Generalized A.D.Alexandrov Inequalities

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Let $a^{(1)}(\vec{x}), \dots, a^{(1)}(\vec{x})$ to be real quadratic forms defined in R^m, and let $A^{(k)} = 3D\|a_{ij}^{(k)}\|$ are their matrices. The determinant of linear combination $\sum \lambda_k A^{(k)}$ is homogeneous form of mth degree over variables $\lambda_1, \dots, \lambda_m$. The coefficients $D(A^{(1)}, \dots, A^{(m)})$ by mixed monomials containing each of λ_i in degree one vere called by A.D.Alexandrov "mixed discriminants" of quadratic forms having matrices $A^{(1)}, \dots, A^{(m)}$.

Theorem 1. (A.D.Alexandrov). *Let $A^{(1)}, \dots, A^{(m-2)}$ and B are positive definite matrices of order m, and let C is a (not necessarily positivve definite) matrix of the same order. Then the following inequality holds:*

$$\|D_{i_1+i_2}\| \leq 0, \tag{1}$$

where $D_{i_1+i_2} = 3DD(A^{(1)}, \dots, A^{(m-2)}, B^{(i_1)}B^{(i_2)})$,
$i_1, i_2 = 3D0 : 1, B^{(0)} = 3DB, B^{(1)} = 3DC.$

In this paper we represent inequalities that generalize the determinantal inequality (1) to hyperdeterminants of arbitrary dimensionality. Namely, if we use notations
$D_{i_1+\dots+i_k} = 3DD(A^{(1)}, \dots, A^{(m-k)}, B^{(i_1)}, \dots, B^{(i_k)})$,
$i_1, i_2 = 3D0 : 1, B^{(0)} = 3DB, B^{(1)} = 3DC$,
Then we have following results:

Teorem 2. *For positive definite matrices $A^{(1)}, \dots, A^{(m-2l)}, B$ and for arbitrary matrix C (all matrices of same order m) there holds inequality*

$$(-1^l)|D_{i_1+\dots+i_{2l}}|^{(\pm,\dots,\pm)} \geq 0.$$

Teorem 2'. *For positive definite matrices $A^{(1)}, \dots, A^{(m-2l-1)}, B, C$ (all matrices of same order m) there holds inequality*

$$(-1^{l+1})|D_{i_1+\dots+i_{2l+1}}|^{(\pm,\dots,\pm,0)} \geq 0.$$