Isospectral Flat 3-Manifolds

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This work deals with spectra of the Laplace–Beltrami operator $\Delta = -\mathrm{div}\ \mathrm{grad}$ on compact Riemannian manifold without boundary. Let us say that two manifolds M and M' are *isospectral* if their spectra of the Laplace–Beltrami operator on manifolds M and M' coincide.

There is a well-known problem about isospectrality of Riemannian manifolds: whether isospectral manifolds are isometric. The answer is negative in general (see [1] for details). In 1997 Schiemann [2] proved that any two 3-dimensional flat tori are isospectral if and only if they are isometric.

In this work we give a positive answer to this problem for compact flat 3-manifolds. It is known [3] that there are 10 classes of pair-wise homeomorphic (or affine diffeomorphic) compact flat 3-manifolds (6 classes are orientable and 4 classes are non-orientable). To describe the spectrum of manifold M we use a trace function of the manifold M $\mathrm{tr}(H_M) = \int_M H_M(x,x,t)\,dM$, where $H_M(x,y,t)$ is a fundamental solution of the heat equation on manifold M, dM is the volume element. Using a method developed in [5] we have the following

Theorem. *Any two homeomorphic compact flat* 3*-manifolds are isospectral if and only if they are isometric.*

In order to prove Theorem we use a relationship between the fundamental solution of the heat equation on manifold M and on finite-fold regular covering of M [4] and also the following statements:

(i) Spectrum of manifold M determines exactly the trace function $\mathrm{tr}(H_M)$ and, conversely, spectrum of manifold M can be determined by the trace function $\mathrm{tr}(H_M)$.

(ii) The compact flat 3-manifold M can be determined up to isometry by the trace function $\mathrm{tr}(H_M)$.

In the case of 3-dimensional flat torus we use a result obtained by Schiemann [2]. All other cases are proved by completely different methods.

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