The Topological Analysis of the Integrable Problems of Celestial Mechanics in the Lobachevsky space.

Vozmichtcheva T.G.
Izhevsk State Technical University, tavo@mail.ru

The problem on the motion of a mass point on a pseudosphere (with the metric of constant negative curvature) in the field generated by two fixed attracting centers is considered in this work. The problem on the motion of a mass point under the action of Newtonian attraction of a fixed center and force which is constant in magnitude and direction is also studied. This problem is the limit case of the two-center problem. In the process of the passage to the limit one of the centers is moved to infinity in the force direction (its mass tends to infinity in such a way that the perturbing acceleration is constant, i.e., the mass should increase in proportion to the squared distance). These problems are the generalizations of the classical plane problem of the two-center problem and its limit case integrated by Euler and Lagrange. However the integrability of the analogous problems on a sphere and in the Lobachevsky space was noted not long ago [1, 2, 3]. So the topological properties of these problems are not enough studied yet.

The statement of problems of dynamics in spaces of constant curvature is often rather nontrivial, there exist several approaches to the generalization of the classical problems for curved spaces. The systems under consideration in this work have the singularities and, formally speaking, are not integrable in the sense of Liouville (the vector fields generated by the Hamiltonians are not complete). In the work some regularization of the systems is described after that the vector fields become complete and smooth.

The topological analysis of the two-center problem on the two-dimensional sphere was carried out in the paper [4]. The topological invariants characterizing the system up to the Liouville equivalence are constructed. It turns out that the answer is sufficient interesting. In particular, some of the invariants obtained in this case did not appear in integrable cases investigated earlier. The topology of the isoenergy surfaces is also strongly different from what authors investigated earlier.

In the present work the topological analysis of the integrable problems of celestial mechanics in the case of noncompact phase space (that is for the two-center problem and the Lagrange problem on a pseudosphere) is carried out. In the work the theorem on the Liouville equivalence of the two-center problem under the condition that a mass point moves on the upper hemisphere at all time of motion, on a pseudosphere under the condition $h < -\dfrac{\gamma_1 + \gamma_2}{R}$ and on a plane under the condition $h < 0$, i.e., in the case of bounded motion, is proved. The bifurcation for the noncompact phase space are constructed, the analysis of these bifurcations and comparing with the bifurcations for the compact cases is carried out. The connection between integrable systems describing some problems of celestial mechanics in spaces of constant curvature is investigated as well. It is shown that these problems are transformed one to another as the curvature varies. In particular, it is proved that the Kepler problem and the two center problem on spaces of nonzero constant curvature λ turn to the corresponding classical problems on the plane.

Thus, some new topological effects in the spaces of constant curvature are obtained.

References

[1] Kozlov V. V., Harin O. A.: 'Kepler's problem in constant curvature spaces', Celest. Mech. and Dyn. Astr. 1992. V. 54. P. 393–399.

[2] Vozmischeva T.G.: 'Classification of motions for generalization of the two-center problem on a sphere', Celest. Mech. & Dyn. Astr. 2000. V. 77. P. 37-48.

[3] Vozmischeva T.G.: 'The Lagrange and Two-center Problems in the Lobachevsky Space', Celest. Mech. & Dyn. Astr. (published)

[4] Vozmischeva T.G., Oshemkov A.A.: 'The topological analysis of the two-center problem on the two-dimensional sphere', Sbornik: mathematics, 2001. (published)