Gluing of Solutions of Seiberg-Witten Equations

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We will give a report of a gluing theorem for Seiberg-Witten moduli spaces in a quite general context, which encompasses earlier results. These moduli spaces are associated to a 4-manifold M together with a Spin^c-structure and are essentially the solution space to a certain system of PDE's on the manifold modulo the gauge action. We consider the case of a closed 4-manifold M which is split into two cylindrical-end pieces M^+ and M^- along a closed, oriented submanifold N and we are interested in describing the SW moduli space of M in terms of those of M^+ and M^-. We will describe a gluing procedure for the moduli spaces of M^+ and M^- over the open subset of smooth and irreducible points, provided that the boundary maps to the character variety are transversal.

The construction is done in several steps. We first define an approximate gluing map and then we perturb it with a suitable term to construct an accurate solution. To obtain the perturbation term, we need to delicately choose an appropriate *weight* for our Sobolev completions, so that we can construct an inverse for a certain operator. We show that this is possible when the cylindrical ends are sufficiently elongated.