On spontaneous surgeries on the Borromean rings.

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In the paper we consider a cone manifolds that can be obtained by spontaneous surgery at the components of Borromean rings. For arising cone manifolds we discuss the existence of geometrical structures, trigonometrical rules and volume formula. The analogous results for the orbifold surgery one can find in [1] and [2]. We illustrute the obtained results by the following theorems.

Let $O_{k,\,l,\,m} = B((0,k),(0,l),(0,m))$ be a cone manifolds obtained by spontaneous surgery $(0,k),(0,l),(0,m)$ on the three components of Borromean rings. Let also $\alpha = 2\pi/k, \beta = 2\pi/l, \gamma = 2\pi/m$.

THEOREM 1. *The cone manifold* $O_{k,\,l,\,m} = B((0,k),(0,l),(0,m))$ *is:*

(i) *hyperbolic, if* $0 \le \alpha, \beta, \gamma < 2\pi$

(ii) *Euclidean, if* $\alpha = \beta = \gamma = 2\pi$

(iii) *spherical,if* $2\pi < \alpha, \beta, \gamma < 4\pi$

THEOREM 2. *Let* $O_{k,\,l,\,m} = B((0,k),(0,l),(0,m))$ be a hyperbolic cone manifold and L_k, L_l, L_m *are lengths of the singular geodesics of* $O_{k,\,l,\,m}$. *Then*

$$\frac{\tanh(L_k/2)}{\tan(\pi/2k)} = \frac{\tanh(L_l/2)}{\tan(\pi/2l)} = \frac{\tanh(L_m/2)}{\tan(\pi/2m)} = T,$$

where T *is a positive root of the equation*
$A^2B^2C^2T^4 + (1 + A^2 + B^2 + C^2)T^2 - 1 = 0,$
$A = \tan(\pi/2k), B = \tan(\pi/2l),$ *and* $C = \tan(\pi/2m)$.

THEOREM 3. *Let* $O_{k,\,l,\,m} = B((0,k),(0,l),(0,m))$ *be a hyperbolic cone manifold. Then*

$$Vol\, O_{k,\,l,\,m} = 2\int_T^{+\infty} \log\Big(\frac{(t^2 - A^2)(t^2 - B^2)(t^2 - C^2)}{(1 + A^2)(1 + B^2)(1 + C^2)T^2}\Big)\frac{dt}{t^2 + 1}$$

where T *is the same as above.*

The spherical versions of Theorem 2 and 3 are also obtained.

References

[1] H.M.Hilden, M.T.Lozano, J.M.Montesinos-Amilibia, On the Borromean orbifolds: geometry and arithmetic, in:Topology'90, eds. B.A.Apanasov, W.Newmann, A.Reid, L.Siebenmann, de Gruyter Berlin,133–167,1992.

[2] D.A.Derevnin, A.D.Mednykh, On the volume of spherical Lambert cube, Preprint, 2001.