CONFORMAL FIELD THEORY AND FROBENIUS ALGEBRAS IN MODULAR TENSOR CATEGORIES

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Jürgen Fuchs

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KARLSTADS UNIVERSITET
Jürgen Fuchs

Plan

- Aim:

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- Aim: Understand two-dimensional conformal field theory

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- Aim: Understand "holomorphic factorization" in RCFT

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- Step 1: Thicken the world sheet

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  Connecting manifold , ribbon graphs , modular tensor categories

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Holomorphic factorization

Free boson QFT in $d = 2$:

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eq. of motion $\left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial t^2}\right)\phi(x,t) = 0$ solved by $\phi = \phi_+(x+t) + \phi_-(x-t)$

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"left- and right-movers"

"holomorphic / antiholomorphic"

"chiral / antichiral"

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Observation: Same phenomenon for various aspects
of a large class of 2-d QFT models
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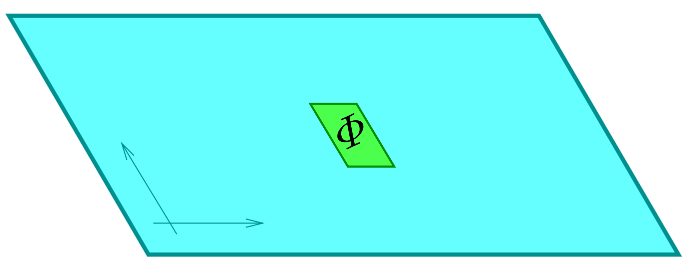
In particular: Bulk fields Φ of a RCFT



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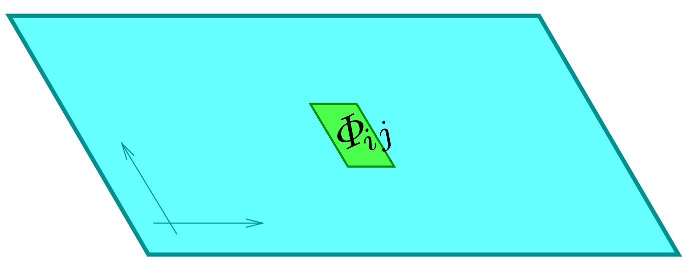

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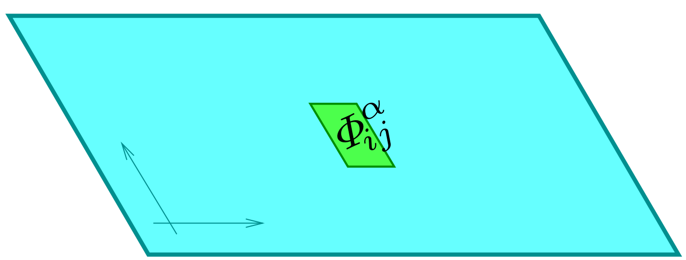

Bulk fields

CFT $\Longrightarrow$ Conformal symmetry / extensions

2-d $\Longrightarrow$ Virasoro algebra / affine Lie algebras / W-algebras / ...

$\Longrightarrow$ conformal vertex algebra [Borcherds 1986 , . . .]

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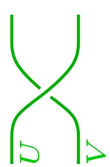
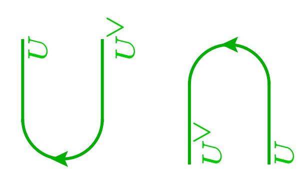
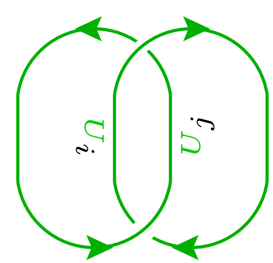
Rep category $\mathcal{R}ep(\mathcal{V})$: a modular tensor category

- Tensor product $U \otimes V$ of $\mathcal{V}$-modules and of intertwiners , $\mathcal{V} = \mathbf{1} =: U_0$
- Abelian, $\mathbb{C}$-linear, semisimple, finite $U \cong \bigoplus_{i \in \mathcal{I}} U_i^{\oplus n_i}$
- Braiding 

- Twist 

- Duality 

- Braiding maximally non-degenerate: $\det$  $\neq 0$

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Bulk field $\Phi = \Phi_{ij}$

$\Longrightarrow$ two simple objects $U_i \, , U_j \; \in \mathcal{C} := \mathcal{R}ep(\mathcal{V})$

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Recall: Free boson CFT: geometric separation of left- and right-movers

Bulk fields

Separate locally left- and right-movers :

- double $\widehat{\mathrm{X}}$ of world sheet X

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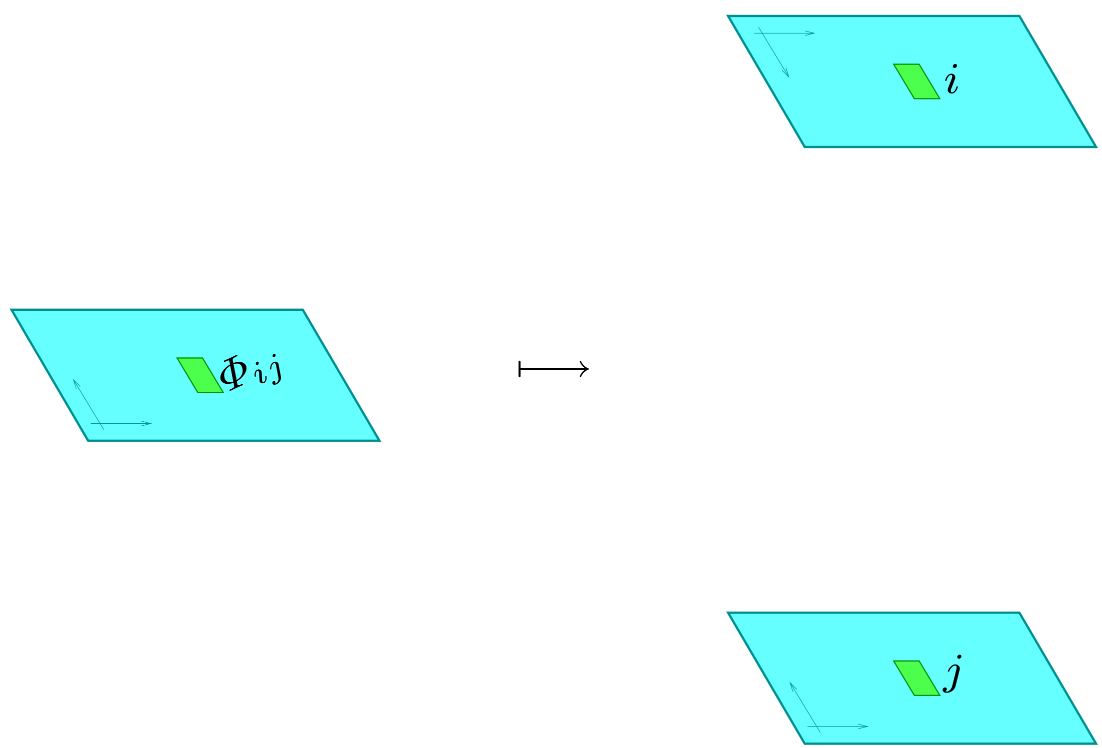
$\widehat{\mathrm{X}}$ = orientation bundle over X modulo identification of points in fibers over $\partial\mathrm{X}$



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To connect to X: Fatten the world sheet:

- connecting manifold M_X $\qquad \partial M_X = \widehat{X}$

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$\mathrm{M_X}$ $=$ interval bundle over X modulo identification over $\partial\mathrm{X}$

$$= (\widehat{\mathrm{X}} \times [-1, 1])/\sim \quad \text{with} \quad ([x, \mathrm{or}_2], t) \sim ([x, -\mathrm{or}_2], -t)$$

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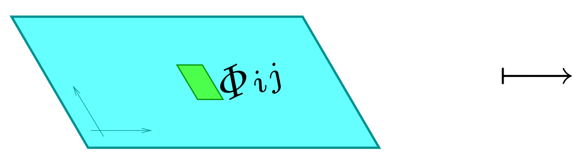
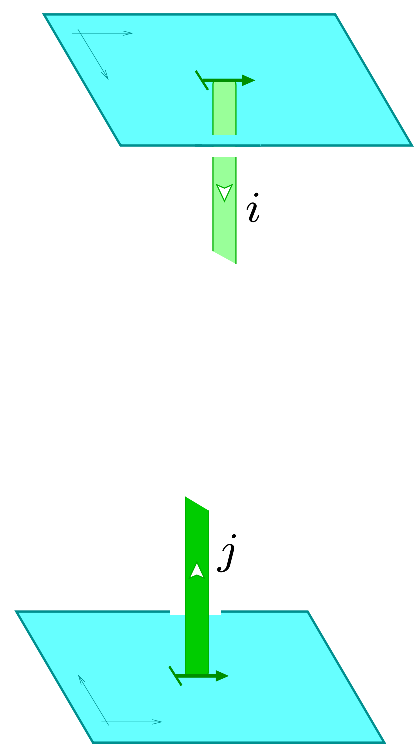
Field theory on $\mathrm{M_X}$ non-dynamical

$\rightsquigarrow$ 3-d TFT

$\rightsquigarrow$ ribbon graphs





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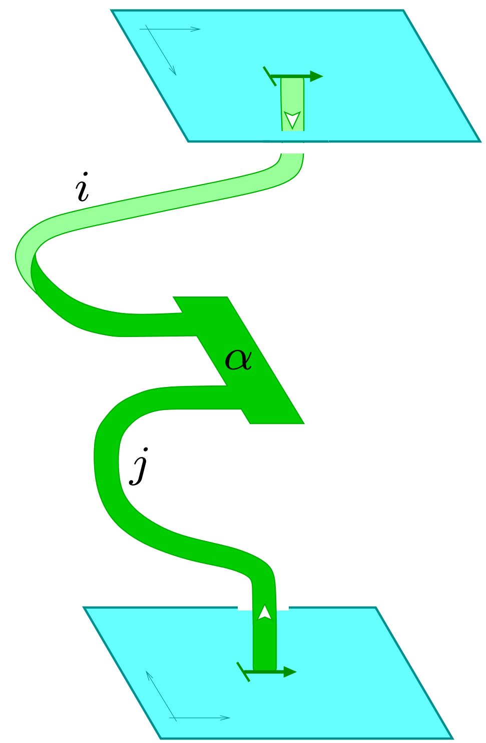

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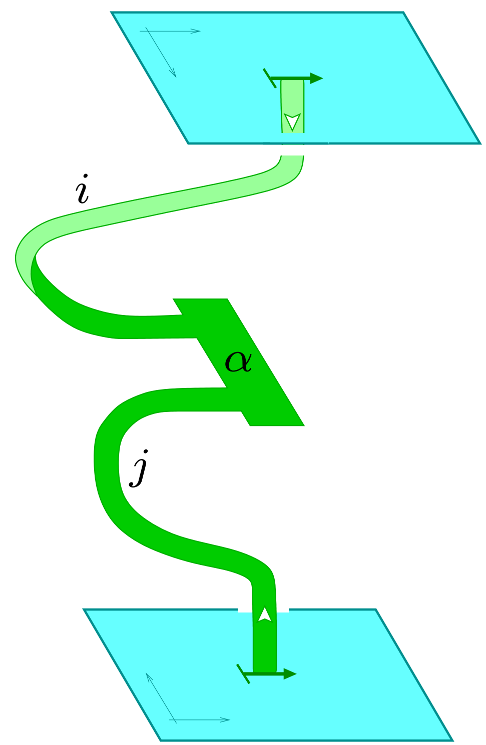

- X embedded as $M_X \supset X \times \{t=0\}$

Bulk fields

- double $\widehat{\mathrm{X}}$
  $($ orientation bundle over $\mathrm{X}\,)/\!\sim$
- connecting manifold $\mathrm{M_X}$
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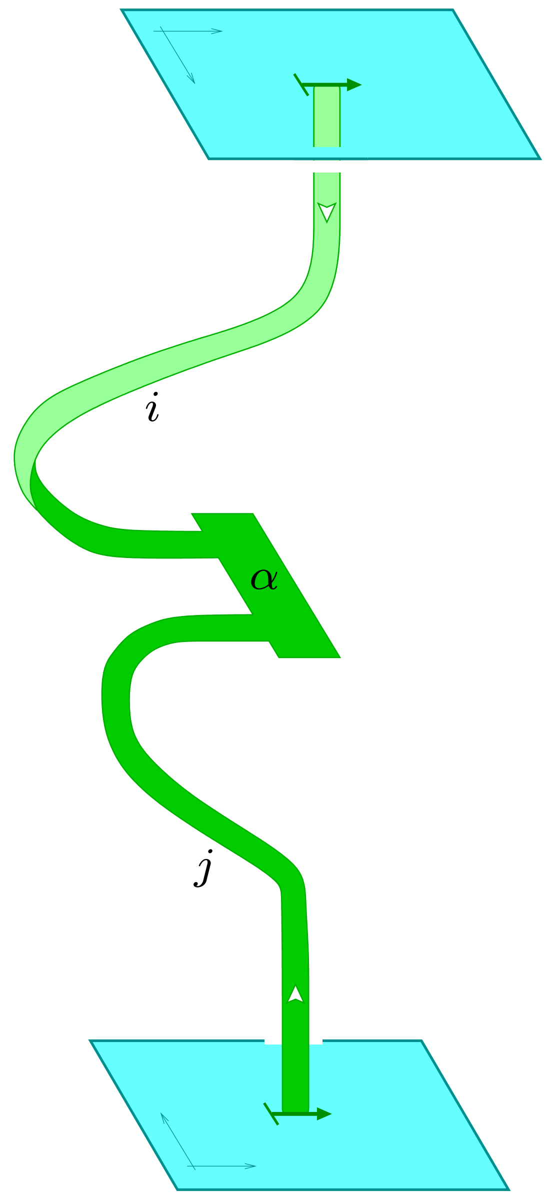
- embedding $\mathrm{X} \times \{t{=}0\} \subset \mathrm{M_X}$



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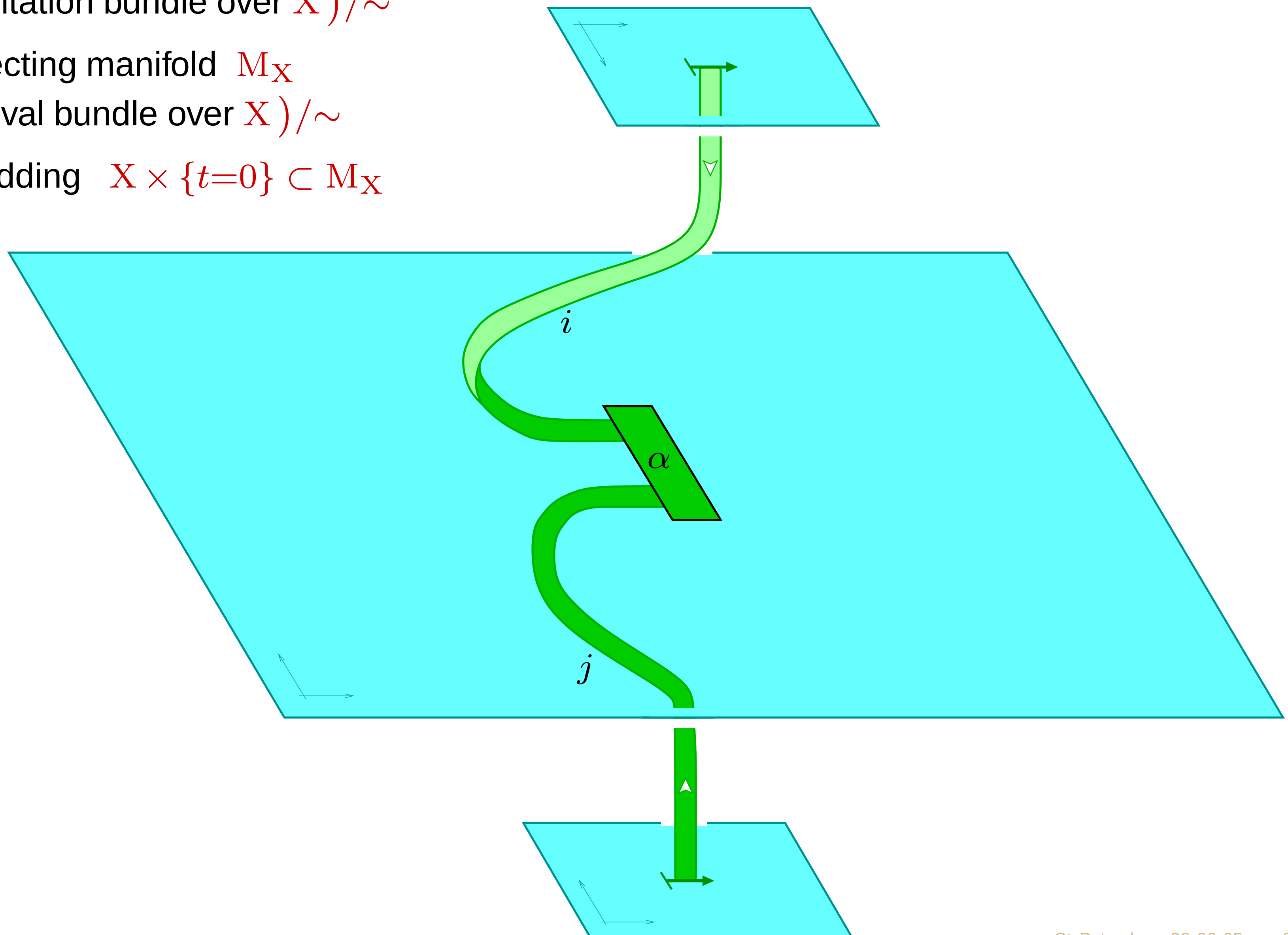

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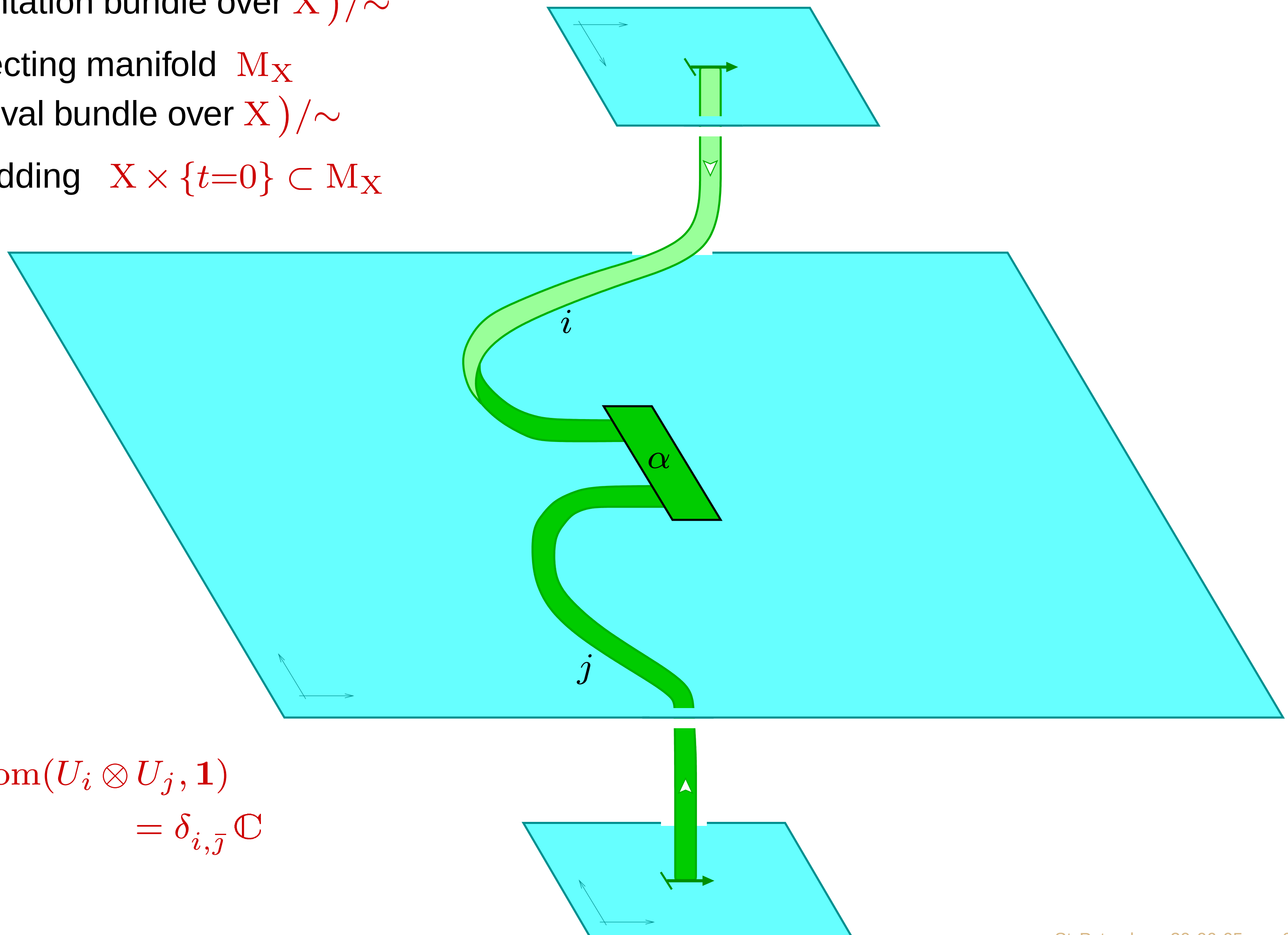

$$\alpha\in\mathrm{Hom}(U_i\otimes U_j\,,\mathbf{1})$$
$$=\delta_{i,\bar{j}}\,\mathbb{C}$$

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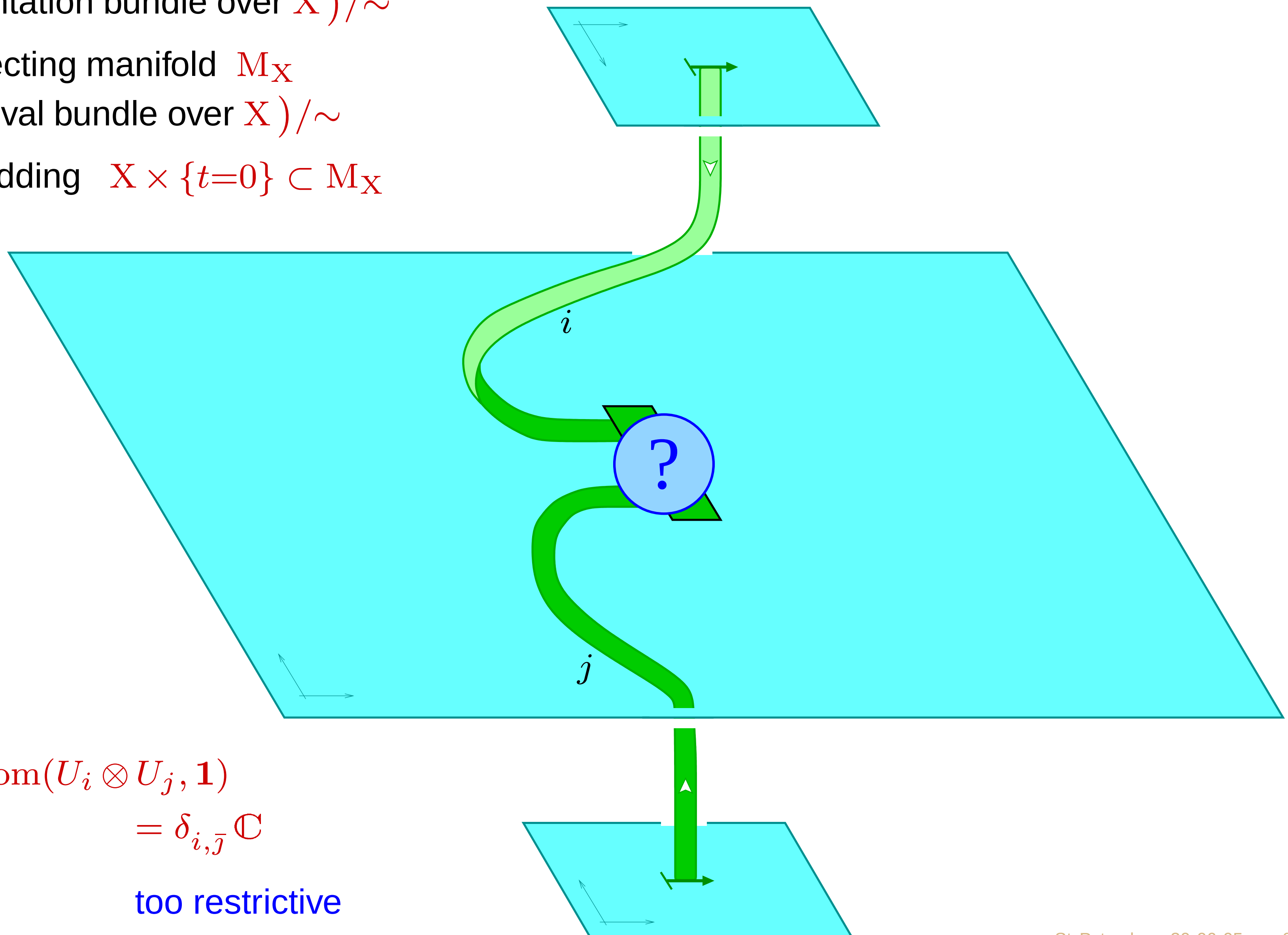

$$\alpha \in \mathrm{Hom}(U_i \otimes U_j, \mathbf{1}) \\ = \delta_{i,\bar{\jmath}}\, \mathbb{C}$$

too restrictive

Frobenius algebras

Special case: meromorphic CFT

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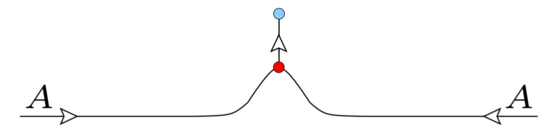
Special case: meromorphic CFT

- $\mathcal{C} = \mathcal{V}ect_{\mathbb{C}}$
- 2-d lattice topological CFT
- corresponds to separable Frobenius algebra A in $\mathcal{V}ect_{\mathbb{C}}$
  properties of $A \longleftrightarrow$ triangulation independence

Frobenius algebras

Prescription for general RCFT:

- triangulate the world sheet (trivalent vertices)
- label edges (ribbons) by a Frobenius algebra A in $\mathcal{C}$



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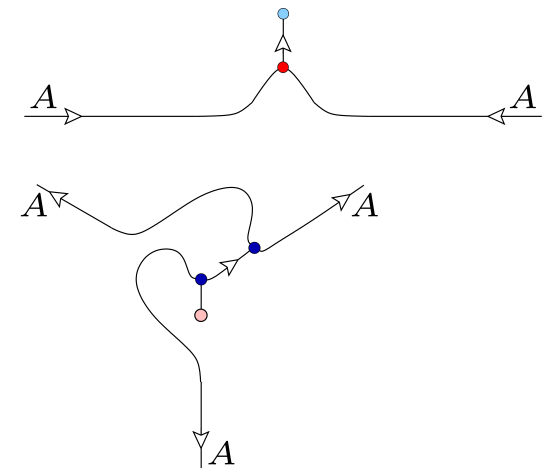

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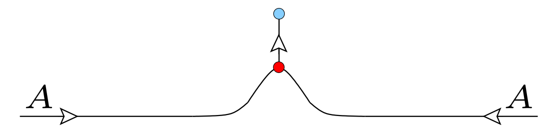

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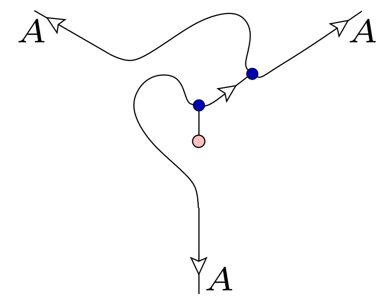
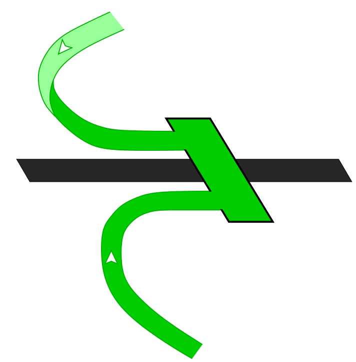


- connect to bulk field ribbons by
  suitable morphisms in $\mathrm{Hom}(U_i \otimes A \otimes U_j, A)$



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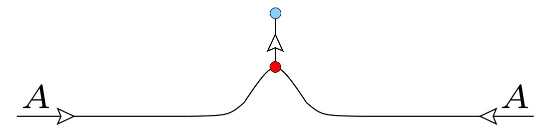
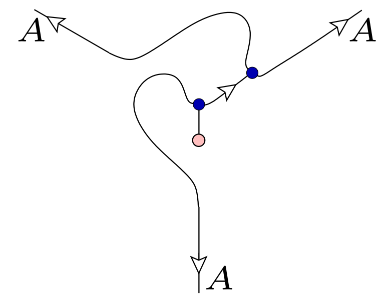
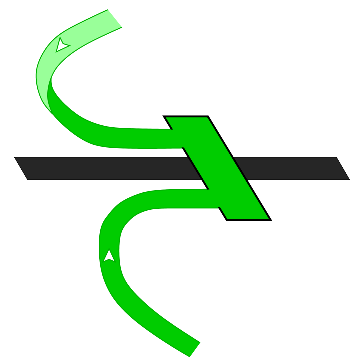




- label vertices (coupons) in $\mathrm{X}\backslash\partial\mathrm{X}$ by product / coproduct morphisms
- connect to bulk field ribbons by suitable morphisms in $\mathrm{Hom}(U_i{\otimes}A{\otimes}U_j, A)$



More precisely: A a symmetric special Frobenius algebra

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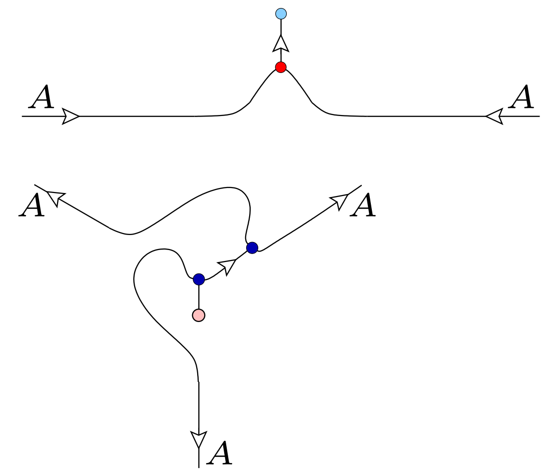

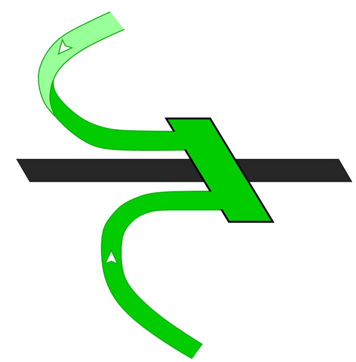

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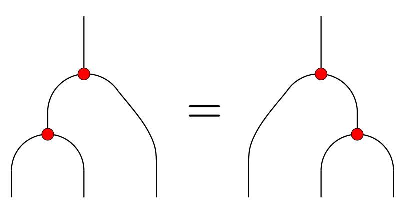

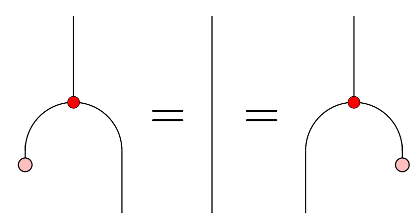

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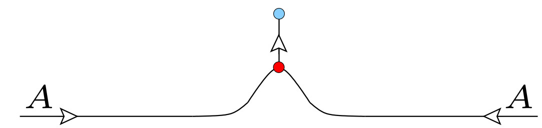

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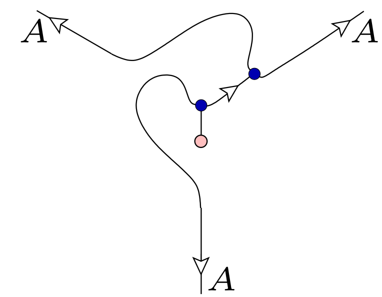

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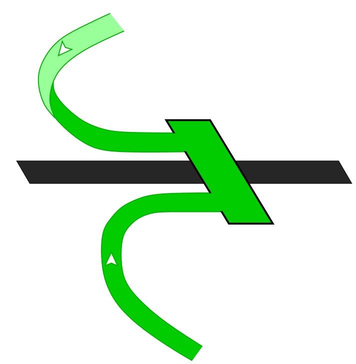
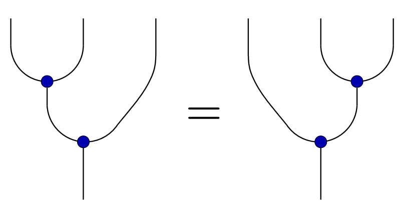
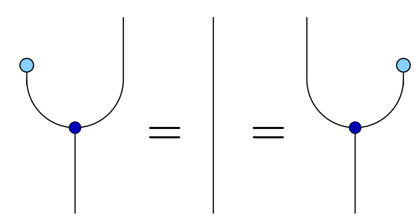


More precisely: A a symmetric special Frobenius co algebra

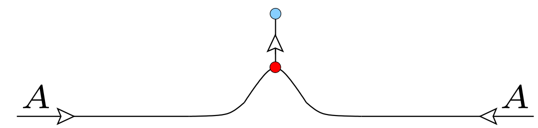




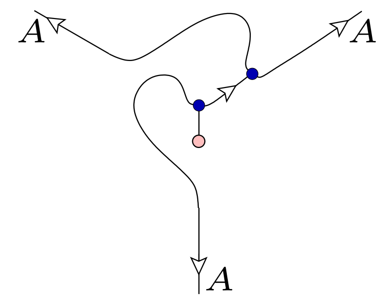
Frobenius algebras

Prescription for general RCFT:

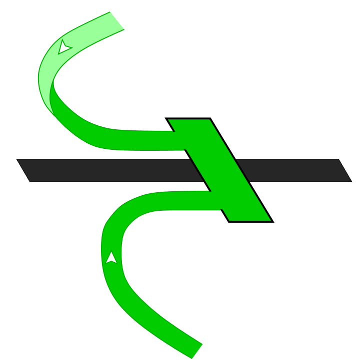
- triangulate the world sheet (trivalent vertices)
- label edges (ribbons) by a Frobenius algebra A in $\mathcal{C}$



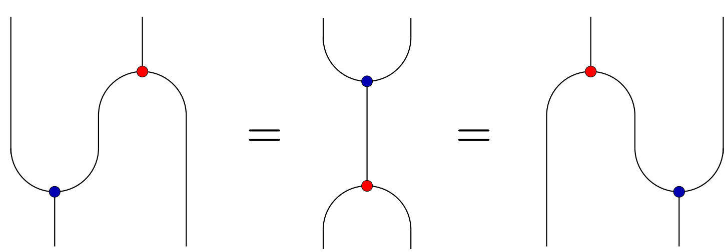
- label vertices (coupons) in $\mathrm{X}\backslash\partial\mathrm{X}$
  by product / coproduct morphisms



- connect to bulk field ribbons by
  suitable morphisms in $\mathrm{Hom}(U_i \otimes A \otimes U_j, A)$



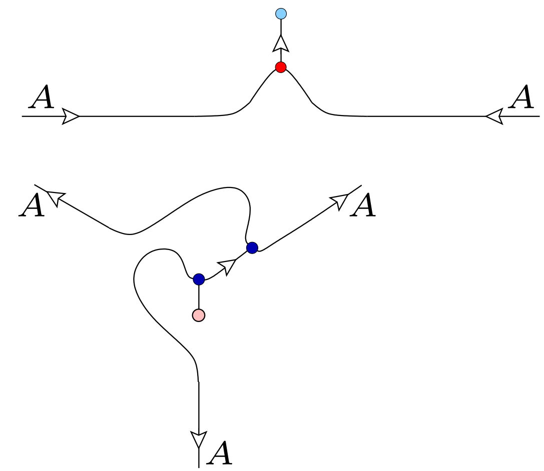
More precisely: A a symmetric special Frobenius algebra



Frobenius algebras

Prescription for general RCFT:

- triangulate the world sheet (trivalent vertices)
- label edges (ribbons) by a Frobenius algebra A in $\mathcal{C}$
- label vertices (coupons) in $\mathrm{X}\setminus\partial\mathrm{X}$
  by product / coproduct morphisms



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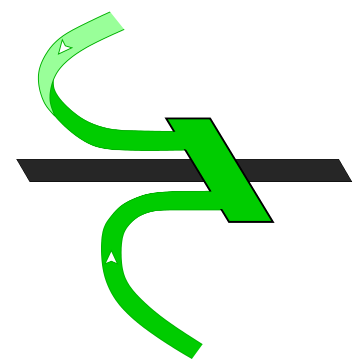

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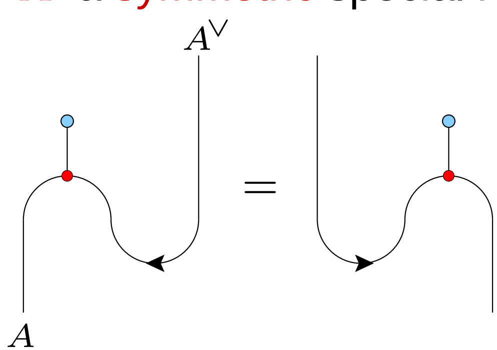

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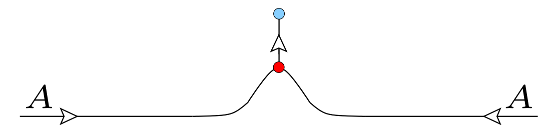
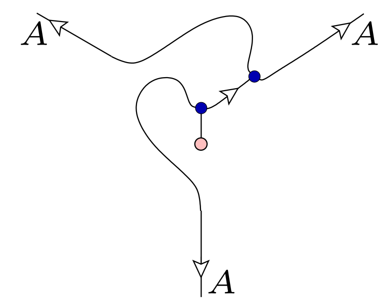


- label edges (ribbons) by a Frobenius algebra A in $\mathcal{C}$



- label vertices (coupons) in $\mathrm{X}\backslash\partial\mathrm{X}$
  by product / coproduct morphisms

- connect to bulk field ribbons by
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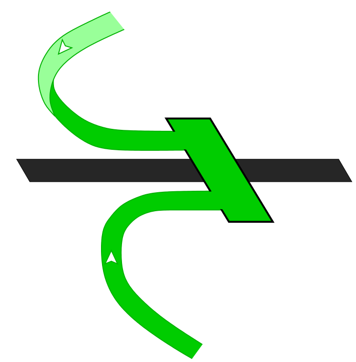
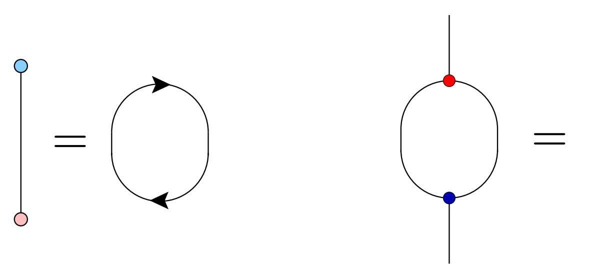


More precisely: A a symmetric special = strongly separable Frobenius algebra



Frobenius algebras

Prescription for general RCFT:

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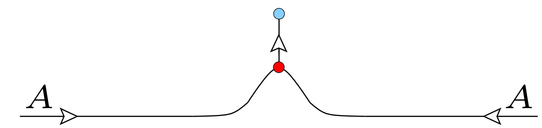

- label vertices (coupons) in $\mathrm{X}\backslash\partial\mathrm{X}$
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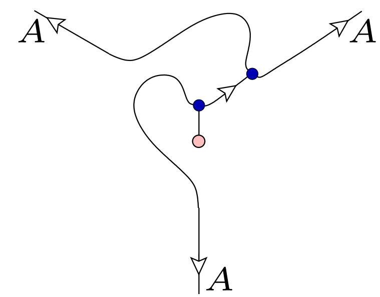

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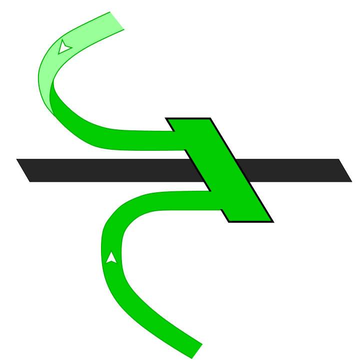

More precisely: A a symmetric special Frobenius algebra

In addition:
- reversion $=$ isomorphism $A \leftrightarrow A^{\mathrm{opp}}$ squaring to twist
  when X is unoriented
- prescription for vertices of triangulation on $\partial\mathrm{X}$ when $\partial\mathrm{X} \neq \emptyset$

Bulk fields II

Prescription involves choices:

- Triangulation
- Insertion of ribbon graph fragment for edge (two possibilities)
- Insertion of ribbon graph fragment for vertex (three possibilities)

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Properties of algebra A $\Longleftrightarrow$ correlation functions (*see below*)
independent of all choices

Bulk fields II

Prescription involves choices:

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Properties of algebra A $\Longleftrightarrow$ correlation functions without field insertions independent of all choices (*see below*)

Bulk fields II

Prescription involves choices:

- Triangulation
- Insertion of ribbon graph fragment for edge (two possibilities)
- Insertion of ribbon graph fragment for vertex (three possibilities)

Properties of algebra A $\Longleftrightarrow$ correlation functions (*see below*)
$+$ restriction of morphism spaces — independent of all choices

Bulk fields II

Prescription involves choices:

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Properties of algebra A $\Longleftrightarrow$ correlation functions (*see below*)
$+$ restriction of morphism spaces — independent of all choices

Restriction of $\mathrm{Hom}(U_i \otimes A \otimes U_j, A)$ to a subspace:

- A is A-bimodule

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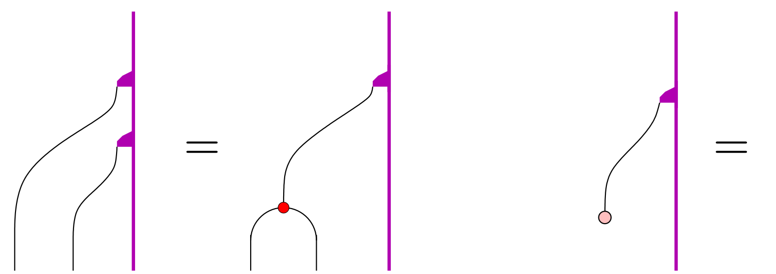
left module properties:



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Properties of algebra A $\Longleftrightarrow$ correlation functions (*see below*)
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Restriction of $\mathrm{Hom}(U_i \otimes A \otimes U_j, A)$ to a subspace:

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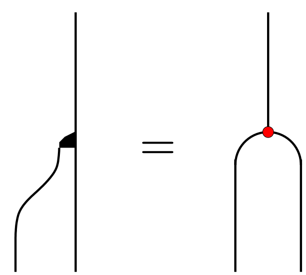
A as a left module:



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Restriction of $\mathrm{Hom}(U_i{\otimes}A{\otimes}U_j\,,A)$ to a subspace:

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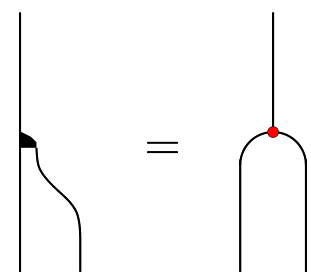
A as a right module:



left and right actions commute by associativity

Bulk fields II

Prescription involves choices:

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Properties of algebra A $\Longleftrightarrow$ correlation functions (*see below*)
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Restriction of $\mathrm{Hom}(U_i \otimes A \otimes U_j, A)$ to a subspace:

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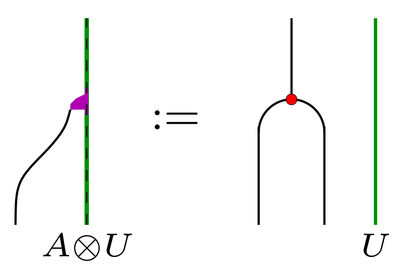
Induced left module:



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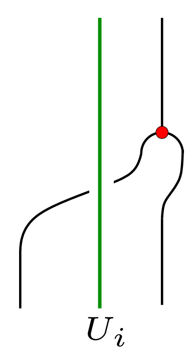
$\otimes^{+}$*-induced left module:*



(use braiding)

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Restriction of $\mathrm{Hom}(U_i \otimes A \otimes U_j, A)$ to a subspace:

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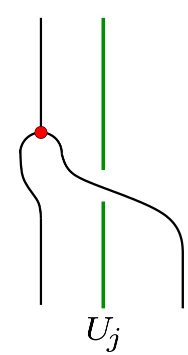
$\otimes^{-}$-induced right module:



cf alpha-induction of subfactors [Longo-Rehren]

Bulk fields II

Prescription involves choices:

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Restriction of $\mathrm{Hom}(U_i{\otimes}A{\otimes}U_j, A)$ to a subspace:

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  $U_i{\otimes}^+A{\otimes}^-U_j \quad \in \mathcal{O}bj(\mathcal{C}_{A|A})$

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- Subspace $=$ space of bimodule morphisms

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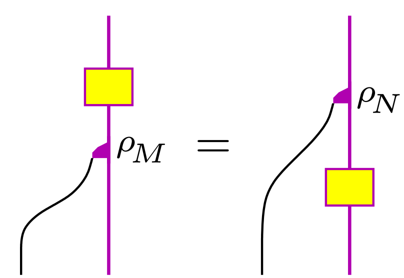
Left-module intertwiner:



Bulk fields *II*

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- Subspace $=$ space of bimodule morphisms

  Prescription:

  Space of bulk fields $= \mathrm{Hom}_{A|A}(U_i{\otimes^+}A{\otimes^-}U_j, A)$
  with chiral labels i,j

Bulk fields II

Prescription: Space of bulk fields

$$\{\, \Phi_{ij}^{\alpha} \,\} \;=\; \mathrm{Hom}_{A|A}(U_i \otimes^{+} A \otimes^{-} U_j, A)$$

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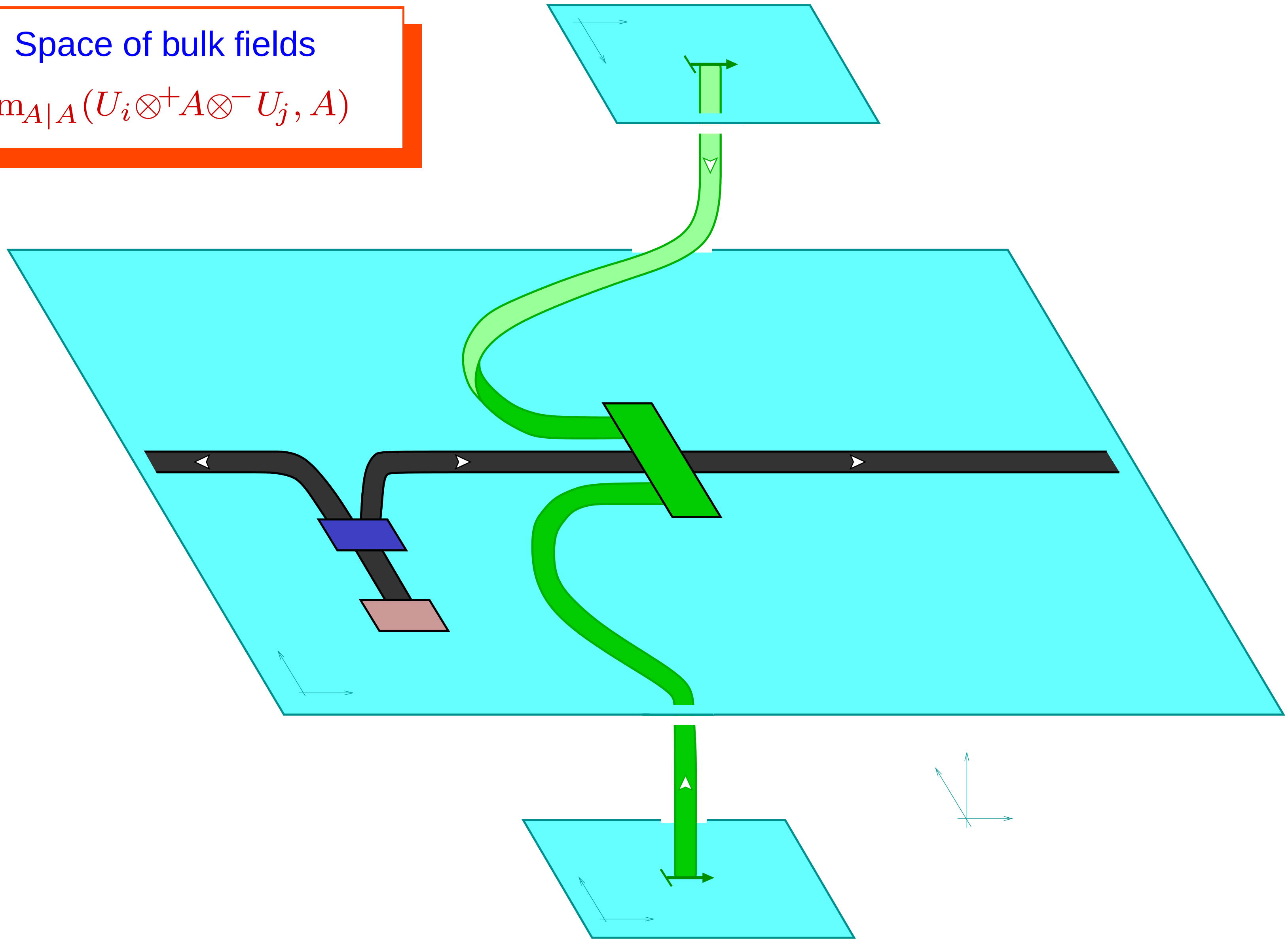

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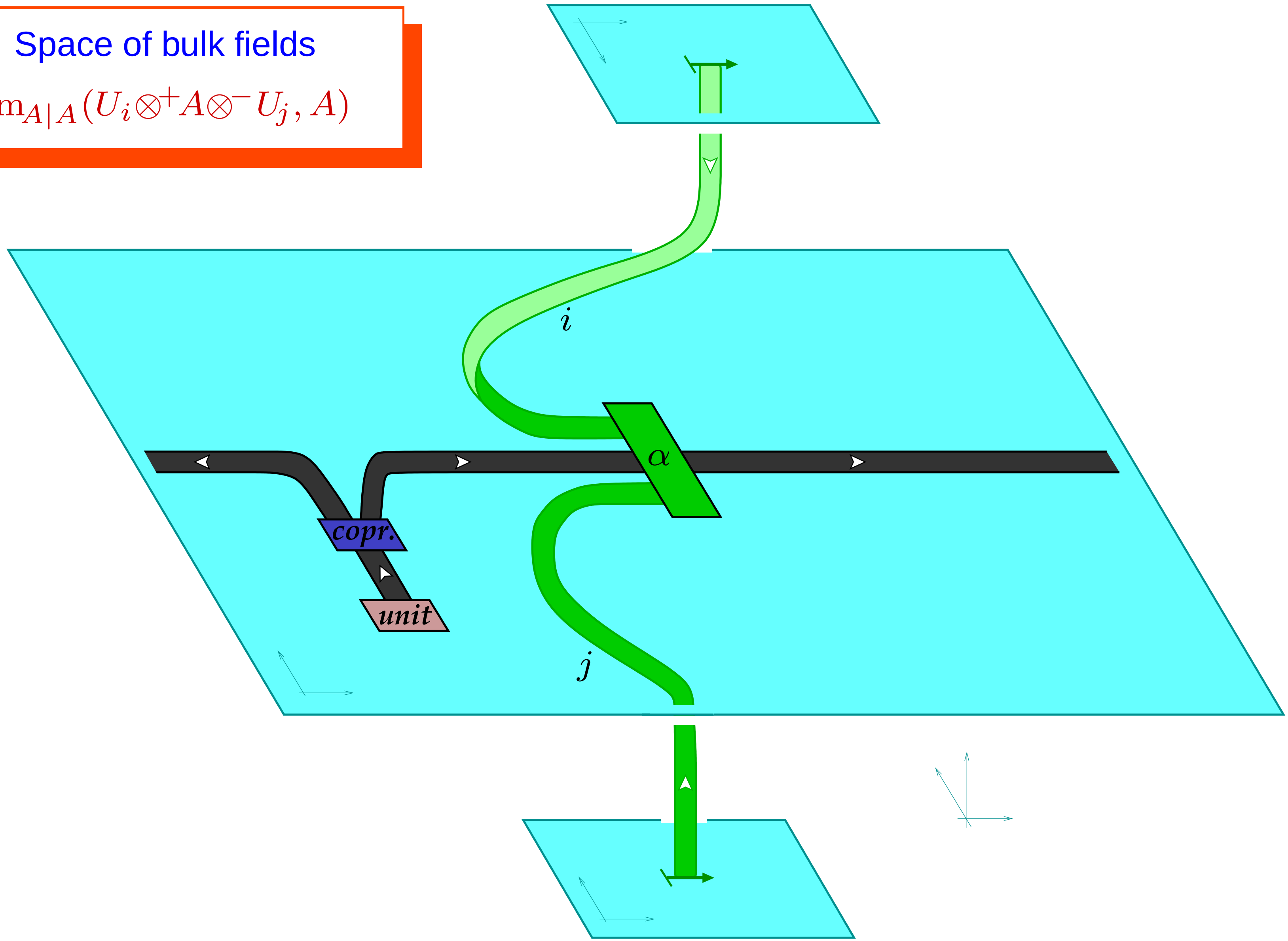

Correlation functions

Ribbon graphs in 3-manifolds vs morphisms in $\mathcal{C}$:

Correlation functions

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- RCFT $\rightsquigarrow$ vertex algebra $\mathcal{V}$

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$$\Sigma \mapsto \mathrm{tft}_\mathcal{C}(\Sigma) =: \mathcal{H}(\Sigma)$$

$$\mathcal{H}(\emptyset) = \mathbb{C} \qquad \mathrm{tft}_\mathcal{C}(\mathrm{Hom}(\emptyset, \Sigma)) \cong \mathcal{H}(\Sigma)$$

Correlation functions

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  state spaces $\mathcal{H}(\Sigma)$ $=$ spaces of conformal blocks of RCFT

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Correlation function $C(\mathrm{X})$ for world sheet X with field insertions

$=$ " expectation value of product of field operators "

depending on position of insertions and on moduli of X

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Holomorphic factorization for correlation functions: $\boxed{C(\mathrm{X}) \in \mathcal{H}(\widehat{\mathrm{X}})}$

Correlation functions

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Holomorphic factorization for correlation functions: $C(\mathrm{X}) \in \mathcal{H}(\widehat{\mathrm{X}})$

in words: a correlation function on X is a vector in the state space of the double $\widehat{\mathrm{X}}$

Correlation functions

Ribbon graphs in 3-manifolds vs morphisms in $\mathcal{C}$:

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Holomorphic factorization for correlation functions: $\boxed{C(\mathrm{X}) \in \mathcal{H}(\widehat{\mathrm{X}})}$

Idea: Specify the vector $C(\mathrm{X})$ as a concrete cobordism $\emptyset \to \widehat{\mathrm{X}}$
(3-manifold + ribbon graph)

Correlation functions

Prescription for cobordism $C(\mathrm{X})$:

Correlation functions

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- 3-manifold: Connecting manifold $\mathrm{M_X}$

  $\partial \mathrm{M_X} = \widehat{\mathrm{X}}$ $\quad$ X embedded as $\mathrm{X} \times \{t{=}0\}$

Correlation functions

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- 3-manifold: Connecting manifold $\mathrm{M_X}$

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- Ribbon graph: Triangulation of $\mathrm{X} \times \{t{=}0\}$ labeled by A

  connected to bulk field ribbons labeled by U_{i_s} & U_{j_s}

  by coupons labeled by $\mathrm{Hom}_{A|A}(U_{i_s} \otimes^{+} A \otimes^{-} U_{j_s}, A)$

Correlation functions

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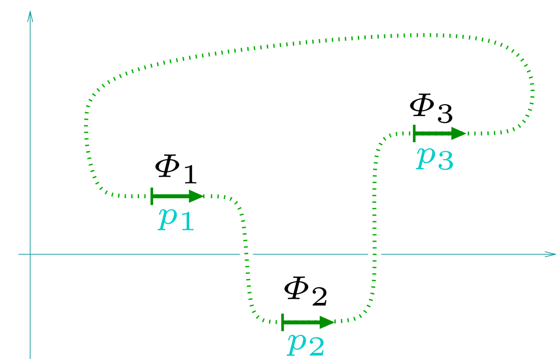
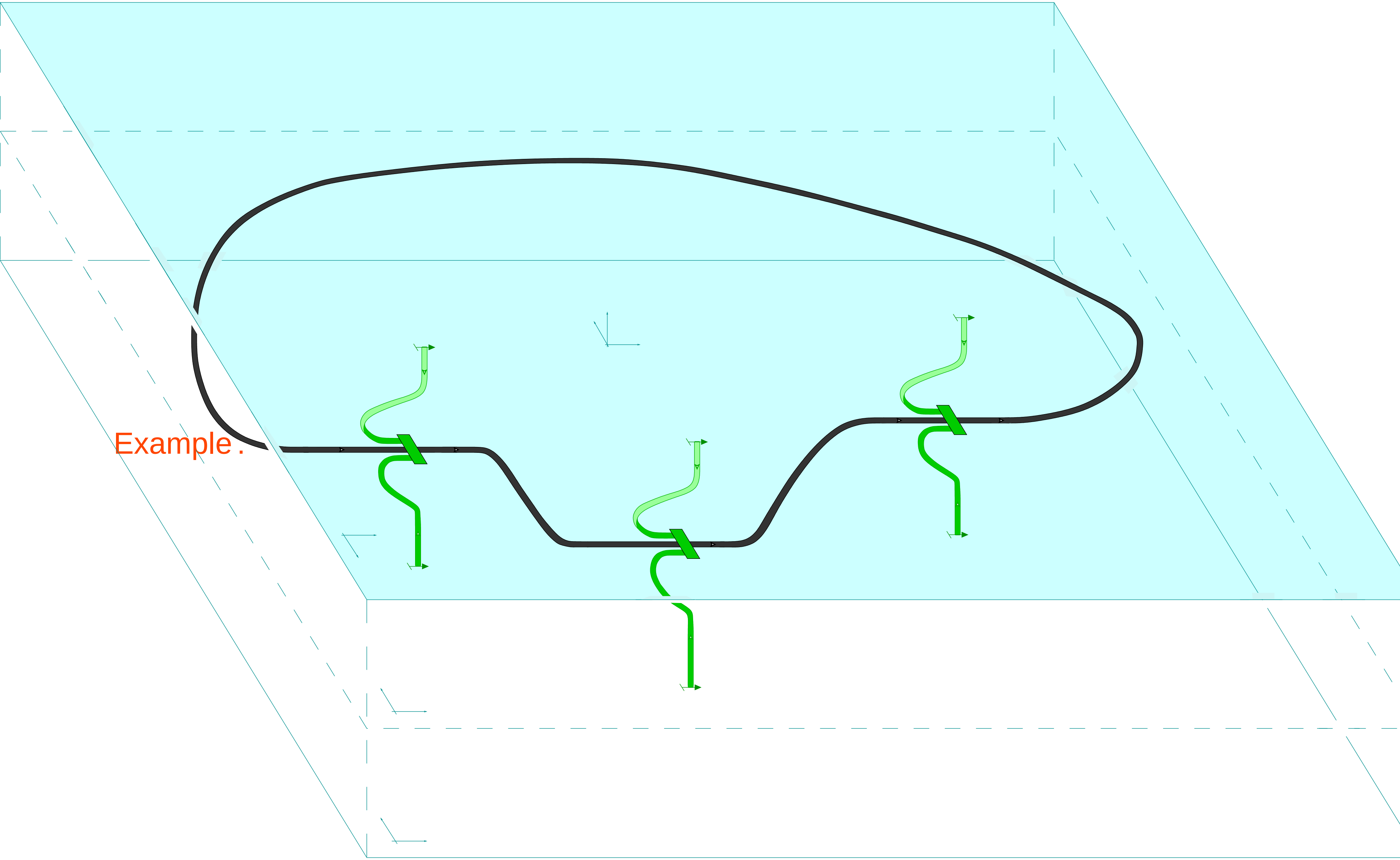
Example : Three bulk fields on the sphere $S^2 = \mathbb{R}^2 \cup \{\infty\}$:



Correlation functions

Example :



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  $\partial\mathrm{M_X} = \widehat{\mathrm{X}}$ $\quad$ X embedded as $\mathrm{X} \times \{t{=}0\}$

- Ribbon graph: Triangulation of $\mathrm{X} \times \{t{=}0\}$ labeled by A

  connected to bulk field ribbons labeled by U_{i_s} & U_{j_s}

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Example : Three bulk fields on the sphere $S^2 = \mathbb{R}^2 \cup \{\infty\}$:

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- Defect lines and defect fields

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Prescription for cobordism $C(\mathrm{X})$:

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- Boundary fields – can change boundary condition
- Defect lines and defect fields
- Be careful with 1- and 2-orientations of ribbons , . . .

Correlation functions

DICTIONARY

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chiral labels $\longleftrightarrow$ $\mathcal{C} = \mathcal{R}ep(\mathcal{V})$, simple objects $\cong U_i$, $i \in \mathcal{I}$

full non-diagonal CFT $\longleftrightarrow$ symmetric special Frobenius algebra A in $\mathcal{C}$

actually: Morita class $[A]$

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internal symmetries	$\longleftrightarrow$	Picard group $\mathrm{Pic}(\mathcal{C}_{A\|A})$
Kramers-Wannier like dualities	$\longleftrightarrow$	duality bimodules Y: $Y^\vee \otimes_A Y \in \mathcal{P}ic(\mathcal{C}_{A\|A})$

Results

See:

JF , Ingo Runkel , Christoph Schweigert :

TFT construction of RCFT correlators

I:	*Partition functions*	Nucl. Phys. B 646 (2002) 353–497	`hep-th/0204148`
II:	*Unoriented world sheets*	Nucl. Phys. B 678 (2004) 511–637	`hep-th/0306164`
III:	*Simple currents*	Nucl. Phys. B 694 (2004) 277–353	`hep-th/0403157`
IV:	*Structure constants and correlation functions*	Nucl. Phys. B 715 (2005) 539–638	`hep-th/0412290`

& Jens Fjelstad :

V: *Proof of modular invariance and factorisation* `hep-th/0503194`

& Jürg Fröhlich :

Correspondences of ribbon categories Adv. Math. ... (2005) ... `math.CT/0309465`

Results

- Finding the possible structures of symmetric special Frobenius algebra (if any) on an object of a modular tensor category $\mathcal{C}$ is a finite problem
  (and only one of the equations to be solved is nonlinear)
- For any symmetric special Frobenius algebra A in $\mathcal{C}$ constructing $\mathcal{C}_A$ and $\mathcal{C}_{A|A}$ is a finite problem
- In any modular tensor category $\mathcal{C}$ there is only a finite number of Morita classes of simple symmetric special Frobenius algebras
  (simple: A a simple A-bimodule)
- A symmetric special Frobenius algebra can be reconstructed from the operator product of boundary fields Ψ_i^{MM}
  (for any full CFT with at least one consistent boundary condition M)

Results

For A a symmetric special Frobenius algebra in a modular tensor category $\mathcal{C}$:

- $C(\mathrm{X})$ is independent of the choices involved in the prescription

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Choices to be made:

- Triangulation T_{X} Prop. V: 3.1, V: 3.7 (fusion and bubble moves)
- Local orientations at vertices of T_{X} for unoriented X Sec. II: 3.1, IV: 3.2, IV: 3.3
- Insertion of ribbon graph fragments for vertices and edges of T_{X} Sec. I: 5.1, II: 3.1

Results

For A a symmetric special Frobenius algebra in a modular tensor category $\mathcal{C}$:

- Thm. I: 5.1 (Torus partition function)

  The coefficients Z_{ij} of $C(\mathrm{T};\emptyset) = \sum_{i,j\in\mathcal{I}} Z_{ij}\,|\chi_i,\mathrm{T}\rangle \otimes |\chi_j,-\mathrm{T}\rangle$

  satisfy $[\,\Gamma, Z\,] = 0$ for $\Gamma \in \mathrm{SL}(2,\mathbb{Z})$

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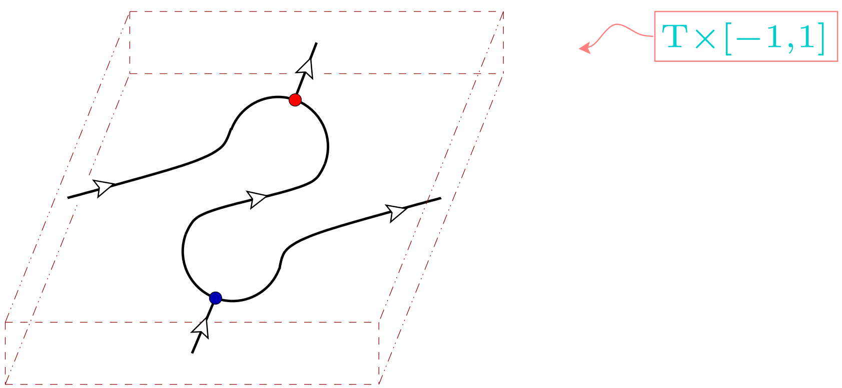
$C(\mathrm{T};\emptyset) =$



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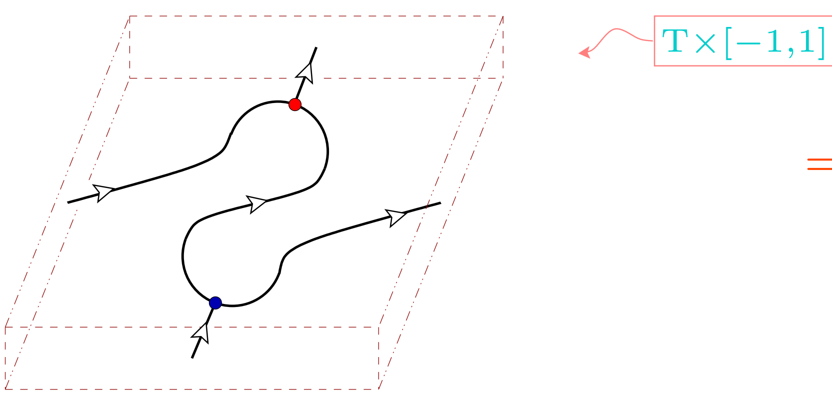
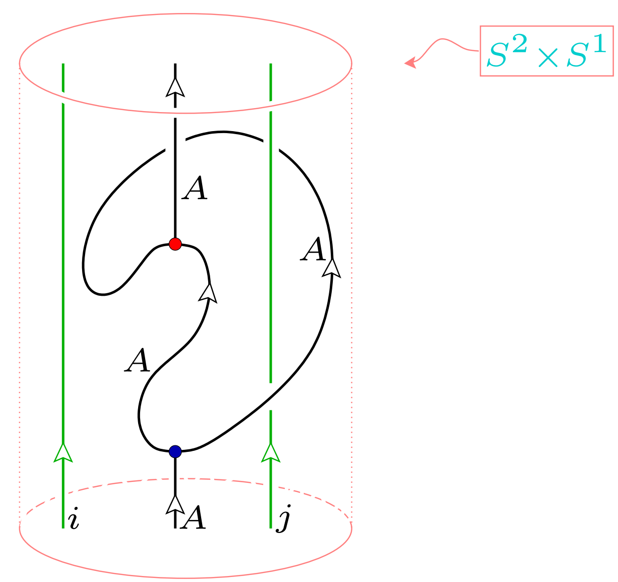


$\Longrightarrow$ $Z_{ij} =$



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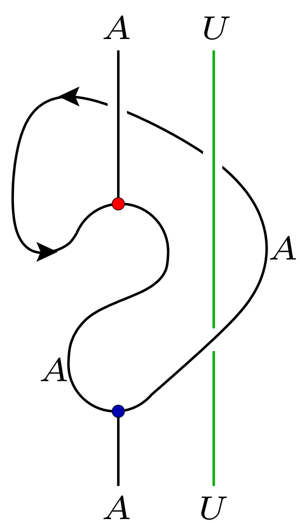
Lemma I: 5.2 : 

is an idempotent

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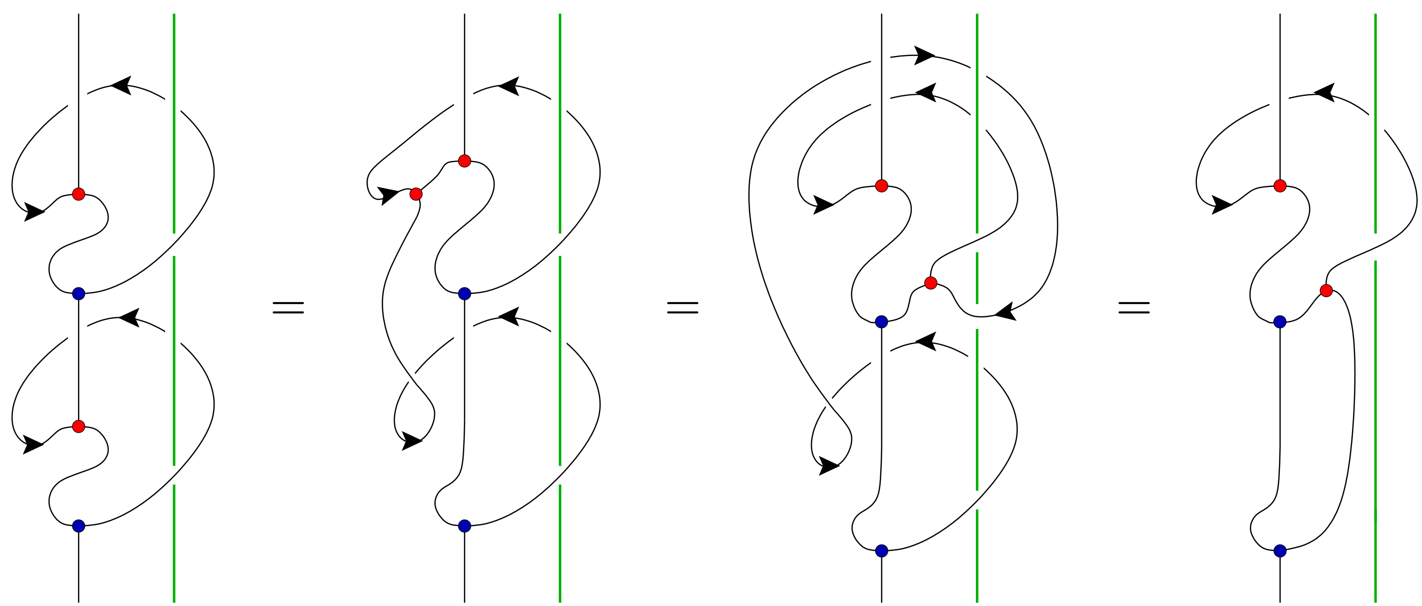
Proof :



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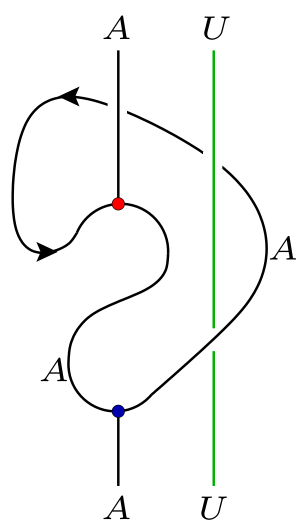

is an idempotent

Composition with the idempotent projects $\mathrm{End}(A \otimes U) \to \mathrm{End}_{A|A}(A \otimes^- U)$

Analogously: $\mathrm{End}(V \otimes A) \to \mathrm{End}_{A|A}(V \otimes^+ A)$

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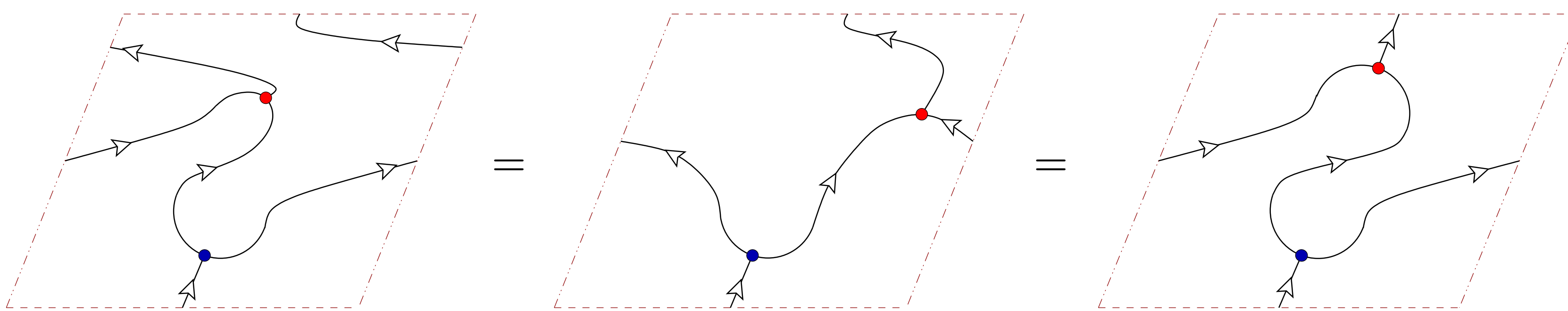
Proof of $[T, Z] = 0$:



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- Prop. I: 5.3 (Torus partition function)

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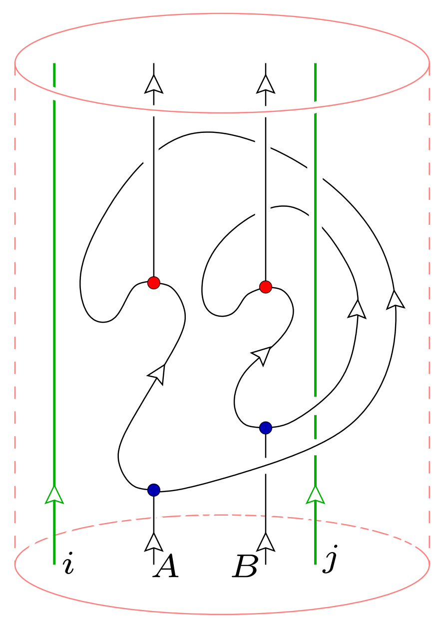
$\widetilde{Z}^{A\otimes B}_{ij} =$ 

$\widetilde{Z}_{ij} \equiv Z_{\bar{\imath} j}$

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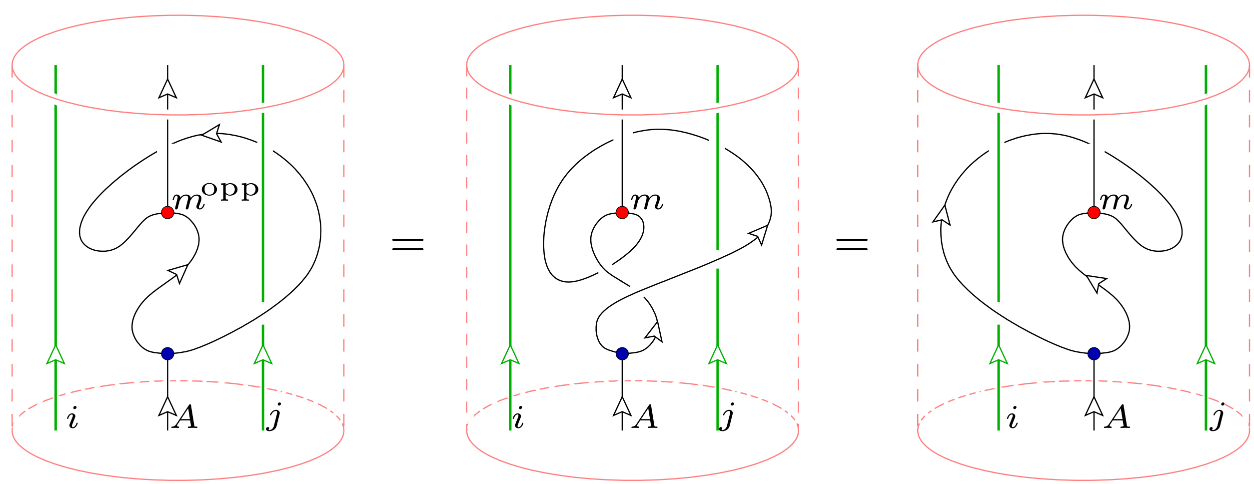

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- Thm. II: 3.7 (Klein bottle partition function)

  The coefficients K_j of $C(\mathrm{K};\emptyset) = \sum_{j\in\mathcal{I}} K_j\, |\chi_j, \mathrm{T}\rangle$

  satisfy $K_j \in \mathbb{Z}$ $\qquad K_j = K_{\bar{j}}$ $\qquad \frac{1}{2}\,(Z_{jj} + K_j) \in \{0, 1, \ldots, Z_{jj}\}$

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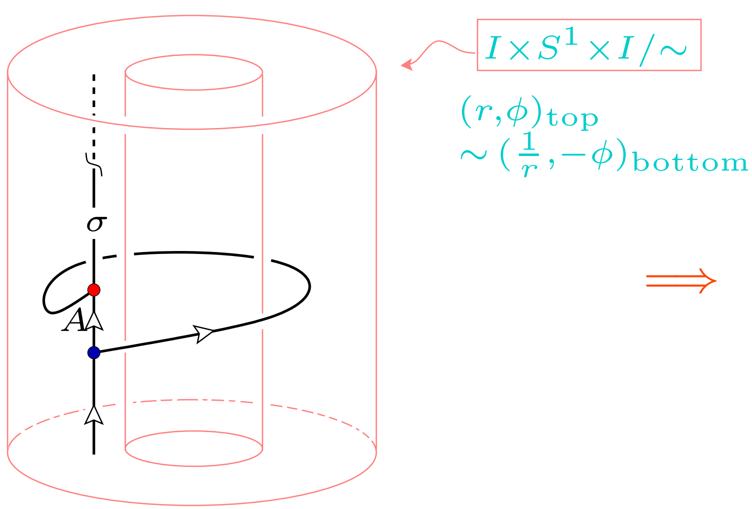
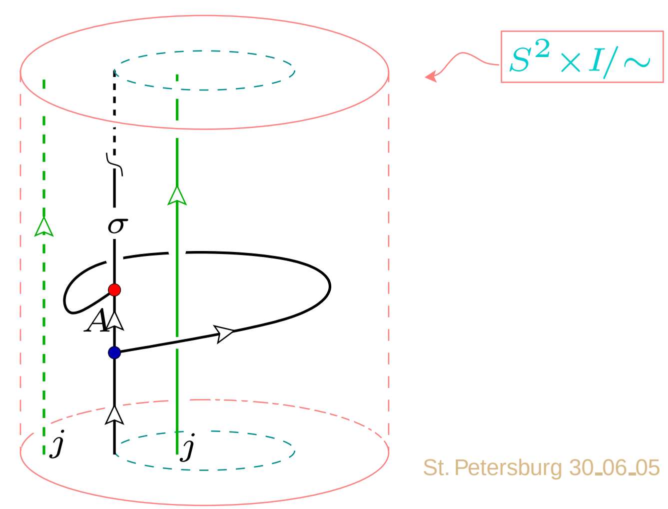
  $K_j \in \mathbb{Z} \qquad K_j = K_{\bar{j}} \qquad \frac{1}{2}\,(Z_{jj} + K_j) \in \{0, 1, \ldots, Z_{jj}\}$

$C(\mathrm{K};\emptyset) =$



$\Longrightarrow \qquad K_j =$



Results

For A a simple symmetric special Frobenius algebra in a modular tensor category $\mathcal{C}$:

- Thm. I: 5.1 & Prop. I: 5.3 (Torus)

  $[\,\Gamma, Z\,] = 0 \quad (\Gamma \in \mathrm{SL}(2,\mathbb{Z})) \qquad Z_{ij} = \dim \mathrm{Hom}_{A|A}(U_i \otimes^+ A \otimes^- U_j, A)$

  $Z_{00} = 1 \qquad Z^{A\oplus B} = Z^A + Z^B \qquad \widetilde{Z}^{A\otimes B} = \widetilde{Z}^A\, \widetilde{Z}^B \qquad Z^{A^{\mathrm{opp}}} = \big(Z^A\big)^{\mathrm{t}}$

- Thm. II: 3.7 (Klein bottle) $\quad K_j \in \mathbb{Z} \qquad K_j = K_{\bar{\jmath}} \qquad \frac{(Z_{jj}+K_j)}{2} \in \{0, \dots, Z_{jj}\}$

Results

For A a simple symmetric special Frobenius algebra in a modular tensor category $\mathcal{C}$:

- Thm. I: 5.1 & Prop. I: 5.3 (Torus)

  $[\,\Gamma, Z\,] = 0 \quad (\Gamma \in \mathrm{SL}(2,\mathbb{Z})) \qquad Z_{ij} = \dim \mathrm{Hom}_{A|A}(U_i \otimes^+ A \otimes^- U_j, A)$

  $Z_{00} = 1 \qquad Z^{A\oplus B} = Z^A + Z^B \qquad \widetilde{Z}^{A\otimes B} = \widetilde{Z}^A\, \widetilde{Z}^B \qquad Z^{A^{\mathrm{opp}}} = (Z^A)^{\mathrm{t}}$

- Thm. II: 3.7 (Klein bottle) $\qquad K_j \in \mathbb{Z} \qquad K_j = K_{\bar\jmath} \qquad \frac{(Z_{jj}+K_j)}{2} \in \{0, ... , Z_{jj}\}$

Special case: $\quad A = \mathbf{1} \quad \Longrightarrow \quad Z_{ij} = \delta_{i\bar\jmath} \qquad K_j = \begin{cases} \pm 1 & \text{if } j = \bar\jmath \\ 0 & \text{else} \end{cases}$ (FS indicator)

Results

For A a simple symmetric special Frobenius algebra in a modular tensor category $\mathcal{C}$:

- Thm. I: 5.1 & Prop. I: 5.3 (Torus)

  $[\,\Gamma, Z\,] = 0 \quad (\Gamma \in \mathrm{SL}(2,\mathbb{Z})) \qquad Z_{ij} = \dim \mathrm{Hom}_{A|A}(U_i \otimes^+ A \otimes^- U_j, A)$

  $Z_{00} = 1 \qquad Z^{A\oplus B} = Z^A + Z^B \qquad \widetilde{Z}^{A\otimes B} = \widetilde{Z}^A\, \widetilde{Z}^B \qquad Z^{A^{\mathrm{opp}}} = \big(Z^A\big)^{\mathrm{t}}$

- Thm. II: 3.7 (Klein bottle) $\quad K_j \in \mathbb{Z} \quad K_j = K_{\bar{\jmath}} \quad \frac{(Z_{jj}+K_j)}{2} \in \{0, ... , Z_{jj}\}$

- OPE coefficients for the fundamental correlation functions

Results

For A a simple symmetric special Frobenius algebra in a modular tensor category $\mathcal{C}$:

- Thm. I: 5.1 & Prop. I: 5.3 (Torus)

  $[\,\Gamma, Z\,] = 0 \quad (\Gamma \in \mathrm{SL}(2,\mathbb{Z})) \qquad Z_{ij} = \dim \mathrm{Hom}_{A|A}(U_i \otimes^+ A \otimes^- U_j, A)$

  $Z_{00} = 1 \qquad Z^{A\oplus B} = Z^A + Z^B \qquad \widetilde{Z}^{A\otimes B} = \widetilde{Z}^A\, \widetilde{Z}^B \qquad Z^{A^{\mathrm{opp}}} = (Z^A)^{\mathrm{t}}$

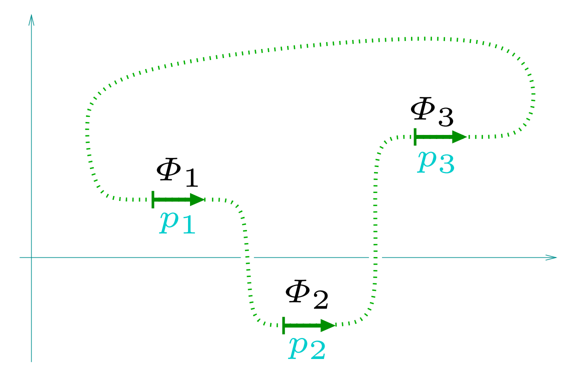
- Thm. II: 3.7 (Klein bottle) $\quad K_j \in \mathbb{Z} \qquad K_j = K_{\bar{\jmath}} \qquad \frac{(Z_{jj}+K_j)}{2} \in \{0, ... , Z_{jj}\}$

- OPE coefficients for the fundamental correlation functions

  Three bulk fields on the sphere

  Sec. IV: 4.4

  $C = \sum_{\mu,\nu} c(\Phi_1 \Phi_2 \Phi_3)_{\mu\nu}\, B(p_1, p_2, p_3)_{\mu\nu}$



Results

For A a simple symmetric special Frobenius algebra in a modular tensor category $\mathcal{C}$:

- Thm. I: 5.1 & Prop. I: 5.3 (Torus)

  $[\,\Gamma, Z\,] = 0 \quad (\Gamma \in \mathrm{SL}(2,\mathbb{Z})) \qquad Z_{ij} = \dim \mathrm{Hom}_{A|A}(U_i \otimes^+ A \otimes^- U_j, A)$

  $Z_{00} = 1 \qquad Z^{A\oplus B} = Z^A + Z^B \qquad \widetilde{Z}^{A\otimes B} = \widetilde{Z}^A\, \widetilde{Z}^B \qquad Z^{A^{\mathrm{opp}}} = (Z^A)^{\mathrm{t}}$

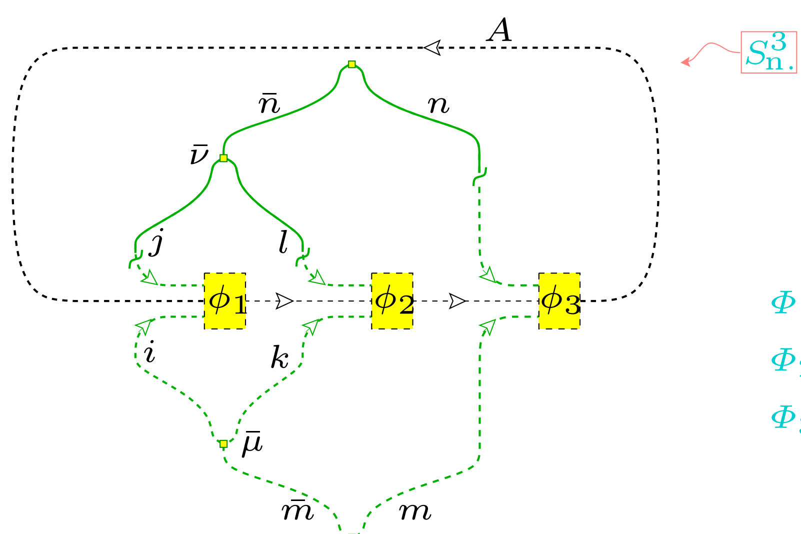
- Thm. II: 3.7 (Klein bottle) $\quad K_j \in \mathbb{Z} \qquad K_j = K_{\bar{j}} \qquad \frac{(Z_{jj}+K_j)}{2} \in \{0, \dots, Z_{jj}\}$

- OPE coefficients for the fundamental correlation functions

  Three bulk fields on the sphere

  Sec. IV: 4.4

$$c(\Phi_1\Phi_2\Phi_3)_{\mu\nu} = \frac{1}{S_{0,0}}$$



$\Phi_1 = (i, j, \phi_1, p_1, [\gamma_1], \mathrm{or}_2)$
$\Phi_2 = (k, l, \phi_2, p_2, [\gamma_2], \mathrm{or}_2)$
$\Phi_3 = (m, n, \phi_3, p_3, [\gamma_3], \mathrm{or}_2)$

Results

For A a simple symmetric special Frobenius algebra in a modular tensor category $\mathcal{C}$:

- Thm. I: 5.1 & Prop. I: 5.3 (Torus)

  $[\,\Gamma, Z\,] = 0 \quad (\Gamma \in \mathrm{SL}(2,\mathbb{Z})) \qquad Z_{ij} = \dim \mathrm{Hom}_{A|A}(U_i \otimes^+ A \otimes^- U_j, A)$

  $Z_{00} = 1 \qquad Z^{A\oplus B} = Z^A + Z^B \qquad \widetilde{Z}^{A\otimes B} = \widetilde{Z}^A\,\widetilde{Z}^B \qquad Z^{A^{\mathrm{opp}}} = (Z^A)^{\mathrm{t}}$

- Thm. II: 3.7 (Klein bottle) $\quad K_j \in \mathbb{Z} \qquad K_j = K_{\bar{\jmath}} \qquad \frac{(Z_{jj}+K_j)}{2} \in \{0, \dots, Z_{jj}\}$

- OPE coefficients for the fundamental correlation functions

  Three bulk fields on the sphere

  Sec. IV: 4.4

  $C = \sum_{\mu,\nu} c(\Phi_1\Phi_2\Phi_3)_{\mu\nu}\, B(p_1, p_2, p_3)_{\mu\nu}$

  $$c(\Phi_1\Phi_2\Phi_3)_{\mu\nu} \;=\; \frac{\dim(A)}{S_{0,0}} \sum_{\beta} \mathsf{F}[A|A]^{(ki0jl)0}_{\alpha_1 0\alpha_2\,,\,\beta\bar{m}\bar{n}\mu\nu}\, \mathsf{F}[A|A]^{(m\bar{m}0\bar{n}n)0}_{\beta 0\alpha_3\,,\,\cdot 00\cdot\cdot}$$

  $\phi_1 = \xi^{\alpha_1}_{(i0j)0} \qquad \phi_2 = \xi^{\alpha_2}_{(k0l)0} \qquad \phi_3 = \xi^{\alpha_3}_{(m0n)0}$

Results

For A a simple symmetric special Frobenius algebra in a modular tensor category $\mathcal{C}$:

- Thm. I: 5.1 & Prop. I: 5.3 (Torus)

  $[\,\Gamma, Z\,] = 0 \quad (\Gamma \in \mathrm{SL}(2,\mathbb{Z})) \qquad Z_{ij} = \dim \mathrm{Hom}_{A|A}(U_i \otimes^+ A \otimes^- U_j, A)$

  $Z_{00} = 1 \qquad Z^{A\oplus B} = Z^A + Z^B \qquad \widetilde{Z}^{A\otimes B} = \widetilde{Z}^A\, \widetilde{Z}^B \qquad Z^{A^{\mathrm{opp}}} = (Z^A)^{\mathrm{t}}$

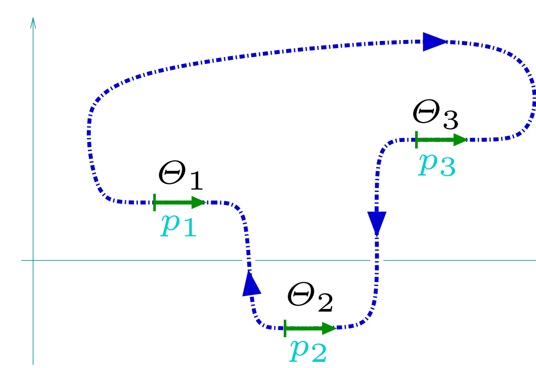
- Thm. II: 3.7 (Klein bottle) $\qquad K_j \in \mathbb{Z} \qquad K_j = K_{\bar{\jmath}} \qquad \frac{(Z_{jj}+K_j)}{2} \in \{0, \ldots, Z_{jj}\}$

- OPE coefficients for the fundamental correlation functions

  Three bulk fields on the sphere
  Sec. IV: 4.4

  Three defect fields on the sphere
  Sec. IV: 4.5

  $C = \sum_{\mu,\nu} c(X, \Theta_1, Y, \Theta_2, Z, \Theta_3, X)_{\mu\nu}\, B(p_1, p_2, p_3)_{\mu\nu}$



Results

For A a simple symmetric special Frobenius algebra in a modular tensor category $\mathcal{C}$:

- Thm. I: 5.1 & Prop. I: 5.3 (Torus)

  $[\,\Gamma, Z\,] = 0 \quad (\Gamma \in \mathrm{SL}(2,\mathbb{Z})) \qquad Z_{ij} = \dim \mathrm{Hom}_{A|A}(U_i \otimes^+ A \otimes^- U_j, A)$

  $Z_{00} = 1 \qquad Z^{A\oplus B} = Z^A + Z^B \qquad \widetilde{Z}^{A\otimes B} = \widetilde{Z}^A\, \widetilde{Z}^B \qquad Z^{A^{\mathrm{opp}}} = (Z^A)^{\mathrm{t}}$

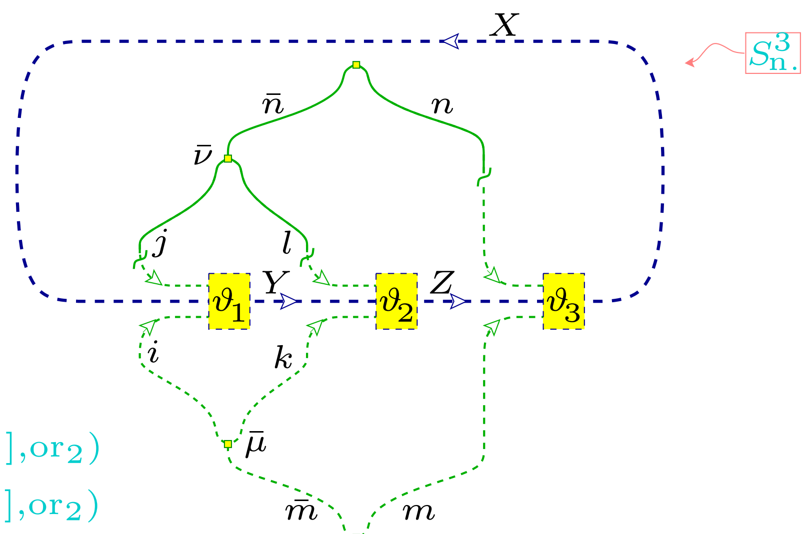
- Thm. II: 3.7 (Klein bottle) $\quad K_j \in \mathbb{Z} \qquad K_j = K_{\bar{j}} \qquad \frac{(Z_{jj}+K_j)}{2} \in \{0, \dots, Z_{jj}\}$

- OPE coefficients for the fundamental correlation functions

  Three bulk fields on the sphere
  Sec. IV: 4.4

  Three defect fields on the sphere
  Sec. IV: 4.5

$$c(X,\Theta_1,Y,\Theta_2,Z,\Theta_3,X)_{\mu\nu} = \frac{1}{S_{0,0}}$$



$\Theta_1 = (X,\mathrm{or}_2,Y,\mathrm{or}_2,i,j,\vartheta_1,p_1,[\gamma_1],\mathrm{or}_2)$

$\Theta_2 = (Y,\mathrm{or}_2,Z,\mathrm{or}_2,k,l,\vartheta_2,p_2,[\gamma_2],\mathrm{or}_2)$

$\Theta_3 = (Z,\mathrm{or}_2,X,\mathrm{or}_2,m,n,\vartheta_3,p_3,[\gamma_3],\mathrm{or}_2)$

Results

For A a simple symmetric special Frobenius algebra in a modular tensor category $\mathcal{C}$:

- Thm. I: 5.1 & Prop. I: 5.3 (Torus)

  $[\,\Gamma, Z\,] = 0 \quad (\Gamma \in \mathrm{SL}(2,\mathbb{Z})) \qquad Z_{ij} = \dim \mathrm{Hom}_{A|A}(U_i \otimes^+ A \otimes^- U_j, A)$

  $Z_{00} = 1 \qquad Z^{A\oplus B} = Z^A + Z^B \qquad \widetilde{Z}^{A\otimes B} = \widetilde{Z}^A\, \widetilde{Z}^B \qquad Z^{A^{\mathrm{opp}}} = (Z^A)^{\mathrm{t}}$

- Thm. II: 3.7 (Klein bottle) $\quad K_j \in \mathbb{Z} \qquad K_j = K_{\bar{\jmath}} \qquad \frac{(Z_{jj}+K_j)}{2} \in \{0, \dots, Z_{jj}\}$

- OPE coefficients for the fundamental correlation functions

  Three bulk fields on the sphere
  Sec. IV: 4.4

  Three defect fields on the sphere
  Sec. IV: 4.5

$$C = \sum_{\mu,\nu} c(X, \Theta_1, Y, \Theta_2, Z, \Theta_3, X)_{\mu\nu}\, B(p_1, p_2, p_3)_{\mu\nu}$$

$$c(X, \Theta_1, Y, \Theta_2, Z, \Theta_3, X)_{\varepsilon\varphi} = \frac{\dim(\dot{X}_\mu)}{S_{0,0}} \sum_\beta \mathsf{F}[A|A]^{(ki\mu jl)\kappa}_{\alpha_1\nu\alpha_2\,,\,\beta\bar{m}\bar{n}\varepsilon\varphi}\, \mathsf{F}[A|A]^{(m\bar{m}\mu\bar{n}n)\mu}_{\beta\kappa\alpha_3\,,\,\cdot 00 \cdot\cdot}$$

$$X = X_\mu \quad Y = X_\nu \quad Z = X_\kappa \quad \vartheta_1 = \xi^{\alpha_1}_{(i\mu j)\nu} \quad \vartheta_2 = \xi^{\alpha_2}_{(k\nu l)\kappa} \quad \vartheta_3 = \xi^{\alpha_3}_{(m\kappa n)\mu}$$

Results

For A a simple symmetric special Frobenius algebra in a modular tensor category $\mathcal{C}$:

- Thm. I: 5.1 & Prop. I: 5.3 (Torus)

  $[\Gamma, Z] = 0 \quad (\Gamma \in \mathrm{SL}(2,\mathbb{Z})) \qquad Z_{ij} = \dim \mathrm{Hom}_{A|A}(U_i \otimes^+ A \otimes^- U_j, A)$

  $Z_{00} = 1 \qquad Z^{A\oplus B} = Z^A + Z^B \qquad \widetilde{Z}^{A\otimes B} = \widetilde{Z}^A \, \widetilde{Z}^B \qquad Z^{A^{\mathrm{opp}}} = (Z^A)^{\mathrm{t}}$

- Thm. II: 3.7 (Klein bottle) $\quad K_j \in \mathbb{Z} \qquad K_j = K_{\bar{\jmath}} \qquad \frac{(Z_{jj}+K_j)}{2} \in \{0, \dots, Z_{jj}\}$

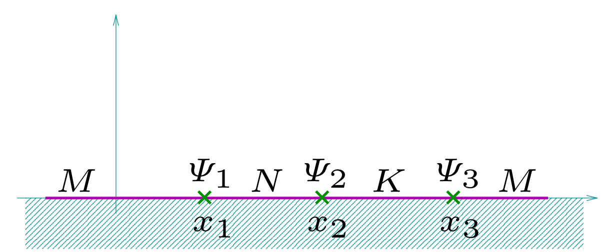
- OPE coefficients for the fundamental correlation functions

  Three bulk fields on the sphere
  Sec. IV: 4.4

  Three defect fields on the sphere
  Sec. IV: 4.5

  Three boundary fields on the disk
  Sec. IV: 4.2

$$C = \sum_{\delta=1}^{N_{ij}{}^{\bar{k}}} c(M\Psi_1 N\Psi_2 K\Psi_3 M)_\delta \, B(x_1,x_2,x_3)_\delta$$



Results

For A a simple symmetric special Frobenius algebra in a modular tensor category $\mathcal{C}$:

- Thm. I: 5.1 & Prop. I: 5.3 (Torus)

  $[\,\Gamma, Z\,] = 0 \quad (\Gamma \in \mathrm{SL}(2,\mathbb{Z})) \qquad Z_{ij} = \dim \mathrm{Hom}_{A|A}(U_i \otimes^+ A \otimes^- U_j, A)$

  $Z_{00} = 1 \qquad Z^{A\oplus B} = Z^A + Z^B \qquad \widetilde{Z}^{A\otimes B} = \widetilde{Z}^A\, \widetilde{Z}^B \qquad Z^{A^{\mathrm{opp}}} = (Z^A)^{\mathrm{t}}$

- Thm. II: 3.7 (Klein bottle) $\quad K_j \in \mathbb{Z} \qquad K_j = K_{\bar{\jmath}} \qquad \frac{(Z_{jj}+K_j)}{2} \in \{0, \dots, Z_{jj}\}$

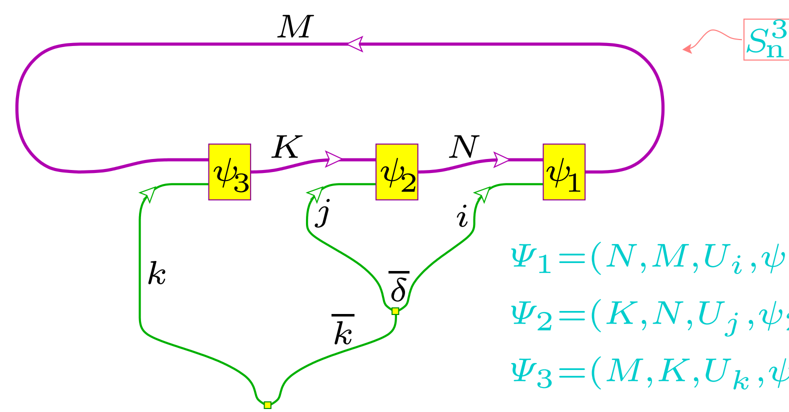
- OPE coefficients for the fundamental correlation functions

  Three bulk fields on the sphere
  Sec. IV: 4.4

  Three defect fields on the sphere
  Sec. IV: 4.5

  Three boundary fields on the disk
  Sec. IV: 4.2

$c(M\Psi_1 N\Psi_2 K\Psi_3 M)_\delta \;=$



$\Psi_1 = (N, M, U_i, \psi_1, x_1, [\gamma_1])$

$\Psi_2 = (K, N, U_j, \psi_2, x_2, [\gamma_2])$

$\Psi_3 = (M, K, U_k, \psi_3, x_3, [\gamma_3])$

Results

For A a simple symmetric special Frobenius algebra in a modular tensor category $\mathcal{C}$:

- Thm. I: 5.1 & Prop. I: 5.3 (Torus)

  $[\,\Gamma, Z\,] = 0 \quad (\Gamma \in \mathrm{SL}(2,\mathbb{Z})) \qquad Z_{ij} = \dim \mathrm{Hom}_{A|A}(U_i \otimes^+ A \otimes^- U_j, A)$

  $Z_{00} = 1 \qquad Z^{A\oplus B} = Z^A + Z^B \qquad \widetilde{Z}^{A\otimes B} = \widetilde{Z}^A\, \widetilde{Z}^B \qquad Z^{A^{\mathrm{opp}}} = (Z^A)^{\mathrm{t}}$

- Thm. II: 3.7 (Klein bottle) $\quad K_j \in \mathbb{Z} \quad K_j = K_{\bar{\jmath}} \quad \frac{(Z_{jj}+K_j)}{2} \in \{0, \dots, Z_{jj}\}$

- OPE coefficients for the fundamental correlation functions

  Three bulk fields on the sphere
  Sec. IV: 4.4

  Three defect fields on the sphere
  Sec. IV: 4.5

  Three boundary fields on the disk
  Sec. IV: 4.2

$$c(M\Psi_1 N\Psi_2 K\Psi_3 M)_\delta \;=\; \dim(\dot{M}_\mu) \sum_{\beta=1}^{\mathrm{A}_{\bar{k}M_\kappa}{}^{M_\mu}} \mathsf{G}[A]^{(\kappa j i)\mu}_{\alpha_2\nu\alpha_1\,,\,\beta\bar{k}\delta}\, \mathsf{G}[A]^{(\mu k\bar{k})\mu}_{\alpha_3\kappa\beta\,,\,\cdot 0 \cdot}$$

$M = M_\mu \quad N = M_\nu \quad K = M_\kappa \qquad \psi_1 = \psi^{\alpha_1}_{(\nu i)\mu} \quad \psi_2 = \psi^{\alpha_2}_{(\kappa j)\nu} \quad \psi_3 = \psi^{\alpha_3}_{(\mu k)\kappa}$

Results

For A a simple symmetric special Frobenius algebra in a modular tensor category $\mathcal{C}$:

- Thm. I: 5.1 & Prop. I: 5.3 (Torus)

  $[\,\Gamma, Z\,] = 0 \quad (\Gamma \in \mathrm{SL}(2,\mathbb{Z})) \qquad Z_{ij} = \dim \mathrm{Hom}_{A|A}(U_i \otimes^+ A \otimes^- U_j, A)$

  $Z_{00} = 1 \qquad Z^{A\oplus B} = Z^A + Z^B \qquad \widetilde{Z}^{A\otimes B} = \widetilde{Z}^A\, \widetilde{Z}^B \qquad Z^{A^{\mathrm{opp}}} = (Z^A)^{\mathrm{t}}$

- Thm. II: 3.7 (Klein bottle) $\quad K_j \in \mathbb{Z} \qquad K_j = K_{\bar{\jmath}} \qquad \frac{(Z_{jj}+K_j)}{2} \in \{0, \ldots, Z_{jj}\}$

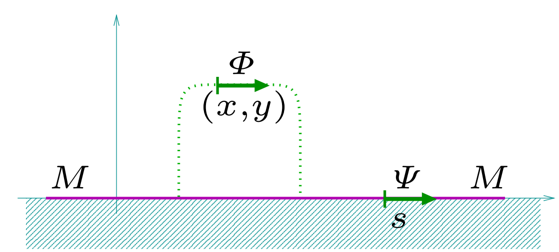
- OPE coefficients for the fundamental correlation functions

  Three bulk fields on the sphere
  Sec. IV: 4.4

  Three defect fields on the sphere
  Sec. IV: 4.5

  Three boundary fields on the disk
  Sec. IV: 4.2

  One bulk and one boundary field on the disk
  Sec. IV: 4.3



$$C = \sum_\delta c(\Phi; M\Psi)_\delta\, B(x,y,s)_\delta$$

Results

For A a simple symmetric special Frobenius algebra in a modular tensor category $\mathcal{C}$:

- Thm. I: 5.1 & Prop. I: 5.3 (Torus)

  $[\Gamma, Z] = 0 \quad (\Gamma \in \mathrm{SL}(2,\mathbb{Z})) \qquad Z_{ij} = \dim \mathrm{Hom}_{A|A}(U_i \otimes^+ A \otimes^- U_j, A)$

  $Z_{00} = 1 \qquad Z^{A\oplus B} = Z^A + Z^B \qquad \widetilde{Z}^{A\otimes B} = \widetilde{Z}^A \, \widetilde{Z}^B \qquad Z^{A^{\mathrm{opp}}} = (Z^A)^{\mathrm{t}}$

- Thm. II: 3.7 (Klein bottle) $\quad K_j \in \mathbb{Z} \qquad K_j = K_{\bar{\jmath}} \qquad \frac{(Z_{jj}+K_j)}{2} \in \{0, \dots, Z_{jj}\}$

- OPE coefficients for the fundamental correlation functions

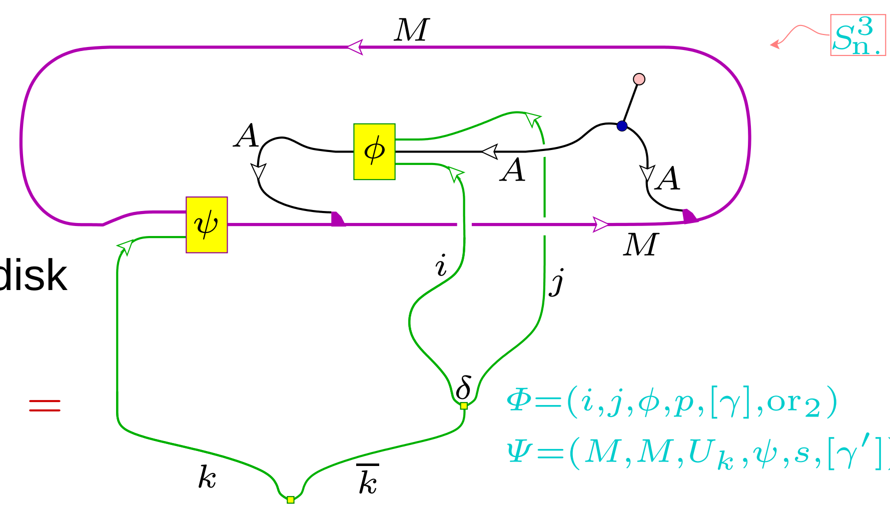
  Three bulk fields on the sphere
  Sec. IV: 4.4

  Three defect fields on the sphere
  Sec. IV: 4.5

  Three boundary fields on the disk
  Sec. IV: 4.2

  One bulk and one boundary field on the disk
  Sec. IV: 4.3

$c(\Phi; M\Psi)_\delta \; =$



Results

For A a simple symmetric special Frobenius algebra in a modular tensor category $\mathcal{C}$:

- Thm. I: 5.1 & Prop. I: 5.3 (Torus)

  $[\Gamma, Z] = 0 \quad (\Gamma \in \mathrm{SL}(2,\mathbb{Z})) \qquad Z_{ij} = \dim \mathrm{Hom}_{A|A}(U_i \otimes^+ A \otimes^- U_j, A)$

  $Z_{00} = 1 \qquad Z^{A\oplus B} = Z^A + Z^B \qquad \widetilde{Z}^{A\otimes B} = \widetilde{Z}^A \, \widetilde{Z}^B \qquad Z^{A^{\mathrm{opp}}} = (Z^A)^{\mathrm{t}}$

- Thm. II: 3.7 (Klein bottle) $\quad K_j \in \mathbb{Z} \quad K_j = K_{\bar{\jmath}} \quad \frac{(Z_{jj}+K_j)}{2} \in \{0, \dots, Z_{jj}\}$

- OPE coefficients for the fundamental correlation functions

  Three bulk fields on the sphere
  Sec. IV: 4.4

  Three defect fields on the sphere
  Sec. IV: 4.5

  Three boundary fields on the disk
  Sec. IV: 4.2

  One bulk and one boundary field on the disk
  Sec. IV: 4.3

$M = M_\mu \qquad \psi = \psi^{\alpha}_{(\mu k)\mu} \qquad \phi = \xi^{\beta}_{(i0j)0}$

$N_{ij}^{\;\;k} \in \{0,1\} \qquad \langle U_i, A\rangle \in \{0,1\}$

$$c(\Phi; M\Psi)_{\bullet} = \sum_{m,n,a,p,\rho,\sigma} [\xi^{\beta}_{(i0j)0}]^{0ja} \rho^{M_\mu\,(m\rho)}_{a\,(n\sigma)} [\psi^{\alpha}_{(\mu k)\mu}]^{mn}_{\rho\sigma} \, \mathsf{R}^{(n\,a)\,m} \, \frac{\dim(U_m)\,\theta_n\theta_i}{\theta_p} \, \mathsf{G}^{(mk\bar{k})m}_{n0} \, \mathsf{F}^{(n\,i\,j)\,m}_{a\,p} \, \mathsf{G}^{(n\,i\,j)\,m}_{p\,\bar{k}}$$

Results

For A a simple symmetric special Frobenius algebra in a modular tensor category $\mathcal{C}$:

- Thm. I: 5.1 Torus — Thm. I: 5.20 Annulus
  Thm. II: 3.7 Klein bottle — Thm. II: 3.5 Möbius strip

- OPE coefficients for the fundamental correlation functions

Sec. IV: 4.4	Three bulk fields on the sphere
Sec. IV: 4.5	Three defect fields on the sphere
Sec. IV: 4.2	Three boundary fields on the disk
Sec. IV: 4.3	One bulk and one boundary field on the disk
Sec. IV: 4.6 / 7	One bulk / defect field on the cross cap

Results

For A a simple symmetric special Frobenius algebra in a modular tensor category $\mathcal{C}$:

- Thm. I: 5.1 Torus — Thm. I: 5.20 Annulus

  Thm. II: 3.7 Klein bottle — Thm. II: 3.5 Möbius strip

- OPE coefficients for the fundamental correlation functions

- Thm. V: 2.9 Bulk factorisation:

  $C(\mathrm{X}) = \sum_{i,j\in\mathcal{I}} \sum_{\alpha,\beta} \dim(U_i)\dim(U_j)\, ({\mathrm{c}^{\mathrm{bulk}}_{i,j}}^{-1})_{\beta\alpha}\, G^{\mathrm{bulk}}_{f,ij}\big(C(\Gamma^{\mathrm{bulk}}_{f,ij,\alpha\beta}(\mathrm{X}))\big)$

  with α, β bases of $\mathrm{Hom}_{A|A}(U_i \otimes^+ A \otimes^- U_j, A)$ and $\mathrm{Hom}_{A|A}(U_{\bar\imath} \otimes^+ A \otimes^- U_{\bar\jmath}, A)$

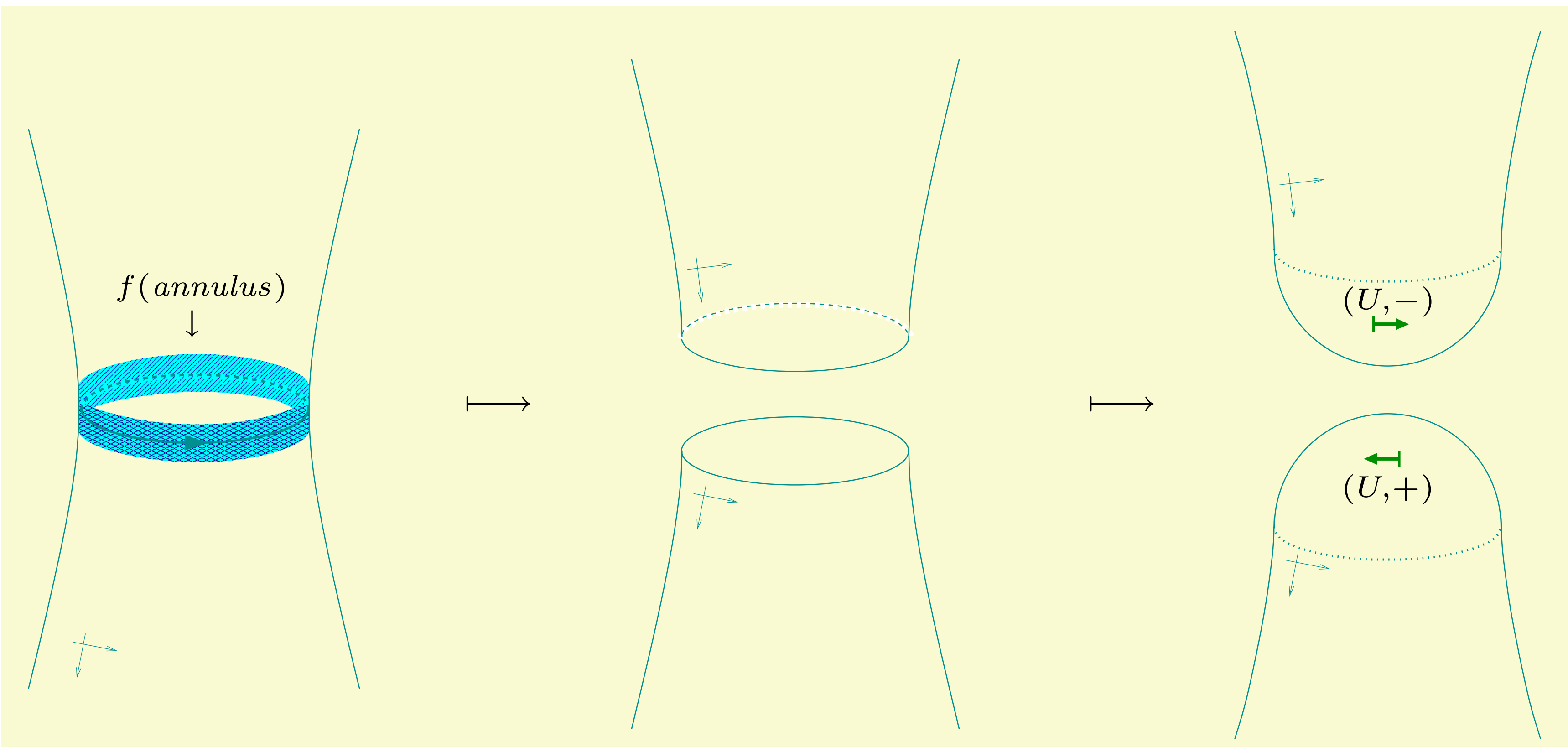
  (f injective continuous orientation preserving map $\mathit{open\ annulus} \to \mathrm{X}$

  no defect field insertions)

Results

For A a simple symmetric special Frobenius algebra in a modular tensor category $\mathcal{C}$:

- Thm. I: 5.1 Torus — Thm. I: 5.20 Annulus
  Thm. II: 3.7 Klein bottle — Thm. II: 3.5 Möbius strip
- OPE coefficients for the fundamental correlation functions



Results

For A a simple symmetric special Frobenius algebra in a modular tensor category $\mathcal{C}$:

- Thm. I: 5.1 Torus — Thm. I: 5.20 Annulus
  Thm. II: 3.7 Klein bottle — Thm. II: 3.5 Möbius strip
- OPE coefficients for the fundamental correlation functions
- Thm. V: 2.9 Bulk factorisation:
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  with α, β bases of $\mathrm{Hom}_A(M_l \otimes U_k, M_r)$ and $\mathrm{Hom}_A(M_r \otimes U_{\bar{k}}, M_l)$
  $\big($ f injective, continuous 2-orientation preserving map $\;$ *strip* $\mathrm{R}_\varepsilon \to \mathrm{X}$
  $f(\partial\mathrm{R}_\varepsilon \cap \mathrm{R}_\varepsilon) \subset \partial\mathrm{X}$ $\quad$ $M_{l/r}$ boundary conditions at left/right end $\big)$

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  $$C(\mathrm{Y}) = \hat{f}_\sharp(C(\mathrm{X}))$$
  $\big(\, [f] \in \mathrm{Map}(\mathrm{X},\mathrm{Y})$ orientation preserving
  $f_\sharp = \mathrm{tft}_{\mathcal{C}}\big(\mathrm{X}\times[-1,0] \sqcup \mathrm{Y}\times[0,1]\big)/\!\sim\, \big)$

Proof : consequence of triangulation independence

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