector in $\mathscr{H}$ on $\mathscr{K}$. We propose a new functional model for L_0 constructed using operators and spaces of the dynamical systems α^T, $T>0$:

$$\begin{aligned}
&u''+L_0^*u=0 && \text{in } \mathscr{H},\ 0<t<T\leqslant\infty,\\
&u(0)=u'(0)=0 && \text{in } \mathscr{H},\\
&\Gamma_1u(t)=f(t)\in\mathscr{K}, && 0\leqslant t\leqslant T.
\end{aligned}$$

Let $u=u^f(t)$, $0\leqslant t\leqslant T$, be the trajectory. The basic element of the construction is the triangular factorization of the connecting operator $C^T:=(W^T)^*W^T$ in the space $L_2([0,T];\mathscr{K})$, where $W^T: f\mapsto u^f(T)$ is the control operator of the dynamical system. The factorization is provided by the so-called *diagonal* of the control operator expressed as an operator integral along a nest of subspaces. The resulting model has the form of a Schrödinger operator in $L_2([0,\infty);\mathcal{K})$ with the potential being a decomposable operator, which is an operator-valued (in $\mathcal{K}$) function.

Metamaterials in Russia and former USSR

Pavel Belov
School of Physics and Engineering, ITMO University, St. Petersburg, Russia
e-mail: `belov@metalab.ifmo.ru`

The artificial media with electromagnetic properties not available in natural materials were studied for more than 80 years. These days such substances are called as metamaterials. The discovery of left-handed metamaterials is usually associated with work of Viktor Georgievich Veselago (1929–2018) [1]. However, the media with simultaneously negative permittivity and permeability supporting backward waves and demonstrating negative refraction were discussed in the works of academician Leonid Isaakovich Mandelshtam (1879–1944) [2–5] and Dmitry Vasilievich Sivukhin (1914–1988) [6] much earlier. More details about the early history of metamaterials are available in [7] and [8].

The negative refraction and backward waves are known to be observed in photonic crystals and the publications of Masaya Notomi about this effects are dated by 2000 year [9]. It is interesting that the phenomena were known in USSR since 1970s. The key person who was studying these effects was Robert Andreevich Silin (1927–2018): he published papers [10, 11] and even books [12–14], the last one was a table book for microwave engineers in USSR.

The special kinds of metamaterials are known as bianisotropic media. These structures were actively studied within development of STEALTH technologies. The major banch of studies of bianisotropic media in USSR was done by the school of academician Fedor Ivanovich Fedorov (1911–1994) [15] in Belarus. This contribution was well appreciated by international bianisotropics and metamaterials community [16].

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Glancing points and asymptotic solutions

I. Bogaevskii
The University of Liverpool, UK
Scientific Research Institute for System Analysis of RAS, Russia
e-mail: ibogaevsk@gmail.com

M. Rouleux
Aix Marseille Univ, Université de Toulon, CPT, CNRS, France
e-mail: michel.rouleux@univ-tln.fr

We investigate asymptotic solutions to the equation

$$\widehat{H}u_h(x) = f_h(x), \quad \widehat{H} = H(\widehat{p}, x), \quad H \in C^\infty(T^*\mathbb{R}^n), \quad \widehat{p} = -ih\partial_x, \quad x \in \mathbb{R}^n,$$

with a right hand side f_h microlocalized on a smooth Lagrangian submanifold Λ_0:

$$f_h(x) = [\mathcal{K}_{\Lambda_0}(a)](x), \quad a : \Lambda_0 \to \mathbb{R},$$

where a is a smooth amplitude on Λ_0 and $\mathcal{K}_{\Lambda_0}$ denotes the Maslov canonical operator on Λ_0.

The paper [1] gives a formula (under some extra conditions) in terms of the Maslov canonical operator for the asymptotic solution in the case when the Hamiltonian vector field is not tangent to the Lagrangian submanifold Λ_0 at the points of the level hypersurface

$$\Sigma = \{H(p, x) = 0\}.$$

In other words, the hypersurface Σ is assumed to be smooth and transversal to Λ_0.

The points of tangency Λ_0 and the smooth hypersurface Σ are called *glancing*, they have been considered in [2]. It turns out that in the simplest case the asymptotic solution can be expressed via the Airy function along the Hamiltonian trajectory starting at a glancing point.

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Inverse spectral problem for higher-order differential operators

Bondarenko N.P.
Samara National Research University, Moskovskoye Shosse 34, Samara 443086, Russia
e-mail: bondarenkonp@sgu.ru

Consider the differential equation

$$y^{(n)} + \sum_{k=0}^{\lfloor n/2\rfloor - 1} (\tau_{2k}(x)y^{(k)})^{(k)} + \sum_{k=0}^{\lfloor (n-1)/2\rfloor - 1} \left((\tau_{2k+1}(x)y^{(k)})^{(k+1)} + (\tau_{2k+1}(x)y^{(k+1)})^{(k)}\right) = \lambda y, \quad x \in (0,1), \quad (1)$$

where $n \geq 2$, the notation $\lfloor a \rfloor$ means rounding a real number a down, $\tau_\nu \in W_2^{\nu-1}[0,1]$, $\nu = \overline{0, n-2}$, $i^{n+\nu}\tau_\nu$ are real-valued functions, λ is the spectral parameter.

This talk is concerned with the recovery of the coefficients $\{\tau_\nu\}_{\nu=0}^{n-2}$ from the eigenvalues $\{\lambda_{l,k}\}_{l\geq 1}$ and the weight numbers $\{\beta_{l,k}\}_{l\geq 1}$ of the boundary value problems $\mathcal{L}_k$, $k = 1, 2, \dots, n-1$, for equation (1) with the separated boundary conditions

$$y^{(j-1)}(0) = 0, \quad j = 1, \dots, k, \qquad y^{(s-1)}(1) = 0, \quad s = 1, \dots, n-k.$$

The necessary and sufficient conditions for solvability of this inverse spectral problem are obtained in [1] by developing the method of [2].

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On negative eigenvalues of toy graph with small edge

Borisov D.I.
Institute of Mathematics, Ufa Federal Research Center, RAS, 450008, Russia, Ufa, Chernyshevsky str. 112,
e-mail: borisovdi@yandex.ru

We consider a simple Schrödinger operator with zero potential on a toy graph consisting of two edges. One edge has the unit length, while the other is assumed to be small and its length is a small positive parameter ε. At the vertex joining theses edges a general boundary condition is imposed. We show that as the small edge shrinks, the considered operator can have eigenvalues of three types. The eigenvalues of the first type are holomorphic in ε. The eigenvalues of two other types tends to the negative infinite and they behave as $\lambda \sim \varepsilon^{-1}\Lambda(\varepsilon^{\frac{1}{2}})$ or $\lambda \sim \varepsilon^{-\frac{2}{3}}\Lambda(\varepsilon^{\frac{2}{3}})$, where Λ is some holomorphic function. The symbol $\sim$ means that we have an identity up to a certain exponentially small error term.

This is a joint work with G. Berkolaiko (Texas A&M University) and M. King (Texas A&M University).

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Hot spots and field focusing in dielectric ring resonators

Bronnikov K.A., Solodovchenko N.S., Chetverikova A.P., Limonov M.F.
ITMO University, 191002, St. Petersburg, Russia
e-mail: kirill.bronnikov@metalab.ifmo.ru, n.solodovechenko@metalab.ifmo.ru, alina.chetverikova@metalab.ifmo.ru, m.limonov@metalab.ifmo.ru

The photonic effects accompanying the transition between two dielectric structures having different topology — a disk and a ring — have been investigated in simulation and experiment. It was found that an intense electric hot spot appears inside a coaxial hole when it is introduced into a disk resonator. With an increase of the hole radius the hot spot quickly fades out. The dependence of this effect on permittivity of the ring was studied. In turn, the photonic modes experience shift to higher frequencies upon the increase of the hole and gather to a separate sets of resonances according to their radial and azimuthal indices forming “ring gallery modes” [1].

For a ring resonator with a relatively large hole (the ratio of the inner to outer radius of the ring is about 0.9), an interesting effect is observed for certain modes: in the center of the hole, the electric field is enhanced, effectively realizing the focusing of a plane wave. A detailed analysis of this effect was carried out.

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Miniature Mobius strip inspired scatterer

V.D. Burtsev, T.S. Vosheva, A.P. Sedov, A.V. Nikulin, D.S. Filonov
Center for Photonics and 2D Materials Moscow Institute of Physics and Technology, Dolgoprudny, Russia
e-mail: burtsev.vd@phystech.edu

In this paper, we investigate a novel subwavelength scatterer, the layout of which is inspired by the Mobius strip. The characteristic size of the proposed particle is about 10 mm. So that we consider the frequency range from 1 to 30 GHz in numerical simulation for field distribution calculation of scattering task using finite element method. The results of spectral analysis of multipolar coefficients of far electric field allows us to conclude that with certain specified geometric parameters of the scatterer, co-directional electric and magnetic dipole moments are simultaneously excited in it. Due to this behavior at multiple frequencies, the proposed particle is transparent to one circular polarization, while the other is effectively scattered. Due to the reciprocity principle, such scatterer can also be used as an antenna with a predictable radiation pattern in several frequency ranges.

Nonstandard perturbation approach to find nonlinearized solutions in a problem of electromagnetic wave propagation in a plane waveguide with nonlinear filling

Chalyshov G.V., Dyundyaeva A.A., **Valovik D.V.**
Penza State University, 440026, Russia, Penza, Krasnaya Str., 40
e-mail: 19gordey99@gmail.com, andyundyaeva@gmail.com, dvalovik@mail.ru

We consider a monochromatic electromagnetic TE wave $(\mathbf{E},\mathbf{H})e^{-i\omega t}$ that propagates in the plane waveguide Σ of thickness h, where $\mathbf{E}=\big(0,\mathrm{E}_y(x),0\big)e^{i\gamma z}$, $\mathbf{H}=\big(\mathrm{H}_x(x),0,\mathrm{H}_z(x)\big)e^{i\gamma z}$ are electric and magnetic fields, ω is circular frequency, γ is an unknown real parameter, and at $x=0$ and $x=h$ the waveguide has absolutely conducted walls.

The permittivity ε in layer Σ has the form $\varepsilon=\varepsilon_1+\beta_1\varepsilon_2+(\alpha_1+\beta_2\alpha_2)|\mathbf{E}|^2$, where ε_1, α_1 are positive constants, $\beta_{1,2}$ are real constant such that $|\beta_{1,2}|$ are assumed to be sufficiently small, $\varepsilon_2\equiv\varepsilon_2(x)$ and $\alpha_2\equiv\alpha_2(x)$ are continuous functions for $x\in[0,h]$.

The fields satisfy Maxwell's equations $\operatorname{rot}\mathbf{H}=-i\omega\varepsilon_0\varepsilon\mathbf{E}$, $\operatorname{rot}\mathbf{E}=i\omega\mu\mathbf{H}$, where ε_0 is a constant (permittivity of free space). Tangential component E_y of the electric field vanishes at the absolutely conducted walls. Besides this, we assume that tangential component H_z of the magnetic field is fixed at the boundary $x=0$. The permeability $\mu=\mu_0$, where μ_0 is the permeability of free space.

Substituting the fields into Maxwell's equations, performing the normalization in accordance with the formulas $\widetilde{x}=k_0x$, $\widetilde{\gamma}=k_0^{-1}\gamma$, $\widetilde{h}=k_0h$, where $k_0^2=\omega^2\varepsilon_0\mu_0$, using the notation $E_y:=u$ and omitting the tilde symbol, after some algebra one finds

$$u''(x)=-(\varepsilon_1+\beta_1\varepsilon_2(x)-\gamma^2)u(x)-(\alpha_1+\beta_2\alpha_2)u(x)^3. \tag{1}$$

From the conditions imposed on the fields, one gets the following boundary conditions $u(0)=0$, $u'(0)=A$, $u(h)=0$, where $A\neq 0$ is a real constant.

From the mathematical point of view the physical problem is equivalent to the problem of finding $\gamma = \widehat{\gamma} > 0$ such that there exists function $u(x)$ satisfying equation (1) and the above listed conditions. The looked-for values $\gamma = \widehat{\gamma}$ we call propagation constants (or *eigenvalues*) and the corresponding functions $u(x; \widehat{\gamma})$ we call eigenmodes (or *eigenfunctions*) of the problem.

If $\beta_{1,2} = 0$, one gets a simpler nonlinear problem [1]. We use the main properties of its solutions to develop a perturbation approach to solve the main problem. Since the problem with zero $\beta_{1,2}$ has nonlinearizable solutions, then we expect that the main problem also has nonlinearizable solutions as soon as $|\beta_{1,2}|$ are suffuciently small. To be more precise, we prove that if $\gamma = \bar{\gamma}$ is a solution to the problem with $\beta_{1,2} = 0$, then there exists a constant $\beta_0 > 0$ such that for any $\beta_{1,2}$, where $|\beta_{1,2}| < \beta_0$, the main problem has at least one solution $\gamma = \widehat{\gamma}$ in the vicinity of $\bar{\gamma}$ and $\lim_{\beta_{1,2} \to 0} \widehat{\gamma} = \bar{\gamma}$. The case of $\beta_2 = 0$ and $\beta_1 \neq 0$ see in [2].

If $\alpha_{1,2} = 0$ then the main problem degenerates into the linear problem, where β_1 is not necessarily equals to zero. This problem is classical in the linear waveguide theory [3].

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Inverse problem for a quasilinear model of complex heat transfer with internal thermal radiation

Chebotarev A.Yu., Mikolosyuk E.A., Kovtanyuk A.E.
Far Eastern Federal University, Vladivostok, Russia
e-mail: chebotarev.ayu@dvfu.ru

The process of endovenous laser ablation accounting for the internal thermal radiation (black body radiation) in a bounded domain $\Omega \subset \mathbb{R}^3$ is modeled by the following quasilinear system of complex heat transfer [1]:

$$\sigma \partial\theta/\partial t - \operatorname{div}(k(\theta)\nabla\theta) + b(\theta^3|\theta| - \varphi) = q(t)f(x), \tag{1}$$

$$-\operatorname{div}(\alpha\nabla\varphi) + \beta(\varphi - \theta^3|\theta|) = 0, \quad x \in \Omega, \quad 0 < t < T, \tag{2}$$

$$k(\theta)\partial_n\theta + c(\theta - \theta_b)|_\Gamma = 0, \quad \alpha\partial_n\varphi + \gamma(\varphi - \theta_b^4)|_\Gamma = 0, \quad \theta|_{t=0} = \theta_0. \tag{3}$$

Here, θ is the normalized temperature, φ is the normalized intensity of radiation. Positive physical parameters σ, b, α, β, c, and γ describing properties of the media are determined in a standard way [1], $k(\theta)$ is the coefficient of thermal conductivity, σ is the product of the specific heat capacity and the volume density.

Unknown function q model the intensity of heat sources. The inverse problem is to find the intensity $q = q(t)$ and the corresponding solution θ, φ of system (1)–(3) under additional integral conditions:

$$\int_\Omega f(x)\theta(x,t)\,\mathrm{d}x = r(t), \quad 0 < t < T. \tag{4}$$

Previously [2], similar inverse problems were considered for quasilinear models of complex heat transfer that do not take into account internal thermal radiation. Analysis of the presented inverse problem containing a polynomial nonlinearity requires obtaining new a priori estimates.

The unique solvability of the inverse problem is proven without any smallness assumptions on the model parameters. Further, we study the Tikhonov regularization in the framework of a PDE constrained optimization problem and show that the approximating sequence converges to the solution of the inverse problem.

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On the scattering problem for the nonhomogeneous ultrahyperbolic equation

Maxim N. Demchenko
St. Petersburg Department of V. A. Steklov Institute of Mathematics of the Russian Academy of Sciences, Fontanka 27, St. Petersburg, Russia
e-mail: demchenko@pdmi.ras.ru

We consider the scattering problem for the ultrahyperbolic equation

$$(\Delta_y - \Delta_x)u = f \tag{1}$$

in $\mathbb{R}^d \times \mathbb{R}^n$, in which a solution $u(x, y)$ is to be determined from $f(x, y)$ and the scattering data $F(\theta, \omega, p)$ prescribing the behavior of a solution at the infinity:

$$u(s\theta, s\omega + p) = \frac{1}{s^{N/2-1}} F(\theta, \omega, p)(1 + o(1)), \quad s \to +\infty, \tag{2}$$
$$(\theta, \omega, p) \in Z := S^{d-1} \times S^{n-1} \times \mathbb{R}, \quad N := d + n.$$

In this set-up, the problem is overdetermined, since there is a certain class of regular solutions of homogeneous equation (1) (with $f = 0$), which exhibit tha behavior (2) with the asymptotic coefficient F satisfying the following relation [1]

$$F(-\theta, -\omega, -p) = (H^{d-n} F(\theta, \omega, \cdot))(p), \tag{3}$$

where H is the Hilbert transform in p. One way to eliminate this overdetermination is to impose condition (2) for (θ, ω, p) in some part of Z. For example, in the case of the wave equation with $d = 3$, $n = 1$, the manifold Z is a disjoint union of two closed manifolds corresponding to $\omega = \pm 1$. The data F given in one of them uniquely determines a solution u. This provides a well-posed scattering problem for the homogeneous wave equation. Furthermore, this set-up can be generalized to the case of nonhomogeneous wave equation [2], when relation (3) does not hold. However, in the case $d, n > 1$, the manifold Z is connected, and so the restriction of the data F to some part Z_0 of Z gives rise to the issue of behavior of F in the vicinity of the boundary ∂Z_0.

In the scattering problem for the homogeneous ultrahyperbolic equation, the overdetermination can also be eliminated by assuming that the data F satisfies condition (3). This formal approach can not be generalized to the case $f \neq 0$, when the asymptotics of solutions may not satisfy (3). This motivates us to consider a different set-up of the scattering problem connected with vector-valued solutions u. Namely, we assume that u and f take values in $\mathbb{C}^2$, and replace condition (2) with a less restrictive one:

$$P(\theta, \omega)u(s\theta, s\omega + p) = \frac{1}{s^{N/2-1}} \mathcal{F}(\theta, \omega, p)(1 + o(1)), \quad s \to +\infty,$$

where $P(\theta,\omega)$ is a family of orthogonal projections of rank one in $\mathbb{C}^2$ satisfying

$$P(\theta,\omega)+P(-\theta,-\omega)=I.$$

In this talk, we discuss a fundamental solution for the ultrahyperbolic equation that corresponds to such a scattering problem.

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Long nonlinear water waves in "whispering galleries" formed by gentle shores

S.Yu. Dobrokhotov, D.S. Minenkov, M.M. Votiakova
Ishlinsky Institute for Problems of Mechanics of the Russian Academy of Sciences
e-mail: `s.dobrokhotov@gmail.com`

In problems of acoustics, the asymptotic eigenfunctions of a two-dimensional Laplace operator in a bounded domain with a smooth convex boundary G with Dirichlet or Neumann boundary conditions are well known. Such functions are localized in the vicinity of the boundary G and in some sense are related to classical "degenerate" billiards. In this talk we discuss time-periodic asymptotic solutions of a nonlinear system of shallow water equations that have the property of localization in the vicinity of the coastline (boundaries of the region) and generalizing (linear) Stokes and Ursell waves. The essential difference is that, firstly, the problem under consideration is a nonlinear one with a free boundary and, secondly, that the boundary of the domain is associated with the degeneration of the coefficients of the equations at the boundary of the domain in which the limiting linear problem is studied. Therefore, the "degenerate" billiard tables that arise in the situation under consideration are formed by so-called "semi-rigid" walls, which allows us to consider situations when the coastline is not convex. The resulting asymptotic formulas have a simple and effective form, which allows us to establish, for example, the relationship between the local amplitudes of the studied waves with the angle of inclination of the shore and the curvature of the coastline.

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Ultrasonic inspection of fluid-loaded anisotropic laminate plates

Ermolenko O.A., Glushkov E.V., Glushkova N.V.
Institute for Mathematics, Mechanics and Informatics, Kuban State University, Krasnodar, 350040, Russian Federation
e-mail: o.ermolenko.a@gmail.com

The ultrasonic sounding of anisotropic laminate plates immersed in acoustic fluid is considered. Mathematical and computer models for simulating wave processes in the coupled system source — acoustic medium — multilayer isotropic plate have been already developed [1]. The present work aims at extending this model on the anisotropic case and analyzing, on this basis the effect of anisotropy on the guided wave dispersion properties, their angular directivity, and the directivity of the scattered (reflected and transmitted) acoustic waves. The developed model is based on the explicit integral representations of the excited waves through the Fourier symbol of Green's matrix of the coupled system a distributed source — an anisotropic laminate plate — acoustic fluid. In the far-field, the bulk acoustic and guided elastic waves are described by asymptotic representations derived from path integrals using the stationary phase method and residue technique. The developed computer model is validated against the finite element simulation (Comsol Multiphysics 5.6).

In the numerical analysis, a three-layer transversally isotropic composite plate immersed in water is considered to study the influence of anisotropy on the directivity of the excited wave fields. The numerical examples illustrate angular dependence of the dispersion characteristics of guided waves, their attenuations, and amplitude factors. For the bulk acoustic waves reflected from and transmitted through the composite plate, the angular and frequency dependences of the reflection and transmission amplitude factors are also discussed and analyzed. The possibility of determining the optimal source tilt angles and frequency ranges is studied to solve the problems of selective mode excitation and maximization. The choice of optimal parameters for the source operation is presented and discussed.

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Scattering of fundamental elastic guided waves by localized 3D obstacles in bimaterial isotropic laminates

Evdokimov A.A., Vareldzhan M.V., **Eremin A.A.**
Institute for Mathematics, Mechanics and Informatics, Kuban State University, Krasnodar, 350040, Russian Federation
e-mail: eremin_a_87@mail.ru

Laminate structures fabricated from isotropic materials such as aluminium, steel or glass through adhesive bonding are widely used in automotive industry, building construction, microdevice fabrication, etc. When exposed to mechanical loads or environmental factors, localized damage, e.g., pitting corrosion or delaminations, may develop in them. Evaluation of such defects could be achieved through various non-destructive testing (NDT) and structural health monitoring (SHM) strategies including those which rely on elastic guided waves (EGWs) as a physical basis. Since spatial distribution of amplitudes of EGWs scattered by any inhomogeneity depends on its geometric characteristics, these data could be used not only for damage localization employing a distributed sensor network but might allow evaluating its extent and severity. For the practical implementation of such approach, prior corresponding theoretical studies might be useful.

In the current contribution, for the simulation of EGW interaction with 3D localized obstacles in bimaterial laminate structures, a hybrid numerical scheme [1] is adopted. Within this approach finite element method (FEM) is used for the discretization of a bounded domain with complex-shaped defects, and perfectly matched layers as absorbing elements are employed to simulate energy outflow to infinity. Corresponding scattering diagrams for each specific normal mode are further evaluated by projecting the obtained numerical data on explicit analytically-based representations for EGWs within the far-field assumption. The latter, in turn, are derived employing semi-analytical integral approach and take into account multi-layering of the considered waveguide relying on the corresponding Green's matrix [2].

In numerical studies, interaction of fundamental EGWs with a single localized thickness change or delamination in bilayered aluminium-steel structures is considered, and the influence of geometric characteristics and sublayer thickness on EGW scattering diagrams is investigated. In general, they depend on frequency rather smoothly. However, strong changes in the behaviour of scattered wave-fields are observed for deep flat-bottom holes and delaminations near the resonance frequencies of such defects (i.e., almost real complex spectral points of the elastodynamic boundary value problem for an open waveguide with an obstacle [3]). It is revealed that at such frequencies EGW directivity patterns are governed by corresponding eigenforms, which, in turn, might be used to refine damage evaluation results employing a distributed piezosensor network within the SHM concept.

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Some new results of applying the separation of variables method to spheroids

Farafonov V.G.
St. Petersburg University of Aerospace Instrumentation, Morskaya 67, St. Petersburg 190000, Russia
e-mail: far@aanet.ru

Turichina D.G.
Main (Pulkovo) Astronomical Observatory, Pulkovskoe sh. 65/1, St. Petersburg 196140, Russia
e-mail: t.dasha5@mail.ru

Il'in V.B., Laznevoi S.I.
St. Petersburg State University, Universitetskaya nab. 7-9, St. Petersburg 199034, Russia
e-mail: v.b.ilin@spbu.ru, st079380@student.spbu.ru

Last year, two papers were published [1, 2], where the exact solution to the classical problem of light scattering by a spheroid was at last extended to allow its detailed comparing with the ray-tracing results, i.e. for diffraction parameters larger than 200. In the first paper, the scattering problem was solved by the extended boundary condition method with a sophisticated spheroidal basis equivalent to use of both Debye potentials and those involved to solve the problem for infinite cylinders. In the second work, the separation of variables method (that is known to provide less complicated algebraic systems) was applied along with a simple spheroidal basis.

In this paper, we combine the separation of variables method with the sophisticated basis used in [1]. We also apply various new approaches suggested in [1], namely the problem formulation in terms of normalised spheroidal functions defined according to Meixner & Schäfke, the use of T-matrix transformations to skip solving the problem in the complicated case of the TE-mode, separation of the arising systems in two smaller parts, transition from the spheroidal T-matrix to the standard spherical one useful for particle ensembles, etc. We present some results of our numerical calculations. They illustrate that when using the basis from [1], the number of terms taken into account in the field expansions to reach a given accuracy of the results rather weakly depends on the aspect ratio of the spheroid, while for the simple basis of [2] this number grows strongly with increasing degree of particle asphericity and it leads to a significant increase in the time of computations.

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On cylindrically symmetrical exact transparent boundary condition in cylindrical computational domain

Feshchenko R.M.
P. N. Lebedev Physical Institute of RAS, Russia, Moscow, 53 Leninski Prospect
e-mail: rusl@sci.lebedev.ru

Numerical solution of the 3D parabolic equation with any finite-difference scheme requires an appropriate (e.g. transparent, absorbing, etc.) boundary condition to truncate the computational domain. Earlier we obtained such an exact transparent boundary condition (TBC) in the rectangular computational domain [1]. It should be noted that there exists a class of problems with cylindrical symmetry, for which the use of full TBC in the rectangular domain will lead to unnecessary computational costs. On the other hand the exact TBC in the cylindrical computational domain, while is known, is formulated in a very complicated analytical form requiring extensive evaluation of complex kernels containing Hankel functions [2]. This however proved to be unworkable in practice. In this work we demonstrate how the known exact TBC in the rectangular computational domain can be simplified in the presence of cylindrical symmetry and how a computationally efficient exact TBC for the cylindrically symmetrical problem in the cylindrical computational domain can be obtained. Furthermore, using a number of numerical experiments we demonstrate how the derived TBC works in practice.

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Axisymmetric vibrations of a cylindrical elastic shell with inhomogeneous Poisson's ratio

Filippenko G.V.
Institute for Problems in Mechanical Engineering of the Russian Academy of Science (IPME RAS), Bolshoy pr. V.O., 61, St. Petersburg, Russia, 199178
e-mail: g.filippenko@gmail.com, fgv@ipme.ru

Actual for modern industry are the problems in the design and operation of various pipelines, supports, structural elements, which are modelled by inhomogeneous cylindrical shells. The inhomogeneity of elastic properties of such shells can be inherent from the very beginning, but can also arise as a result of operation (wear, hydrogen embrittlement, etc.). This can lead to premature wear of structures and accidents, for example, due to oscillations reaching dangerous modes.

Axisymmetric vibrations are of particular interest due to their frequent occurrence and convenience of analytical analysis. These wave processes arise from axisymmetric excitation of waves by the source of vibrations or they become dominant due to damping of other types of the vibrations, e.g. by structural elements or the medium in contact with the shell.

The paper analyses a shell whose inhomogeneous elastic properties depend only on the radial variable. The heterogeneity can be either a piecewise constant (homogeneous layers) or a continuously varying function, both throughout the thickness of the shell and within each of the layers. Free harmonic vibrations of the shell are considered.

The asymptotic approaches described in the literature to account for the variation of the mechanical properties of the shell along its thickness are too complicated to obtain sufficiently simple estimation formulae. Therefore, in this paper, the averaged shell parameters are calculated using the formulas given in [1], but with the introduction of the necessary corrections.

The area of applicability of different models of averaging of inhomogeneous shell parameters is analysed on the example of a bilayer shell with inhomogeneity of Poisson's ratio. On the basis of an exact analytical solution, the influence of inhomogeneity on the shell natural frequencies for axisymmetric modes is investigated.

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Guided wave propagation control in multilayered piezoelectric waveguide by system of electrodes connected via electrical circuit

Fomenko S.I., Golub M.V., Usov P.E.
Institute for Mathematics, Mechanics and Informatics, Kuban State University, Krasnodar, 350040, Russia
e-mail: sfom@yandex.com, m_golub@inbox.ru, lyova-pavel.usov@yandex.ru

The application of multi-field coupled materials is a very promising direction in the design of tunable phononic crystals and acoustic metamaterials (AMMs), whose band structures are tuned by using the sensitivity to the external stimulus, such as a temperature, magnetic or electric field [1].

In this study, an extended boundary integral equation method (BIEM) [2] is applied for simulating elastic guided wave excitation and propagation in a multi-layered piezoelectric laminate (so-called bimorph) with a system of electrodes connected via an electrical circuit with external components (for example, capacitors, inductors, etc.). The method is based on the integral representation in terms of the Fourier transform of the Green's matrix for the whole layered waveguide [3]. The derived boundary integral equations are solved numerically using the Bubnov–Galerkin method with

Chebyshev polynomials of the first kind. The present method is also compared with the finite element method (FEM), and the efficiency and convergence of the method are studied and demonstrated by several representative examples. The numerical parametric analysis of the influence of the circuit parameters (electrical impedance) on wave propagation is discussed. The developed semi-analytical method allows us to investigate the spectral properties of the bimorph with circuited electrodes and it is also can be extended for the analysis of periodic arrays of connected electrodes, which will be further employed to construct AMM for active control of wave propagation [4].

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Consistency of the Bayes method for the inverse scattering problem

Takashi Furuya
Simane University
e-mail: takashi.furuya0101@gmail.com

In this work, we consider the inverse scattering problem of determining an unknown refractive index from the far-field measurements using the nonparametric Bayesian approach. We use a collection of large “samples”, which are noisy discrete measurements taking from the scattering amplitude. We will study the frequentist property of the posterior distribution as the sample size tends to infinity. Our aim is to establish the consistency of the posterior distribution with an explicit contraction rate in terms of the sample size. We will consider two different priors on the space of parameters. The proof relies on the stability estimates of the forward and inverse problems. Due to the ill-posedness of the inverse scattering problem, the contraction rate is of a logarithmic type. We also show that such contraction rate is optimal in the statistical minimax sense.

This is based on joint work [1] with Pu-Zhao Kow (National Chengchi University), Jenn-Nan Wang (National Taiwan University).

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A conditionally exactly solvable 1D Dirac pseudoscalar interaction potential

Ghazaryan A.M., Ishkhanyan A.M.
Institute for Physical Research, Ashtarak, 0204 Armenia
e-mail: astghik.ghazaryan1@edu.isec.am, aishkhanyan@gmail.com

We study an analytically solvable pseudoscalar interaction potential for the one-dimensional stationary Dirac equation, which consists of power terms proportional to x^{-1}, $x^{-1/3}$, and $x^{1/3}$. This potential is classified as conditionally exactly solvable due to the fixed strength of the first term at a specific constant (see [1–3]). We present the general solution to the Dirac equation in terms of non-integer index Hermite functions, which are distinct from the conventional integer index Hermite polynomials. We analyze the energy spectrum of the bound states and the eigenfunctions and compare the results with the case without the term.

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Source energy distribution and wave energy streamlines in an elastic anisotropic half-space

Glushkov E.V., Glushkova N.V., Kiselev O.N.
Institute for Mathematics, Mechanics and Informatics, Kuban State University, Krasnodar, 350040, Russian Federation
e-mail: `evg@math.kubsu.ru`

The time-averaged wave energy supplied by a time-harmonic surface source in an elastic half-space is distributed among the excited surface and body waves. In the isotropic case, the energy partition is calculated using explicit integral and asymptotic representations in terms of Green's matrix of the substrate considered and the source parameters. The fast algorithms of Green's matrix calculation developed for layered arbitrarily anisotropic elastic media [1] made it possible to derive and computer implement similar explicit representations for source-generated guided and body waves in anisotropic substrates. They are described by the contribution of residues from the poles of the integrand and stationary points of the phase functions to the path Fourier integral representation of the total wave field. The dependence of the poles and stationary points on the direction of wave propagation is more complicated here than in the isotropic case, up to the appearance of multiple points and folds giving rise to additional waves and caustics [2]. Nevertheless, the derived semi-analytical solutions allow us to study the energy balance and the pattern of energy fluxes, the same as in the isotropic case.

In the present talk we discuss the influence of anisotropy on the source energy partition, radiation diagrams and wave energy fluxes. The latter are visualized by the time-averaged Umov–Poynting vectors and tangential to them energy streamlines. Wave fields generated by the normal and tangential point loads are considered. In the isotropic case, the ratio between the Rayleigh wave and bulk wave parts of the source power depends only on the Poisson's ratio of the half-space material. In the case of cubic anisotropy inherent in heat-resistant crystalline metals used in the aerospace and automotive industry (e.g. nickel alloys for turbine blades), the dependence on the ratio of the third independent element C_{12} of the elastic modulus matrix to C_{11} and C_{44} is added. Numerical examples illustrate the variation of radiation diagrams, energy characteristics and streamline patterns depending on C_{12}. Examples for other materials, primarily five-constant transversely isotropic composites, are also discussed.

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Guided waves and resonance effects in piezoelectric laminate structures

Glushkov E.V., **Glushkova N.V.**, Tatarkin A.A.

Institute for Mathematics, Mechanics and Informatics, Kuban State University, Krasnodar, 350040, Russian Federation

e-mail: `nvg@math.kubsu.ru`

Due to the coupling of the electromagnetic and mechanical fields, piezoelectric materials make it possible to excite traveling waves in layered elastic waveguides by an applied electric charge. This property is used in mechatronics, microelectronics, sensorics and other high-tech industries. Examples here are micro-electromechanical systems (MEMS), surface acoustic wave (SAW) devices, piezo transducers, and piezo wafer active sensors (PWAS). Of particular interest in their design is the use of specific properties and wave effects inherent in certain excited modes (quasi-longitudinal polarization, backward waves, zero group velocity (ZGV) modes, ZGV resonances, etc.). To exploit these effects, an adequate mathematical modeling of the wave processes occurring in such layered structures is required.

To compute the wavefield generated by a source in such media, software packages implementing direct mesh-based numerical methods, such as the finite element method (FEM), are commonly used. However, the FEM is usually too computationally expensive for wave simulation, and additional post-processing is required to extract specific wave modes from the total numerical solution. On the other hand, the explicit integral representation via the Green's matrix of the structure and the source parameters provides a quantitative solution, the same as the FEM simulation, while the asymptotics of these path integrals provide closed physically clear representations for the source-generated waves of various types.

This semi-analytical scheme has been implemented for piezo materials with arbitrary anisotropy, and the excited guided waves (GW) are obtained using the residue technique. In this talk we plan to illustrate this general scheme by applying it to the problems related to 1) dimond-based SAW devices [1], and 2) water/ice sensors [2]. Surface and pseudo-surface acoustic waves (PSAW) generated in AlN/Diamond and AlN/Diamond/Gamma-TiAl structures by an electric interdigital transducer are analyzed. Attention is focused on the effect of PSAW-to-SAW degeneration at certain discrete values of the film-thickness-to-wavelength ratio, which makes it possible to excite high-velocity SAWs propagating with practically negligible attenuation. In the second problem, the developed model was verified by the results obtained for the quartz/ice and quartz/water two-layer waveguides [2]. In addition, to get rid of parasitic modes arising in the bottom layer of finite thickness, corresponding two-layer half-space models of infinite depth were considered. As a special case, an interesting effect of SH backward modes caused by the piezo effect [3] is considered. Manifestations of the related ZGV resonances and mode repulsion effects are also numerically analyzed.

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Semi-analytical methods for modelling elastic wave conversion, trapping and focusing in layered elastic metamaterials with arrays of voids

Golub M.V., Fomenko S.I., Doroshenko O.V., Kozhevnikov V.V., Okoneshnikova E.A., Kanishchev K.K.

Institute for Mathematics, Mechanics and Informatics, Kuban State University, Krasnodar, 350040, Russian Federation

e-mail: m_golub@inbox.ru, sfom@yandex.ru, oldorosh@mail.ru, vvk@math.kubsu.ru, owljaneok@yandex.ru, kanishchevkir9@gmail.com

Elastic and/or acoustic metamaterials are able to exhibit unusual enhanced wave properties such as waveguiding, wave focusing and lensing, energy conversion etc. Various numerical methods have already been applied to determine the performance range of designing elastic metamaterials. Thus, it was shown that matching layers between piezoelectric active elements and substrate could increase the transducer's sensitivity by optimizing the structure of ultrasonic transducers [1]. Using elastic metamaterials as a bulk wave mode converter and to enhance the wave energy transfer, this paper proposes a novel ultrasonic transducer configuration [2], where an elastic metamaterial with voids is inserted to change the impedance and manipulate the elastic wave excitation.

In this study, several semi-analytical and purely numerical methods [3, 4] are employed to simulate and analyze wave propagation in layered elastic metamaterials with and without arrays of crack-like voids. The analysis has shown that the proposed configurations can sufficiently modify the wave energy transmission from a piezoelectric active element for various frequency ranges (relatively low frequencies as well as higher ones). Some examples of experimental validation of the advanced properties of elastic metamaterials with voids are also presented. The sensitivity of ultrasonic transducers/sensors to receive elastic waves on the surfaces of some materials is relatively low in certain frequency ranges [5], therefore, the proposed configurations can also be considered as sensors with higher sensitivity in certain frequency ranges or as demultiplexers for various kinds of elastic waves.

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Anomalously high diffraction efficiency of low-frequency sinusoidal and lamellar gratings for neutron optics

Goray L.I.
Alferov University, Khlopin str. 8/3, let. 'A', St. Petersburg, 194021, Russia;
Institute for Analytical Instrumentation, St. Petersburg, 190103 Russia;
Moscow Institute of Physics and Technology, 9 Institutskiy per., Dolgoprudny, Moscow Region, 141701, Russia;
Saint Petersburg Electrotechnical University, 5 Professor Popov str., St. Petersburg, 197022, Russia;
University under IPA of EurAsEC, St. Petersburg, 199106, Russia
e-mail: lig@pcgrate.com, ligoray@mail.ru

For the reflection and dispersion of cold and ultra-cold neutrons, low-frequency diffraction gratings with a small ratio of the neutron wavelength λ to the period $d \sim 10^{-5}$–10^{-2} are most often used in strongly grazing beams with angles $\theta \sim 87.5°$–$89.975°$ beyond limits of total external reflection. The corresponding refractive indices $n = 1 - \delta + i\beta$ of various reflective materials are $\delta \sim 10^{-7}$–10^{-3} and $\beta \sim 10^{-12}$–10^{-5} [1]. The scalar diffraction theory and the Born approximation, valid for low-frequency gratings with $\lambda/d \ll 1$ (usually less than 0.2 at near-normal incidence) and the ratio groove depth h to $d \ll 1$ (usually ~ 0.1–0.05) [2, 3], predicts optimal depths for grooves of sinusoidal, lamellar or triangular profile $h \approx \lambda/[k\cos(D/2)]$, where D is the deviation angle (between incident and diffracted beams) and $k = 4, 3.4$ or 2, respectively [4]. The corresponding values of the maximum diffraction efficiency (in the approximation of ideal conductivity of the grating material) of the minus first order, $\eta(-1)$, are determined by the values of the coefficients in the Rayleigh expansion: $\eta(-1) \approx 33.8\%$, $\eta(-1) \approx 40.5\%$ or $\eta(-1) = 100\%$, respectively.

As was previously shown by direct numerical calculations using computer programs based on rigorous differential and integral formalisms, for high-frequency gratings with grazing incidence of X-ray radiation, the known approximations do not work, since they are not able to accurately take into account the effects of absorption, shadowing, multiple reflection, etc. [2, 5]. For low-frequency gratings with typical values of $\lambda/d \sim 10^{-5}$ and $h/d \sim 10^{-3}$, until now it was believed that the scalar or perturbation theory works and the maximum η for different groove profiles cannot exceed the specified values both for X-rays and cold neutron beams. Computations of η performed in this work using two commercial electromagnetic codes valid in above mentioned range of parameters, namely PCGrate™ and GSolver™ [6], show that the maximum achievable efficiencies of sinusoidal and lamellar gratings can significantly exceed known analytical estimates. For example, for a sinusoidal grating with $d = 50$ µm, $h = 53.4$ nm, $\delta = 3.43 \cdot 10^{-5}$, $\beta = 1.65 \cdot 10^{-10}$ at $\theta = 89.72°$ and $\lambda = 1$ nm (critical angle $\theta = 89.53°$) was obtained $\eta(-1) \approx 46.8\%$ for both polarizations, i.e. 38.5% greater than the maximum scalar efficiency. For a similar lamellar grating with $d = 50$ µm, $h = 43.3$ nm, $\delta = 3.43 \cdot 10^{-5}$, $\beta = 1.65 \cdot 10^{-10}$ and duty cycle of 0.5 at $\theta = 89.73°$ and $\lambda = 1$ nm, $\eta(-1) \approx 44.5\%$, i.e. almost 10% more than the analytical limit. The data for $\eta(-1)$ in both solvers converge well and coincide with an accuracy of $\sim 0.1\%$. A physical explanation was given for this.

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An improved numerical algorithm for the wave equation optimization problems

Goray L.I.[1,2], **Dashkov A.S.**[1,2], Kostromin N.A.[2,3], Barykin D.A.[3]
[1]Alferov University, Khlopin str. 8/3, let. 'A', St. Petersburg, 194021, Russian Federation
[2]Saint Petersburg Electrotechnical University, str. Professor Popov, 5, St. Petersburg, 197022
[3]Peter the Great St. Petersburg Polytechnic University, str. Polytechnicheskaya, 29, St. Petersburg, 195251, Russian Federation
e-mail: `lig@pcgrate.com, dashkov.alexander.om@gmail.com, nik.kostromin.00@inbox.ru`

A refined algorithm for solving optimization tasks, which include the wave equation direct problems is suggested. The proposed method incorporates the accuracy of the rigorous numerical methods with the computational power of modern graphical processing units (GPUs) and the flexibility of the variety of numerical optimization techniques. The developed approach is applied to the solving of the optimization problem of the whole structure of the quantum cascade detector (QCD) device. The optimization problem is narrowed to the two maximization tasks: maximization of the efficiency of the diffraction gratings, which is used as a waveguide structure in the QCD; and maximization of the estimated responsivity of the cascade of the QCD. The initial computational problem is subdivided into two parts. The first part includes solving the direct Helmholtz equation for obtaining the efficiency of the grating (the waveguide part) and the stationary Schrodinger equation for determining the energy-band diagram of the cascade of the QCD (the active region part). The second part consists of optimization tasks, i.e. maximizations of the corresponding efficiency and responsivity. The first part of the problem is solved using conventional central processing unit (CPU) algorithms and modern computation schemes on GPU. The boundary integral equation method ([1], Ch. 12) is applied to solve the Helmholtz equation, and the symmetrical transfer matrix approach [2] is utilized to solve the stationary Schrodinger equation. One-periodical gratings with arbitrary conductivity are considered. The cascade of the QCD consists of the AlGaAs/GaAs, InGaAs/AlInAs materials with no more than 50 layers. The second part of the problem is solved using a variety of numerical optimization techniques, including simplex, gradient descent, genetic algorithm, and Bayes optimization methods. Overall the method significantly reduced the computation time and can be used to improve and enhance the efficiency and performance of the QCDs and other optical devices [3]. This work was supported by Russian Scientific Foundation (project № 23-29-00216).

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Transparent scatterers and transmission eigenvalues

Grinevich P.G.[1], **Novikov R.G.**[2,3]
[1]Steklov Mathematical Institute, RAS, 119991 Moscow, Russia
[2]CMAP, CNRS, École polytechnique, 91128 Palaiseau, France
[3]IEPT RAS, 117997 Moscow, Russia
e-mail: `grinev@mi-ras.ru, novikov@cmap.polytechnique.fr`

We give a short review of old and recent results on scatterers with transmission eigenvalues of infinite multiplicity, including transparent scatterers. Historically, these studies go back to the works of Tulio Regge (1959), Roger Newton (1962), and Pierre Sabatier (1966).

This talk is based on the publications [1–4], where our examples include potentials from the Schwartz class and multipoint potentials of Bethe–Peierls–Thomas type.

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Solutions to difference Riccati equation via continued fractions

Ivanov A.V.
Saint-Petersburg State University, 199034, Russian Federation, Saint-Petersburg, Universitetskaya nab. 7/9
e-mail: a.v.ivanov@spbu.ru

We study a difference Riccati equation

$$\Phi(x) + \rho(x)\frac{1}{\Phi(x-\omega)} = v(x), \tag{1}$$

where ρ and v are known smooth 1-periodic functions and $\omega \in \mathbb{T}^1 = \mathbb{R}/\mathbb{Z}$ is irrational.

A formal solution to this equation can be represented as a continued fraction (see e.g. [1])

$$\Phi(x) = b_0(x) + \cfrac{1}{b_1(x) + \cfrac{1}{b_2(x) + \cdots}} = b_0(x) + \mathrm{K}_{j=1}^{\infty}\frac{1}{b_j(x)}, \tag{2}$$

where $b_0(x) = v(x)$ and $b_j(x) = -\dfrac{v(x-(j+1)\omega)}{\rho(x-j\omega)}$ for $j \in \mathbb{N}$.

If continued fraction (2) converges, it defines a true solution of (1). Unfortunately, the continued fraction theory gives in general case very rough conditions sufficient for the convergence. For example, one of such condition reads (see e.g. [2])

$$|b(x)| > 2, \quad \forall\, x \in \mathbb{T}^1$$

and seems to be rather restrictive.

Denote by $\Phi_n(x) = b_0 + \mathrm{K}_{j=1}^{n}\dfrac{1}{b_j(x)}$ a finite continued fraction and introduce a transformation $A_n : \mathbb{T}^1 \to SL(2,\mathbb{R})$ as

$$A_n(x) = \begin{pmatrix} b_{n+1}(x) & 1 \\ -1 & 0 \end{pmatrix}.$$

This transformation generates a cocycle $M_n(x) = A_1(x)\cdots A_n(x)$. Then it is known (see e.g. [3]) that

$$\Phi_n(x) = b_0(x) + \frac{(M_n(x)\vec{e}_1, \vec{e}_2)}{(M_n(x)\vec{e}_1, \vec{e}_1)},$$

where $\{\vec{e}_1, \vec{e}_2\}$ is the standard basis in $\mathbb{R}^2$.

Using the critical set method ([4]) we study hyperbolic properties of the cocycle M and formulate sufficient conditions for the convergence of continued fraction (2).

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Wave processes in conducting media from the point of view of classical and extended electrodynamics

Ivanova E.A.
Higher School of Theoretical Mechanics and Mathematical Physics, Peter the Great St. Petersburg Polytechnic University, Polytechnicheskaya, 29, Saint-Petersburg, 195251, Russia;
Institute for Problems in Mechanical Engineering of the Russian Academy of Sciences, Bolshoy pr. V.O., 61, Saint-Petersburg, 199178, Russia
e-mail: `elenaivanova239@gmail.com`

Müller W.H., Rickert W.
Technische Universität Berlin, Institut für Mechanik, Fachgebiet Kontinuumsmechanik und Materialtheorie, Einsteinufer 5, 10587 Berlin, Germany
e-mail: `wolfgang.h.mueller@tu-berlin.de`

Vilchevskaya E.N.
Flugsnapparegatan 6, Mölndal, Sweden
e-mail: `vilchevska@gmail.com`

We discuss various aspects of the behavior of electromagnetic waves near the interface between a dielectric medium and a conducting medium. We focus our attention on unresolved issues, on the contradictions between experimental data and theoretical results obtained in the framework of classical electrodynamics, as well as on some conclusions repeated in many literature sources that clearly contradict common sense. In particular, we show that the experimental and theoretical definitions of plasma frequency are not in agreement with each other. We discuss the problem of experimental determination of permittivity of metals, which has not yet been resolved, and give a simple asymptotic formula expressing permittivity in terms of the reflection coefficient. We show that experiments with metals can be described in the framework of classical electrodynamics only if we use the values of conductivity that are very far from the known values. We also show that extended electrodynamics [1–3] allows us to avoid this contradiction with experimental data.

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Explicit inversion formulas for normal operators of momentum ray transforms

Shubham R. Jathar[1], Manas Kar[2], **Venky P. Krishnan**[3], Vladimir A. Sharafutdinov[4]
[1]Computational Engineering, School of Engineering Sciences, Lappeenranta-Lahti University of Technology LUT, Lappeenranta, Finland
[2]Indian Institute of Science Education and Research (IISER) Bhopal, India
[3]Centre for Applicable Mathematics, Tata Institute of Fundamental Research, India
[4]Sobolev Institute of Mathematics, 4 Koptyug Av., 630090, Novosibirsk, Russia
e-mail: shubham.jathar@lut.fi, manas@iiserb.ac.in,
vkrishnan@tifrbng.res.in, sharaf@math.nsc.ru

We study certain weighted ray transforms of symmetric tensor fields called momentum ray transforms. In the absence of weights, the ray transform reduces to the standard ray transform of symmetric tensor fields, and full inversion of the ray transform recovering the symmetric tensor field is not possible due to the presence of an infinite dimensional null space. The supplementary data in the form of higher order moments of the ray transform leads to explicit recovery of the entire tensor field. In this talk, instead of the momentum ray transforms, we will focus our attention on normal operators associated to momentum ray transforms (the composition of the transform with its formal adjoint), and introduce an approach for the explicit reconstruction of the entire symmetric tensor field from this data.

Hybrid semi-analytical method for modelling in-plane wave motion of elastic laminates with periodic and localized volumetric inhomogeneities

Khanazaryan A.D., **Golub M.V.**, Makarenko A.A.
Institute for Mathematics, Mechanics and Informatics, Kuban State University, Krasnodar, 350040, Russian Federation
e-mail: artur97.10@mail.ru, m_golub@inbox.ru, makarenko.alex02@mail.ru

In recent decades, the proportion of composite materials has significantly increased in various fields. Guided waves propagate in composites and are employed for inspecting thin-walled composite structures. On the other hand, artificially manufactured composites such as elastic metamaterials are used as components of various devices. Therefore, efficient mathematical models and algorithms are needed. Since the simulated physical processes are described by partial differential equations in various domains it is usually impossible to obtain an analytical solution. Hybrid approaches have been intensively developed recently to solve various wave problems, in which areas of complex geometry are described using purely numerical methods, and wave propagation in elongated layered structures is described using semi-analytical approaches.

In this paper, a hybrid approach based on the spectral finite element method (SFEM) [1] and the semi-analytical finite element method (SAFEM) [2–4] is proposed to study the dynamic behavior of a structure consisting of an elongated layered waveguide of finite length and an adjacent domain of dissimilar shape. The implementation of this approach in the in-plane case is described here. The SAFEM is applied to simulate wave motion in a layered waveguide. The latter is similar to normal mode expansion, whereas adjacent regions are discretized via the SFEM [5]. Then, the boundary conditions at the boundary common to the two subdomains must be satisfied. To solve the coupled problem, an auxiliary displacement function is introduced at the common interface. The unknown function is approximated via Lagrange interpolation polynomials at Gauss–Legendre–Lobatto nodal points, which are also used in the SFEM and the SAFEM. The unknowns are determined via the Galerkin method and the collocation method, and a comparison is provided. The simulation results are compared with calculations in COMSOL Multiphysics and good matching is demonstrated.

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Parabolic-equation approach in high-frequency grazing diffraction theory: from Fock to nowadays

Kiselev A.P.[1,2,3], Zlobina E.A.[1]
[1]St. Petersburg State University, St. Petersburg, Russia
[2]St. Petersburg Department of Steklov Mathematical Institute, St. Petersburg, Russia
[3]Institute for Problems in Mechanical Engineering RAS, St. Petersburg, Russia
e-mail: `kiselev@pdmi.ras.ru, ezlobina2@yandex.ru`

An overview of the progress in the theory of high-frequency diffraction under grazing incidence is given, starting with Fock's pioneering research on scattering by a smooth convex body, see [1, 2]. Diffraction by boundary inflection [3, 4], by a jump of curvature [5–8], and several other problems approachable through the parabolic-equation method, are reviewed.

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Mortar coupling between FEM and BAE with non-conforming meshes and its applications to diffraction modeling

Korolkov A.I.
Department of Mathematics, University of Manchester, Manchester, UK

Shanin A.V., **Laptev A.Y.**
Faculty of Physics, M.V.Lomonosov Moscow State University, Moscow, Russia
e-mail: `laptev97@bk.ru`

A huge number of numerical methods was developed to model acoustic wave processes. The most popular of them form two groups: finite element methods (FEM) and boundary element methods (BEM). Both of these groups have advantages and disadvantages. In [1], a singularity free boundary algebraic equation (BAE) method was introduced. In [2], a coupling between FEM and BAE was introduced. This method requires the FEM and BAE meshes to be conforming. Unfortunately, such meshes cannot be generated to the authors knowledge by available meshing software. In this work this issue is solved by allowing the FEM and BAE meshes to be non-conforming. A coupling of a non-conforming mesh is introduced with the help of mortar element method [3].

A 3D stationary exterior acoustic wave diffraction problem on a sufficiently smooth rigid scatterer is considered. Let the Helmholtz equation be satisfied in the infinite domain Ω:

$$\Delta u(\mathbf{x}) + k^2 u(\mathbf{x}) = f(\mathbf{x}), \quad \mathbf{x} \equiv (x, y, z) \in \Omega \tag{1}$$

with the Neumann boundary conditions on the scatterer boundary $\partial\Omega$:

$$\frac{\partial u(\mathbf{x})}{\partial n} = 0, \quad \mathbf{x} \in \partial\Omega. \tag{2}$$

Here $f(\mathbf{x})$ is the source function, $k = \omega/c$ is a wavenumber, and $u(\mathbf{x})$ may correspond to acoustical pressure or acoustical potential.

The domain Ω is then divided into two domains with a boundary of a simple shape. The first domain is modeled by FEM, the second is modeled by BAE method. The system (1,2) is then transformed into the following equation system that is BAE/FEM coupled formulation with non-conforming meshes:

$$\begin{pmatrix} \mathrm{Q}_{11}^{\mathrm{T}}\mathrm{L}_1\mathrm{Q}_{11} & \mathrm{Q}_{11}^{\mathrm{T}}\mathrm{L}_1\mathrm{Q}_{12} \\ \hat{\mathrm{G}}\mathrm{Q}_{12}^{\mathrm{T}}\mathrm{L}_1\mathrm{Q}_{11} & \hat{\mathrm{G}}\mathrm{Q}_{12}^{\mathrm{T}}\mathrm{L}_1\mathrm{Q}_{12} + \mathrm{G}\mathrm{L}_2 \end{pmatrix} \begin{pmatrix} \mathrm{u}_1 \\ \mathrm{u}_s \end{pmatrix} = \begin{pmatrix} \mathrm{Q}_{11}^{\mathrm{T}}\mathrm{f}_1 \\ \mathrm{G}_{\mathrm{f}}\mathrm{f}_2 \end{pmatrix}, \tag{3}$$

where u_1 and u_s are the vectors of u nodal values in interior nodes of FEM domain and in boundary nodes of BAE domain correspondingly, f_1 and f_2 are the vectors of source function values in FEM and BAE domains correspondingly, L_1 and L_2 are the Helmholtz equation approximations in the FEM domain and in the boundary nodes and neighbors to boundary nodes of the BAE domain correspondingly, $\hat{\mathrm{G}}$, G and G_{f} are matrices composed of some Green's lattice functions, Q_{11} is the projection matrix between all FEM domain nodal values and its interior nodal values, Q_{12} matrix represents the coupling between FEM and BAE domains.

Several numerical demonstrations are also provided.

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Inverse extremal problem for an anti-tumor therapy model

Kovtanyuk A.E.[1,2], Kuttler C.[1], Koshel K.O.[2], Chebotarev A.Y.[2]
[1]Technical University of Munich, Munich, Germany
[2]Far Eastern Federal University, Vladivostok, Russia
e-mail: kovtanyu@ma.tum.de

Consider the following model describing the change in the density of tumor cells under the influence of a drug:

$$S'(t) = -M(S) + u(t), \quad t \in (0, T), \tag{1}$$

$$V_t = \nabla \cdot (D\nabla V) + \delta(S)V, \quad x \in \Omega, \tag{2}$$

$$V|_\Gamma = 0, \tag{3}$$

$$S(0) = 0, \quad V(x, 0) = V_0. \tag{4}$$

Here, Ω is a bounded model domain with boundary Γ; S is the drug concentration; V is the density of viable tumor cells; $M(S)$ describes the rate of drug clearance or degradation; $\delta(S)$ is a decreasing function describing the summarized effect of tumor cell growth and their death due to drug exposure, in more details, $\delta(S) > 0$ if $S < S_m$ and $\delta(S) < 0$ if $S > S_m$, where S_m is the minimal effective level of drug concentration. The function $u = u(t)$ is a control describing the drug delivery into the domain Ω.

The *inverse extremal problem* consists in the minimization of the quality functional J defined on solutions of system (1)–(4) with the constraint $S(x) \le S_\star$ for $x \in B \subset \Omega$, that is

$$J(S, V, u) = \frac{1}{2}\int_0^T \int_D V^2(x, t)\,\mathrm{d}x\,\mathrm{d}t + \frac{\lambda}{2}\int_0^T u^2(t)\,\mathrm{d}t \to \min; \quad S(x) \le S_\star, \quad x \in B \subset \Omega. \tag{5}$$

Here, D is the subdomain of the tumor location, B is the subdomain of healthy tissue where the drug concentration is controlled (see Figure 1), $S_\star$ is the maximal level of drug concentration that is not too toxic for healthy cells, and $\lambda > 0$ is the regularization parameter.

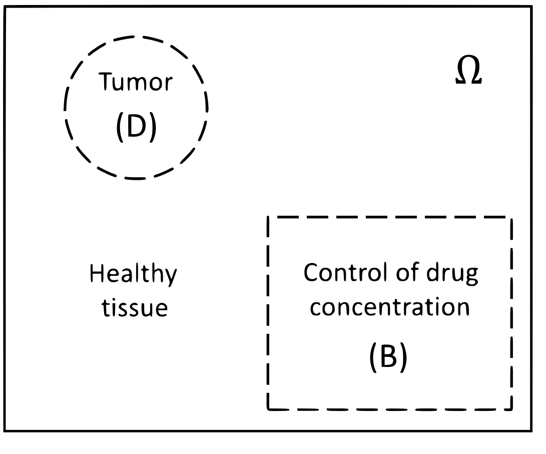

Fig. 1: Scheme of the computational domain.

The solvability of the inverse problem is proved. A numerical algorithm based on the finite element method is constructed and implemented. The convergence of the numerical algorithm is established.

Electrodynamic characteristics of a multigap loop antenna with phased excitation located on the surface of an anisotropic cylinder

Kudrin A.V.[1], **Zaitseva A.S.**[1], Zaboronkova T.M.[1,2]
[1]University of Nizhny Novgorod, 23 Gagarin Ave., Nizhny Novgorod 603022, Russia
[2]R. E. Alekseev Technical University of Nizhny Novgorod, 24 Minin St., Nizhny Novgorod 603950, Russia
e-mail: kud@rf.unn.ru, zaitseva@rf.unn.ru, t.zaboronkova@rambler.ru

Electrodynamic characteristics of loop antennas located in the vicinity or on the surface of anisotropic cylindrical structures attract increased interest because of numerous applications of such systems [1, 2]. Despite this fact, only a few theoretical works present closed-form solutions of the related problems in view of their complicatedness. As practically important examples of such structures, an axially magnetized plasma cylinder and an anisotropic dielectric or metamaterial cylinder can be mentioned. The magnetized plasma generally possesses both anisotropic and gyrotropic properties. However, it becomes a uniaxially anisotropic medium without gyrotropic properties in the limiting case of an infinitely strong external magnetic field. Therefore, the characteristics of antennas in the presence of cylindrical structures filled with the above-mentioned media can be analyzed using a common approach. Special attention should be paid to resonant media whose dielectric tensors have diagonal elements of opposite sign. Then the refractive index surface for one of the normal waves of such media has unbounded branches corresponding to vanishingly short wavelengths [1].

In the present work, we employ an integral equation method to find the current distribution of a circular loop antenna having the form of a perfectly conducting narrow strip which is coiled into a ring and excited by phased voltages applied to several gaps of the antenna conductor. The antenna is located coaxially on the surface of an infinitely long anisotropic cylinder surrounded by a homogeneous isotropic medium. The anisotropy axis of the medium filling this cylindrical structure is aligned with its symmetry axis, being parallel to an external static magnetic field in the case of a magnetized plasma inside the cylinder. We derive integral equations for azimuthal harmonics of the antenna current and find the singular parts of the kernels of these equations in explicit form. The emphasis is placed on the case of a resonant medium inside the cylinder. It turns out that in both the presence and absence of gyrotropic properties of this medium, the singular parts of the kernels are identical, whereas their regular parts are different. The latter parts can be neglected, as a first approximation, if the strip conductor of the antenna is sufficiently narrow. This allows us to solve the integral equations analytically and derive closed-form expressions for the current distribution of the antenna. Based on the solution for the current, we also determine the partial powers radiated from the antenna into guided and unguided waves of the cylinder, which have different azimuthal indices. It is established that, by appropriately choosing the phases of the voltages applied across the feeding gaps of the antenna, as well as their number and location, one can efficiently control the distribution of the radiated power over the azimuthal harmonics of the excited field.

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PINN method and solution of inverse problems of complex heat transfer

Kuznetsov K.S., Amosova E.V., Afanasyeva S.D., Dranov S.V.
Institute of Applied Mathematics FEB RAS, Radio str. 7, Vladivostok, 690041, Russia;
Far Eastern Federal University, Ajax Bay 10, Russky Island, Vladivostok, 690922, Russia
e-mail: kuznetsovks17@gmail.com, el_amosova@mail.ru

The Physics Informed Neural Networks (PINN) method was proposed in [1] and has become popular in recent years for solving inverse problems. The method is based on approximation of unknown functions using neural networks by minimizing quadratic functional, which includes terms for residual of equations, boundary and initial conditions, and, in case of the inverse problem, additional information. Classic machine learning method to solve complex heat transfer optimal control problem was considered in [2]. Compared to the the other methods, PINN has shown more potential in solving inverse problems.

Unsteady complex heat transfer in a two-dimensional Ω region is described by a system of differential equations consisting of the heat equation and the quasi-stationary P_1 approximation of the radiative transfer equation [3]:

$$\partial\theta/\partial t - a\Delta\theta + b\kappa_a(|\theta|\theta^3 - \varphi) = 0, \tag{1}$$

$$-\alpha\Delta\varphi + \kappa_a(\varphi - |\theta|\theta^3) = 0, \quad x \in \Omega, \quad t \in (0, T), \tag{2}$$

where θ is the normalized temperature, φ is the normalized radiant intensity averaged over all directions.

Boundary and initial conditions have the following form:

$$\theta|_{t=0} = \theta_0, \quad x \in \Omega, \tag{3}$$

$$a\partial_n\theta + \beta(\theta - \theta_b) = 0, \quad \alpha\partial_n\varphi + \gamma(\varphi - \theta_b^4) = 0, \quad x \in \Gamma, \quad t \in (0, T), \tag{4}$$

Various problems of controlling the boundary coefficients γ, β are considered by minimizing the functional J using the PINN method:

$$J[\gamma, \beta] = \int_0^T \int_\Omega [\theta - \theta_d]^2 \,\mathrm{d}\Omega\,\mathrm{d}T \quad \to \inf, \tag{5}$$

where θ_d is the given function, $\gamma = \gamma(x, t)$, $\beta = \beta(x, t)$ are boundary control functions.

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Optical vortices in multihelicoidal fibers with torsional mechanic stress

Lapin B.P., Barshak E.V., Yavorsky M.A., Alexeyev C.N.
V. I. Vernadsky Crimean Federal University, Vernadsky Prospekt, 4, Simferopol, 295007, Crimea, Russian Federation
e-mail: lapinboris@gmail.com

We study the structure of higher order modes in multihelicoidal optical fibers (MHFs) in the presence of torsional mechanic stress (TMS). We show that at some values of pitch such modes present circularly polarized non-degenerate optical vortices robust to external perturbations of the cross section's form. Basing on analytical expressions for propagation constants of such vortex modes we investigate polarization, topological and hybrid dispersions of vortex modes. We also demonstrate that based on MHF with TMS one can implement a universal CCNOT logic gate (also known as "Toffoli gate"). This property could be useful for classical emulation of quantum computations.

History of diffraction in natural and synthetic opals

Limonov M.F., Samusev K.B.
Ioffe Institute, 194021, St. Petersburg, ITMO University, Department of Physics and Engineering, 191002, St. Petersburg, Russia, Russia
e-mail: m.limonov@mail.ioffe.ru, k.samusev@mail.ioffe.ru

We present an overview of optical diffraction studies in natural and synthetic opals, as well as in various opal-like photonic structures. Our story covers a wide geography: from the underground settlement Coober Pedy of opal miners in the Australian desert to the country house of Prof. Alexander Kaplyanskii and his apple orchard located near St. Petersburg. This story begins thirty years ago, when A. Kaplyanskii listened to a plenary paper by E. Yablonovitch about photonic crystals [1] and immediately realized that the opals that adorn the earrings and beads of the fair sex are nothing more than natural photonic crystals. A certain line under these studies is summed up by the fundamental book "Optical properties of photonic structures: interplay of order and disorder", published by CRC Press in 2012 [2]. Photonic crystals are structures formed by dielectric elements periodically located in space, so light is not simply reflected from them, but diffracts with a frequency determined by the period of these elements. A classic example is butterflies with intricate patterns on their wings, which are covered with colorless scales with a size in the visible light range. Opals are built up of quasi-spherical particles of amorphous silica a-SiO_2, each particle has a fairly hard shell and a porous core, and the particle sizes cover the visible and are in the range of 0.3–1 microns. Ideally grown synthetic opals have a face-centered cubic (FCC) lattice and are formed by a-SiO_2 particles with a very small size variation. Accordingly, they determine optical diffraction patterns analogous to the diffraction patterns of x-rays from ordered three-dimensional crystals. Disordered natural opals are formed by a-SiO_2 particles of different sizes, so they create an amazing play of light and are highly prized by jewelers.

The pioneering works in which it was first written that "opals are three-dimensional photonic crystals for visible light" are the publications of A. Kaplyanskii and his colleagues from Ioffe Institute, St. Petersburg [3, 4]. This team has published many papers on the study of opals and opal-like structures, we present only a small part [3–11], including an article in Nature [6], which has been cited more than 1500 times to date. Diffraction experiments made it possible to determine the orientation of the opal sample, its homogeneity, lattice parameters and the structure of the a-SiO_2 particle itself with a thin hard shell and a porous core.

To draw attention to the problems of opal inhomogeneity, Prof. Kaplyansky used apples from his garden, placing them in a pyramid with an FCC lattice. Indeed, through diffraction studies of opals, it has been demonstrated that photonic crystals comprised of inhomogeneous components bring new opportunities to photonics due to the discovered quasiperiodic resonant behavior of their

(hkl) photonic stop bands as a function of the reciprocal lattice vector. A resonant stop band cannot be switched off for any permittivity of structural components. Tuning the permittivity or structural parameters allows the selective on-off switching of nonresonant (hkl) stop bands. This independent manipulation of light at different Bragg wavelengths provides a new degree of freedom to design selective optical switches and waveguides [11].

The demonstration of Bragg photonic effects in synthetic and natural opals a-SiO_2 was a powerful impetus for the search and creation of new objects formed by spherical particles with new optical and transport properties. In particular, we note the observation of multiple Bragg wave coupling in photonic fcc crystals consisting of air spheres in titania TiO_2 [12]. An intriguing change in the symmetry of the diffraction pattern was observed when changing the refractive index contrast of the photonic crystal [13]. Next, inverted opal structures were created and studied [14, 15]. It was noted that the carbon inverse opals provide examples of both dielectric and metallic optical photonic crystals. They strongly diffract light and may provide a route toward photonic band-gap materials [14].

A new page in the history of opals has become photonic glasses — disordered materials obtained using a modified self-assembly technology from polymer microspheres. These random materials are solid thin films which exhibit rich novel light diffusion properties originating from the optical properties of their building blocks [16–18]. Another page is an experimental and theoretical study of disordered optical materials, called Lévy glasses, in which light transfer is superdiffusive [19, 20]. Lévy glasses can be made with ease by embedding transparent spheres with diameters varying by orders of magnitude in a diffusive medium. Lévy glasses offer the opportunity to investigate superdiffusive transport processes in a laboratory with light.

We believe that these are far from the last pages in the study of the optical, in particular diffraction, properties of various phononic structures, which began with jewel opals.

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Explicit solution to Birman's problem for Schrödinger operators

Mark Malamud
RUDN University, Moscow
e-mail: malamud3m@gmail.com

In early 2000s M. S. Birman posed the following problem:

Problem. *Let A be a closed non-negative symmetric densely defined differential operator in a Hilbert space $\mathfrak{H}$ and let $\mathfrak{H}_1 := \mathrm{ran}(A+I)$. Assume that the operator $(A+I)^{-1} : \mathfrak{H}_1 \to \mathfrak{H}$ is compact. Is it true that the resolvent of the Friedrichs' extension A_F of A is also compact?*

We will discuss a solution to this Birman problem for certain restrictions of Schrödinger operators $H(q) = -\Delta + q \geq 0$ in $\mathbb{R}^n$ with $\mathrm{dom}(-\Delta) = W^{2,2}(\mathbb{R}^n)$.

Namely, for any sequence $Y = \{Y_k\}_1^\infty$ of disjoint surfaces of dimension $n-2$ in $\mathbb{R}^n$, define the restriction $-\Delta_Y$ of the Laplace operator $-\Delta$ to the domain

$$\mathrm{dom}(\Delta_Y) = W_Y^{2,2}(\mathbb{R}^n) := \{u \in W^{2,2}(\mathbb{R}^n) : u \restriction Y_k = 0,\ k \in \mathbb{N}\}. \tag{1}$$

Starting with such a set Y, we determine minimal symmetric Schrödinger operator $A = H_Y(q) \geq 0$ being a restriction of $H(q)$ to the domain $\mathrm{dom} A = \mathrm{dom}(H_Y(q)) = \mathrm{dom}(\Delta_Y)$.

These restrictions meet the following properties:

1. The inverse operator $(A+I)^{-1}$ is compact;

2. Under the additional assumption that Y has zero $(1,2)$-capacity, $C_{1,2}(Y) = 0$, and a certain assumption on a potential $q \geq 0$, the Friedrichs extension A_F of A has continuous (sometimes absolutely continuous) spectrum filling the whole semiaxes $\mathbb{R}_+$.

Next, assuming A to be positive definite we define the reduced Krein extension A'_K by setting $A'_K := A_K \restriction \mathfrak{H}_0$, where $\mathfrak{H}_0 := \operatorname{ran} A$ and P_0 is the orthoprojection in $\mathfrak{H}$ onto $\mathfrak{H}_0$.

The solution to the Birman problem shows that the Friedrichs extension A_F of A does not inherit the compactness property of $(I_{\mathfrak{H}} + A)^{-1}$. In opposite to A_F, the reduced Krein extension A'_K inherits the compactness property of $(I_{\mathfrak{H}} + A)^{-1}$. Moreover, the following result holds.

Theorem. Let $0 \in \widehat{\rho}(A)$. Then for any two-sided ideal $\mathfrak{S}$ in $\mathcal{B}(\mathfrak{H})$ the following equivalence is valid

$$P_0(A)^{-1} \in \mathfrak{S}(\mathfrak{H}_0) \Longleftrightarrow (A'_K)^{-1} \in \mathfrak{S}(\mathfrak{H}_0). \tag{2}$$

As a byproduct of the above construction of the operators $A = H_Y(q)$ gives surprising explicit examples of symmetric Schrödinger operators whose squares A^2 are densely defined nonnegative symmetric operators with the following properties:

(i) The Friedrichs' extension $(A^2)_F$ of the operator A^2 is A^*A and its spectrum is discrete, i.e. the inverse operator $((A^2)_F)^{-1}$ is compact;

(ii) The operator $(A_F)^2$ is not discrete. Moreover, its spectrum is

$$\sigma((A_F)^2) = \sigma_{\text{ess}}((A_F)^2) = [0, \infty).$$

Partially the results of the talk are announced in [1, 2].

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Fractional diffusion-wave modification of Landau–Khalatnikov model applied to polarization switching in ferroelectric nanowires

Maslovskaya A.G., Moroz L.I.
Amur State University, Ignatyevskoe Shosse 21, 675027 Blagoveshchensk
e-mail: `maslovskayaag@mail.ru, lubovep@mail.ru`

In current decades, special attention is paid to research of ferroelectric nanostructures: thin films, nanowires, nanotubes, nanorings, etc. The ferroelectric nanomaterials are promising materials in nanoelectronics such as side-gated NW photodetectors, high-density nonvolatile memories, piezoelectric transducers and actuators. Many researchers have noted that the reduction in size is crucial to the behavior of ferroelectrics and disappearance of their features at a critical size [1]. With the increasing demand for the miniaturization of ferroelectric electronic devices, it is important to know polarization behavior in low-dimensional materials. The Landau–Ginzburg theory provides a fundamental approach to describing polarization switching processes and characteristics of phase transitions in ferroelectrics. To take into account the memory effect (the proper time nonlocality) in ferroelectrics, we can introduce a time-fractional subdiffusion modification of the Landau–Khalatnikov equation [2]. Furthermore, to describe polarization fluctuations in low-dimensional ferroelectric structures, the wave analogue of the Landau–Khalatnikov equation becomes more relevant. The research was undertaken to develop an effective numerical scheme for implementation of a time-fractional diffusion-wave modification of the Landau–Khalatnikov model and apply computational techniques to simulate polarization switching in ferroelectric low-dimensional structures.

In the present study we consider a ferroelectric nanowire with a first-order phase transition. Here we suppose 2D geometry of the model of nanowire with a radius R and infinite length. In this case, we can use the cylindrical coordinates with z axis along the wire axis and the polarization distribution is axisymmetric. The time-fractional modification of the diffusion-wave Landau–Khalatnikov model is given as follows

$$\xi \frac{\partial^\eta P}{t^* \partial \tau^\eta} = \psi \frac{\partial^2 P}{\partial r^2} + \frac{\psi}{r}\frac{\partial P}{\partial r} + \alpha P + \beta P^3 - \gamma P^5 + E, \quad 0 < r < R, \quad 0 < \tau < \tau_{ob}, \tag{1}$$

$$P\big|_{\tau=0} = P_0, \quad 0 \le r \le R, \tag{2}$$

$$P\big|_{r=0} = 0, \quad \left.\frac{\partial P}{\partial r}\right|_{r=R} = -\frac{P}{\lambda}, \quad 0 \le \tau \le \tau_{ob}, \tag{3}$$

where $P(r,\tau)$ is the space-temporal distribution of polarization; α, β, γ, ψ, ξ are the positive constants; η is the order of the time-fractional Caputo derivative, $0 < \eta < 2$, $\eta \neq 1$, $\tau = t/t^*$ is the dimensionless time; t^* is the characteristic time of the process; λ is the extrapolated length.

To solve the problem (1)–(3) numerically, we derived an implicit finite-difference scheme based on approximation Caputo derivative and combine it with an iterative procedure. The program application was designed in Matlab to perform a series of computational experiments for $BaTiO_3$ nanowire.

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On elastic waves in topographic waveguides

Andrey Matskovskiy, **German Zavorokhin**
St. Petersburg Department of Steklov Mathematical Institute, Fontanka emb., 27, St. Petersburg 191023, Russia
e-mail: `amatskovskiy@mail.ru, zavorokhin@pdmi.ras.ru`

The existence of elastic waves with displacements localized at the tip with aperture angle less than $\frac{\pi}{2}$ of isotropic topographic waveguides was rigorously proven by V. M. Babich in [1]. This proof, which is based on a variational approach, is extended to rectangular topographic waveguides. We supplement the V. M. Babich result and prove the existence of localized elastic waves at the rectangular tip of topographic waveguides with different shapes of transverse cross sections.

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Non-uniqueness in the water wave problem for partially submerged bodies and an ice plate floating on the surface

Oleg V. Motygin
Laboratory for Mathematical Modelling of Wave Phenomena, Institute for Problems in Mechanical Engineering, Russian Academy of Sciences, V.O., Bol'shoy pr. 61, 199178 St. Petersburg, Russia
e-mail: `o.v.motygin@gmail.com, mov@ipme.ru`

In this work we study a two-dimensional linear problem describing time-harmonic oscillations of an ideal incompressible fluid of infinite depth in the presence of partially submerged fixed bodies and an ice plate of finite size covering part of the fluid's surface. The ice is assumed to behave like a thin elastic plate. The motion of fluid and ice is described in terms of a velocity potential $\mathrm{Re}\{u(x,y)\mathrm{e}^{-\mathrm{i}\omega t}\}$, where ω is the circular frequency.

We denote by B the submerged part of bodies, the domain occupied by fluid is $W = \mathbb{R}^2_- \setminus \overline{B}$, where $\mathbb{R}^2_- = \{(x,y) : x \in \mathbb{R},\, y < 0\}$ (the y-axis is directed upwards). Besides, the ice plate is defined as $P = \{(x,y) : 0 < x < L,\, y = 0\}$ and $F = \partial\mathbb{R}^2_- \setminus \left(\overline{P} \cup \overline{B}\right)$ is the free surface.

Consider the following set of conditions for $u : W \mapsto \mathbb{C}$ (see, e.g., [1] and references therein):

$$-\nabla^2 u = 0, \quad (x,y) \in W, \tag{1}$$

$$\partial_y u - \nu u = 0, \quad (x,y) \in F, \tag{2}$$

$$\alpha\partial_x^4\partial_y u + \beta\partial_y u - \nu u = 0, \quad (x,y) \in P, \tag{3}$$

$$\partial_{\vec{n}} u = f, \quad (x,y) \in S = \partial B \cap \overline{W}, \tag{4}$$

where $\nu = \omega^2/g$, $g \approx 9.8\ \mathrm{ms}^{-2}$ is the acceleration due to gravity. The third condition (on the ice-covered part of boundary) includes the parameters $\alpha = \frac{Ed^3}{12\rho g(1-\mu^2)}$ and $\beta = 1 - \frac{\rho_+}{\rho}\nu d$, where E is Young's modulus, μ is Poisson's ratio, ρ is the density of fluid, ρ_+ is the density of the plate, d is its thickness. Besides, the free-edge conditions (vanishing the bending moment and the crosscutting force)

$$\partial_x^n \partial_y u\big|_{y=0} \to 0 \ \text{ as } \ x \to +0, \quad x \to L - 0, \quad n = 2, 3, \tag{5}$$

must be satisfied by a solution at the points separating the free and the ice-covered parts. It is assumed that $|\nabla u| \to 0$ as $y \to -\infty$. We also need the radiation condition guaranteeing that the wave field at infinity consists of outgoing waves ($u \sim C_\pm \mathrm{e}^{\nu y}\mathrm{e}^{\mathrm{i}\nu|x|}$ as $x \to \pm\infty$):

$$\partial_{|x|} u - \mathrm{i}\nu u \to 0 \ \text{ as } \ |x| \to \infty. \tag{6}$$

In the Neumann condition on the wetted part of bodies' surface, we will fix $f = 0$ because we intend to find solutions of the homogeneous problem. We prove the following statement.

A solution of the homogeneous ($f = 0$) *boundary-value problem* (1)–(6) *is localized, i.e. both* u *and* $|\nabla u|$ *decay to zero as* $|x + \mathrm{i}y| \to \infty$.

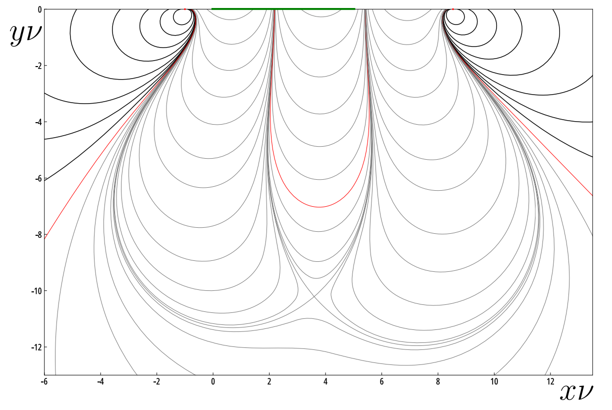
For construction of the localized solutions (also known as trapped modes), we use a semi-inverse procedure, where we fix a potential satisfying all conditions except the Neumann one and then use this condition to define bodies' geometry (see [2, 3]). Developing the procedure suggested in [5], the potential is constructed as a combination of singular potentials [4] — Green's function of the problem (1)–(6) $G(x,y;\xi,\eta)$, satisfying equation $-\nabla^2_{x,y}G = \delta(x-\xi)\delta(y-\eta)$ for $(x,y) \in \mathbb{R}^2_-$, where $\delta(\cdot)$ is Dirac's delta-function, and x-dipole $\mathfrak{X}(x,y;\xi,\eta)$, subject to $-\nabla^2_{x,y}\mathfrak{X} = \delta'(x-\xi)\delta(y-\eta)$ for $(x,y) \in \mathbb{R}^2_-$.

Namely, we look for a potential with singularities located at two points $(r_1, 0)$ and $(r_2, 0)$ on $\partial\mathbb{R}^2_-$:

$$u_*(x,y) = a_1 G(x,y;r_1,0) + b_1\mathfrak{X}(x,y;r_1,0) + a_2 G(x,y;r_2,0) + b_2\mathfrak{X}(x,y;r_2,0).$$

For a given r_1, we elaborate a procedure which allows us to determine r_2 and the coefficients $a_{1,2} \in \mathbb{C}$, $b_{1,2} \in \mathbb{R}$ in such a way that the potential u_* has no waves propagating to infinity. Then, we define $u(x,y) = \mathrm{Re}\{u_*(x,y)\}$, which satisfies (1) in $\mathbb{R}^2_-$, (2) on $\partial\mathbb{R}^2_- \setminus P$, (3) on P, (5) and (6). Now we should find a geometry of S such that the homogeneous Neumann condition (4) is satisfied and S contains the singularities inside, separating them from the outer (fluid) domain.

It is not difficult to write a stream function $v(x,y)$ — harmonically conjugate to $u(x,y)$. On its streamlines, defined as $v(x,y) = \mathrm{const}$, in view of Cauchy–Riemann equations, we have $\partial_{\vec{n}} u = 0$. So, if streamlines are found that connect free surface on different sides of the singularity $(r_1, 0)$ or $(r_2, 0)$, then this curve can be fixed as body's contour. Taking into account local behaviour of x-dipoles and the fact that $b_{1,2} \in \mathbb{R}$ by construction, arguments of [5] allow us to prove rigorously the existence of streamlines enclosing each of the singularities. A picture of streamlines is shown in figure —



this example is constructed for $d\nu = 0.01$, $L\nu = 5$, $\alpha\nu^4 = 0.056066$, $\beta = 0.99100$ (corresponding to parameters of ice $E = 6$ GPa, $\mu = 0.3$). The nodal lines $v = 0$ are shown in red. Bold lines present two families of streamlines enclosing the singularities at $(r_1, 0)$ and $(r_2, 0)$, where $r_1\nu = -1$, $r_2\nu \approx 8.5341$. In these two families, any bold line can be fixed as a contour of body; two such curves constitute the set S — for this geometry u is a solution to the homogeneous problem (1)–(6).

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Relaxation tensor for extended Burgers model

Gen Nakamura
Dept. Math. and Research Institute of Electronic Science, Hokkaido University, Japan
e-mail: gnaka@math.sci.hokudai.ac.jp

We provide a new method for deriving the anisotropic relaxation tensor and its exponentially decaying property for the extended Burgers model (abbreviated by EBM), which is an important model in rheology, and Earth and planetary sciences. Upon having this tensor, the EBM can be converted to a Boltzmann-type viscoelastic system of equations (abbreviated by BVS). Historically, the relaxation tensor for the EBM is derived by solving the constitutive equation using the Laplace transform. We refer to this by L-method. Since inverting the inverse Laplace transform needs the partial fraction expansion, the L-method needs to assume the commutativity for the elastic tensors of the EBM. This is a very strong assumption. The new method not only avoids using this assumption but also can derive several important properties of the relaxation tensor, which are the positivity, C^∞-smoothness with respect to the time variable, the exponentially decaying property together with its derivative, and the causality. Further, we will show that for the initial boundary value problem for the BVS with homogeneous boundary data and source term, any solution to this problem decays exponentially with respect to time as it goes to infinity.

Asymptotics of spectrum of the Dirichlet problem with a row of small concentrated masses

Nazarov S.A.
Institute for Problems in Mechanical Engineering of the Russian Academy of Sciences, 199178, Saint-Petersburg, V.O., Bol'shoy avenue, 61
e-mail: srgnazarov@yahoo.co.uk

In the domain $\Omega = \{x = (x_1, x_2, x_3) \in \mathbb{R}^3 : z = x_3 \in (-1,1), |\varphi| < \alpha, r < R(\varphi, z)\}$ we consider the spectral problem about a row of small concentrated masses between walls of a dihedral angle

$$-\Delta_x u^\varepsilon(x) = \lambda^\varepsilon\big(\rho + \varepsilon^{-3} X^\varepsilon(x)\big)u^\varepsilon(x),\ x \in \Omega, \quad u^\varepsilon(x) = 0,\ x \in \partial\Omega. \tag{1}$$

Here, (r, φ, z) is the cylindrical coordinate system, $R \in C^1([-\alpha,\alpha] \times [-1,1])$ is a positive profile function, $\varepsilon = (1+2N)^{-1}$ is a small parameter and $N \in \mathbb{N} := \{1,2,3,\dots\}$ is a big number while X^ε is the set function of the union of small inclusions $\omega_k^\varepsilon = \{x : (\varepsilon^{-1}x_1, \varepsilon^{-1}x_2, \varepsilon^{-1}x_3 - k) \in \omega\}$, $k = -N, \dots, N$, where $\omega \in \mathbb{R}^3$ is a domain with a piecewise smooth boundary $\partial\omega$ and the compact closure $\overline{\omega} = \omega \cup \partial\omega \subset \Xi$. The layer sector Ξ is given by formula $\{\xi : \rho > 0, |\varphi| < \alpha, |\zeta| < 1/2\}$, where $\rho = \varepsilon^{-1} r$ and $\zeta = \varepsilon^{-1} z$. It has the bases $\Sigma_\pm = \Sigma \times \{\pm 1/2\}$ and the lateral side $\Upsilon = \partial\Xi \setminus \big(\overline{\Sigma_+} \cup \overline{\Sigma_-}\big)$, where $\Sigma = \{\eta = (\xi_1, \xi_2) : \rho > 0, |\varphi| < \alpha\}$ is a planar angle of opening $2\alpha \in (0, 2\pi]$.

The first eigenvalue $\mu_1 > 0$ of the auxiliary problem in the layer sector

$$-\Delta_\xi w(\xi) = 0,\ \xi \in \Xi \setminus \overline{\omega}, \quad -\Delta_\xi w(\xi) = \mu w(\xi),\ \xi \in \omega, \quad w(\xi) = 0,\ \xi \in \Upsilon,$$
$$w\Big(\eta, +\frac{1}{2}\Big) = w\Big(\eta, +\frac{1}{2}\Big),\ \frac{\partial w}{\partial \zeta}\Big(\eta, +\frac{1}{2}\Big) = \frac{\partial w}{\partial \zeta}\Big(\eta, +\frac{1}{2}\Big),\ \eta \in \Sigma,$$

is simple and the corresponding eigenfunction w_1, which is normalized in $L^2(\omega)$ and fixed positive in $\Xi \cup \Sigma_+ \cup \Sigma_-$, behaves at infinity as $K_1 \rho^{-\pi/2\alpha} \cos(\pi\varphi/2\alpha) + O(\rho^{-\pi/\alpha})$, where $K_1 > 0$.

Asymptotic expansions of the eigenvalues

$$0 < \lambda_1^\varepsilon < \lambda_2^\varepsilon \le \lambda_3^\varepsilon \le \dots \le \lambda_p^\varepsilon \le \dots \to +\infty$$

of problem (1) crucially depends on the opening 2α.

If $\alpha \in (0, \pi/2)$, then

$$\lambda_p^\varepsilon = \varepsilon\big(\mu_1 + \varepsilon^2 \beta_p + O\big(\varepsilon^{\min\{3, \pi/\alpha\}}\big)\big),$$

where $\{\beta_p\}_{p\in\mathbb{N}}$ is the ordered sequence of eigenvalues of the Dirichlet problem for an ordinary differential operator $-Bd^2/dz^2$ in the interval $(-1,1) \ni z$ and $B = \|w_1; L^2(\Xi)\|^2$.

If $\alpha \in (\pi/2, \pi]$, the eigenvalue asymptotics becomes

$$\lambda_p^\varepsilon = \varepsilon\big(\mu_1 + \varepsilon^{\pi/\alpha}\beta_p + O\big(\varepsilon^{\min\{2, 3\pi/2\alpha\}}\big)\big)$$

but $\{\beta_p\}_{p\in\mathbb{N}}$ is the eigenvalue sequence for the Dirichlet problem in the interval $(-1,1)$ for a certain classical pseudo-differential operator with the main symbol $M|\theta|^{\pi/\alpha}$, where θ is the dual variable of the Fourier transform $\mathcal{F}_{z\to\theta}$ on the applicate axis while the coefficient $M > 0$ is proportional to K_1^2.

Finally, for $\alpha = \pi/2$, the asymptotics takes the form

$$\lambda_p^\varepsilon = \varepsilon\big(\mu_1 + \varepsilon^2 |\ln\varepsilon| \beta_p + O\big(\varepsilon^2\big)\big),$$

and again involves eigenvalues of the Dirichlet problem for an ordinary differential operator.

Asymptotics of eigenfunctions u_p^ε in all three cases is derived as well.

The problem (1) belongs to the class of problems on concentrated masses and uses technics of boundary homogenization. Vast literature in these topics will be given during the talk.

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Some solutions of nonlinear wave equations with first derivatives

Pavlov Yu.V.
Institute for Problems in Mechanical Engineering of the Russian Academy of Sciences, Saint Petersburg, Russia
e-mail: yuri.pavlov@mail.ru

Gai'kov A.L.
Mikhailovsky Military Artillery Academy, Saint Petersburg, Russia
e-mail: 2304andy@mail.ru

In this work we consider methods for solving the equations

$$\sum_{i=1}^{n} \frac{\partial^2 U}{\partial x_i^2} - \frac{1}{c^2}\frac{\partial^2 U}{\partial t^2} + \sum_{i=0}^{n} \lambda_i \frac{\partial U}{\partial x_i} = p(\mathbf{x}, t)\, F(U), \tag{1}$$

where c and $\lambda_0, \dots, \lambda_n$ are constants, $x_0 = ct$, $F(U)$ is some given (in general nonlinear) function of U and $p(\mathbf{x}, t)$ is some function of variables $\mathbf{x} = (x_1, \dots, x_n)$ and t. If $\lambda_0 = \ldots = \lambda_n = 0$, $p = 1$ and $F(U) = \sin U$, then equation (1) is the known sine-Gordon equation, for variable $p(\mathbf{x}, t)$ is the sine-Gordon equation with variable amplitude, for $F(U) = \exp U$ — the Liouville equation, for $F(U) = \sinh U$ — the sinh-Gordon equation. Equations with first derivatives and variable amplitudes $p(\mathbf{x}, t)$ appear in many areas of mechanics and physics, for example, in the theory of liquid crystals [1], in the theory of wave propagation in inhomogeneous media or when taking into account wave dissipation.

The first derivatives can be eliminated in the equation (1) using the standard substitution, but in the case of a nonlinear function $F(U)$, the equation (1) will not be simplified. Here we generalize the method of functionally invariant solutions of wave equation [2], previously used for the sine-Gordon equation [3] and non-autonomous nonlinear wave equation [4], to the case of a nonlinear wave equation with first derivatives. In this work, new exact solutions of nonlinear equations of type (1) are obtained, their plots are presented and a set of solutions, which can be found in this way, is analyzed.

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Asymptotic normal modes for the Schrödinger-type equation with two-scale time dependence of the Hamiltonian

Perel M.V.
Department of Higher Mathematics and Mathematical Physics, Saint Petersburg State University, 7/9 Universitetskaya Emb., St. Petersburg 199034, Russia
e-mail: m.perel@spbu.ru

Different problems of wave propagation in nonhomogeneous media can be studied after reduction of the governing equations to the Schrödinger-type equation, see [1–5]. We obtain asymptotic normal waves for this equation in the case of two-scale inhomogeneity

$$\mathbf{K}\Big(\varepsilon x, \frac{x}{\varepsilon}\Big)\Psi = -i\mathbf{\Gamma}\frac{\partial\Psi}{\partial x}, \tag{1}$$

where $\mathbf{K}$ ia a selfadjoint operator in some Hilbert space, $\mathbf{\Gamma}$ is a self-adjoint matrix, Ψ is a vector function. The example is the Dirac equations in 2D [1], describing graphene in the external electric and magnetic fields dependent on the variable x. Equations of the same type can be obtained by studying elastic waves in a plane inhomogeneous waveguide resting on the Winkler foundation. In this case, the operator $\mathbf{K}$ is a matrix differential operator with respect to the distance across the waveguide, and this operator depends on x, i.e., on the distance along the waveguide axis, as a parameter. In both cases, $\mathbf{\Gamma}$ is a matrix. We assume that the operator $\mathbf{K}$ is periodic with respect to the fast variable τ:

$$\mathbf{K}(\rho, \tau + 1) = \mathbf{K}(\rho, \tau), \ \ \rho = \varepsilon x, \ \ \tau = \frac{x}{\varepsilon}. \tag{2}$$

We construct formal asymptotic solutions as

$$\Psi = \exp\left(i\int^{\rho}\frac{\beta(\rho')}{\varepsilon}\,d\rho'\right)\sum_{j=0}^{\infty}\varepsilon^j\Phi^{(j)}(\rho,\tau), \qquad \Phi^{(j)}(\rho,\tau+1) = \Phi^{(j)}(\rho,\tau). \tag{3}$$

The correction term to the Berry phase is found.

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On a variant of Acoustic Imaging based on the Boundary Control Method

L. Pestov
Immanuel Kant Baltic Federal University, Kaliningrad, Nevskogo 14, 236016, Russia
e-mail: `lpestov@kantiana.ru`

We propose a regularized variant of Acoustic Imaging based on the Boundary Control method and Singular Value Decomposition. Some results of numerical testing are presented.

Ray chaos and its short wave footprints — old and new paradigms

Innocenzo M. Pinto
Retired Professor, University of Naples Federico II, INFN, LVK, Italy
e-mail: `pinto@sa.infn.it`

Electromagnetic boundary value problems exhibiting exponential divergence of nearby-incident scattered rays have been known since long. Their short-wave asymptotic properties have also been investigated by several Authors, paralleling the development of so called Quantum Chaology. In this communication we review the subject, including original work from the Author's group, with an eye to possible applications.

Isospectral quantum graphs

Mats-Erik Pistol
Solid State Physics, Lund University, Sweden
e-mail: mats-erik.pistol@ftf.lth.se

Quantum graphs are defined by having a Laplacian defined on the edges of a metric graph with boundary conditions (typically Neumann–Kirchhoff) on each vertex such that the resulting operator is self-adjoint. There are few known examples of pairs of non-isomorphic but isospectral quantum graphs. We have found all sets of isospectral but non-isomorphic equilateral connected quantum graphs with at most nine vertices. This includes thirteen isospectral triplets and one isospectral set of four. One set has the loop as a member. We also present several different combinatorial methods to generate arbitrarily large sets of isospectral graphs, including infinite graphs in different dimensions. As part of this we have found a method to determine if two vertices have the same Titchmarsh–Weyl M-function.

All of this has been done using computer algebra which is done symbolically, and thus exactly. Our program also allows visualisation of eigenfunctions and can handle a large set of self-adjoint boundary conditions.

We find that several sets of graphs that are isospectral under both Neumann and Dirichlet boundary conditions as well as under more general, δ-type and $\delta_{s'}$-type, boundary conditions.

Our software is open-source and can be inspected by the community. Most of our results can be verified by hand.

Unidirectional pulses*

Plachenov A.B.
MIREA — Russian Technological University, 78 Vernadsky Avenue, Moscow 119454 Russia
e-mail: a_plachenov@mail.ru

In recent years, there has been an increased interest in localized solutions of the wave equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = 0, \tag{1}$$

having the property of unidirectionality. One of the formulations of unidirectionality (see, for example, [1]) consists in the requirement that only homogeneous plane waves traveling in directions forming an angle with a certain chosen direction not exceeding $\frac{\pi}{2}$, are present in the decomposition of the solution in plane waves. This property expresses the requirement, natural from a physical point of view, that the mathematical model of the pulse describe its propagation strictly from the source.

The first results on the construction of unidirectional pulses were based on the consideration of axisymmetric solutions of the equation (1) in the form of Fourier–Bessel integrals

$$u = u(\rho, z, t) = \int_0^\infty \mathrm{d}\omega\, e^{i\omega t} \int_0^{\omega/c} \mathrm{d}k_z\, A(k_z, \omega) e^{-ik_z z} J_0(\rho\sqrt{\omega^2/c^2 - k_z^2}), \tag{2}$$

where $\rho = \sqrt{x^2 + y^2}$, with fairly arbitrary weight functions A [2, 3]. A proper choice of such a weight allowed finding several solutions expressed in terms of elementary functions. The simplest localized

*Invited talk

unidirectional solution, however, was found differently and was based on a lucky trick [4], which used a partial fraction decomposition of the well-known splash pulse.

On the other hand, Besieris and Saari [5] (see also [3, 6]) noted that a special class of *relatively undistorted waves* is important in the description of unidirectional wave propagation, namely,

$$u = \frac{f(S - z - ib)}{S}\,, \tag{3}$$

where the *waveform* f is an *arbitrary* function. Here,

$$S = S(t, \rho) = \sqrt{(ct + ib)^2 - \rho^2}, \tag{4}$$

where $b > 0$ is a free parameter, the square root branch in (4) is chosen so that $S|_{x=y=0} = ct + ib$.

The equivalence of representations of cylindrically symmetric pulses described by Fourier–Bessel integrals (2), by relatively undistorted quasi-spherical waves (3) and in the form of a superposition of plane waves with wave vectors having positive projections on a given direction, is established. The approach based on a technique developed by Blagoveshchenskii [7] and Moses–Prosser [8] is systematically used. It rests upon on formulas expressing the solution through its asymptotic behavior in the far zone at large time. The approch allowed integral representations for solutions which belong to considered classes are obtained and those unidirectionality is proved.

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Fixed angle inverse scattering for Riemannian metrics

Rakesh
University of Delaware, USA
e-mail: `rakesh@udel.edu`

An inhomogeneous acoustic medium, modeled by a Riemannian metric, is probed by a finite number of plane waves and the resultant time dependent waves are measured on the boundary of a ball enclosing the inhomogeneous part of the medium. We describe our partial results about the recovery of the Riemannian metric from the boundary measurements. This is a formally determined inverse problem for the operator $\partial_t^2 - \Delta_g$ for a Riemannian metric g on $\mathbb{R}^n$, consisting of the recovery of the metric g from the boundary measurements. We show we can distinguish between g and the Euclidean metric using boundary data corresponding to $n(n+1)/2$ different plane wave sources. This talk is based on work done with Lauri Oksanen and Mikko Salo.

An inverse problem for a quasilinear wave equation

Romanov V.G.
Sobolev Institute of Mathematics, 4, Koptyug prosp., Novosibirsk 630090, Russia
e-mail: romanov@math.nsc.ru

Consider the equation

$$u_{tt} - \Delta u + \sigma(\mathbf{x})(u_t)^m + q(\mathbf{x})u^2 = 0,\ (\mathbf{x}, t) \in \mathbb{R}^4; \quad u|_{t<0} = g(t - \mathbf{x}\cdot\nu - R), \tag{1}$$

where $\mathbf{x} = (x_1, x_2, x_3)$, $\sigma(\mathbf{x})$ and $q(\mathbf{x})$ are smooth finite functions with support in the ball $B(R) = \{\mathbf{x} \in \mathbb{R}^3 | \, |\mathbf{x}| < R\}$, $m > 1$ is a real number and ν is a unite vector. The function $g(t)$ is supposed such that $g(t) = 0$ for $t < 0$ and $g(0) = 0$, $g'(+0) = a > 0$, $g \in C^\infty[0, \infty)$. If $\sigma(\mathbf{x}) = 0$ and $q(\mathbf{x}) = 0$ then $u(\mathbf{x}, t) = g(t - \mathbf{x}\cdot\nu - R)$ is the plane wave running in direction ν. A solution of problem (1) corresponds to falling down of the wave on the inhomogeneity located in $B(R)$. The wave front $t = \mathbf{x}\cdot\nu + R$ tangents to the boundary of ball $B(R)$ at point $\mathbf{x} = -R\nu$ in the moment of time $t = 0$. It is supposed that $\nu = \nu(\varphi) = (\cos\varphi, \sin\varphi, 0)$, $\varphi \in [0, \pi)$, and φ is the variable parameter of the problem. Related to this, by $u(\mathbf{x}, t, \varphi)$ is denoted a solution of problem (1).

Let $S(R, \nu) = \{\mathbf{x} \in \mathbb{R}^3 | \, |\mathbf{x}| = R,\ \mathbf{x}\cdot\nu > 0\}$ be a part of the boundary of $B(R)$ and $g(t)$ be the given function.

For the wave equation (1) the following inverse problem is studied.

The inverse problem. Find $\sigma(\mathbf{x})$ and $q(\mathbf{x})$ for $\mathbf{x} \in B(R)$ from the given trace of solutions to problem (1) on $S(R, \nu)$ for variable ν and for an interval of time, i.e. from the given function

$$p(\mathbf{x}, t, \varphi) = u(\mathbf{x}, t, \varphi),\ \forall\varphi \in [0, \pi),\ \forall x \in S(R, \nu),\ t \in [\mathbf{x}\cdot\nu + R - \eta, \mathbf{x}\cdot\nu + R + \eta], \tag{2}$$

where $\eta > 0$ is an arbitrary small number.

The main result is concluded in a reduction of the original inverse problem for coefficient $\sigma(\mathbf{x})$ to the usual X-ray tomography problem and for $q(\mathbf{x})$ to a new problem of the integral geometry. A stability estimate for solutions of the latter problem is stated.

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"Violations" and violations of relations for the electrical area of short electromagnetic pulses

N.N. Rosanov
Ioffe Institute, Saint-Petersburg, Russia
e-mail: nnrosanov@mail.ru

The electric area of extremely short pulses, defined as the integral over time t of the electric field $\mathbf{E}$, $\mathbf{S}_E = \int_\infty^\infty \mathbf{E}\,\mathrm{d}t$, determines the efficiency of their interaction with micro- and nanoobjects [1]. Assuming that after exposure to a pulse the object returns to its original state, it follows directly from Maxwell's equation, which expresses Faraday's law of induction, the irrotational nature of the vector field of the electric area follows: $\mathrm{rot}\,\mathbf{S}_E = 0$. In effectively one-dimensional schemes, in which $\mathbf{E}$ depends only on one Cartesian coordinate (along the direction of propagation of the pulse z) and time, this property turns into a rule for the conservation of electrical area: $\frac{d}{dz}\mathbf{S}_E = 0$ [1]. In particular, it follows from the conservation rule that it is impossible to amplify the electrical area of pulses in a medium with laser gain.

The indicated properties of the electric area are largely universal; in their derivation, only Maxwell's equation is used, regardless of the constitutive relations for the medium or object. Thus, they are valid for homogeneous and inhomogeneous, linear and nonlinear, transparent and absorbing media. Due to their seeming counterintuitiveness, the validity of these rules is questioned by a number of researchers. For example, in [2] a violation of the conservation rule is reported for media with electric conductivity.

In this talk we will show that this "violation" is caused by a departure from the strict Maxwell equations and the use of the unidirectional propagation approximation in [2], see also [3]. In addition, we will show that the property of the irrotational nature of the electric area can indeed be violated in media with magnetic hysteresis, when a pulse revers the object magnetization, see also [4]. This circumstance removes the prohibition on the impossibility of amplifying the electrical area of extremely short pulses.

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Spectral invariants for Schrödinger operators on periodic discrete graphs

Saburova N.Yu.
Northern (Arctic) Federal University, Northern Dvina emb. 17, Arkhangelsk, 163002, Russia
e-mail: n.saburova@gmail.com

We consider Schrödinger operators with real periodic potentials on periodic discrete graphs. Their spectrum consists of a finite number of bands. We construct a complete system of the Floquet spectral invariants. These spectral invariants are polynomials with respect to the potential and are obtained as coefficients of the Fourier series for the traces of the fiber Schrödinger operators on quotient graphs. We apply these invariants to determine isospectral potentials for some periodic graphs.

Spectral series of the Schrödinger operator with double delta potential on a three-dimensional spherically symmetric manifold

Shafarevich A.I., **Rykhlov V.V.**
Lomonosov Moscow State University, Moscow, Russia
e-mail: shafarev@yahoo.com, vladderq@gmail.com

The Schrödinger operators with delta potentials, as one described in our study, are commonly used to model physical systems with localized impurities or defects. These operators are vital for understanding how quantum particles behave in the presence of such perturbations, providing insights into the influence of localized perturbations on the system dynamics as a whole.

We consider the spectral problem for the Schrödinger operator of the form

$$H\psi = E\psi, \qquad H = -\frac{h^2}{2}\Delta + \alpha_1\delta_{x_1}(x) + \alpha_2\delta_{x_2}(x), \qquad x \in M, \tag{1}$$

where $\alpha_j \in \mathbb{R}$, $M \cong \mathbb{S}^3$, Δ denotes the Laplace–Beltrami operator on M, and δ_{x_j} represents the Dirac delta function centered at x_j, in the semiclassical limit as $h \to +0$. In this context, the operator H is conceived as a self-adjoint extension of the Laplace–Beltrami operator restricted to the functions from $W_2^2(M)$ vanishing at x_1 and x_2 (extensions of that kind are parametrized by a pair $(\alpha_1, \alpha_2) \in \mathbb{R}^2$).

We obtain an explicit form of the Bohr–Sommerfeld–Maslov quantization conditions that allow one to establish the asymptotic behavior of the spectrum of H. As for asymptotic eigenfunctions, we derive a representation for them in terms of the Bessel functions of half-integer order. This representation offers a comprehensive understanding of the structural characteristics and behavior of the eigenfunctions as h approaches zero.

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Isotropic surfaces and complex vector bundles corresponding to the Schrödinger equation with a delta potential

Shafarevich A.I., **Shchegortsova O.A.**
Lomonosov Moscow State University, Moscow, Russia
e-mail: olga.shchegortsova@gmail.com

We study modifications of isotropic manifolds and complex vector bundles corresponding to asymptotic solutions of the non-stationary Schrödinger equation with a delta potential localized on a surface of codimension 1. The initial data are localized in a small neighborhood of the surface and can be represented as a wave function with a complex phase. The Schrödinger operator with a delta-potential is defined using the theory of extensions. Its domain consists of functions that satisfy boundary conditions on the carrier of the singularity.

To construct an asymptotic solution, we use a modification of the Maslov's complex germ method. This approach is deeply connected with the learning of special geometric objects — Lagrangian surfaces and complex vector bundles over isotropic manifolds.

The effect of a delta potential is illustrated by the fact that an incident wave, after interacting with a delta potential, is split into two parts — reflected and transmitted waves. Geometric objects corresponding to the problem need to be modified at the points of the surface carrier of the delta function.

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Diffraction by a Dirichlet $\pi/3$ wedge on a hexagonal lattice

Shanin A.V.[1], Korolkov A.I.[2], **Makarov O.I.**[1]
[1]M. V. Lomonosov Moscow State University, Faculty of Physics, Department of Acoustics, Leninskie Gory, 1-2, 119991, Moscow, Russia
[2]Department of Mathematics, University of Manchester, Oxford Road, Manchester, M13 9PL, UK
e-mail: `olegmakarovlip@gmail.com`

The problem of a diffraction by a $\pi/3$ wedge cut from a triangular lattice is studied in the talk. Following our previous works [1–3] we show that the set of all plain waves on the lattice forms a complex one-dimensional surface $\mathbf{R}$ with a topology of a torus. The diffraction field can be represented as an integral over a family of contours on this torus $\mathbf{R}$. This integral is the Sommerfeld integral.

By using the A. Sommerfeld's method of reflections, a branched surface $\boldsymbol{S_5}$ can be introduced, containing the original physical lattice surface $\mathbf{S}$ and 4 reflected copies of it. Then, the diffraction problem on the physical surface with boundaries can be reformulated as a diffraction problem on the boundary-free branched surface.

It is shown that of the Sommerfeld transformant belongs to the field of meromorphic functions of an 5-sheet covering of $\mathbf{R}$ denoted $\mathbf{R_5}$. There exists a functional problem that uniquely defines the integrand. Once the integrand is obtained, straightforward integration enables one to compute the diffracted field.

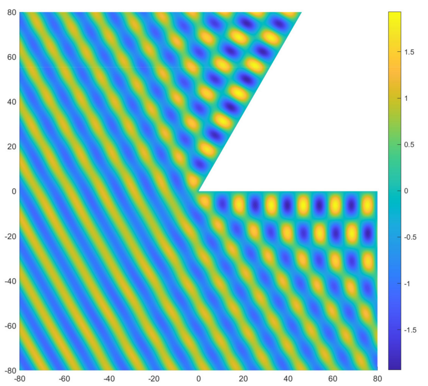

Fig. 1: The real part of the diffraction field on a hexagonal lattice for an incident plane wave with an angle of incidence $\pi + \pi/6$.

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Electroacoustic tomography: inverse problems and deep learning

Shishlenin M.A., Novikov N.S., Kabanikhin S.I.
Sobolev Institute of Mathematics, Institute of Computational Mathematics and Mathematical Geophysics, Novosibirsk
e-mail: `m.a.shishlenin@mail.ru, novikov-1989@yandex.ru, ksi52@mail.ru`

The inverse problems of electro acoustic tomography [1] are considered, the mathematical model of which is based on conservation laws. A combined formulation of the inverse problem for a system of equations of acoustics and electrodynamics for determining acoustic and electromagnetic parameters of the medium from measurements at the boundary of the studied region is investigated [2]. A mathematical model has been developed that allows monitoring compliance with conservation laws and allows parallelization in solving direct and inverse problems on a supercomputer.

The inverse problem is reduced to minimizing the target functional by the gradient descent method [3]. The results of numerical calculations using machine learning technology are presented. A comparative analysis of model-driven and data-driven methods is carried out.

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Numerical implementation of multipoint formulas in inverse problems

Sivkin V.N.
Lomonosov MSU, Moscow
e-mail: `sivkin96@yandex.ru`

Novikov R.G.
École polytechnique de Paris
e-mail: `roman.novikov@polytechnique.edu`

We present the first numerical study of multipoint formulas for finding leading coefficients in asymptotic expansions arising in scattering theory. In particular, we implement different formulas for finding the Fourier transform of potential from the scattering amplitude at several high energies. We show that the aforementioned approach can be used for essential numerical improvements of classical results including the slowly convergent Born–Faddeev formula for inverse scattering at high energies. The approach of multipoint formulas can be also used for recovering the X-ray transform of potential from boundary values of the scattering wave functions at several high energies. In addition, we show that the aforementioned multipoint formulas admit an efficient regularization for the case of random noise. This talk is based on [1]. In particular, we proceed from theoretical works [2, 3].

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Utilizing the viscosity approximation and Carleman weight-based quasi-reversibility method for acoustic imaging

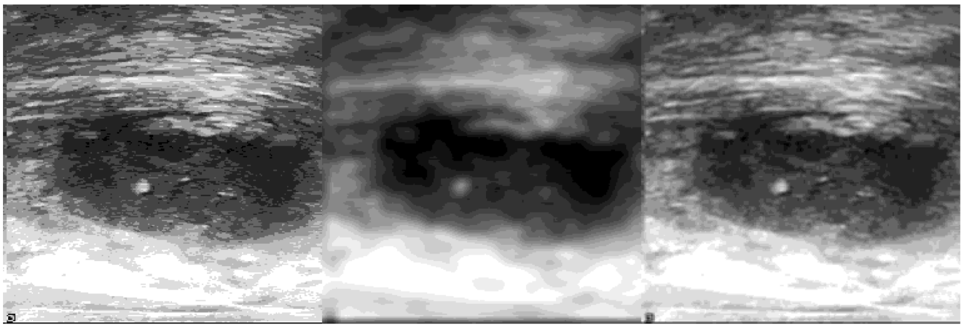
Timonov A.
Steklov Mathematical Institute of the RAS (St. Petersburg branch);
University of South Carolina Upstate, USA
e-mail: altim@pdmi.ras.ru

The Cauchy problem for an acoustic wave equation is considered as a mathematical model that describes the wave propagation and scattering.

$$q(x)u_{tt} - \Delta u = 0 \quad \text{in} \quad \mathbb{R}^2 \times (0, \infty), \tag{1}$$

$$u(x, 0) = 0, \quad u_t(x, 0) = \delta(x - x_0), \quad x, x_0 \in \mathbb{R}^2, \tag{2}$$

Let $\Omega \subset \mathbb{R}^2$ be a bounded and simply connected domain with the Lipschitz boundary, and $\mathrm{supp}(q) \subset \Omega$. Let $\mathcal{C} \subset \mathbb{R}^2 \setminus \overline{\Omega}$ be a smooth curve, which an impulse source runs over.



An inverse problem is formulated as follows. Suppose $q \geq 1$ in Ω and $q = 1$ in $\mathbb{R}^2 \setminus \Omega$. Given the Dirichlet and Neumann boundary conditions (the latter may be given only on a part of $\partial\Omega$) for each $x_0 \in \mathcal{C}$, determine some approximations of q in Ω. Using the Lavrentiev approach, this nonlinear inverse problem can be transformed to a linear integral equation of the first kind at sufficiently low frequencies. Then, we construct a viscosity approximation of the inverse of Lavrentiev's operator associated with the inverse problem and reduce solving two resulting coupled differential equations to a variational problem by applying the Carleman weight-based quasi-reversibility method. Once a unique solution of the variational problem is found, the sought approximations of q is determined. To demonstrate the computational effectiveness of the proposed technique, several results of numerical experiments are shown. One such result is given below. From left to right: the ground truth image, which is a real sonogram of woman breast that contains a malignant tumor; an approximate image reconstructed from simulated data; an image obtained from the previous one by the post-processing procedure based on the weighted mean curvature flow model.

Localized Gaussian beams and asymptotics of solutions of the Helmholtz equation

Tsvetkova A.V.
Ishlinsky Institute for Problems in Mechanics RAS, Russia, Moscow
e-mail: annatsvetkova25@gmail.com

We discuss a geometric approach based on the Maslov canonical operator theory to constructing the asymptotics of Gaussian beams. In particular, we consider the Laguerre–Gauss beams, which are the solution of the three-dimensional Helmholtz equation in the paraxial approximation (which can be considered as the Schrödinger equation). The considered beams are the product of the Gaussian exponent and the Laguerre polynomials.

The discussed approach based on the semiclassical approximation and the study of the dynamics of Lagrangian manifolds, which makes it possible to obtain an effective asymptotics of the considered beams in terms of Airy and Bessel functions of compound argument. Obtained formula gives a good approximation even for small indices of the corresponding polynomials [1].

One of the advantages of the discussed approach is that it is quite universal. In particular, it allows us to abandon the paraxial approximation and consider the original Helmholtz equation. In the talk, the global asymptotics in terms of special functions of the solution of the Helmholtz equation with "initial" conditions generated by Laguerre–Gauss beams will be given.

The work is supported by RSF (project № 24-11-00213).

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M. Kac problem with augmented data

A.I. Vakulenko
St. Petersburg Department of Steklov Mathematical Institute, St. Petersburg, Russian Federation
e-mail: vak@pdmi.ras.ru

Let Ω be a smooth compact n-dimensional Riemannian manifold with boundary. The (generalized) M. Kac problem is the question: to what extent does the spectrum $0 < \lambda_1 < \lambda_2 \leqslant \dots$ of its Dirichlet Laplacian $L = -\Delta \restriction H^2(\Omega) \cap H_0^1(\Omega)$ determine Ω?

To give the spectrum is to represent L in the form $\tilde{L} = \Phi L \Phi^* = \operatorname{diag}\{\lambda_1, \lambda_2, \dots\}$ in $\mathbf{l}_2$, where $\Phi : L_2(\Omega) \to \mathbf{l}_2$ is the Fourier transform. Let $\mathscr{K} = \{h \in L_2(\Omega) \mid \Delta h = 0 \text{ in } \Omega \setminus \partial\Omega\}$ be the harmonic function subspace, $\tilde{\mathscr{K}} = \Phi\mathscr{K} \subset \mathbf{l}_2$ be its Fourier image. We show that, in a generic case, the pair $\{\tilde{L}, \tilde{\mathscr{K}}\}$ determines Ω up to isometry. The isospectral but not isometric Ω and Ω' have the same $\tilde{L} = \tilde{L}'$ but differ by $\tilde{\mathscr{K}} \neq \tilde{\mathscr{K}}'$.

Thus, $\tilde{\mathscr{K}} \subset \mathbf{l}_2$ augments the spectrum, making the problem uniquely solvable not only for the plain domains (drums) but for the manifolds of arbitrary dimension and topology.

These results are obtained in collaboration with M. I. Belishev.

3D optical vector soliton with crossed vortex lines

Veretenov N.A., Fedorov S.V., Rosanov N.N.
Ioffe Institute, Saint-Petersburg, Russia
e-mail: torrek@gmail.com

We predict a new interesting type of optical dissipative vector 3D solitons in a medium with laser amplification and saturable absorption without additional trapping potentials. The quasioptical nonlinear equation for the slow varying complex amplitude (generalized Ginzburg–Landau equation) was solved by numerical integration using split-step and Crank–Nicolson methods (see for example, [1]). Saturating absorption is modeled by a two-level scheme [2] and a four-level spin-flop scheme is adopted for amplification [3, 4]. These fully localized structures could be generated in continuous medium with layers with gain and absorption from initial distibution with specially oriented phase dislocations. The circular polarization components have the toroidal intensity distribution and a topological charge $m = 1$. The angle between the components is $90°$, vortex lines of two components intersect in the central point and this point has zero intensity. The total intensity profile is almost spherical. We investigate the stability and bifurcations of this type of soliton and the effects of anisotropy in transverse dimensions.

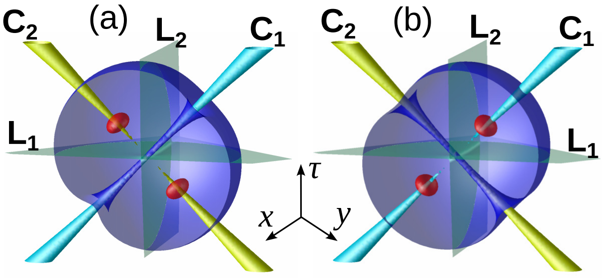

Fig. 1: Stable cruciform soliton: isointensities of two circular polarization components and polarisation singularities: L-plates with linear polarisation and C-lines with pure circular polarisation.

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All-fiber generation and control of higher-order optical beams via lowest-order flexural acoustic wave

Vikulin D.V., Barshak E.V., Alexeyev C.N., Lapin B.P., Yavorsky M.A.
V. I. Vernadsky Crimean Federal University, Vernadsky Prospekt, 4, Simferopol 295007, Russia
e-mail: `vikulindmitriy@mail.ru`

In this paper, we have studied the acousto-optic interaction in circular optical fibers endowed with the lowest-order linearly and circularly polarized flexural acoustic waves. We took into account the nonlinear modification of fiber's permeability under the influence of acoustic waves. We demonstrate the existence of new acoustic resonances resulting from nonlinear modification of fiber permittivity by acoustic waves. The resonance optical fiber modes and the propagation constants are found. Acoustically-controlled tunable generation of higher-order optical regular and topologically-charged beams directly from a Gauss-like mode is predicted. Our results can be useful in different photonics applications.

On distinguishing between bending edge wave and Lamb wave A0 in the wave-fields exited by an edge load

Wilde M.V., Pleshkov V.N.
Saratov State University, 83, Astrakhanskaya str., 410012, Saratov, Russia
e-mail: `mv_wilde@mail.ru, vnp98@yandex.ru`

The bending edge wave, or Konenkov wave [1], is usually investigated as time-harmonic wave, for which the exponential decay in the interior of the plate is prescribed by the form of the sought solution. This approach separates the edge wave from Lamb waves and allows to study its dispersion properties. However, in the practice the excitation of an edge wave in particular is hardly possible. The problem becomes complicated by closeness between the dispersion curves of Konenkov wave and Lamb wave A0. By applying the frequency-wavenumber analysis or the matrix pencil method to the experimental data, one can observe a dispersion curve which could be attributed either to Konenkov wave or to bending edge wave.

In this work, the transient bending waves excitation is considered on the basis of the refined theory of plate bending with the refined boundary conditions, obtained in [2]. For comparison, 3D solution is also obtained. The method of contour integral, analogous to that used for classical Lamb problem (see, e.g., [3]), allows us to separate the field of Konenkov wave, represented by the pole. The pseudo-experimental data are acquired by numerical evolution of this field and the one resulting from the branch cuts, which contain the field of A0 wave. The matrix pencil method is applied to these field separately and to their sum in order to investigate the possibility of distinguishing between bending edge wave and Lamb wave A0.

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Implementation of the Peano and WKB approaches to find vibrational modes and frequencies of a thin rod with variable parameters

D.D. Zakharov, I.S. Nikitin
Institute of Computer Aided Design of Russian Academy of Sciences, 123056 Moscow, 2nd Brestkaya Str., 19/18
e-mail: dmitrii.zakharov@gmail.com

Boundary value problems of vibration of a thin rod under various conditions at the ends of the rod are considered. The parameters of stiffness and inertia may depend on the longitudinal coordinate. The respective solutions to the Bernoulli–Euler equation are obtained using Peano and WKB methods. Dispersion equations are considered. As shown, the rarely used Peano method is an efficient tool for this kind of problems and permits one to find modes and frequencies. The comparison shows that WKB method satisfactorily predicts frequencies even by leading order term and despite the error in the calculation of modes. The presented approaches can be used to calculate rods with different profiles as well as for the case of functionally graded materials when the Young's modulus and mass density may depend on the longitudinal coordinate. The examples are presented in context of practical assessment of falling strength at the experimental study of alloys subjected to the cyclic point-load at resonance of bending vibrations.

Surface waves generation by array of thin wires in the presence of impedance surface of metallic grid

V. Zalipaev
ITMO University, St. Petersburg, 197101, Russia
e-mail: v.zalipaev@metalab.ifmo.ru

The problem of signal transmission on the basis of electromagnetic wave propagation over a long distances has been vital since the beginning of the twentieth century (see for example [1]) in various areas of radio-science theoretical activity and engineering. Thus, in this paper we study an excitation of the electromagnetic wave-modes localized along 1D periodic finite array of thin metallic parallel wires in the presence of impedance surface of metallic grid. The array is located close to the grid and is excited by one of elements of the array. In practice it is a vibrator antenna that is thin wire of finite length with a gap in the middle with applied voltage of AC. The grid screen is a very dense mesh of two sets of orthogonal metallic parallel wires (much thinner than wires of the array) with square element and described by impedance boundary condition. We apply analysis based on Pocklington system of integral equations (see, for example, [2]) to compute resonance frequencies and construct excited wave-modes localised within the array as well as generated Sommerfeld surface waves localised in the neighbourhood of the grid screen that could propagate over a long distance. This approach is based on the long-wave approximation of thin wires. It is worth remarking that recently the approach based on the Pocklington type integral equation was successfully applied to studying the electromagnetic localized modes of linear periodic arrays of thin metallic wires (see [3]). As far as the electric currents of the thin wires of the array have been computed numerically we evaluate the exited near field in the form of Hertz vector integral representations of Sommerfeld exact solution. In the far field zone, applying the steepest descend method of the short-wave approximation to these integral representations, for the wave field we obtain asymptotic expansions of the expanding bulky spherical waves in both half-spaces as well as the poles contributions of excited surface waves. Thus, developing the asymptotic analysis we derive approximate formulas for the surface waves excitation coefficients incorporating the electric currents of the array wires. In the numerical analysis we pay a particular attention to the power of the surface wave generated by the array of thin wires.

The description of the described above wave-modes is directly connected with another problem of guided localized electromagnetic waves propagating along 1D periodic infinite array of thin metallic wires in the presence of impedance surface of metallic grid. For the second problem the quasi-periodic wave field is constructed as a superposition of vector wave fields irradiated by linear electric currents. The approach is also based on the method of integral equations of Pocklington type. Dispersion curves are presented for the first lowest complex half-wave resonance of a single thin vibrator well-known in radio physics. Finally, an approximate quantization condition is being discussed that helps to compute resonance frequencies of the excited wave-modes for the finite array located over the metallic grid and generated surface waves amplitudes. The data of the dispersion dependencies of the infinite array are incorporated into this quantization condition.

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Diffraction of a video pulse

V. Zalipaev, I. Bubenchikov, A. Tityapkin, S. Kozlov
ITMO University, St. Petersburg, 197101, Russia
e-mail: `v.zalipaev@metalab.ifmo.ru`

In recent years, great interest among researchers has been attracted to the theoretical study of the effects of extremely short “unipolar” electromagnetic pulses (video pulses) on matter (see, for example, [1]). In this case, the analysis of the interaction of radiation with matter is usually carried out in the approximation of plane transversely homogeneous waves having infinite energy. In this work, it is analytically shown that the limitation of a video pulse to the case of being bounded in transverse spatial size, respectively, and in energy, leads to the splitting of its spatial spectral components into two parts: travelling and evanescent waves. In the case of a bell-shaped (for example, Gaussian) temporal structure of a video pulse, the set of travelling waves represents a single-cycle wave. Therefore, after the evanescent component of the video pulse decays with distance, its waveform becomes a single-cycle wave. The ratio of the energies of travelling and evanescent waves in a video pulse is determined by the ratio of its transverse to longitudinal scales. It is shown that when they are equal, the evanescent components decay already at a distance of only about three of these sizes. Thus, the pulse ceases to be a video pulse. The work shows that transversely bounded video pulses are not unipolar. Part of the radiation in the domain of existence of the video pulse is significantly non-paraxial and the longitudinal component of the wave-field may turn out to be comparable in magnitude to its component transverse to the axis of wave propagation. With a bell-shaped (for example, Gaussian) spatial structure of the beam for the transverse field component, its longitudinal part has a ring distribution. This longitudinal component of the video pulse field is also divided into travelling and evanescent waves.

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Scattering of surface waves by linear discontinuities in elastic half-space: approximate approach

Zhuchkova M.G.
Saint-Petersburg State University of Aerospace Instrumentation, SUAI, 67 Bolshaya Morskaia street, Saint-Petersburg, 190000, RUSSIA
e-mail: zhuchkova.guap@gmail.com

We study the effect of surface defects in elastic half-space on the incidence of surface waves. Such defects affect the propagation of surface waves generating scattering wave fields [1]. We put forward the approximate methods of calculation of these problems. We compare our results with those previously published [2] and the experimental data by Viktorov [3]. Good agreement of the results is received.

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Paraxial wave propagation along a delta potential

Zlobina E.A.[1], Kiselev A.P.[1,2,3], Fedorov N.S.[1]
[1]St. Petersburg State University, St. Petersburg, Russia
[2]St. Petersburg Department of Steklov Mathematical Institute, St. Petersburg, Russia
[3]Institute for Problems in Mechanical Engineering RAS, St. Petersburg, Russia
e-mail: ezlobina2@yandex.ru, kiselev@pdmi.ras.ru

Diffraction of high-frequency waves by delta singularities of refractive index, localized on codimension 1 surfaces, has recently attracted attention of various researchers. Mathematicians were interested in the development of Maslov's theory for non-tangential incidence in an arbitrary dimension [1, 2]. Physicists considered a 2D problem to simulate the effect of a thin absorbing layer [3].

Keeping in mind the Leontovich–Fock paraxial approach that reduces the high-frequency Helmholtz equation to the 'parabolic' Schrödinger equation, we investigate the following problem

$$\begin{cases} 2iku_x + u_{yy} + 2i\nu\delta(y)u = 0, \quad x > 0,\ -\infty < y < \infty; \\ u|_{x=0} = u_0 \end{cases} \tag{1}$$

with the given function $u_0 = u_0(y)$. Here, the constant $k > 0$ is large, and the parameter ν is complex that enables to model absorbing, or active, or conservative thin layer. The problem (1) allows a closed-form, though somewhat cumbersome, solution. In the simple case $u_0 = 1$ associated with the plane-wave propagation along the singularity, the solution takes the following form:

$$u = 1 - 2\Phi\left(-\sqrt{\frac{k}{2x}}|y|\right) + 2\exp\left(-i\nu|y| - i\frac{x}{2k}\nu^2\right)\Phi\left(-\sqrt{\frac{k}{2x}}\left(|y| + \frac{x}{k}\nu\right)\right). \tag{2}$$

Here,

$$\Phi(Z) = \frac{e^{-i\pi/4}}{\sqrt{\pi}} \int_{-\infty}^{Z} e^{it^2} dt \tag{3}$$

is the classical Fresnel integral.

We will report on the results of asymptotic study of the solution to the problem (1).

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