ON MINKOWSKI DECOMPOSITIONS OF POLYTOPES

G.PANINA

The paper derives conditions for virtual polytope to be decomposable in the Minkowski sum of lower dimensional polytopes. If such decomposition exists, its explicit form is given. Moreover, a collection of mutually orthogonal projectors on cylinder subgroups is explicitly constructed.

Virtual polytope group was introduced originally by Khovanskii, Pukhlikov [1] (see also McMullen [2] and Morelli [3]).

Virtual polytopes in the real space $\mathbb{R}^n$ form a group $\mathcal{P}^*$ with respect to the Minkowski summation.

We single out its subgroups of k -cylinders as follows.

A virtual polytope $K \in \mathcal{P}^*$ is called a k *-cylinder* ($k = 1, \dots, n+1$), if it is representable as the Minkowski sum of $n-k+1$ -dimensional polytopes: $K = \otimes_i K_i$, $dim K_i \leq n-k+1$. The groups of k -cylinders Cyl_k form the filtration.

$$\mathcal{P}^* = Cyl_1 \supset \cdots \supset Cyl_{n+1} = \{E\},$$

where E is the unite element of $\mathcal{P}^*$.

We explicitly construct a collection of mutually orthogonal projectors , group homomorphisms

$$\delta_k : \mathcal{P}^* \to Cyl_k,$$

whose sum is the identity operator. These projectors induce (in the usual way) the direct sum decompositions of the groups $\mathcal{P}^*$ and Cyl_k .

$$\mathcal{P}^* = \delta_1\mathcal{P}^* \oplus \delta_2\mathcal{P}^* \oplus \cdots \oplus \delta_n\mathcal{P}^*$$

and

$$Cyl_k = \delta_k\mathcal{P}^* \oplus \cdots \oplus \delta_n\mathcal{P}^*.$$

References

[1] A. Khovanskii, A. Pukhlikov, *Finitely additive measures of virtual polytopes.* St. Petersburg Math. J. 4 (1993), 337-356.

[2] P. McMullen , *The polytope algebra*, Adv. Math. 78, (1989), 1, 76–130.

[3] Morelli R. *A theory of polyhedra*, Adv. Math. 97, (1993) 1, 1–73.

[4] G.Yu.Panina, *The structure of virtual polytope group with respect to cylinders subgroups filtration.* (in russian) St. Petersburg Math. J. 13(2001), N3, 179-197.

Gajane Panina, Institute for Informatics and Automation, V.O. 14 line 39, St.Petersburg, 199178, RUSSIA

E-mail address: panina@iias.spb.su