On Hermite-Liouville structures on hermitian manifolds

Masayuki IGARASHI

It is widely known that Jacobi researched on the ellipsoid and that Liouville formulated it in the form of the Liouville line elements in 19th century. In 1991 K.Kiyohara began to study the global structure of Liouville manifolds, and then Kiyohara and the author have researched Kähler-Liouville manifolds([1][2]). Recently, Kiyohara and the author research Hermite-Liouville manifolds.

An Hermite-Liouville structure on an n-dimensional complex manifold (M, J) is defined as a couplet $(g, \mathfrak{F})$ of a hermitian metric g and an n-dimensional real vector space $\mathfrak{F}$ of the first integrals (on the cotangent bundle) of the geodesic flows with respect to g such that (1) $\mathfrak{F}$ contains the energy function with respect to g; (2) every first integral in $\mathfrak{F}$ is a fiberwise homogeneous polynomial of degree 2 and is hermitian; (3) $F_p = F|_{T_p^*M}$, $F \in \mathfrak{F}$, are simultaneously normalizable for each $p \in M$. We say that $(M, g, J, \mathfrak{F})$ is an Hermite-Liuoville manifold if (M, g, J) is a hermitian manifold and if $(g, \mathfrak{F})$ is an Hermite-Liouville structure on (M, J). An Hermite-Liouville manifold $(M, g, J, \mathfrak{F})$ is called Kähler-Liouville manifold if g is a Kähler metric. In this case this structure $(g, \mathfrak{F})$ is called Kähler-Liouville structure. It is known that, on the standard complex projective space $(\boldsymbol{C}P^n, g, J)$, the typical Kähler-Liouville strucutres are constructed by means of a certain systematic method, and their geodesic flows are completely integrable by certain sets of the first integrals in the structures. Kähler-Liouville manifolds in a certain class can be constructed from real Liouville manifolds by means of a certain method of the complexification, and their geodesic flows are completely integrable. We here see that Hermite-Liouville structures also can be constructed on the complex projective space $(\boldsymbol{C}P^n, J)$, where J is the standard complex structure, by means of the similar method, and their geodesic flows are completely integrable. We moreover see that, for an Hermite-Liouville manifold $(M, g, J, \mathfrak{F})$, under a certain additional condition, n vector fields defined locally can be derived from the Hermite-Liouville structure $(g, \mathfrak{F})$ and its geodesic flow is completely integrable by the first integrals in $\mathfrak{F}$ and the derived n vector fields.

References [1] K.Kiyohara, *Two classes of Riemannian Manifolds Whose Geodesic Flows Are Integrable*, MEMOIRS of the American Mathematical Society 619 (1997).
[2] M.Igarashi, *On compact Kähler-Liouvill surfaces*, Journal of the Mathematical Society of Japan, Vol.49 No.2 (1997), 363∼397.