Involutions of Kac-Moody algebras and infinite dimensional symmetric spaces

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A finite dimensional symmetric space G/K is essentially determined by the Lie algebra of G and the involution ϱ on it whose fixed point set is the Lie algebra of K. The isotropy action of the symmetric space is equivalent to the action of K on the (-1)-eigenspace of this involution and the orbits of this action are important submanifolds of euclidean space which are also called flag varieties. Each flag variety determines the symmetric space, at least infinitesimally.

There is an infinite dimensional analogon to all this in the category of affine Kac Moody groups and algebras as has been shown by Terng and others. In particular there are many examples of such symmetric spaces of this type and they give rise to important submanifolds of Hilbert spaces via the isotropy representation.

To classify these symmetric spaces of Kac-Moody type one has to classify the involutions of affine Kac-Moody algebras. This has been attempted by several authors (in particular physicists) more or less successfully and finally be achieved by G. Rousseau and his coworkers in 1995 in a series of papers. In this talk I want to outline a new, elementary classification which moreover relates involutions of affine Kac-Moody algebras to pairs of involutions on finite dimensional compact Lie algebras.