Gaussian measures of surfaces, ellipticity of volume functionals, and volume/distance estimates

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We will discuss the following topics, which happen to be closely related:

1. What are the restrictions on the Gaussian measures of closed surfaces, or of surfaces with planar boundaries (the Gaussian measure of a surface is the push-forward of the surface area under the Gaussian map to the Grassmannian). For instance, under what conditions can one construct a polyhedron (for instance, a two-dimensional polyhedral surface in R^4) with given areas and directions of faces and no boundary (or a boundary lying in a given two-dimensional plane)?

2. What is the relationship between different types of ellipticily for surface area functionals (an area functional is elliptic over Z (resp. R) if regions in affine planes are area minimizers among all Lipschitz chains over Z (R) with the same boundary).

3. Optimal fillings: metrics on a manifold (with boundary)that admit no volume decreasing perturbations that do not decrease distances between boundary points. In other words, we will be looking for situations when an inequality for boundary distance functions of two metrics implies corresponding inequality for the volumes.

4. Asymptotic growth of volume for large balls in a periodic metric (a metric that admits a co-compact isometric action of an abelian group).