Parallel surfaces in affine 4-space

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(Joint work with L. Vrancken)

We study affine immersions as introduced by Nomizu and Pinkall. An immersion $x : (M^n, \nabla) \to (\mathbb{R}^m, D)$ is called *affine* if there exists a transversal $(m-n)$-dimensional bundle σ such that

$$h(X,Y) := D_X x_*(Y) - x_*(\nabla_X Y) \in \sigma \quad \forall X, Y \in \mathfrak{X}(M^n).$$

The σ-valued symmetric bilinear form h is called the *second fundamental form.* Using the Weingarten formula it is possible to define the covariant derivative ∇h ($= C$, the *cubic form*) by

$$(\nabla_X h)(Y,Z) = \nabla_X^{\perp} h(Y,Z) - h(\nabla_X Y, Z) - h(Y, \nabla_X Z).$$

If $\nabla h \equiv 0$, the affine immersion is called *parallel.*
We present a local investigation of surface immersions ($n = 2$), which are linearly full (i. e. $m \in \{2,3,4,5\}$). For each codimension $m - n > 0$ there exists a notion of (non-)degeneracy. We classify the degenerate parallel surface immersions in $\mathbb{R}^4$. They are special types of ruled surfaces. This completes the classification of parallel surfaces of which the first results were obtained in the beginning of this century by Blaschke and his collaborators.