$\mathcal{F}K$-CONVEX FUNCTIONS AND WARPED PRODUCTS OF METRIC SPACES

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By an $\mathcal{F}K$-convex function on a length metric space,we mean one that satisfies $f'' \geq -Kf$ on all unitspeed geodesics. We show that natural $\mathcal{F}K$-convex (-concave) functions occur in abundance on metric spaces of curvature bounded above (below) by K in the sense of Alexandrov. We prove Lipschitz extension and Lipschitz approximation theorems for $\mathcal{F}K$-convex functions on $\mathrm{CAT}(K)$ spaces. Then we give sharp conditions for a warped product of metric spaces to have a given curvature bound, either above or below. The proof (building on our previous work on warped products) makes a close analysis of certain billiards trajectories and their limit behavior, in order to understand the sharp first order constraints on the warping function. As an application of the preceding work, standard cone and suspension constructions are generalized from 1-dimensional base to arbitrary base.