Hyperbolic rank and subexponential corank of metric spaces *

Viktor Schroeder

Abstract

We introduce a new quasi-isometry invariant $\operatorname{corank} X$ of a metric space X called *subexponential corank*. A metric space X has subexponential corank k if roughly speaking there exists a continuous map $g : X \to T$, T is a topological space, such that for each $t \in T$ the set $g^{-1}(t)$ has subexponential growth rate in X and the topological dimension $\dim T = k$ is minimal among all such maps. Our main result is the inequality $\operatorname{rank}_h X \le \operatorname{corank} X$ for a large class of metric spaces X including all locally compact Hadamard spaces, where $\operatorname{rank}_h X$ is maximal topological dimension of $\partial_\infty Y$ among all $\mathrm{CAT}(-1)$ spaces Y quasi-isometrically embedded into X (the notion introduced by M. Gromov in a slightly stronger form). This proves several properties of rank_h conjectured by M. Gromov, in particular, that any Riemannian symmetric space X of noncompact type possesses no quasi-isometric embedding $\mathrm{H}^n \to X$ of the standard hyperbolic space H^n with $n - 1 > \dim X - \operatorname{rank} X$.

*This is joint work with Sergei Buyalo to be published in GAFA