PROJECTIVE PLANES, VERONESE SURFACES AND SPHERES

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ABSTRACT

There is an elementary but very striking result which asserts that the quotient of the complex projective plane CP^2 by complex conjugation is the 4-dimensional sphere. This result has attracted the attention of many geometers over the years, rediscovered afresh each time and with a variety of proofs.

The purpose of the talk is to give two parallel generalizations of this theorem. In the first we view CP^2 as the second member of the family of projective planes over the four normed real division algebras. This is close to Arnold's treatment and completes it by dealing with the octonions. The second generalization views CP^2 as the complex algebraic variety obtained by complexifying the real projective plane RP^2 and hence as the first member of the four algebraic varieties got by complexifying the four projective planes. This family of algebraic varieties has appeared in several contexts. First, in relation to Lie groups and the magic square of Freudenthal, somewhat independently in the characterization by Zak of Severi varieties, and also in the classification by Nakagawa and Takagi of Kähler submanifolds with parallel second fundamental form in complex projective spaces.

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