An infinite sequence of non-realizable weavings

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A *weaving* is a number of lines drawn in the plane so that no three lines intersect in a point, and the intersections are drawn so as to show which of the two lines is "above" the other. A weaving is called *realizable* if it is a projection onto a plane of a number of pairwise skew lines in the 3-space. The problem of realizability of weavings appeared in algebraic knot theory and discrete geometry [Vir85, Whi91, PPW93]. A *subweaving* of a weaving A is a weaving obtained from A by deleting some lines. Our main result solves a problem from [PPW93]:

Theorem. For each integer $n \geq 4$ there exists a nonrealizable weaving, consisting of n lines, all of whose subweavings are realizable.

References

[PPW93] [redacted] Pach, R. Pollack and E. Welzl, *Weaving patterns of lines and line segments in space*, Algorithmica **9** (1993), 561–571.

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[Whi91] W. Whiteley, *Weavings, sections and projections of spherical polyhedra*, Discr. Appl. Math. **32:3** (1991), 275–294.