Geometric description and semi-classical quantization of Calogero – Strocchi systems

Andrei I. Shafarevich

Let $H(x, p)$ be a polynomial in C^2; functions $H_1 = ReH$, $H_2 = ImH$ are in involution in R^4 with coordinates x_1, x_1, p_1, p_2 ($x = x_1 + ix_1, p = p_1 - ip_2$) and standard symplectic form $dp_1 \wedge dx_1 + dp_2 \wedge dx_2$. Thus each of these functions defines integrable Hamiltonian system with two degrees of freedom, called the *Calogero – Strocchi system.* Invariant manifolds $H_1 = c_1, H_2 = c_2$ of the Calogero – Strocchi system are non-compact Riemann surfaces $H = c = c_1 + ic_2$ in C^2 and their topology can differ from that prescribed by the Liouville theorem (note that the corresponding Hamiltonian vector fields are, generally speaking, not complete).

Consider quantum Schroedinger operator $\hat{H} = H_1(x_1, x_2, -ih\partial/\partial x_1, -ih\partial/\partial x_2)$, corresponding to classical Calogero – Strocchi system; as a rule, this operator is not semi-bounded. Maslov semi-classical theory allows to construct asymptotical (as $h \to 0$) eigenvalues and eigenfunctions of $\hat{H}$, corresponding to invariant surface M, if the Bohr-Sommerfeld conditions

$$\frac{1}{2\pi h} \int_\gamma (p, ds) = n + \frac{\mu(\gamma)}{4}, \quad n \in Z$$

are fulfilled. Here γ is an arbitrary cycle on M, $\mu(\gamma)$ – the Maslov index of γ. As the genus of M can be arbitrary, these conditions generically form an overdetermined system of equations on two parameters c_1, c_2.

The aim of the talk is to describe certain modification of the Maslov construction which makes it possible to obtain asymptotical eigenvalues and eigenfunctions for Calogero – Strocchi systems. This modification is based on the study of geometrical properties of trajectories of the corresponding classical systems. As a result, we compute asymptotical eigenvalues by solving the system of equation, determined by the matrix of B-periods of the surface M. Asymptotical eigenfunctions are described in terms of the Maslov canonic operator.