Pompeiu transform for some classes of distributions

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Let $n \geq 2, ISO(n)-$ be the motion group of R^n, $B_r^n = \{x \in R^n : |x| < r\}$, $\varepsilon_m(B_r^n)-$ be the family of integrable functions with compact support in the form of $\varphi(x) = \varphi_0(\rho) \cdot Y^{(m)}(\sigma)$, where $\rho = |x|, \sigma = \frac{x}{\rho} \in S^{n-1}(x \in R^n), Y^{(m)}$-is fixed spherical harmonic of degree $m \in Z$. We define the Pompeiu transform P_φ of function $f \in C^\infty(B_r^n)$ as the map:

$$(P_\varphi f)(\sigma) = \int_{R^n} f(x)\varphi(\sigma x)dx \qquad \forall \sigma \in ISO(n) : \sigma supp\varphi \subset B_r^n,$$

where $\varphi \in \varepsilon_m(B_r^n)$. Let also $N_\varphi(B_r^n)$ be kernel of the transform P_φ.

We define $r(\varphi) = inf\{r_0 > 0 : supp\varphi \subset B_{r_0}^n\}$; $\tilde{\varphi}(z) = \int_{R^n} \varphi_0(x)\frac{J_{m+n/2-1}(z|x|)}{(z|x|)^{n/2-1}}dx$, $z \in C$; $Z_\varphi = \{z \in C : Rez \geq 0, \tilde{\varphi}(z) = 0\}$; n_λ -multiplicity of root $\lambda \in Z_\varphi$;

$$\aleph_2^n(m) = \{\varphi : |Im\lambda| \leq \alpha ln(2 + |\lambda|), |\tilde{\varphi}^{(n_\lambda)}(\lambda)| \geq (2 + |\lambda|)^{n_\lambda - \beta} \quad \forall \lambda \in Z_\varphi\},$$

and $\alpha, \beta > 0$ are independent on λ.

Let also H_k-the space of homogeneous harmonic polynomials of degree k; $\Phi_{z,s}^{k,l}(x) = \frac{d^s}{dz^s}(\frac{(J_{k+n/2-1}(z|x|)}{(z|x|)^{n/2-1}}Y_l^k(\sigma))$, $k \in Z, 1 \leq l \leq d_k$; $Q_m(x) = \sum_{k=0}^{m}\sum_{l=0}^{k-1} a_{k,l}|x|^{2l}P_l^{m-k}(x)$;

$$F_{k,l}(x) = \sum_{\lambda \in Z_\varphi}\sum_{s=0}^{n_\lambda - 1} c_{\lambda,s,k,l}\Phi_{\lambda,s}^{k,l}(x) \qquad \forall x \in B_r^n \qquad (1)$$

where $P_l^{m-k} \in H_{m-k-2l}; c_{\lambda,s,k,l}$, $a_{k,l} \in C$. We obtained the following result.

Theorem The following statements are true.

1). Let $\varphi \in \aleph_2^n(m)$, $f \in C^\infty(B_r^n), r > r(\varphi) > 0$, than

$$f \in N_\varphi(B_r^n) \Longleftrightarrow f(x) = Q_m(x) + \sum_{k \in Z}\sum_{1 \leq l \leq d_k} F_{k,l}(x)$$

$\max_{s=\overline{0,n_\lambda - 1}} |c_{\lambda,s,k,l}| = O(|\lambda|^{-\gamma})$ for $|\lambda| \to \infty$, (2)

where $\gamma > 0$-any fixed number. Moreover the series of the form (1) are convergent in $C^\infty(B_r^n)$.

2). If for $\varphi \in \varepsilon_m(B_r^n)$ the condition $|Im\lambda| \leq \alpha(2 + |\lambda|)$ fails, than $\forall k \in Z, 1 \leq l \leq d_k \quad \exists c_{\lambda,s,k,l} \in C$, which satisfies the condition (2), but the series of the form (1) are not convergent $\forall x \in R^n/\{0\}$.

3). For any function $\beta : C \to R_+^1$, such that $\lim_{z \to \infty} \beta(z) = +\infty$, there exists $\varphi \in \varepsilon_m(B_r^n)$ with conditions:

$$|Im\lambda| \leq \alpha ln(2 + |\lambda|), |\tilde{\varphi}^{(n_\lambda)}(\lambda)| \geq (2 + |\lambda|)^{n_\lambda - \beta(\lambda)} \qquad \forall \lambda \in Z_\varphi$$

and $\forall r > r(\varphi) > 0 \quad \exists f \in N_\varphi(B_r^n)$ with $F_{k,l}(x)$, which are not representable in the form (1) of the convergent in $C^\infty(B_r^n)$ series.