ESTIMATING COMPLEXITY OF 3-MANIFOLDS AS OF BRANCHED COVERINGS

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One of approaches to the classification of 3-manifolds is to define the function which shows how "complicated" is the manifold in some sense. After that one can reduce the problem to infinite number of problems of classification of manifolds with concrete complexity.

Such a complexity function for 3-manifolds was introduced by S. V. Matveev in [1,2]. It arises from the theory of simple spines of 3-manifolds. The *complexity* $c(M)$ of a closed 3-manifold M equals k if M possesses an almost simple spine with k vertices and has no almost simple spines with a smaller number of vertices.

It is a well known method of presenting 3-manifolds as cyclic branched coverings of 3-sphere. It is interesting to know how the complexity of n-fold covering space of 3-sphere branched over link L relates with the number of crossings on a diagram of the link L.

Let L be a link in a 3-manifold M presented by a diagram with k crossing points. We denote by $M_n(L)$ the n-fold cyclic covering space of S^3 branched over link L.

Theorem 1.(Matveev, [1]) $c(M_n(L)) \leq 4nk$.

But for some concrete numbers of folds n this estimation can be improved. For $n = 3D2$ this estimation is decreased in [3]. Here we try to decrease this estimation for $n = 3D3$. It seems to be interesting because of the well-known classical result of Hilden and Montesinos ([4]): any closed 3-manifold can be presented as 3-fold covering of 3-sphere branched over a corresponding knot.

Let L be a knot presented by a diagram with k crossing points.

Theorem 2. $c(M_3(L)) \leq 2k + 2$.

The proof of this theorem uses some methods and ideas of [1,2,5].

References

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