On geometry of double canal hypersurfaces in E^n

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Let us consider a double canal hypersurface M in the Euclidean space E^n: the envelope of an p-parameter family of hyperspheres and of an q -parameter family of hyperspheres $(p + q = n - 1)$. Double canal hypersurfaces form a special case of the cyclides of Dupin : hypercurfaces for which the principial curvatures are constant along the corresponding principal distributions. For a double canal hypersurface the multiplicities of the principal curvatures are p and q. The centers of the hyperspheres describe an p-surface C^p and q-surface $\bar{C}^q$. It is known that in E^3 ([1], P.382) C and $\bar{C}$ are focal second-order curves.

Theorem 1. *If M is a double canal hypersurface, then C^p and $\bar{C}^q$ are conic sections. The p-surface C^p is a $(p+1)$-plane section of a $(p+1)$-dimensional cone, and the q-surface $\bar{C}^q$ is a $(q+1)$-plane section of a $(q+1)$-dimensional cone, or p-surface C^p is a p-plane and q-surface $\bar{C}^q$ is a q-sphere, or p-surface C^p is a p-sphere and q-surface $\bar{C}^q$ is a q-plane.*

p-surface C^p and corresponding $(p+1)$-dimensional cone belong to $(p+2)$-plane. We denote by α the angle formed by $(p+1)$-plane containing C^p and $(p+1)$-plane base of $(p+1)$-dimensional cone. Let us denote by β the angle formed by element of $(p+1)$- dimensional cone and $(p+1)$-plane base of $(p+1)$-dimensional cone. Let us denote by e the eccentricity of p-surface C^p

$$e = \frac{\sin\alpha}{\sin\beta}.$$

Analogously, let us denote by $\bar{e}$ the eccentricity of q-surface C^q.

Theorem 2. *If M is a double canal hypersurface, then $e\bar{e} = 1$.*

Theorem 3. *If M is a double canal hypersurface, then $(p+1)$-planes containing p-spheres curvature have generic p-plane or parallel and $(q+1)$-planes containing q-spheres curvature have generic q-plane or parallel.*

References

[1] Shulikovskii V.I. *Classical differential geometry in tensor form,* GIFML, Moscow 1963.