Cohomogeneity one manifolds with nonnegative curvature

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Several years ago K. Grove and I initiated the study of cohomogeneity one manifolds with nonnegative sectional curvature. In particular we showed that if the singular orbits have codimension two, then such metrics always exist. In the first pat of my talk (joint work with K. Grove), I will discuss some new geometric applications of this result. We will show that all vector bundles, sphere bundles and principal bundles over CP^2 with second Stiefel Whitney class $w_2 \neq 0$ admit a complete metric with nonnegative sectional curvature. In the case of $w_2 = 0$ we show that at least those bundles have this property which are the pullback of bundles over S^4 under a two fold branched cover $CP^2 \to S^4$ with branching locus along the Veronese RP^2 in S^4. The class of bundles with $w_2 \neq 0$ is particularly interesting since it is in that class that one already knows several examples which admit even positive sectional curvature. In the second part of my talk I will report on recent joint work with K.Grove, L.Verdiani, and B.Wilking. Here we examine the situation for cohomogeneity one manifolds where at least one of the singular orbits has codimension larger than two. We will show that in that case not all cohomogeneity one manifolds have an invariant metric with nonnegative curvature, producing counter examples to a conjecture that K. Grove and I made in our initial study several years ago. In particular we will discuss the $(2n-1)$ dimensional Kervaire sphere which in many cases is an exotic sphere and is known to contain all cohomogeneity one manifolds which are homeomorphic but not diffeomorphic to spheres. We will show that as soon as $2n - 1 > 5$, the Kervaire sphere never admits an invariant metric with nonnegative sectional curvature.