Non-equilibrium Kinetics of Disordered Luttinger Liquid

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Outline

- 1D disordered quantum nanowires

  Luttinger-type physics (High T)

  Anderson localization (Low T)

- Kinetic theory of strongly correlated (non-Fermi) disordered 1D liquids

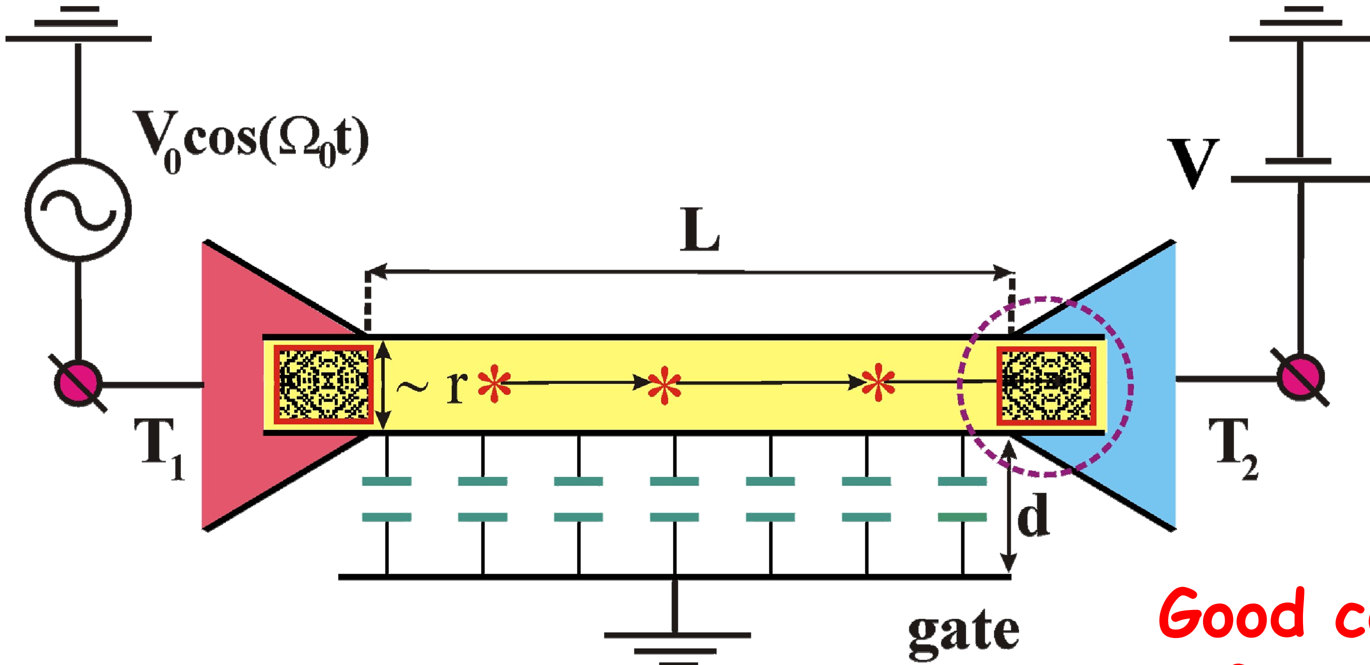
  Basis: Keldysh-type 1D ballistic σ-model.

- Result: coupled "quantum kinetic equations" for electrons and plasmons

  Output: relaxation times.

- Conclusions & Outlook

1-D Quantum Wire



Good contact,
$G_c >> G_Q$

$l \sim L, \quad G \sim G_Q$ - Limit of weak disorder

$\max(eV, T) \gg E_{Th} \sim v_F / L$ - Thouless energy (1D case)

Fermion Action

- **Single-channel wire**

$$\psi(x) = \psi_+(x)e^{ik_F x} + \psi_-(x)e^{-ik_F x} \qquad \rho_\mu(x) \simeq \bar{\psi}_\mu(x)\psi_\mu(x)$$

1-D fermions: left/right movers

$$S = i\int_{C_K} dt \int_0^L dx \sum_{\mu=\pm} \bar{\psi}_\mu (\partial_t - \mu \mathrm{v}_\mathrm{F} \partial_x)\psi_\mu + S_{e-e}$$

linear spectrum !

$$+ \int_{C_K} dt \int_0^L \{U_b(x)\bar{\psi}_L \psi_R + \mathrm{h.c.}\}$$

- **White-noise disorder:** $\left\langle U_b^{\,*}(x_1)U_b(x_2)\right\rangle = \delta(x_1 - x_2)/(2v_F\tau)$

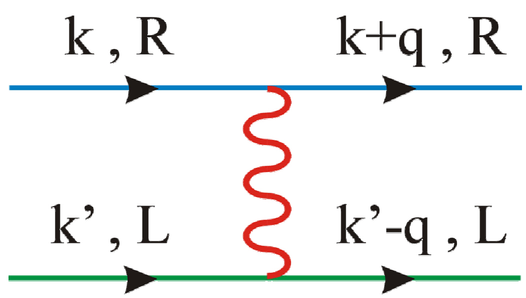
τ – elastic scattering time

Backscattering amplitude !

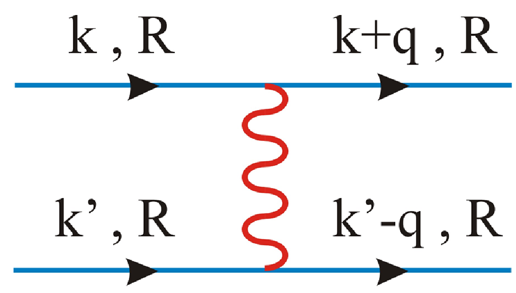
Fermion Action

- **Screened Coulomb interaction**

$$V_q = 2e^2 \ln(2d/r), \quad qd \ll 1$$



$g_2 = V(q \sim 0)$



$g_4 = V(q \sim 0)$

$$S_{e-e} = \frac{1}{2}\sum_{\mu,\mu'}\int_{C_K} dt \int_0^L dx_1 dx_2\, \rho_\mu(x_1) V(x_1 - x_2) \rho_{\mu'}(x_2)$$

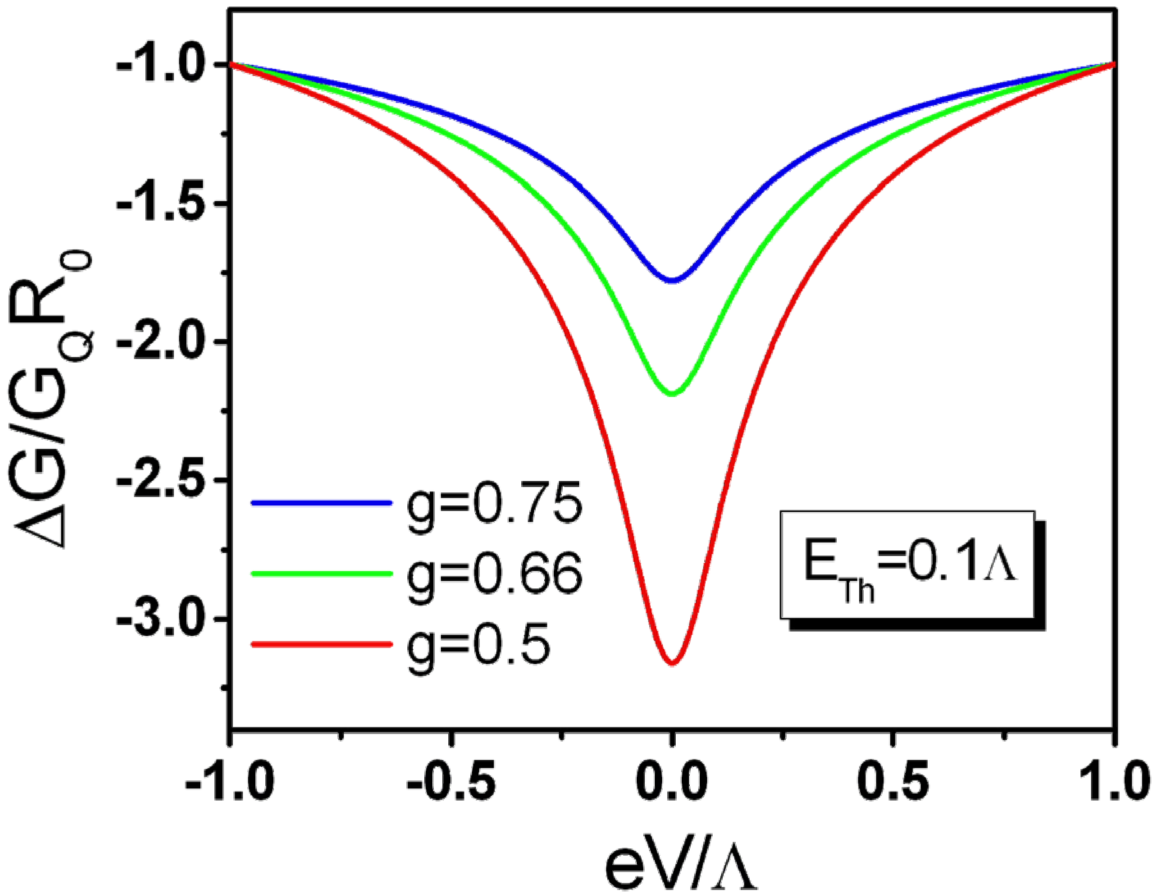
- **Bosonic modes are plasmons (charge density waves) !**

$$u_\rho = \mathrm{v_F}/K$$

$$K = \left(1 + V_q/\pi \mathrm{v_F}\right)^{-1/2} < 1$$

Interaction constant

DC conductance



Conductance

$$\Delta G(E) = -G_Q R(E)$$

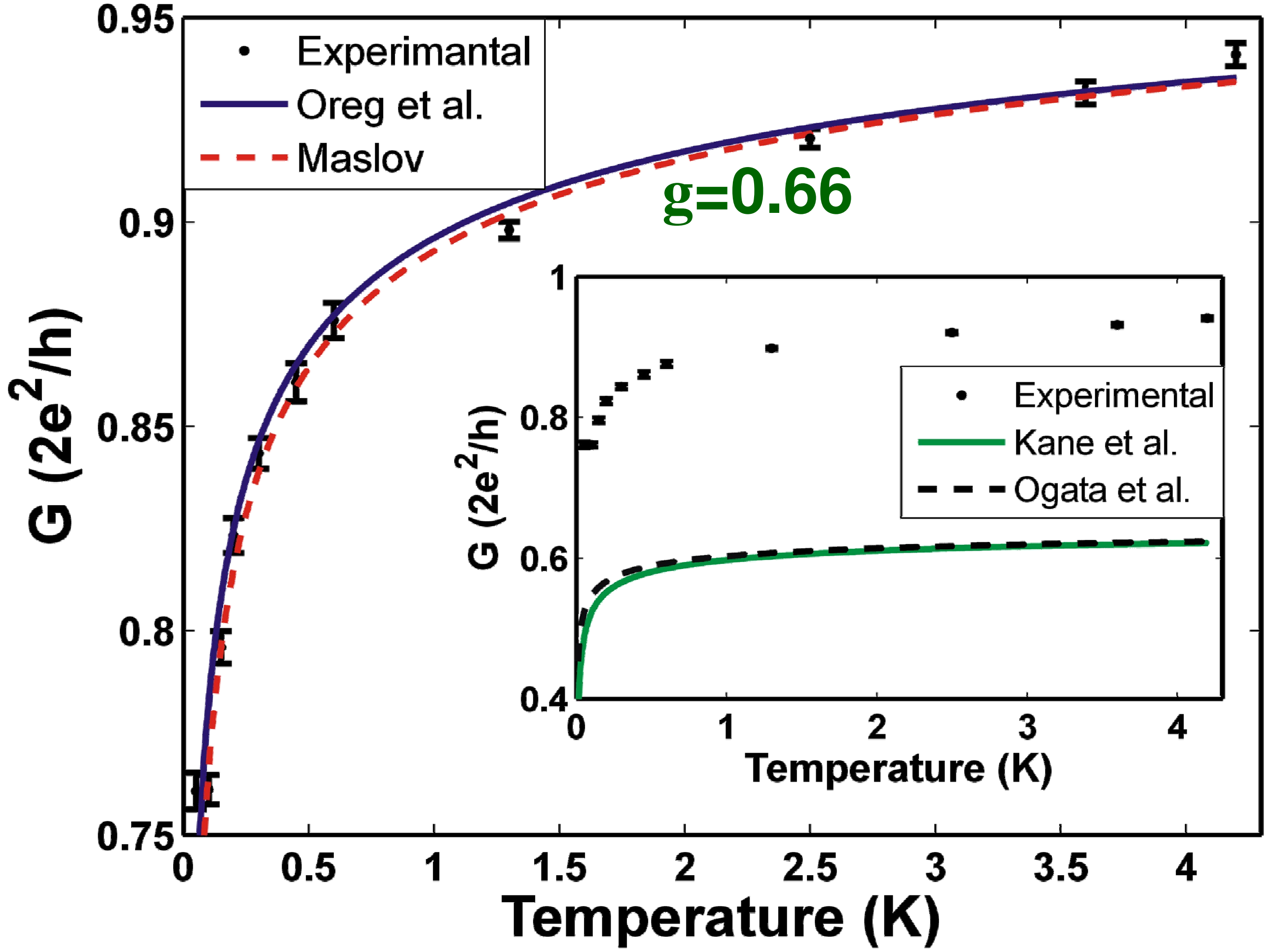
$$R(E) \propto R_0 \left(\frac{\Lambda}{E} \right)^{1-K}$$

Renormalization of reflection coefficient

(Kane & Fisher '92
Ponomarenko '94,
Maslov '94, ...)

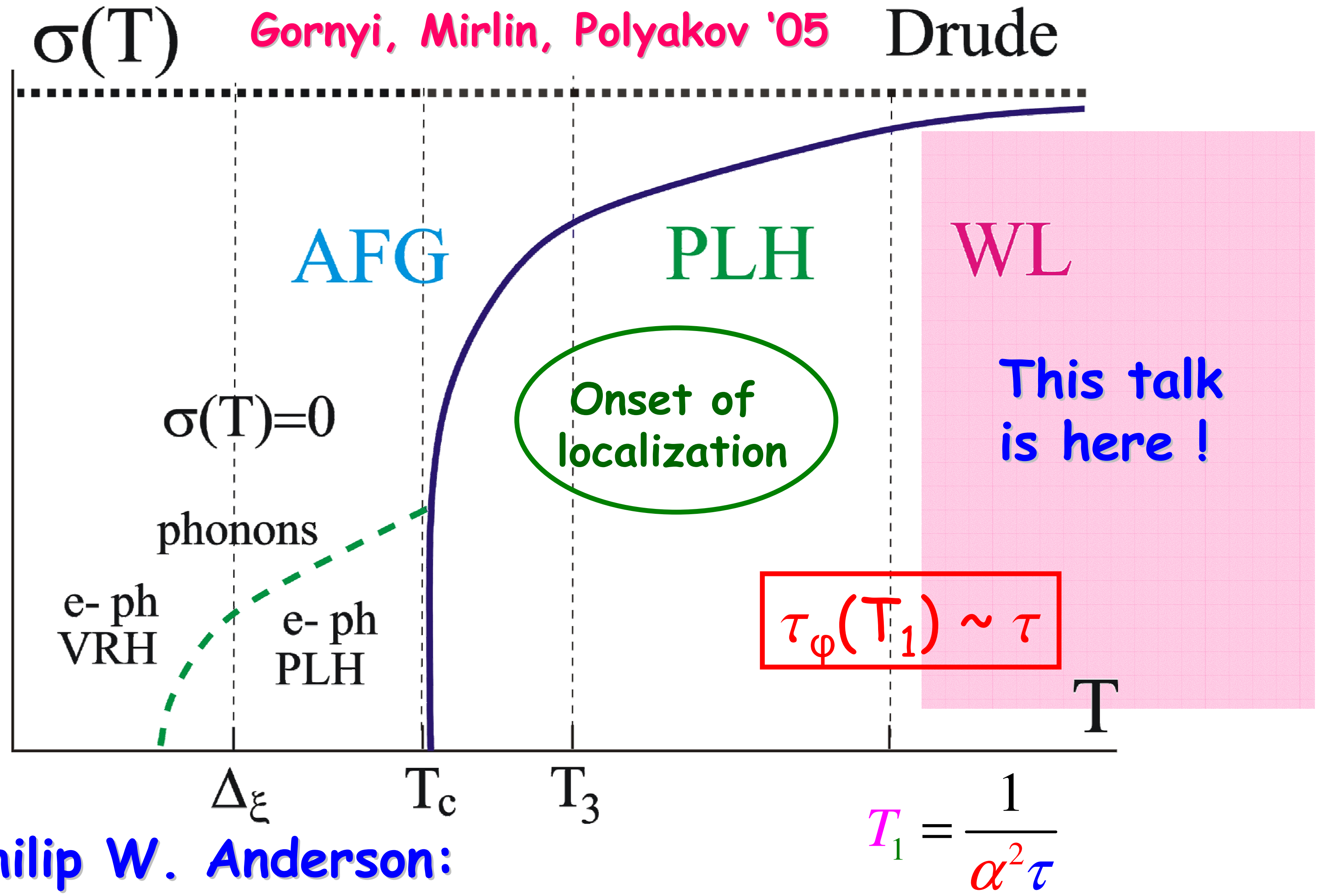
Short wire, $L \ll l(E_{Th})$, weak backscattering limit

Experiment on V-groove wires



E.Levy, A.Tsukernik, M.Karpovsky, A.Palevsky, B.Dwir, E.Pelucchi, A.Rudra, E.Kapon, Y.Oreg, PRL 97, 196802, 2007

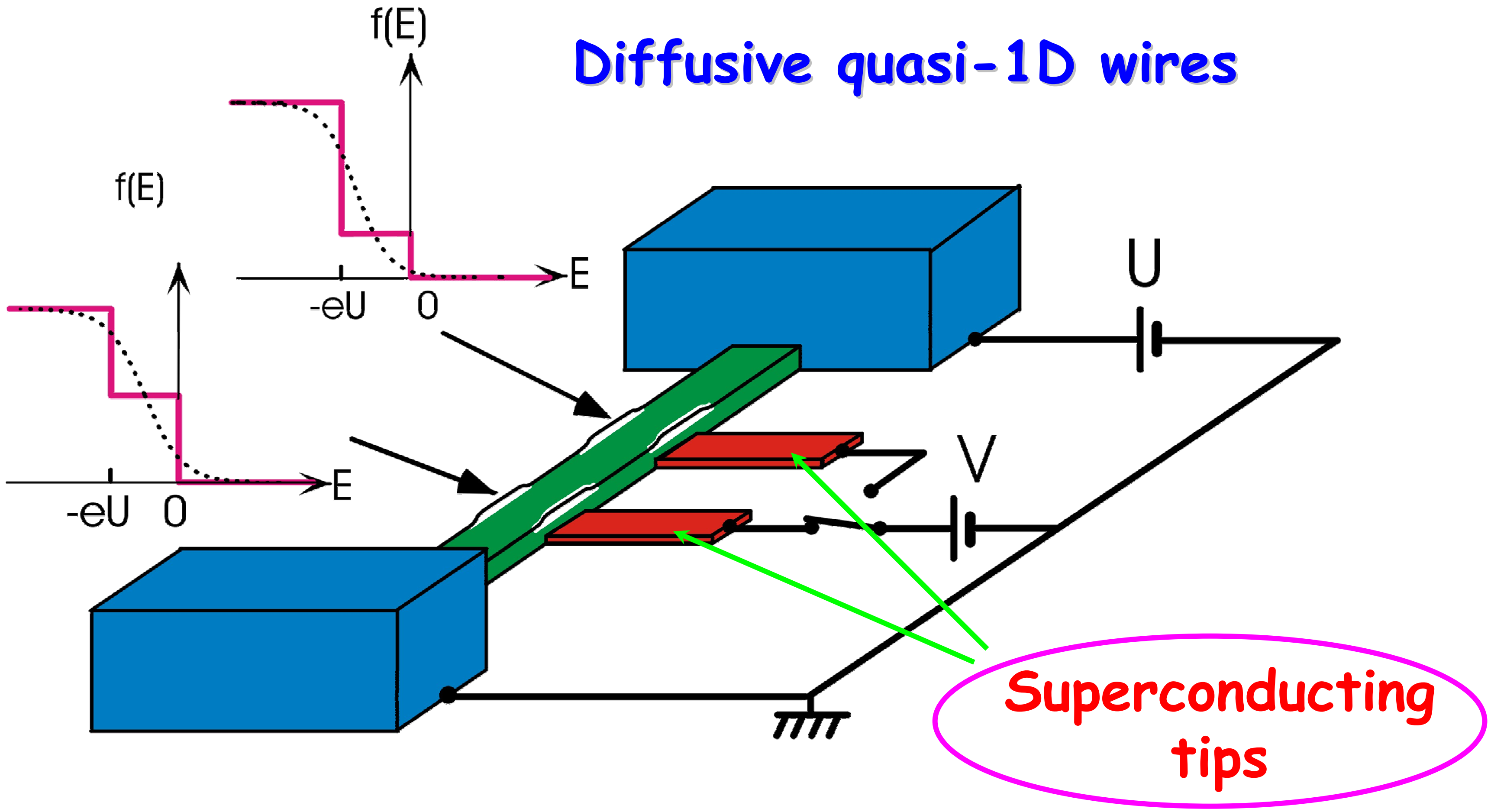
Weak to Strong Localization



Philip W. Anderson:

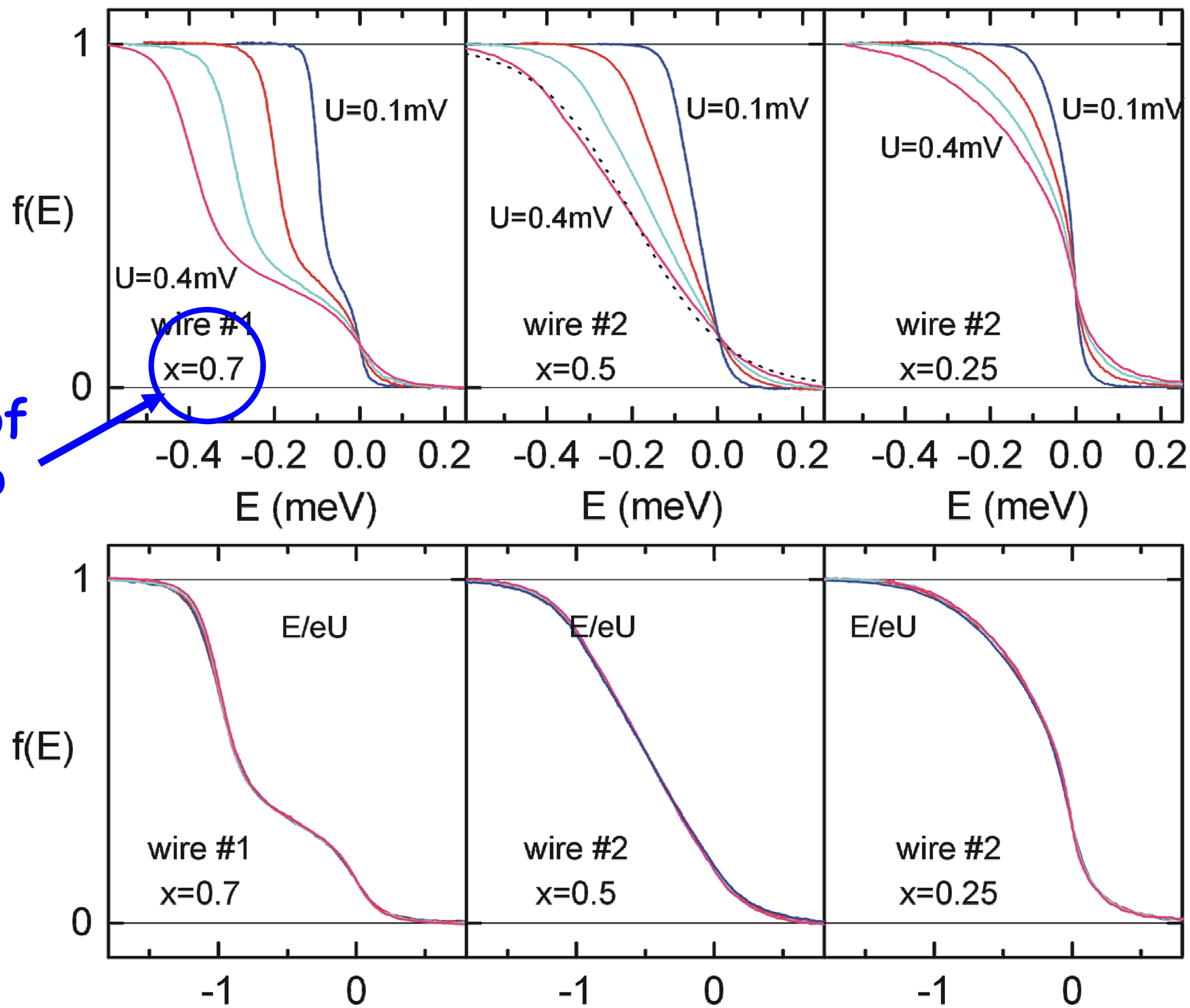
„ At T=0 and D ≤ 2 all electronic states are localized"

Experiments on Energy Relaxation



F. Pierre, H. Pothier, D. Esteve, M.H. Devoret, A.B. Gougam and N.O. Birge, 2000-2003

Distribution function

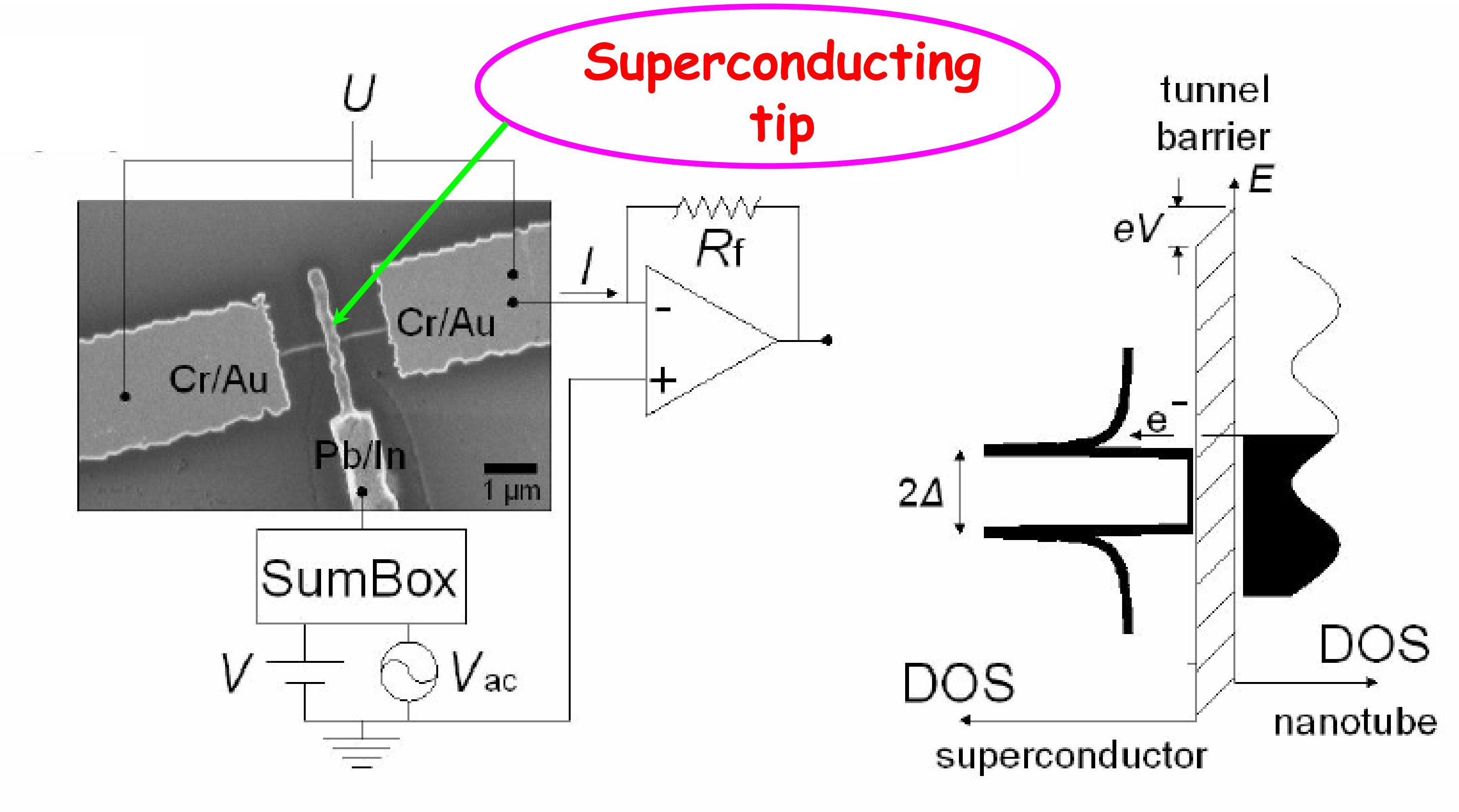


Position of the S-tip

E/eU – normalized energy

Experiment

Non-equilibrium tunneling spectroscopy in carbon nanotube



Y.-F. Chen, T. Dirks, G. Al-Zoubi, N.O. Birge, N. Mason, PRL' 09

Energy Relaxation in 1D

- In the *clean* Luttinger Liquid (integrable model!) there is no energy relaxation!

Non-equilibrium state ⇒ no relaxation

- Disorder ⇒ allows for relaxation to *equilibrium*

(This talk !)

- Energy relaxation *beyond* Luttinger Liquid

Curvature in spectrum + finite range interaction
(A.M. Lunde, K. Flensberg, L. Glazman' 07) ⇒

small in parameter $T/E_F << 1$

How one should describe the *non-equilibrium* state of the Luttinger Liquid?

"Functional" bosonization

We use the Hubbard-Stratonovich decoupling scheme

- Semiclassical Keldysh Green's function at x=x'

$$g(x,t_1,t_2)=iv_F\left[G(x,x+0,t_1,t_2)+G(x,x-0,t_1,t_2)\right]$$

Using the ideas of the non-equilibrium superconductivity!

cf. Shelankov' 85

- Eilenberger equation (exact for linear spectrum in 1D !)

Equation of motion for electron in the fluctuating electic field

$$iv_F\partial_x g+\left[(i\partial_t-\varphi)\tau_z-\tfrac{1}{2}\left(U_b\tau_+ + U_b{}^*\tau_-\right),g\right]=0$$

Hubbard-Stratonovich decoupling field

Impurity backscattering

$$g\circ g=\hat{1}$$

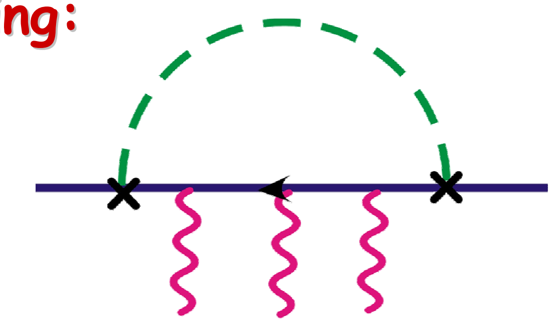
Normalization

1D Keldysh action

Based on ideas of
Muzykantskii, Khmelnitskii '95 & Kamenev, Andreev '99

Born approximation over impurity scattering:

$$i\mu v_F \partial_x \bar{g}^{\mu} + \left[(i\partial_t - \varphi)\tau_z - \frac{i}{4\tau}\bar{g}^{-\mu}, \bar{g}^{\mu} \right] = 0$$



- **Effective action**

Keldysh-type 1D ballistic σ-model

$$S[g,\varphi] = -\frac{1}{2v_F}\mathrm{Tr}(i\partial_t - \varphi)\tau_z g - \frac{i}{2}\mathrm{Tr}\left(g_0 T^{-1}\partial_x T\right) - \frac{i}{8v_F\tau}\mathrm{Tr}(g_R g_L)$$

$$g_0^{\mu} = \begin{pmatrix} \hat{1} & 2(\hat{1} - 2f^{\mu}) \\ 0 & -\hat{1} \end{pmatrix}$$

$$g = T g_0 T^{-1}$$

Ambegaokar-Eckern-Schön action

Electron distribution function

Kinetic equation for electrons

$$\begin{cases} \left(\partial_t - \dot{\varphi}_R \partial_\varepsilon + v_F \partial_x\right) f_\varepsilon^R = -\frac{1}{2\tau}\left(f_\varepsilon^R - f_\varepsilon^L\right) + St_e^R \\ \varphi_R = g_2 \rho_L \\ \rho_L(t,x) = \frac{1}{2\pi v_F}\left[-\varphi_L(t,x) + \int d\varepsilon\, f_\varepsilon^L(t,x)\right] \end{cases}$$

Collision integral

"Poisson" equation

Charge density

cf. the kinetic equations in plasma physics

- **Motion of e⁻ in the dissipative bosonic environment**

$$St_e^\mu(\varepsilon) = \sum_\nu \int d\omega \left\{ I_<^{\mu\nu}(\omega) f_{\varepsilon-\omega}^\nu (1 - f_\varepsilon^\mu) - I_>^{\mu\nu}(\omega) f_\varepsilon^\mu (1 - f_{\varepsilon-\omega}^\nu) \right\}$$

Absorption

Full rate of emission

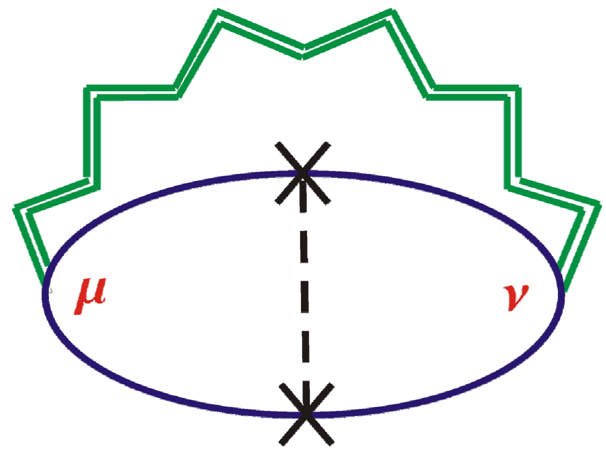
Emission rate (in one-loop)

RPA-like effective e-e interaction: $V_{>}^{\mu\nu}(\omega, q)$

Plasmon propagator,
poles: $q = \pm\omega(1 \pm i/2\omega\tau)/u$

Plasmons exist at $\omega >> T_1 = 1/\alpha^2\tau$ only

Particle-hole propagator,
poles: $q = \pm\omega(1 \pm i/2\omega\tau)/v_F$

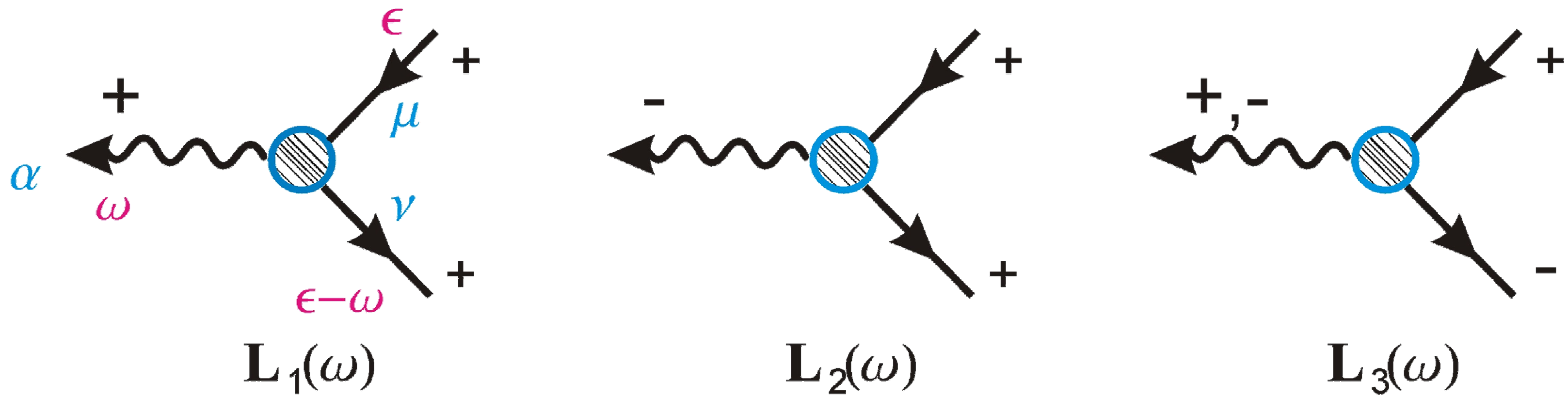


$\mathrm{Re}\, D_R^{\mu\nu}(\omega, q)$

$$I_{>}^{\mu\nu}(\omega) = \frac{i}{\pi}\int \frac{dq}{2\pi} V_{>}^{\mu\nu}(\omega, q)\,\mathrm{Re}\, D_R^{\mu\nu}(\omega, q)$$

Large energy transfer, $\omega >> T_1$

We treat contributions from different poles separately !



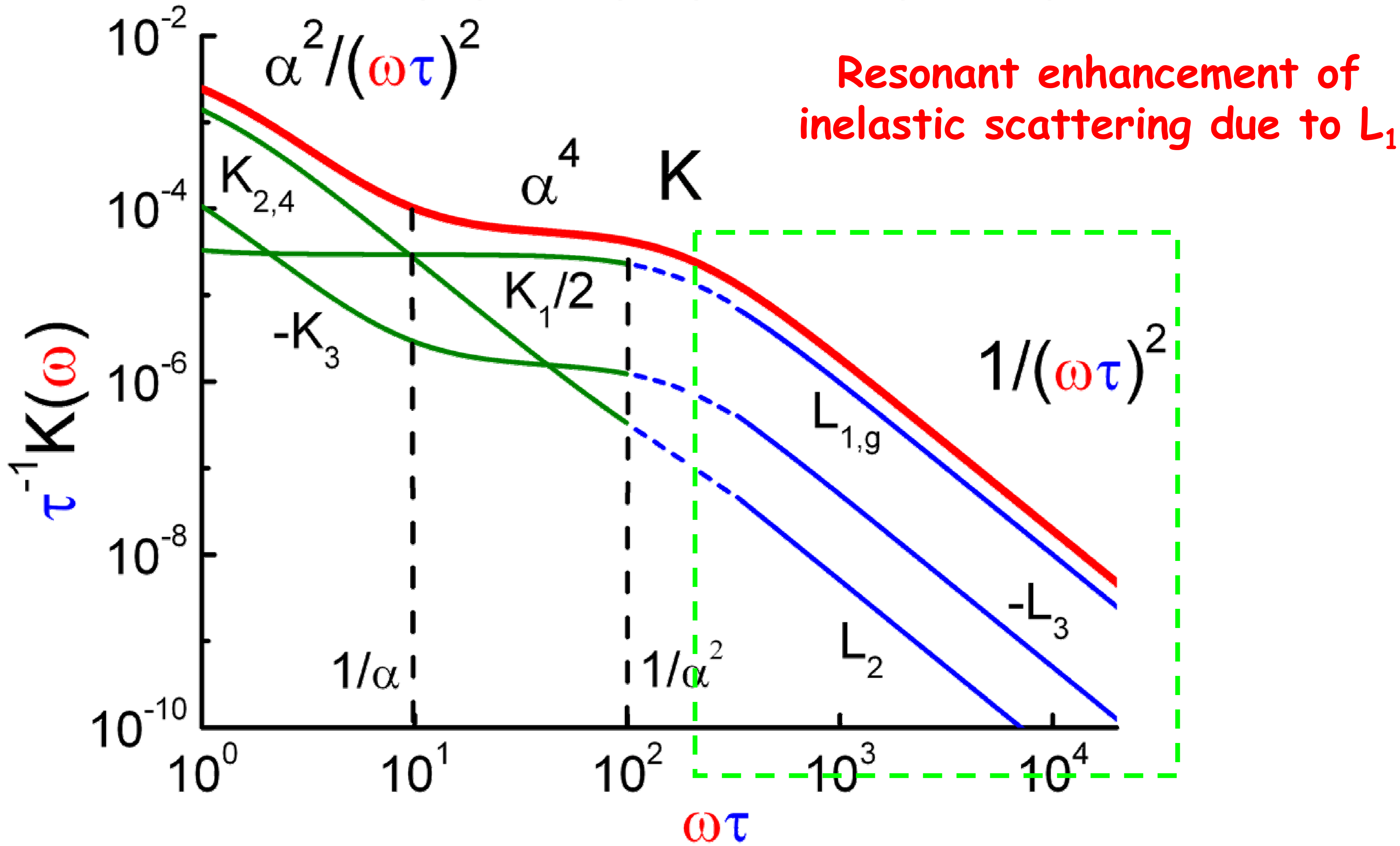
Resonant process!

Emission rate of plasmons:

$$I^{\mu\nu}_{p,>}(\omega) = \sum_{\alpha} L^{\mu\nu}_{\alpha}(\omega)(1+n^{\alpha}_{\omega})$$

$$L_{1,2}(\omega) = \frac{1}{2\omega^2\tau}\left(\frac{v_F}{u} \pm 1\right), \quad L_3(\omega) = -\frac{\alpha}{2\omega^2\tau}\frac{v_F}{u}$$

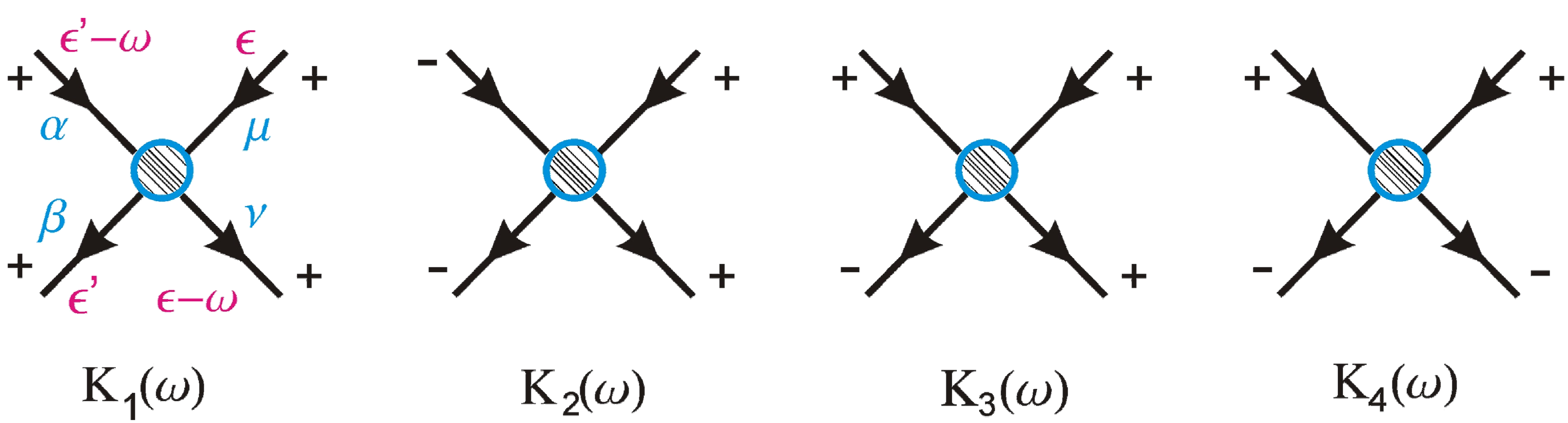
Collision Kernel



Weak interaction limit, $\alpha = V_q/\pi v_F \ll 1$

Small energy transfer, $\omega << T_1$

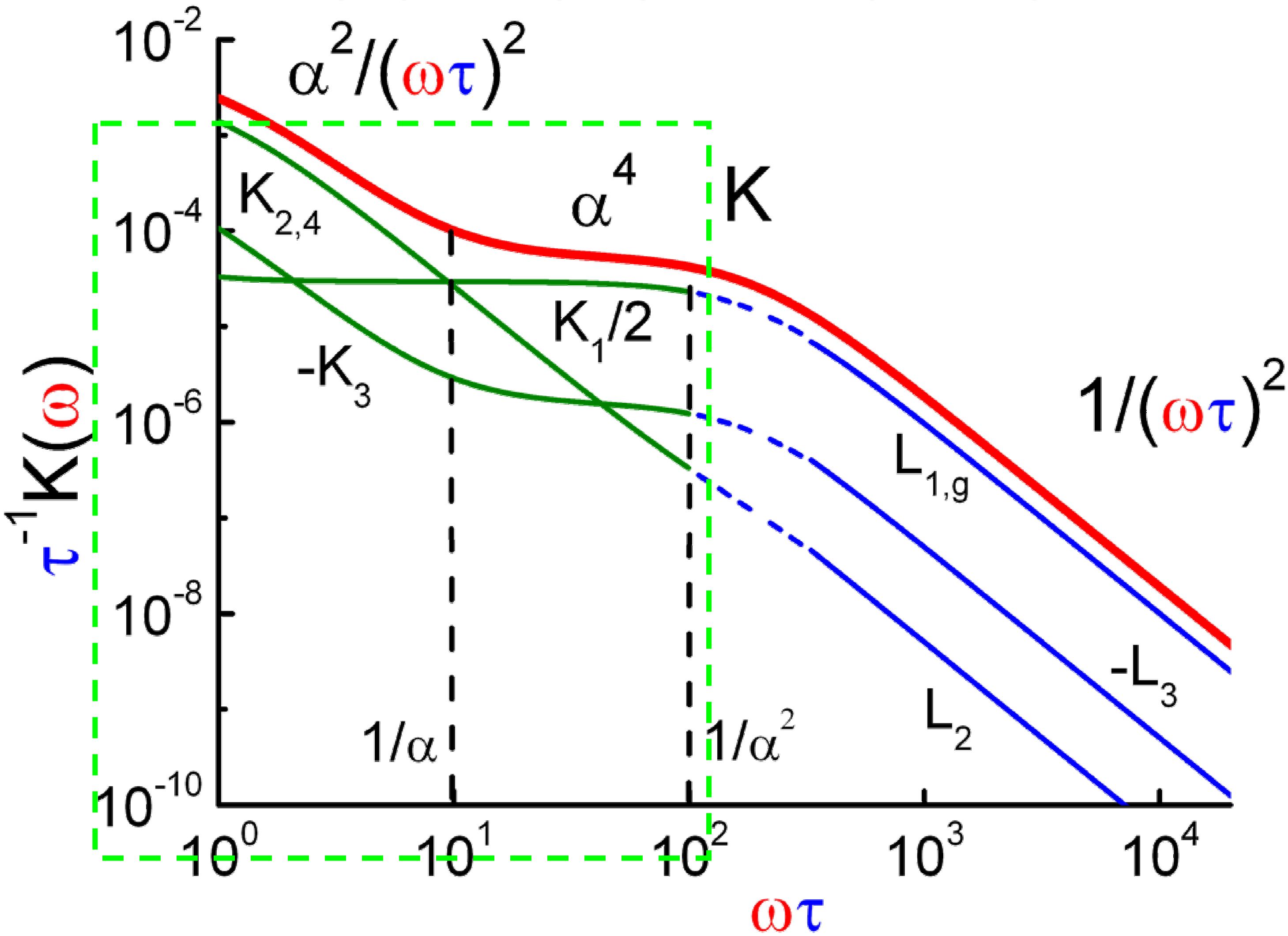
Plasmon-mediated two-fermion scattering



Emission rate:

$$I_{>}^{\mu\nu}(\omega) = \sum_{\alpha\beta} \int d\varepsilon' K^{\alpha\mu}_{\beta\nu}(\omega) f^{\alpha}_{\varepsilon'-\omega} (1 - f^{\beta}_{\varepsilon'})$$

Collision Kernel



Weak interaction limit, $\alpha = V_q/\pi v_F << 1$

Kinetic equation (plasmons)

Decay and recombination of plasmons
from/into electron-hole pairs

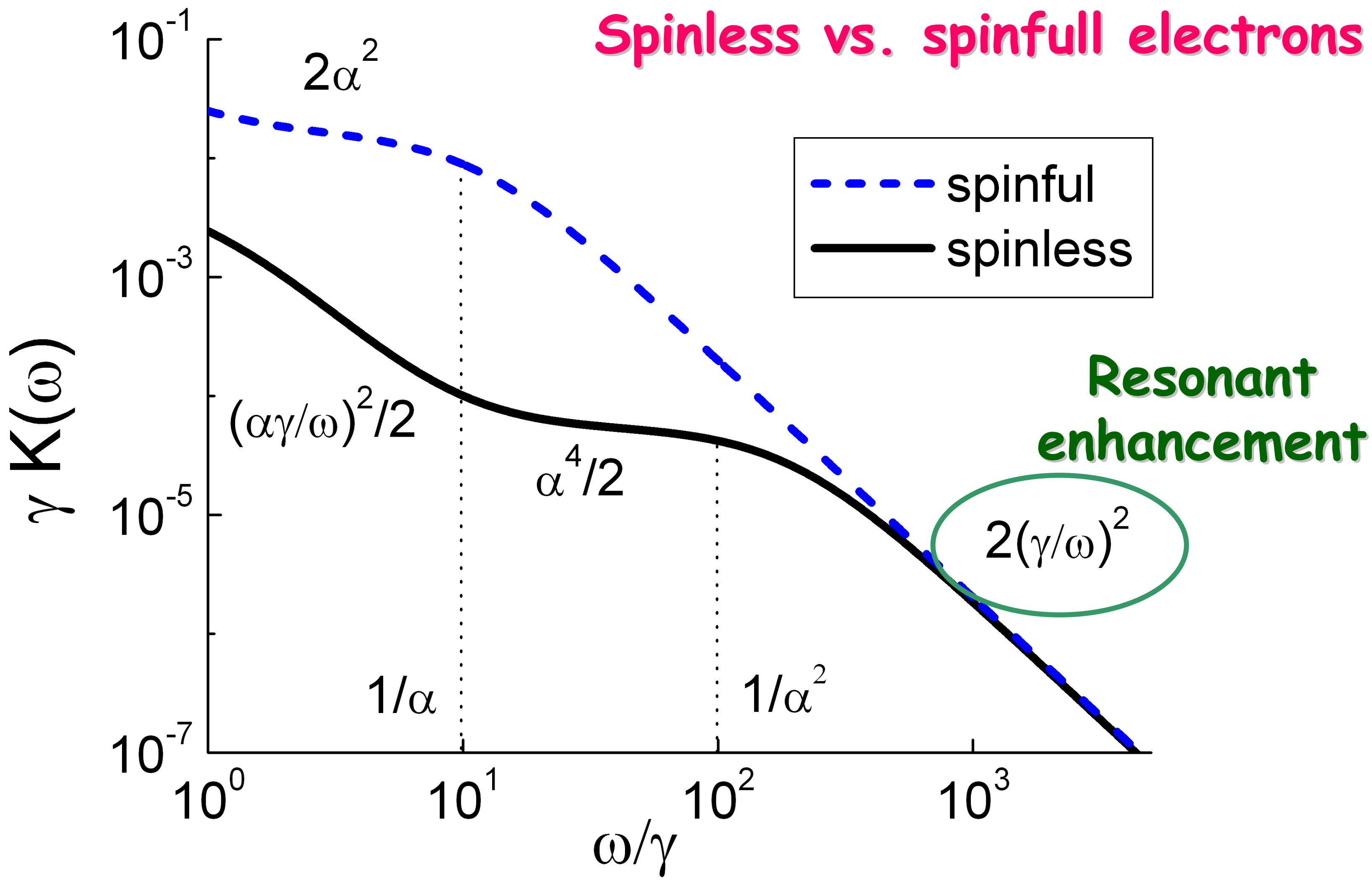
$$\partial_t n_\omega^R + u\,\partial_x n_\omega^R = -\frac{1-\alpha}{\tau} n_\omega^R + St_b^R(\omega)$$

Collision integral:

$$St_b^\mu(\omega) = \frac{1}{2\tau}\left[\left(1+\frac{u}{v_F}\right)N_\omega^{\mu\mu} + \left(1-\frac{u}{v_F}\right)N_\omega^{-\mu,-\mu} - 2\alpha N_\omega^{\mu,-\mu}\right]$$

$$N_\omega^{\mu\nu} = \frac{1}{2\omega}\int dE\, f_{E+\omega}^\mu (1-f_E^\nu) + (\mu \leftrightarrow \nu)$$

Total Collision Kernel

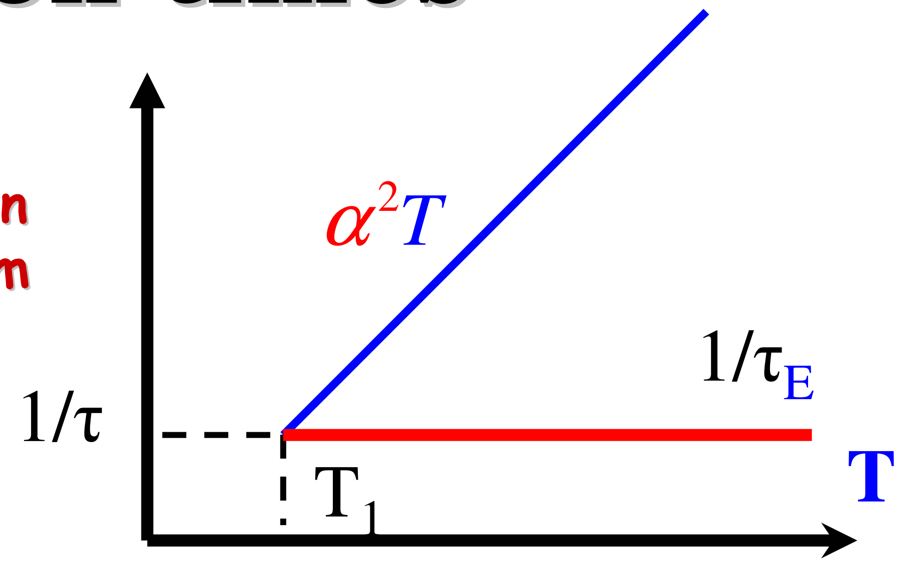


Weak interaction limit, $\alpha = V_q/\pi v_F \ll 1$

Relaxation times

- **Equilibration rate**

  (Relaxation rate of the electron system to the locally equilibrium Fermi distribution)



$$T \gg T_1 = 1/\alpha^2\tau$$

$$\frac{1}{\tau_E(T)} \sim T^{-1}\int_0^T K(\omega)\omega^2 d\omega \sim \frac{1}{\tau}$$

= Impurity scattering rate !!!

(Interaction and T independent!)

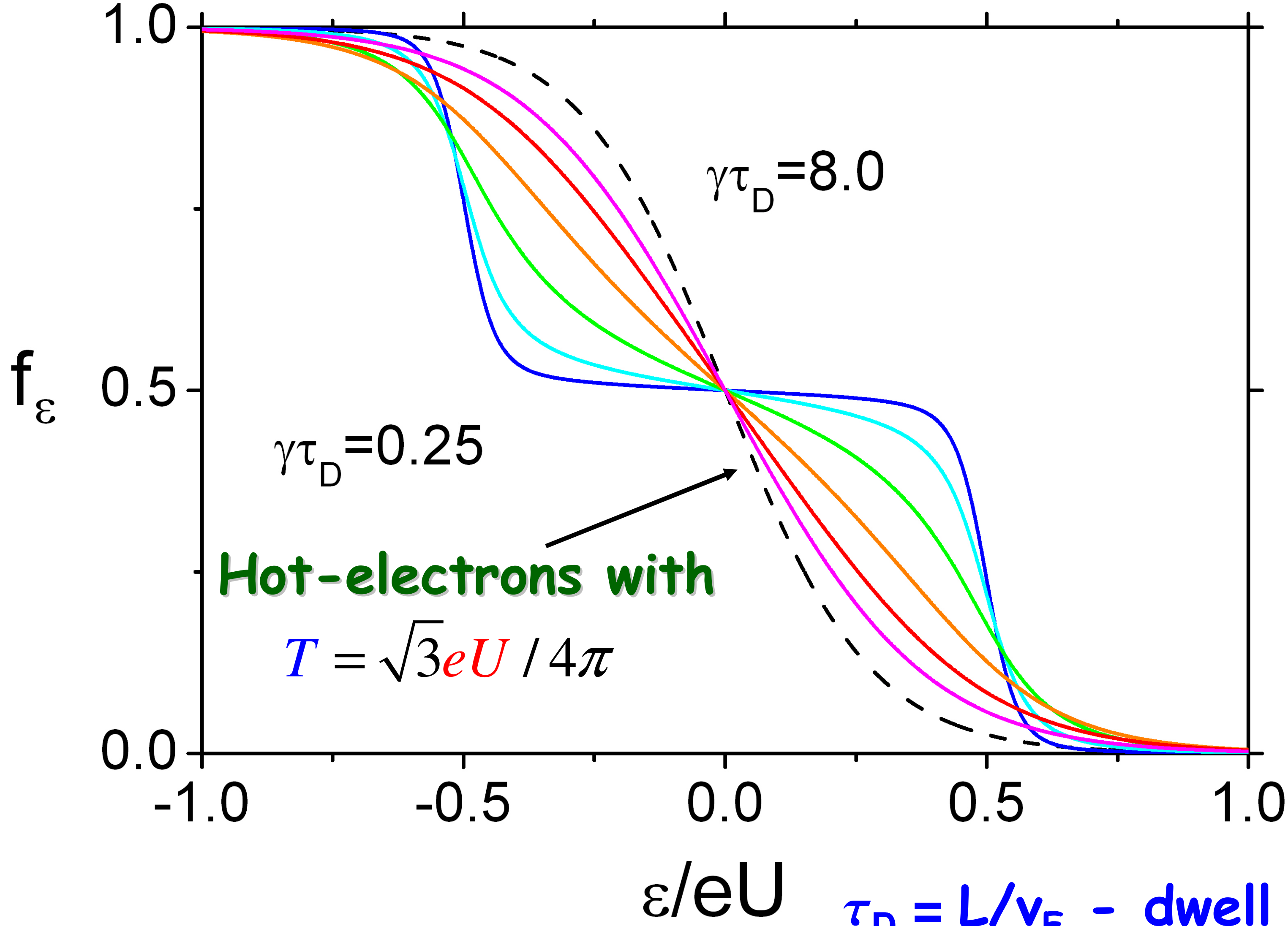
- **Out-scattering rate of electron**

  (It is determined by the out-term of kinetic equation)

$$\frac{1}{\tau_{out}(T)} \sim T\int_0^T K(\omega)d\omega = \alpha^2 T$$

Relevant for zero-bias anomaly, same as in clean limit

Electron distribution function



$\tau_D = L/v_F$ - dwell time

Summary

Kinetic theory approach to non-equilibrium Luttinger Liquids

- Formulated kinetic-equation description of a Luttinger liquid out of equilibrium
- Disorder-induced resonant enhancement of inelastic scattering
- Equilibration rate = elastic scattering rate