Euclidean- and Hyperbolic Rank for Hadamard manifolds

Thomas Foertsch
Institute for Mathematics, University of Zurich

One can define invariants for metric spaces that contain a certain information about their geometrical structure:
The Euclidean rank, for instance, measures the degree of global flatness of a space, whereas the hyperbolic rank, as defined by Gromov, measures its degree of hyperbolicity.
Prof. Gromov describes one of the guiding principles in the asymptotic geometry of spaces of nonpositive curvature X as follows:
"All flatness of X where $K = 0$ is confined to k-flats in X which are the subspaces isometric to $\mathbb{R}^k$. One distinguishes among them maximal flats which are not contained in bigger ones (sometimes maximal refers to the maximal dimension) and then tries to show that X is hyperbolic transversally to maximal flats".

Some aspects of the relation of the Euclidean- and the hyperbolic rank can be understood by generalizing a certain existence theorem for quasi-isometric embeddings of hyperbolic spaces in products of hyperbolic spaces ([BrFa]) in various directions.
This talk will give a survey of recent results of the interaction of the Euclidean- and the hyperbolic rank for Hadamard manifolds, that are achieved by such generalizations ([Leu], [Foe1], [Foe2], [FoeSch]).

References

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