Extended complexity of 3-manifolds

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Any compact 3-manifold M has a complexity $c(M)$, which is a nonnegative integer number and which is defined as the number of true vertices of a minimal almost simple spine P of M. The complexity has many good properties. In particular, it behave well with respect to cutting M along surfaces. Namely, if M_F is obtained from M by cutting along an incompressible surface $F \subset M$, then $c(M_F) \leq c(M)$. However, this useful property has a shortcoming: the inequality is not strong. So we cannot use it in inductive proofs. We improve that by defining extended complexity $\bar{c}(M)$. It is not a number anymore, but a tuple of nonnegative integers. The tuples are considered in lexicographical ordering. We prove that if F is essential, then $\bar{c}(M_F) < \bar{c}(M)$. We apply the extended complexity for proving the algorithmic classification theorem for Haken 3-manifolds.