Soap bubbles and Riemann surfaces

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To construct and classify embedded constant mean curvature (CMC) surfaces in $\mathbb{R}^3$ via meromorphic methods, it would help to know what conformal types of punctured Riemann surfaces are realized. The talk will survey recent progress and open problems in the area, and sketch a proof of the following new result: Let $\mathbb{M}_{g,k}^{CMC}$ be the space of properly (Alexandrov) embedded CMC surfaces of genus g with k ends, and let $\mathbb{M}_{g,k}^{PARA}$ be the moduli space of parabolic structures, that is, the product of $(\mathbb{R}^+)^k$ and the moduli space of Riemann surfaces of genus g with k punctures; then the map which assigns the asymptotic necksizes to the $(\mathbb{R}^+)^k$ factor, and the underlying conformal structure to the other factor, is proper. This result is a key step in showing that every conformal type of k-punctured sphere ($k > 1$) is realized by an embedded genus zero CMC surface with k ends, and that for a generic choice of necksizes, there are an even number of such surfaces with a given conformal structure.