Harmonicity of the hyperbolic Gauss map for cmc-1 surfaces in $H^3(-1)$.

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Let $X(u,v)$ be a regular surface with nonzero mean curvature in the space of Lobachevsky $H^3(-1)$. If we choose direction of unit normal $n(u,v)$ to the surface in such a way that the mean curvature will be positive then unambiguously is defined a geodesic $\gamma(s)$ of $H^3(-1)$ with initial data $\gamma(0) = X(u,v), \gamma'(0) = n(u,v)$.The hyperbolic Gauss map of the surface $X(u,v)$ is the correspondence $g(u,v) : X(u,v) \to lim_{s\to\infty}\gamma(s) \in \partial H^3(-1)$, where $\partial H^3(-1)$ is the ideal boundary of $H^3(-1)$. In the model of Poincare $H^3_0(-1)(y^1,y^2,y^3)$ in the unit ball $\partial H^3_0(-1)$ represents the unit sphere with the standard metric g_0. In the model $\overline{H}^3(-1)(x^0,x^1,x^2,x^3)$ of the Lobachevsky space on the hyperboloid $-(x^0)^2 + (x^1)^2 + (x^2)^2 + (x^3)^2 = -1, x^0 > 0$ in the Minkowski space $R^{3,1}$ the ideal boundary is the light cone$|x| = 0$. There is the isometric correspondence between both models $\phi : \overline{H}^3(-1)(x^0,x^1,x^2,x^3) \to H^3_0(y^1,y^2,y^3)$, where $y^i = \phi^i(x) = \frac{x^i}{1+x^0}, i = 1,2,3$.

Let $X(u,v)$ be a regular surface in $\overline{H}^3_0$. Then the map

$$g(u,v) = \left(\frac{X^1+n^1}{X^0+n^0}, \frac{X^2+n^2}{X^0+n^0}, \frac{X^3+n^3}{X^0+n^0}\right) \in S^2 = \partial H^3_0(-1) \qquad (1)$$

represents the hyperbolic Gauss map of the surface $\phi(X(u,v)) \subset H^3_0(y^1,y^2,y^3)$. We prove the next statement, which is equivalent to harmonicity of the hyperbolic Gauss map of the surface $\phi(X(u,v))$ (with the metric g induced by its immersion in $H^3_0(-1)$) to the $\partial H^3_0(-1) = S^2$ endowed with the standard metric g_0.

Theorem. *The mapping $g(u,v) : (X(u,v), g) \to (S^2, g_0)$, defined by the formula (1) is harmonic if $X(u,v)$ is the surface of constant mean curvature 1 in the hyperbolic space $H^3(-1)$.*

This fact relates to the famous theorem of E.Ruh and J.Vilms [1] which asserts the harmonicity of Gauss map for surfaces F^m with parallel mean curvature vector in the euclidean space R^n. Some concrete examples of harmonic maps to the sphere,based on this theorem,can be presented.

References

[1]. E.Ruh,J.Vilms, The tension field of Gauss map//Trans. Amer. Math. Soc., 1970,y.199,p.569-573.

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