CONVERGENCE OF METRIC SPACES AND VARIATIONAL CONVERGENCE

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Let $L^2(X, Y)$ be the L^2 space of maps from a measure space X to a complete separable Hadamard (or CAT(0)) space Y, where a measure space means a locally compact Polish space with a Radon measure. It is remarkable that $L^2(X, Y)$ is also an Hadamard space (of infinite dimension) with respect to its L^2 metric. Under a convergence $X_i \to X$ of measure spaces, we obtain the convergence $H_i := L^2(X_i, Y) \to H := L^2(X, Y)$ (in some weak sense of Gromov-Hausdorff) and consider *variational convergence* over it (i.e., convergence of energy functionals). Precisely, we define a concept of convergence of a sequence of functionals on H_i to that on H, which we call *Mosco convergence*, as a natural generalization of the original Mosco convergence.

In functional analysis, we usually consider Hilbert spaces instead of L^2 function spaces to clarify the theory. As well as that, we consider general Hadamard spaces H_i, H instead of L^2 mapping spaces and define Mosco convergence under convergence $H_i \to H$.

In the case where H_i, H are Hilbert spaces and F_i, F are quadratic forms on H_i, H respectively, we prove that the Mosco convergence $F_i \to F$ is equivalent to the convergence of the correspoinding spectral measures. U. Mosco formerly proved this for $H_i = H$.

For general H_i, H, we also prove that if F_i, F are lower semi-continuous convex functions on H_i, H and if $F_i \to F$ (Mosco), then the corresponding resolvent operators converges strongly. J. Jost studied something relative to this under $H_i = H$, but his definition is different from ours.

We apply this functional analytic theory to study convergence of Riemannian manifolds (or Alexandrov spaces) and spectrum of Laplacian. For example, we have the following. We first define the energy of a map from an Alexandrov space to a metric space. Then, under a noncollapsing Gromov-Hausdorff convergence $X_i \to X$ of (possibly noncompact) Alexandrov spaces with a uniform lower bound of curvature, we prove the convergence of the energy functionals and consequently convergence of the resolvent operators and the spectral measures associated with the Laplacian.

This talk is on a joint work with K. Kuwae.