MACROSCOPIC DIMENSION OF 3-MANIFOLDS

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DEFINITION [1].*A metric space V has a macroscopic dimension at most k ($\dim_{mc} V \leq k$) if there are k - dimensional polyhedron P and proper continuous map $\phi : V \to P$ such that $Diam(\phi^{-1}(p)) \leq \varepsilon$ for all $p \in P$ and some, possibly large ε. That is the inverse image of any point of P is uniformly bounded. If k is minimal possible we say that $\dim_{mc} V = k$.*

It's clearly that $dim_{mc}V \leq dimV$.

The next question was formulated by M. Gromov in [1]:

QUESTION : Does there exist a compact n-dimensional manifold M^n whose universal covering $\widetilde{M}^n$ is of $\dim_{mc} \widetilde{M}^n = n - 1$?

We give the negative answer to this question in the case $n = 3$:

THEOREM. *The universal covering $\widetilde{M}^3$ of any closed 3-manifold M^3 cannot have the macroscopic dimension equal 2.*

The key idea is to use the decomposition theorem for an orientable 3-manifolds. It is known that an orientable closed 3-manifold can be decomposed as a connect sum (J.Hempel)

$$\Sigma_1 \sharp \ldots \sharp \Sigma_n \sharp k(S^2 \times S^1) \sharp K_1 \sharp \ldots \sharp K_m$$

where Σ_i is covered by a homotopy 3-sphere and each K_j is a $K(\pi, 1)$.

We prove that if K_i in the decomposition of M^3 are absent, than $\dim_{mc} \widetilde{M}^3 \leq 1$. In other case $\dim_{mc} \widetilde{M}^3 = 3$.

As a consequence we have that the macroscopic dimension of the universal covering of a closed 3-dimensional manifold which admits a metric of positive scalar curvature is no greater than 1 (See [2]).

Literature

[1] M.Gromov . Positive curvature, macroscopic dimension, spectral gaps and higher signatures, 1996, Preprint.

[2] M.Gromov and H.B. Lawson. Spin and scalar curvature in the presence of a fundamental group. I, Ann. of Math. 111, N 2, 1980, P. 209-230.