Multiplicative Weierstrass points.

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Let F be a compact Riemann surface of genus $g \geq 2$. Character on $\pi_1(F)$, i.e. homomorphism $\rho : \pi_1(F) \to \mathbf{C}^* = \mathbf{C}\backslash\{0\}$, shall call inessential , if it has on generators $a_j, b_j, j = 1, ..., g$, group $\pi_1(F)$ representation $\rho(a_j) = \exp 2\pi i c_j$, $\rho(b_j) = \exp 2\pi i \sum_{k=1}^{g} \pi_{jk} c_k, j = 1, ..., g$, $(c_1, ..., c_g) \in \mathbf{C}^g$, where (π_{jk}) is a matrix of the order g from $b-$periods for canonical bases of holomorphic abelian differentials on F [1]. The inessential characters on $\pi_1(F)$ form $g-$dimensional Lie subgroup L_g in group $Hom(\pi_1(F), \mathbf{C}^*) \cong [\mathbf{C}^*]^{\mathbf{2g}}$ of all characters. Characters $\rho \in Hom(\pi_1(F), \mathbf{C}^*)\backslash L_g$ shall call essential.

In this paper a multiplicative $q-$Weierstrass points on F for essential characters ρ is introduced, and the theory such points also is constructed, where $q \in \mathbf{N}$. Shown, that in each point P on F exists exactly $g-1$ a multiplicative Weierstrass gaps $n_j(P)$ such that $0 < n_1(P) < n_2(P) < ... < n_{g-1}(P) < 2g$ for every ρ. Also there is exactly $g-1$ a multiplicative Noether gaps for ρ with respect to every sequence points on F. Numbers $N(\rho)$ and $N(\rho, q)$ of multiplicative Weierstrass points and of multiplicative $q-$Weierstrass points for ρ satisfy to two-sided estimate $g - 1 \leq N(\rho) \leq (g-1)^2 g$ and $1 \leq N(\rho, q) \leq (g-1)^2 g(2q-1)^2$ respectively. The new method of a research of multiplicative Weierstrass gaps and of multiplicative Noether gaps for ρ on F through filtration in the Jacobi variety $J(F)$ for F is presented.

Let φ is the Jacobi map from F_n to $J(F)$, where F_n is symmetric $n-$multiple product for $F, n \geq 1$. Set $W_n = \varphi(F_n)$. Then it is well known, that $0 = W_0 \subset W_1 \subset ... \subset W_{g-1} \subset W_g = J(F)$ is filtration in $J(F)$ of length g. Let $(1/2\pi i)(\sum_{j=1}^{g} \log \rho(b_j) e^{(j)} - \sum_{j=1}^{g} \log \rho(a_j) \pi^{(j)}) \equiv \psi(\rho)$ in $J(F)$, where $e^{(1)}, ..., e^{(g)}$ and $\pi^{(1)}, ..., \pi^{(g)}$ is columns of matrixes $a-$periods and $b-$periods, which generate a lattice $L(F)$ for $J(F) = \mathbf{C}^g / L(F)$.

Theorem 1. (about multiplicative Weierstrass gaps). For every essential character ρ and every point P on compact Riemann surface F of genus $g \geq 2$ valid following statements: 1) natural number $j, 1 \leq j \leq g$, will be of multiplicative Weierstrass nongaps in P for ρ on F $\Longleftrightarrow j\varphi(P) + \psi(\rho) \in W_j \backslash (W_{j-1} + \varphi(P))$; 2) point P will not be of multiplicative Weierstrass point on F for $\rho \Longleftrightarrow$ all conditions $j\varphi(P) + \psi(\rho) \notin W_j \backslash (W_{j-1} + \varphi(P)), 1 \leq j \leq (g-1)$ be satisfied.

Theorem 2. (about multiplicative Noether gaps). For every sequence points $P_1, P_2, ..., P_n, ...$ on compact Riemann surface F of genus $g \geq 2$, and every essential character ρ valid following statements: 1) number $j, 1 \leq j \leq g-1$, will be of multiplicative Noether nongaps for ρ on $F \Longleftrightarrow \psi(\rho) + \varphi(P_1 ... P_j) \in W_j$, $\psi(\rho) + \varphi(P_1 ... P_{j-1}) \notin W_{j-1}$; 2) number g will be of multiplicative Noether nongaps for ρ on F, if $r_\rho((P_1 ... P_{g-1})^{-1}) = 0$.

Theorem 3. For fixed point P on compact Riemann surface F of genus $g \geq 2$, and for every essential character ρ $P-$filtration for ρ in group $J(F)$ $0 \subset W_1 - \varphi(P) \subset W_2 - 2\varphi(P) \subset ... \subset W_k - k\varphi(P) \subset ... \subset W_{g-1} - (g-1)\varphi(P) \subset W_g - g\varphi(P) = J(F)$, will be separable exhaustible filtration in $J(F)$ of length g.

This theorem allows to determine of multiplicative Weierstrass gaps and Weierstrass nongaps for essential characters ρ in fixed point P among numbers $\{1, 2, ..., g\}$ through disposition $\psi(\rho)$ in $J(F)$.

1. Chueshev V.V. Vectorials Prym bundle and Gunning bundle over Teichmueller space. // Sibirsk. math. journal. 2001, v.42, N 4, p. 937-951.