Victor Bangert:

Stable systolic inequalities, cohomology products and rigidity
(joint work with M. Katz, Bar Ilan University).

Multiplicative relations in the cohomology ring of a manifold impose constraints upon its stable systoles. Given a compact Riemannian manifold (X, g), its real homology $H_*(X, \mathbb{R})$ is naturally endowed with the stable norm. Briefly, if $h \in H_k(X, \mathbb{R})$ then the stable norm of h is the infimum of the Riemannian k-volumes of real cycles representing h. The stable k-systole $\mathrm{stsys}_k(g)$ is the minimum of the stable norm over nonzero elements in the lattice of integral classes in $H_k(X, \mathbb{R})$. Relying on results from the geometry of numbers due to W. Banaszczyk, and extending work by M. Gromov and J. Hebda, we prove metric-independent inequalities for products of stable systoles, cf. [1].

In particular, we prove a sharp inequality

$$(*) \qquad \mathrm{stsys}_1(g)\mathrm{sys}_{n-1}(g) \leq \gamma'_b \mathrm{vol}_n(g)$$

valid for arbitrary compact, orientable Riemannian manifolds (M, g) with first Betti number $b \geq 1$. Here γ'_b is a constant from the geometry of numbers defined by Bergé/Martinet [3]. In [2] we prove that equality holds in $(*)$ if and only if M admits a Riemannian submersion with connected minimal fibers onto a flat torus $\mathbb{R}^b/L$ where the lattice $L \subseteq \mathbb{R}^b$ is dual-critical in the sense of [3]. All this is due to J. Hebda if $b = 1$. In particular, we conclude that equality in $(*)$ implies $b \leq n$ and $b = n$ if and only if M is a flat torus $\mathbb{R}^b/L$ with L dual-critical. Conversely, given such a flat torus $\mathbb{R}^b/L$ and a submersion $F : M \to \mathbb{R}^b/L$ we show how all metrics g on M can be constructed for which F is a Riemannian submersion with minimal fibers.

[1] V. Bangert, M. Katz: Stable systolic inequalities and cohomology products, to appear in Commun. Pure Appl. Math.

[2] V. Bangert, M. Katz: Riemannian manifolds with harmonic one-forms of constant norm, preprint.

[3] A.M. Bergé, J. Martinet: Sur un problème de dualité lié aux sphères en géométrie des nombres, *J. Number Th.* **32** (1989), 14–42.