# ABOUT ALEKSANDROV SPACES OF CURVATURE BOUNDED ABOVE

V.N. Berestovskiĭ

In this abstract we present our recent results about Aleksandrov spaces $A$ of curvature bounded above.

Every Busemann $G$-space $A$ is Riemannian $C^0$-manifold (with $C^1$-atlas in which the components of metric tensor are continuous). Previously we get two general results. At first we find a necessary and sufficient conditions for isometry of a metric space $A$ to (finite- or infinitedimensional) Euclidean space or unit sphere in Euclidean space. Also we prove that for locally compact geodesically complete inner metric space $A$, the tangent space $A_x$, defined as $O$-cone over space of directions to $M$ at any point $x \in A$, is isometric to Gromov tangent cone $T_xA$, defined as Gromov-Hausdorff limit of scaled space $A$ with the base point $x$.

Then we give two simple examples of $A$ with certain pathological properties. In the first we give a $CAT(-1)$-space $A$, which is homeomorphic to $R^n, n \geq 5$, while its hyperbolic boundary in Gromov sense is not topological manifold. In the second $A$ has curvature $\leq 0$ and entropy dimension (around some point) greater than (equal) topological and Hausdorff dimensions.

Earlier we proved that for every $n \geq 5$, the topological $n$-sphere $S^n$ admits a $CAT(1)$-metric $\rho$ such that the space of directions $\Omega_x(S^n, \rho)$ at some point $x \in S^n$ is not homeomorphic to $S^{n-1}$. Now we show that similar example can be constructed also for $n = 4$, if the famous 3-dimensional Poincare conjecture is not true. This fact is deduced from Theorem which states that the suspension over any closed simply connected topological $3-$manifold is homeomorphic to $S^4$. We prove this Theorem on the ground of very recent paper [1] and known results from geometric topology.

Also we prove that the metric of any locally compact complete geodesically complete $CAT(-1)$-space $A$ can be recovered from the relation $xy \leq r$ (or $xy = r$) for a fixed positive number $r$, if the complement to any closed ball in $A$ is connected.

## References

[1] M.Bestvina, R.J.Daverman, G.A.Venema, and J.J.Walsh, *A 4-dimensional 1-LCC shrinking theorem*, Topology Appl. 110(2001), no.1, 3-20.