On roots of surfaces mappings

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Let M_1, M_2 be two closed (not necessarily orientable) manifolds of the same dimension, $f\colon M_1 \to M_2$ be a continuous map and $c\colon M_1 \to M_2,\ x \mapsto c \in M_2$ a constant map. Two points (roots) x_0, x_1 with $f(x_0) = f(x_1) = c$ belong to the same *root class* if there exists a path $\gamma\colon [0,1] \to M_1, \gamma(0) = x_0,\ \gamma(1) = x_1$ such that $f \circ \gamma \simeq 0$. To each root class one can define the index in the usual way, and a root class is called essential if the index is different from 0. By definition the pair (f, c) has the *Wecken property* if f can be deformed into a map f' such that the number of coincidence points of (f', c) is the same as the number of essential coincidence classes of (f, c) and, hence, every essential coincidence class consists of exactly one point. If the dimension is different from 2 every map has the Wecken property. However for surfaces M_1, M_2 there are cases with and others without the Wecken property. Let $A(f)$ denote the absolute degree as defined by Hopf or Epstein, and let $\ell(f) = [\pi_1(M_2) : f_\#(\pi_1(M_1))]$. Then a map f has the Wecken property if and only if either the Euler characteristic $\chi(M_2) > 0$ or

$$A(f) \cdot (1 - \chi(M_2)) \leq \ell(f) - \chi(M_1).$$

Moreover there are given conditions on the genera to indicate the existence of a map f with given absolute degree $A(f)$ and index $\ell(f)$.

In free groups there are solved certain quadratic equations closely related to the root problem.

(The main part of the talk is based on common research with S. Bogaty, E. Kudryavtseva (Moscow), and D.L. Gonçalves (São Paulo).)