Ceometrical and topological properties of graph manifolds: a unified approach

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The main goal of our talk is to show that some quite different properties of graph manifolds can be described from a unified point of view. These properties are

- to carry a metric of non-positive sectional curvature
- to be (finitely covered by) a surface bundle over the circle
- to be (finitely covered by) a codimensional 1 foliation of a certain kind
- to contain an immersed (embedded) injective surface of negative Euler characteristic

Our approach can be sketched as follows. Each graph manifold M defines a graph which is dual to the canonical splitting of M (by Jaco, Shalen and Jochansson). We consider a difference equation on this graph (with topological invariants of M as coefficients). It turns out that all properties we mentioned above are encrypted in this equation.

In our talk we introduce the equation and give criteria of its solvability in interesting cases. As an application of our approach we obtain the following

Theorem. *Let M be a graph manifold. If M admits a NPC-metric then M is finitely covered by a surface bundle over the circle.*

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