Effective Random Matrix Theory of Finite Density QCD

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with

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review

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Setup

- **Quantum Chromodynamics** (QCD):

  non-Abelian GT of quarks ψ & gluons $\rightarrow n, p, \pi, \rho \ldots$

- **confinement + chiral symmetry breaking (χSB):** strong coupling

  – Lattice GT,

  – effective FT: chiral Perturbation Theory (χPT), RMT, ...

  use cond-mat type concepts: σ-model, resolvent

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- **"sign" Problem**: chem. potential $\times$ density $\boxed{+\mu\psi^\dagger\psi}$

  $\longrightarrow$ Euclidian **QCD**-action complex

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- **Random Matrix Theory (RMT) approach:**

  – variety of predictions depending on $\langle\bar{\Psi}\Psi\rangle$ & F for $\mu \neq 0$

  – controlled approx. = finte-Volume limit of χPT

Chiral Symmetry Breaking

- **chiral symmetry** of kinetic term $\bar{\Psi} \not{D} \Psi$:

$$\Psi = \begin{pmatrix} \psi_{up} \\ \psi_{down} \\ \vdots \end{pmatrix} \rightarrow U\Psi \;, \quad U \in U(N_f)$$

– **independently** for $\Psi_{L/R} = \frac{1}{2}(1 \pm \gamma_5)\Psi$

- **broken:**

$$SU_L(N_f) \times SU_R(N_f) \rightarrow SU(N_f)$$

– explicitly: quark masses $\bar{\Psi}_L M \Psi_R + \bar{\Psi}_R M \Psi_L$

– spontaneously: vev $0 \neq \langle \bar{\Psi}\Psi \rangle$

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$\rightarrow$ study spectral properties

From QCD to effective Field Theory

- **effective FT: Goldstone bosons** $1/L \ll \Lambda$ (in box $V = L^4$) [Weinberg]
- Pions $U(x) = U_0 \exp[i\xi(x)/F] \in SU(N_f)$

$$\boxed{\mathcal{Z}_{\chi PT} \equiv \int_{SU(N_f)}[dU(x)] \; \exp[-\int dx \text{Tr } \mathcal{L}(U,\partial U)]}$$ σ-model of **QCD**

L.O. Lagrangian $\mathcal{L} = \frac{1}{4}F^2 \partial U \partial U^\dagger + \frac{1}{2}\langle\bar{\Psi}\Psi\rangle M \left(U + U^\dagger\right)$

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- **Zero-dim limit:** [Gasser, Leutwyler 87; Leutwyler, Smilga 92]

unphysical: $\boxed{m_\pi \sim \frac{1}{L^2} \ll \frac{1}{L}}$ $\Rightarrow \quad (m_\pi^2 V)^{-1} = \mathcal{O}(1)$ U_0 dominates

$$\boxed{\mathcal{Z}_{\varepsilon\chi \mathrm{PT}} = \int_{U(N_f)} dU_0 \det[U_0]^\nu \mathrm{e}^{-\frac{1}{2}\langle\bar{\Psi}\Psi\rangle V \mathrm{Tr}\left(M(U_0+U_0^\dagger)\right)} \times \int[d\xi]\mathrm{e}^{-\int dx \frac{1}{2}(\partial\xi)^2}}$$

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- same for baryonic chem. potential $\mu_{up} = \mu_{down}$; **different for isospin**

- $\int dU_0$ **equal large-N RMT** $\boxed{\mathcal{Z} = \frac{\det[m_j^{i-1} I_{\nu+i-1}(m_j)]}{\Delta(m)}}$ [Halasz,Verbaarschot 94]

Effective Field Theory with chemical potential

$$\mathcal{Z}_{\varepsilon\chi\text{PT}}(\mu) = \int dU_0 \det[U_0]^\nu \mathrm{e}^{-\frac{1}{2}\langle\bar{\Psi}\Psi\rangle V \mathrm{Tr}\left(M(U_0+U_0^\dagger)\right)+\frac{1}{4}VF^2\mu^2\mathrm{Tr}[\Sigma_3,U_0][\Sigma_3,U_0^\dagger]}$$

- μ-dependence for isospin $\mu_{up/down} = \pm\mu$:

  covariant derivative $\partial \to \partial + \mu[\Sigma_3, \cdot]$

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- **example 2 flavours**: $\Sigma_3 =$diag(1,-1)

$$\mathcal{Z}(N_f = 2) = \int_0^1 dt\, t \exp[-2t^2 V F^2 \mu^2]\ I_\nu(\langle\bar\Psi\Psi\rangle V m t)^2$$

- **general result** $\mathcal{Z}(N_f) \sim \det[\int I_\nu I_\nu \ldots I_\nu]/\Delta\Delta$

  [Splittorff, Verbaarschot 03; Fyodorov, Vernizzi, G.A. 04]

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- **generalised group integral**

$$\mathcal{Z}_{\varepsilon\chi\text{PT}}(\mu) = \int dU_0 \det[U_0]^\nu \mathrm{e}^{-\frac{1}{2}\langle\bar{\Psi}\Psi\rangle V \mathrm{Tr}\left(M(U_0+U_0^\dagger)\right) + \sum_K a_K \mathrm{Tr}\left([\Sigma_3,U_0][\Sigma_3,U_0^\dagger]\right)^K}$$

[Verbaarschot 05; Basile, Lellouch, G.A. 08]

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$$\mathcal{Z}_{MM} \equiv \int dW_{N\times(N+\nu)} \det \begin{bmatrix} 0 & iW \\ iW^\dagger & 0 \end{bmatrix}^{N_f} e^{-N\,\mathrm{Tr}\, W^\dagger W}$$

[Shuryak, Verbaarschot 93]

- block matrix has **same global symmetry** as **QCD**-Dirac operator $\not{D}$ $\{\not{D}, \gamma_5\} = 0$, and ν **exact zero-eigenvalues**
- diagonalise $\boxed{\not{D} = \begin{pmatrix} 0 & iW \\ iW^\dagger & 0 \end{pmatrix}}$, find spectral properties & $\mathcal{Z}$

The Matrix Model of QCD

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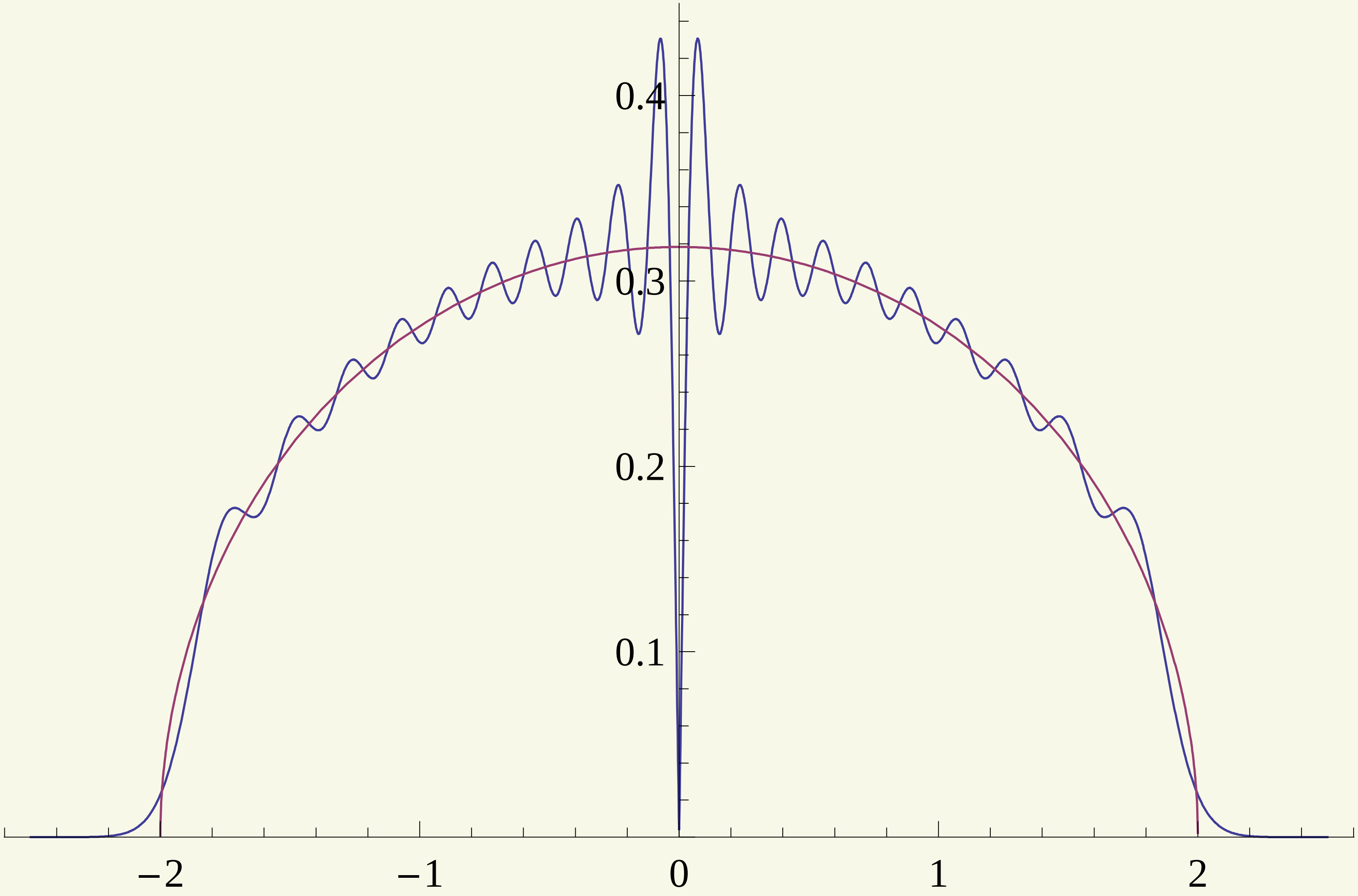
step 1. **Grassmann** $\det[\not{D}+m] = \int d\Psi \exp[\bar{\Psi}[\not{D}+m]\Psi] \rightarrow$ Gauß'

step 2. $\psi^4 \rightarrow$ **Hubbard-Stratonovich**: extra $\int dQ_{N_f\times N_f} \rightarrow$ do $\int d\psi$:

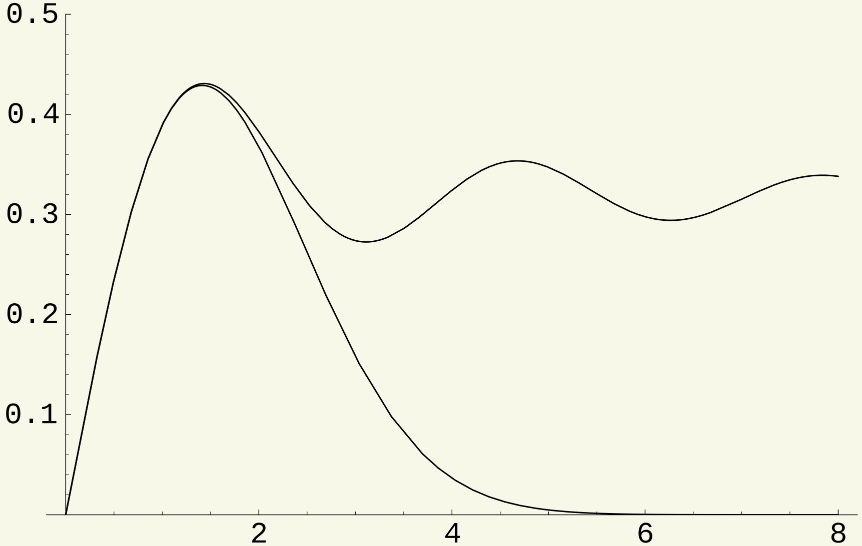
step 3. **Saddle Point**: $\lim_{N\to\infty} \mathcal{Z}_{MM} \sim \int dU_0 \det[U_0]^\nu \mathrm{e}^{N\mathrm{Tr} M(U_0+U_0^\dagger)}$

Example finite-N density

- $\rho(\lambda) = \langle \text{Tr}\delta(\lambda^2 - WW^\dagger)\rangle$



Example real density vs Lattice QCD

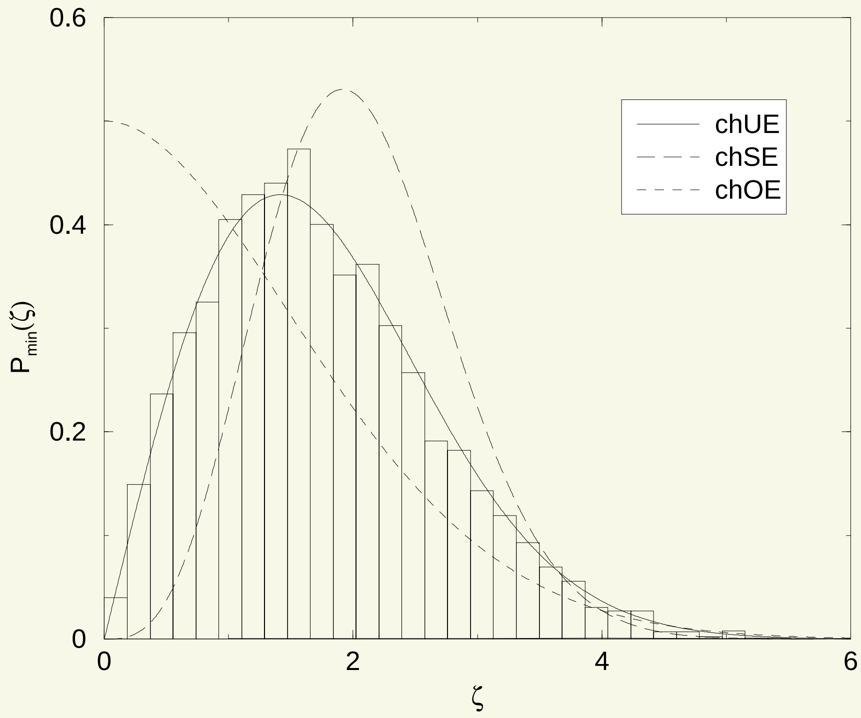


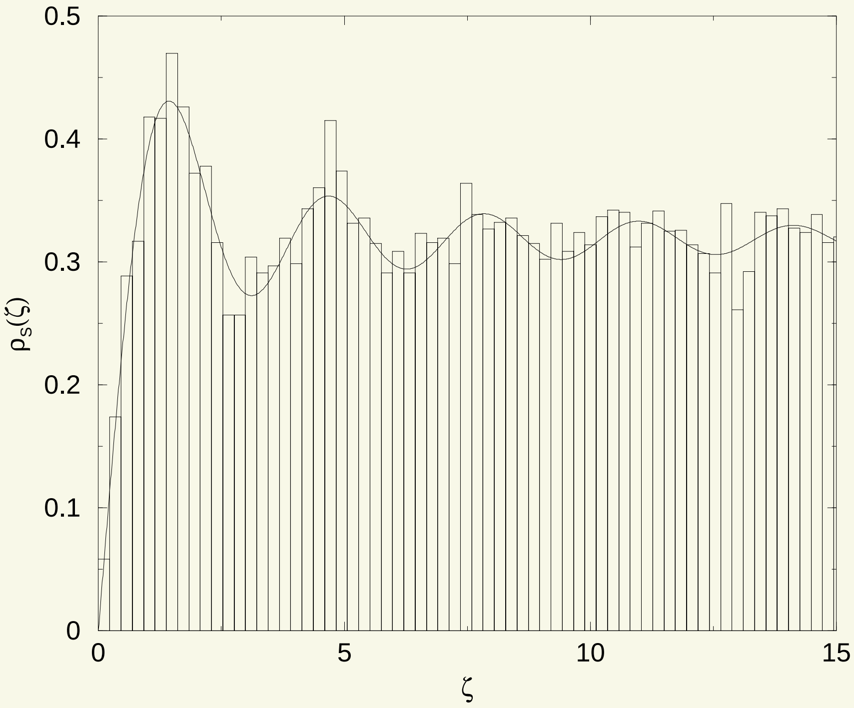
density $\rho(x) = \frac{x}{2}[J_0(x^2) + J_1(x^2)]$

1st eigenvalue $p_1(x) = \frac{x}{2} e^{-x^2/4}$

$x = \lambda V \langle \bar{\Psi}\Psi \rangle$

data: [Damgaard et al.]





The Matrix Model of finite density QCD

$$\mathcal{Z}_{MM} \equiv \int dW_1 dW_2\, \Pi_{f=1}^{N_f} \det \begin{bmatrix} m_f & iW_1 + \mu_f W_2 \\ iW_1^\dagger + \mu_f W_2^\dagger & m_f \end{bmatrix} e^{-N \mathrm{Tr} W_j W_j^\dagger}$$

[Stephanov 96, Osborn 04]

- **jpdf**: $\boxed{\mathcal{Z}_{MM} \sim \int_{\mathbb{C}} \prod_k^N dz_k^2 w(z_k) \prod_f^{N_f} (z^2 + m_f^2)\ |\Delta_N(z^2)|^2} \in \mathbb{R}$
- **weight non-Gauß:** $K_\nu(a|z|^2)\exp[b(z^2 + z^{*\,2})]$ [Osborn 04]

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- **Solution: orthogonal Polynomials on** $\mathbb{C}$ [G.A. 03, 05, + Osborn, Splittorff, Verbaarschot 05]
  - k**-point function**: $\boxed{\rho_k(z_1, \ldots, z_k) = \det_{1,\ldots,k}[K_N(z_j, z_l^*)]}$
  - **Kernel**: **(bi)-OP** on $\mathbb{C}$ $K_N(z,u) = w(z)^{\frac{1}{2}} w(u)^{\frac{1}{2}} \sum_{j=0}^{N-1} P_j(z^2) Q_j(u^2)$

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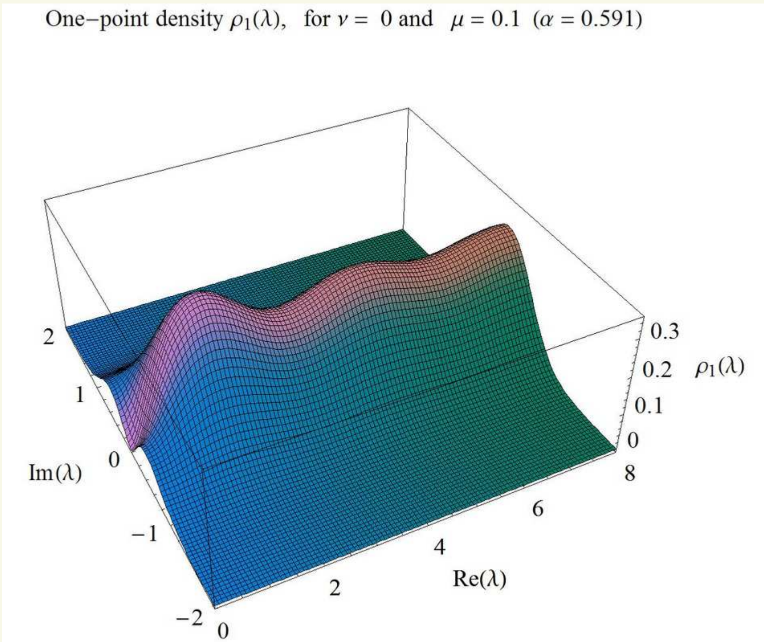
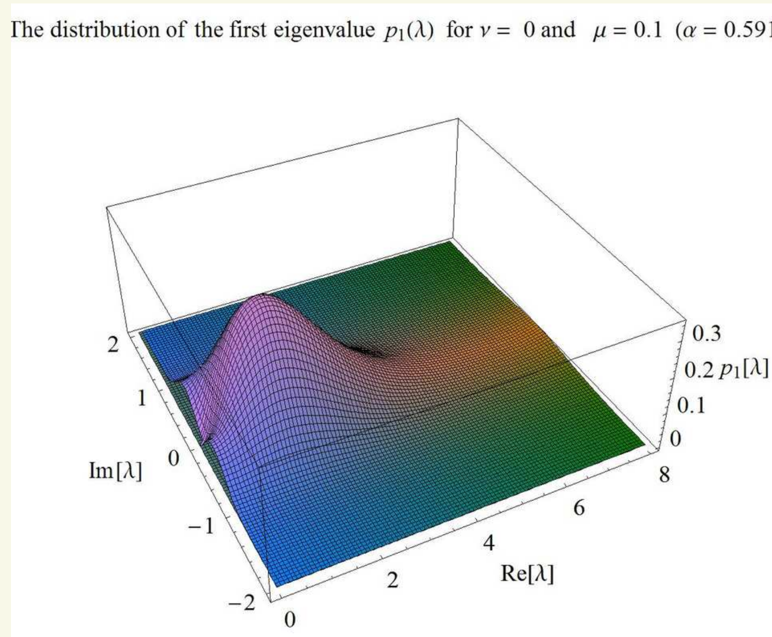
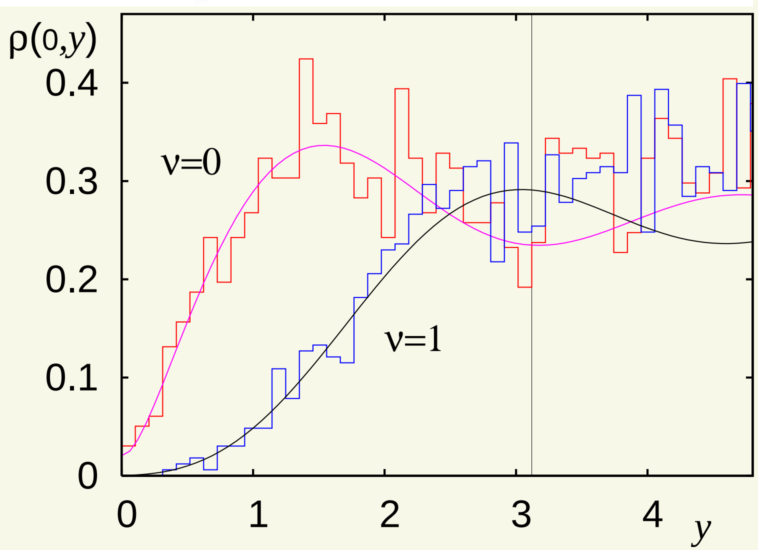
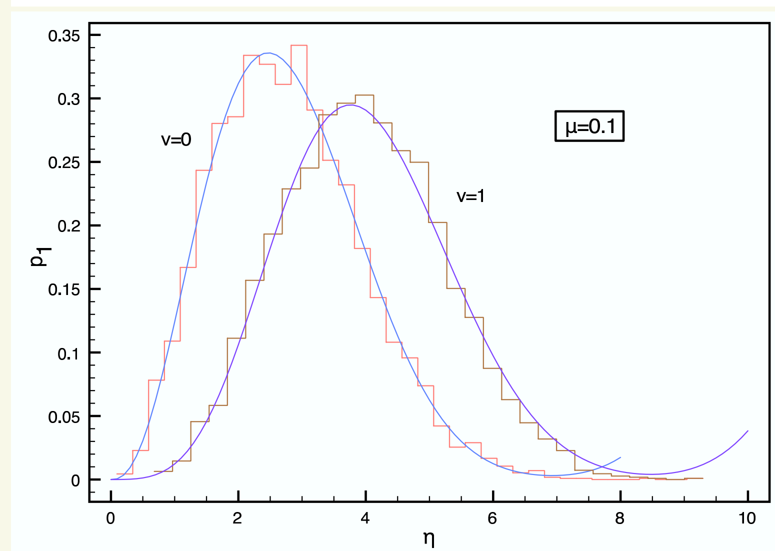
- **example** $N_f = 0$**:** $\boxed{\rho(z) = K_N(z, z^*) \stackrel{N\to\infty}{\longrightarrow} w(z) \int_0^1 dt\, e^{-t^2\mu^2} |J_\nu(tz)|^2}$

Example quenched density vs Lattice QCD



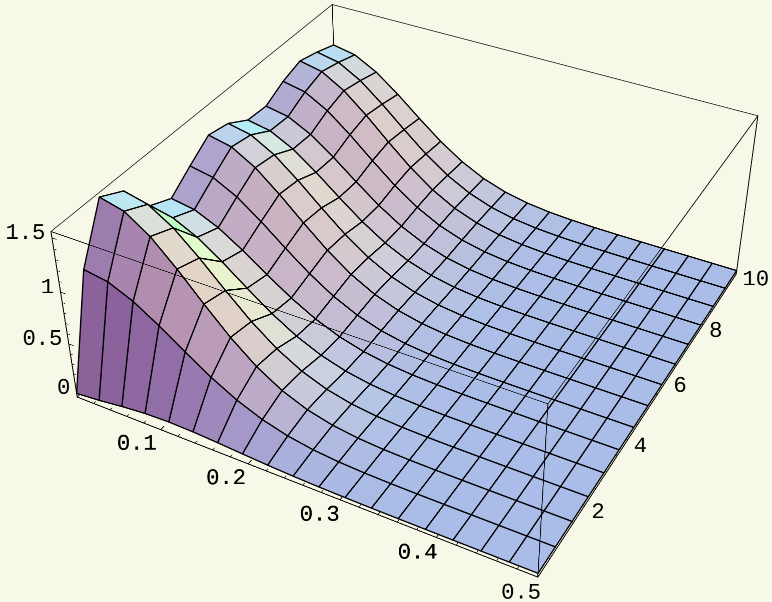


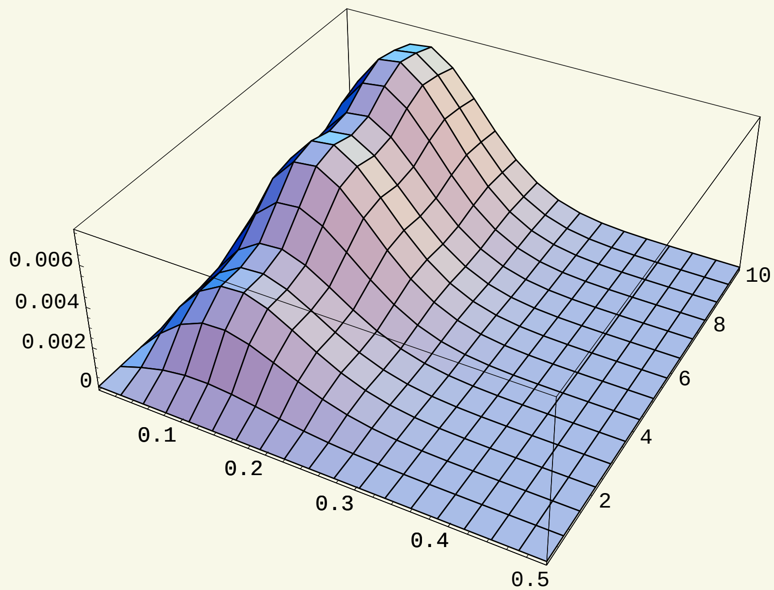




- density (left) [Bloch, Wettig 06], 1st eigenvalue (right) [+ G.A., Shifrin 07]

Example unquenched complex density





- complex density for $N_f = 1$: real- (left) & imaginary part (right) [Osborn, Splittorff, Verbaarschot, G.A. 05]

- verified for **small volumes** using reweighting [Wettig]

Why μ-dependent Density?

- density from resolvent $G(z) \equiv \langle \text{Tr}\frac{1}{\not{D}-z}\rangle = \int d\lambda \rho_{\not{D}}(\lambda)\frac{1}{i\lambda-z}$
- $\underline{\mu = 0}$:

generate $\boxed{G(z) = \partial_m \langle \frac{\det[\not{D}-m]}{\det[\not{D}-z]} \rangle\Big|_{m=z}}$ ($z \in \mathbb{C}$ regularises)

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- $\underline{\mu \neq 0}$: **Hermitization needed in FT** (not in RMT)

$$\boxed{\langle \frac{1}{\det[\not{D}(\mu)-z]} \rangle \equiv \langle \frac{\det[\not{D}(-\mu)+z^*]}{\det[\not{D}(\mu)-z]\det[\not{D}(-\mu)+z^*] + \varepsilon} \rangle}$$

as $\not{D}(\mu)^\dagger = (\not{D} + \mu\gamma_0)^\dagger = -\not{D} + \mu\gamma_0 = -\not{D}(-\mu)$

– generating functional: **extra Fermion/Boson pairs** $\pm\mu$ [Stephanov 96]

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- sum rules from $\mathcal{Z}$ unchanged: $\sum_i' \frac{1}{z_i^2} = \frac{1}{4(N_f+\nu)}$
- all densities from $\varepsilon\chi$PT$\Leftrightarrow$ RMT [Damgaard, et al 98, Basile, G.A. 07]

Beyond RMT

- **expand** $U(x)$ to order $O(\xi^3)$ in

  $\mathcal{L} = \frac{1}{4}F^2\partial U\partial U^\dagger + \frac{1}{2}\langle\bar{\Psi}\Psi\rangle M\left(U + U^\dagger\right)$

- **1-loop calculation:** propagator [Hasenfratz, Leutwyler 90]

  $$<< \xi_{ij}(x)\xi_{kl}(y) >>= (\delta\delta - \frac{1}{N_f}\delta\delta)\bar{\Delta}(x-y) \stackrel{x=y}{\longrightarrow} 1/\sqrt{V}$$

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$\Rightarrow$ same group integral with **renormalised couplings:** [Gasser, Leutwyler 87, Basile, Lellouch, G.A. 08]

$$\langle\bar{\Psi}\Psi\rangle_{eff} = \langle\bar{\Psi}\Psi\rangle\left(1 - \frac{N_f^2-1}{N_f F^2}\right)\bar{\Delta}(0)\ , \quad F_{eff} = F\left(1 - \frac{N_f}{F^2}(\bar{\Delta}(0) - \int\partial\bar{\Delta}^2)\right)$$

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- **scalar and current 2-point functions:** (see also [Hansen 90, Damgaard et al. 01])

  – couple sources (like mass terms) & 1-loop

  $\rightarrow \langle\Pi_a(x)\Pi_a(0)\rangle \sim$ group factor $\times$ kinematic factor

Summary & Discussion

- $\exists$ precise limit QCD$\rightarrow$ RMT! (often RMT heuristic)
- breakdown of RMT approximation and corrections well understood
- vs num. Lattice QCD$\surd$, BUT extremely difficult for $\mu \neq 0$
- <u>Open Problems:</u>
  - relation RMT to other QCD-like theories less understood:
    - ▷ ortho., sympl. RMT correspond to different gauge group & rep
    - ▷ RMT's $\surd$ incl. $\mu \neq 0$ (& Lattice)
    - ▷ Goldstone mfld $\neq$ group: in σ model UU^T with $U \in U(N_f)$ only equal mass $\mathcal{Z}$ known
  - the QCD-sign problem