Quantization of Curvature for Compact Surfaces in S^n

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In this talk, we present joint results with U. Simon (TU Berlin)

Theorem *Let M be a compact surface with the Gauss curvature K and the mean curvature vector $\vec{H}$ in an n-dimensional unit sphere $S^n(1)$. Then we have*

$$\int_M [-2(3K-1)(K-1)+4H^2(2K-1) - \int_M |\mathrm{grad}\vec{H}|^2 \leq 0. \tag{1}$$

As direct applications of Theorem, we get

Corollary 1 *Let M be a compact surface in an n-dimensional unit sphere $S^n(1)$. If Gauss curvature K the mean curvature vector $\vec{H}$ satisfy*

$$\frac{1}{3} \leq K \leq 1, \quad |\mathrm{grad}\vec{H}|^2 \leq 4H^2(2K-1), \tag{2}$$

then either $H=0$,$K=\frac{1}{3}$, $n=4$ and M is the Veronese surface, or $K=1$ and we determine the components of the second fundamental form of M completely. In the latter case, if $n \leq 4$ or $H = \mathrm{constant}$, we determine the surfaces completely.

Corollary 2. *Let M be a compact Willmore surface in $S^n(1)$. Then*

$$\int_M \{(H^2-K)+1\}\{-3(H^2-K)-1\}dv \leq 0. \tag{3}$$

In particular, if $\frac{1}{3} \leq K - H^2 \leq 1$, then either $K - H^2 = 1$ and M is totally umbilic in $S^3(1)$, or $H = 0$, $K = \frac{1}{3}$ and M is the Veronese surface in $S^4(1)$.

Remark Noting that any minimal surface is a Willmore surface, Corollary 1 and Coroally 2 reduce to well-known results of Benko-Kothe-Semmler-Simon in the minimal case.