Surfaces with integrable geodesic flows

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Let M be a two-dimensional riemannian manifold diffeomorphic to the sphere, and suppose that its geodesic flow admits a nontrivial first integral F, which is a homogeneous polynomial of degree k on each cotangent space. If $k = 1$, then M is a surface of revolution and F is a Killing vector field generating the revolution (identified with a function on T^*M). If $k = 2$, then (M, F) is a "Liouville surface". In the cases where $k = 3$ and 4, some finite dimensional families of such (M, F) were found by Bolsinov and Fomenko, and some extensions of those were obtained by Selivanova. However, generally speaking, we do not know much about the cases where $k \geq 3$.

In this talk we shall introduce a family of such (M, F) for every $k \geq 3$, which we found recently. They are parametrized by functions in one variable for each k, and each of them is a $C_{2\pi}$-manifold, namely, all geodesics are closed and have the same length 2π.

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