Non-abelian Vortices in $\mathcal{N}=1$ SQCD

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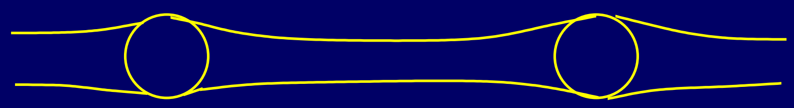
Confinement is customary thought of as medium where (chromo)electric charges are bound by flux tubes via condensation of monopoles.

A number of theories have been proposed, such as the Seiberg-Witten theory, which are shown to have confinement.

However, in these theories, confinement is essentially abelian. This has problems with our experience with QCD. The low-energy group is $U(1)^{N-1}$ which leads to the existence of $N-1$ Abelian strings, and therefore $N-1$ infinite towers of mesons.

In real-world QCD confinement is non-abelian.

Certain supersymmetry SQCD scenarios allow condensation of quarks to cause confinement of the monopoles



Vortex strings are allowed to end on a monopole which they therefore confine. The low-energy dynamics of these strings is described by an effective 2-dimensional worldsheet theory

The strings represent the vacua of this theory, while the monopoles play the role of kinks of the worldsheet theory

For the study of confinement of monopoles, therefore, it is instructive to investigate the dynamics of 2-dim (typically CP(N-1)) effective worldsheet theory

We consider the $\mathcal{N}=2$ SQCD theory with a gauge group $SU(N)\times U(1)$.

Supposedly it emerges from $SU(N+1)$ broken down to $SU(N)\times U(1)$ at the scale

$$m \quad \gg \quad \Lambda_{SU(N+1)}$$

Later we break $\mathcal{N}=2$ supersymmetry to $\mathcal{N}=1$.

$\mathcal{N} = 2$ SQCD

The content of the $\mathcal{N}=2$ SQCD is as follows:

The gauge multiplet includes $\mathcal{N}=1$ gauge multiplets

$$
\begin{array}{cccc}
W_\alpha^{\mathrm{U(1)}} & \ni & A_\mu^{\mathrm{U(1)}} & \lambda^{1\,\mathrm{U(1)}} \\
W_\alpha^{\mathrm{SU(N)}} & \ni & A_\mu^{\mathrm{SU(N)}} & \lambda^{1\,\mathrm{SU(N)}}
\end{array}
$$

and the chiral multiplets

$$
\begin{array}{cccc}
\mathcal{A}^{\mathrm{U(1)}} & \ni & a^{\mathrm{U(1)}} & \lambda^{2\,\mathrm{U(1)}} \\
\mathcal{A}^{\mathrm{SU(N)}} & \ni & a^{\mathrm{SU(N)}} & \lambda^{2\,\mathrm{SU(N)}}
\end{array}
$$

Matter multiplets, in $N_F = N$ quantity

$$
\begin{array}{cccc}
Q^{kA} & \ni & q^{kA} & \psi^{kA} \\
\widetilde{Q}_{Ak} & \ni & \widetilde{q}_{Ak} & \widetilde{\psi}_{Ak}\ .
\end{array}
$$

One also introduces Fayet-Iliopoulos terms:

$$\mathcal{W}_A \quad = \quad - \frac{N}{2\sqrt{2}} \xi \mathcal{A}^{\mathrm{U}(1)} \,, \qquad \text{– F-term}$$

$$\xi \quad = \quad \xi_1 \; - \; i\xi_2 \,,$$

or alternatively (and the only one for $\mathcal{N}=1$)

$$\mathcal{L} \quad \supset \quad - \frac{N}{2} \int d^4\theta \;\xi_3 V^{\mathrm{U}(1)} \,, \qquad \text{– D-term.}$$

ξ_1, ξ_2 and ξ_3 form an $SU_R(2)$ triplet of “generalized FI parameters”, and are completely equivalent for $\mathcal{N}=2$ supersymmetry

they trigger condensation of quarks

condensation of quarks leads to formation of flux tubes

The potential of this theory is

$$
\begin{aligned}
V(q^A, \widetilde{q}_A, a^a, a) \quad = \quad & \frac{g_2^2}{2}\left(\frac{1}{g_2^2} f^{abc}\overline{a}^b a^c \;+\; \overline{q}_A T^a q^A - \widetilde{q}_A T^a \overline{\widetilde{q}}^A\right)^2 \\
+ \quad & \frac{g_1^2}{8}\left(\overline{q}_A q^A \;-\; \widetilde{q}_A \overline{\widetilde{q}}^A \;-\; N\,\xi_3\right)^2 \\
+ \quad & 2\,g_2^2 \left|\widetilde{q}_A T^a q^A\right|^2 \;+\; \frac{g_1^2}{2}\left|\widetilde{q}_A q^A \;-\; \frac{N}{2}\,\xi\right|^2 \\
+ \quad & \frac{1}{2}\sum_{A=1}^{N}\Bigg\{\left|\left(a \;+\; \sqrt{2}m_A \;+\; 2T^a a^a\right) q^A\right|^2 \\
& \qquad + \quad \left|\left(a \;+\; \sqrt{2}m_A \;+\; 2T^a a^a\right) \overline{\widetilde{q}}^A\right|^2\Bigg\}\,.
\end{aligned}
$$

The potential causes condensation of quarks and of the adjoint matter

$$\frac{1}{2}a \quad + \quad T^a a^a \quad = \quad \begin{pmatrix} 0 & \dots & 0 \\ \dots & \dots & \dots \\ 0 & \dots & 0 \end{pmatrix}$$

For quarks one can choose the Colour-Flavour Locked form

$$\left\langle q^{kA} \right\rangle \quad = \quad \left\langle \overline{\widetilde{q}}^{kA} \right\rangle \quad = \quad \sqrt{\frac{\xi}{2}} \begin{pmatrix} 1 & \dots & 0 \\ \dots & \dots & \dots \\ 0 & \dots & 1 \end{pmatrix}$$

This way the diagonal symmetry is unbroken

$$U_C(N) \times SU_F(N) \quad \rightarrow \quad SU_{C+F}(N) \ .$$

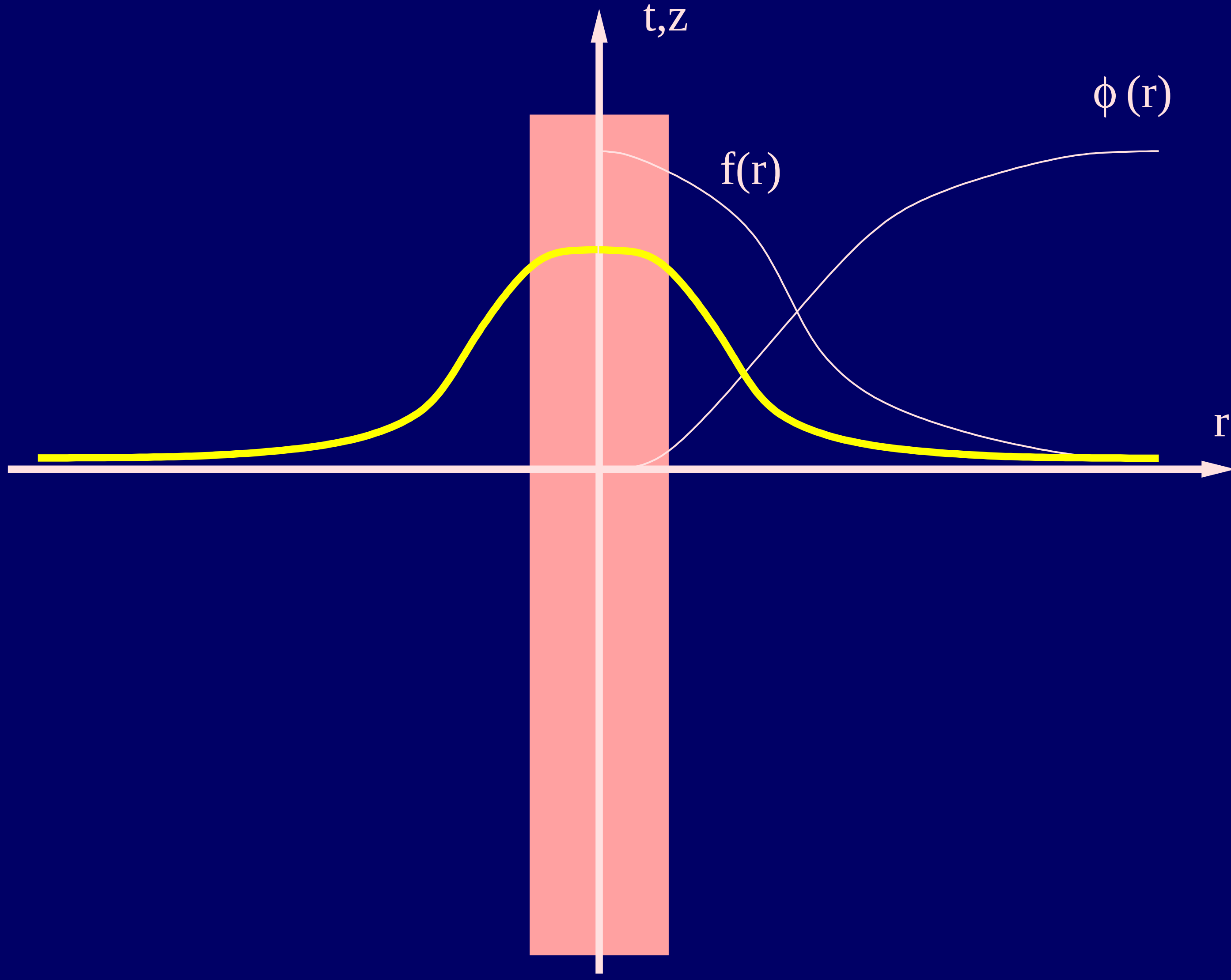
The gauge bosons acquire a mass

$$M_{\mathrm{SU(N)}} \quad = \quad g_2 \sqrt{\xi}$$

and

$$M_{\mathrm{U(1)}} \quad = \quad g_1 \sqrt{\frac{N}{2} \xi}$$

The Z_N string solution



The Z_N string solution

$$\varphi = \begin{pmatrix} \phi_2(r) & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & \phi_2(r) & 0 \\ 0 & 0 & \dots & e^{i\alpha}\phi_1(r) \end{pmatrix}$$

$$A_i^{\mathrm{SU(N)}} = \frac{1}{N} \begin{pmatrix} 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & 1 & 0 \\ 0 & 0 & \dots & -(N-1) \end{pmatrix} \times$$

$$\times\, (\partial_i \alpha) \left(-1 \, + \, f_{NA}(r) \right)$$

$$A_i^{\mathrm{U(1)}} = \frac{1}{N} (\partial_i \alpha) \left(1 \, - \, f(r) \right)$$

The string solution breaks

$$\frac{SU(N)}{SU(N-1)\times U(1)} \sim CP(N-1)\ .$$

The string possesses orientational moduli n^l, in terms of which the bosonic part of the CP(N-1) action reads

$$S^{1+1} = \frac{4\pi}{g_2^2}\int dt\, dz\, \left(|\partial n|^2 + (\overline{n}\,\partial_k\, n)^2 \right)\ .$$

Later we will introduce $\mathcal{N}=2 \to \mathcal{N}=1$ supersymmetry breaking, in order to get closer to the real world, via a deformation

$$\mathcal{W}_{3+1} \;=\; \sqrt{\frac{N}{2}}\,\frac{\mu}{2}\,\mathcal{A}^2 \;+\; \frac{\mu}{2}\,(\mathcal{A}^a)^2$$

and via introduction of a meson-like “mass” field M

$$\begin{aligned}\mathcal{W}_M &= QM\widetilde{Q}\ ,\\ \mathcal{L}_M &= \frac{1}{h}\left|\partial_\mu M^0\right|^2 + \frac{1}{h}\left|\partial_\mu M^a\right|^2\ .\end{aligned}$$

These modifications, change the CP(N-1) model.

Our goal will be to find out the changes of the CP(N-1) model.

The hierarchy of scales is taken as follows:

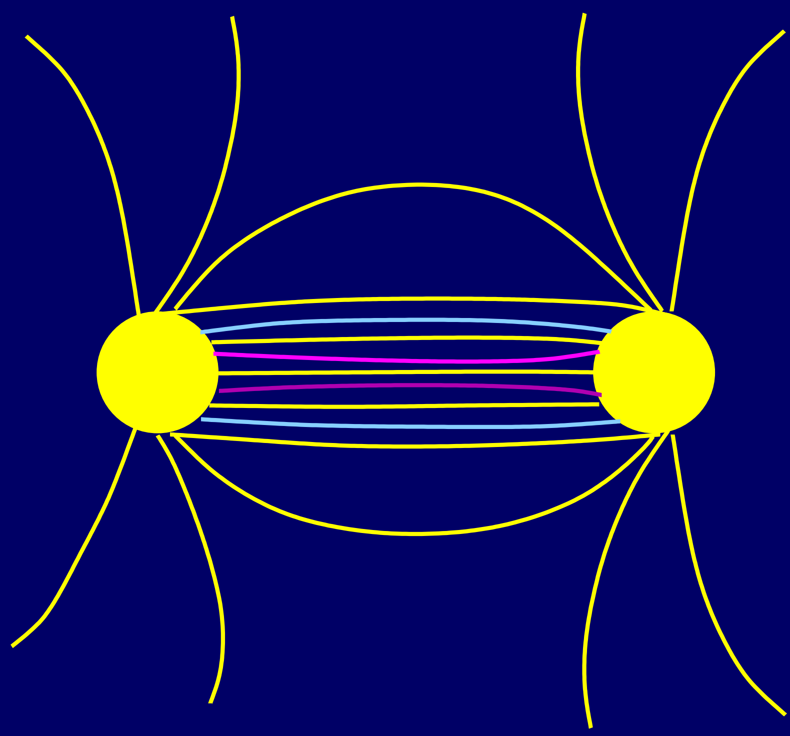
$$\Lambda_{CP(N-1)} \sim \Lambda_{SU(N)} \quad \ll \quad \sqrt{\xi} \quad \ll \quad |\Delta m|$$

The gauge coupling is frozen below the squark condensate scale $\sqrt{\xi}$ and therefore below this scale one is at weak coupling

When $|\Delta m|$ becomes $\sim \sqrt{\xi}$, flux tubes form, whose tension is smaller than the mass of the monopole:

$$M_M \sim \frac{1}{g_2^2}|\Delta m| \gg \sqrt{T} \sim \sqrt{\xi}\ .$$

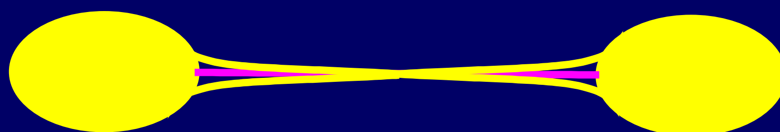
This corresponds to weakly confined monopoles



Further lowering $|\Delta m|$ increases the size of the monopole past the size of the core of the string attached to it — confinement, in quasiclassical regime

As now $|\Delta m|$ is taken to $\Lambda_{CP(N-1)}$ and below to zero, the size of the monopole grows and classically explodes

Classical treatment is unapplicable here, this is a highly quantum regime



However, monopoles are seen as kinks on the worldsheet theory interpolating between vacua which correspond to highly quantum strings

$\mathcal{N} = 1$ SQCD

To break supersymmetry to $\mathcal{N}=1$, we introduce soft mass terms for the adjoint chiral multiplets

$$\mathcal{W}_{3+1} \quad = \quad \sqrt{\frac{N}{2}}\,\frac{\mu}{2}\,\mathcal{A}^2 \quad + \quad \frac{\mu}{2}\,(\mathcal{A}^a)^2$$

One is tempted to take $\mu \to \infty$. The problem arises, however, that $\mathcal{N}=1$ SQCD has a Higgs branch, and, correspondingly, massless states, and an infrared problem develops:

the BPS strings become infinitely thick $\sim \; \mu/\xi$.

CP(N-1) description breaks down

To cure this, an *M model* is introduced

Quarks are given a *dynamical* mass

$$\mathcal{L}_{\rm quark} \quad = \quad \int d^2\theta\, d^2\overline{\theta}\, \frac{2}{h} \mathrm{Tr} \overline{M} M \quad + \quad \int d^2\theta\, Q M \widetilde{Q} \quad + \quad \text{h.c.}$$

One now has two supersymmetry breaking parameters: μ and h.

The meson field M is more natural from the Seiberg's duality standpoint. This field lifts the Higgs branch

This model is *not* the same as Seiberg's theory: gauge group is different, the presence of FI D-term, it etc

The monopoles (kinks) of the CP(N-1) model corresponding to M-model are descendants of the 't Hooft-Polyakov monopoles of $\mathcal{N}=2$.

We track this by starting from undeformed $\mathcal{N}=2$ QCD.

- Coulomb branch:

$$\mu = 0\,, \quad h = 0\,, \quad \xi = 0\,, \quad M \neq 0\,.$$

The M is just then a frozen mass parameter. The adjoint scalar takes the VEV

$$\langle a_l^k \rangle = -\frac{1}{\sqrt{2}}\,\delta_l^k M_l$$

This corresponds to 't Hooft-Polyakov monopoles with masses $|M_A - M_{A+1}|/g_2^2$.

- Now we introduce the FI parameter ξ, bringing the theory into the Higgs phase

$$\mu = 0, \quad h = 0, \quad \xi \neq 0, \quad M \neq 0.$$

Z_N strings are formed, monopoles get confined, semiclassically.

Reducing the masses M_A causes the monopoles to grow, and they get confined by strings, in quantum regime.

$$\Lambda_{CP(N-1)} \ll |M_A| \ll \sqrt{\xi}\,.$$

The strings are seen as neighbouring vacua of the CP(N-1) model.

- Again we bring M_A below $\Lambda_{CP(N-1)}$ and eventually to zero, keeping $\mathcal{N}=2$ supersymmetry

$$\mu = 0\,, \quad h = 0\,, \quad \xi \neq 0\,, \quad M \neq 0\,.$$

The classical monopole size blows up.

The monopoles come into the highly quantum regime and become truly non-Abelian. They do not carry average magnetic flux

$$\langle n^l \rangle = 0.$$

- Now we introduce the $\mathcal{N}=2$ breaking parameters

$$\mu \neq 0\,, \quad h \neq 0\,, \quad \xi \neq 0\,, \quad M = 0\,.$$

In fact, $M=0$ is the vacuum.

The effective worldsheet theory is still given by CP(N-1) model, which has N vacua, being interpreted as N elementary non-Abelian strings.

The kinks of the worldsheet model are interpreted as monopoles or string junctions.

The monopoles are not seen in the semiclassical approximation, as

$$\langle a_l^k \rangle = 0.$$

- Finally, we pass to pure $\mathcal{N}=1$ theory by eliminating the adjoint matter

$$\mu \;\to\; \infty\,, \quad h \;\neq\; 0\,, \quad \xi \;\neq\; 0\,, \quad M \;=\; 0\,.$$

We end up with $\mathcal{N}=1$ SQCD supplemented with the meson M-field.

Again, although the monopoles are not seen in the microscopic theory, their existence is explified in the CP(N-1) theory as kinks.

The monopoles become genuinely non-abelian $\langle n^l \rangle = 0$ and carry global flavour numbers as they are in the fundamental representation of global $SU(N)_{C+F}$.

Worldsheet CP(N-1) Theory

We will mostly discuss $\mu \neq 0$ theory, with $M = 0$.

The draw back will be the presence of the Higgs branch, which will be seen via the long-range tails of the fermionic (and correspondingly, quark) zero-modes.

We expect that the modification of the theory via

$$\hat{\mathcal{W}}_{3+1} \quad = \quad \sqrt{\frac{N}{2}}\,\frac{\mu}{2}\,\mathcal{A}^2 \quad + \quad \frac{\mu}{2}\,(\mathcal{A}^a)^2$$

introduces similar corrections into the CP(N-1) sigma model.

It is anticipated that

$$\hat{\mathcal{W}}_{1+1} \quad = \quad \frac{1}{2}\,\delta\,\Sigma^2$$

The relation between δ and μ we need to find out

The Worldsheet theory can be derived from the microscopic $SU(N) \times U(1)$ theory.

Because half of the supersymmetry is broken, one expects the symmetry of the worldsheet theory to be $\mathcal{N} = (0,2)$.

However, a well-known fact, that the only supersymmetry that CP(N-1) can possess is $\mathcal{N} = (2,2)$.

It was later found out that in fact the worldsheet dynamics is described by $CP(N-1) \times C$ model rather than $CP(N-1)$, [Edalati&Tong].

The former possesses $\mathcal{N} = (0,2)$ supersymmetry.

The string solution in the $\mathcal{N}=1$ case is the same as in the $\mathcal{N}=2$ theory: the bosonic part is not modified

$$q^{kA} \quad = \quad \bar{q}_{Ak} \quad = \quad \varphi(r)$$

with the winding solution

$$\varphi \quad = \quad U \begin{pmatrix} \phi_2(r) & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & \phi_2(r) & 0 \\ 0 & 0 & \dots & \phi_1(r) \end{pmatrix} U^{-1}$$

We have passed to a singular gauge

In the singular gauge, it is the gauge field that winds, now around the origin

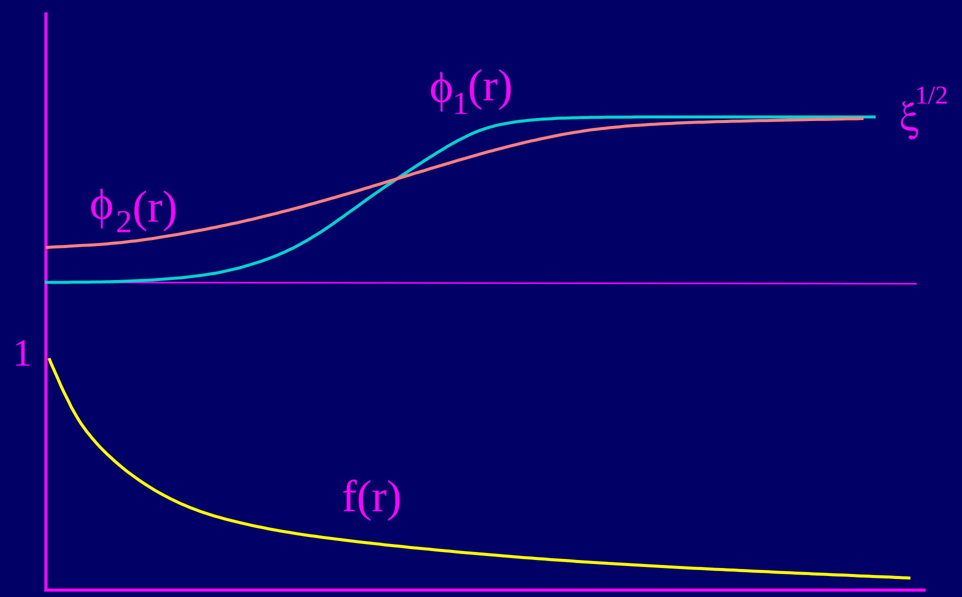
$$A_i^{\mathrm{SU(N)}} \quad = \quad \frac{1}{N}\, U \begin{pmatrix} 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & 1 & 0 \\ 0 & 0 & \dots & -(N-1) \end{pmatrix} U^{-1} \times$$

$$\times\, (\partial_i \alpha)\, f_{NA}(r)$$

$$A_i^{\mathrm{U(1)}} \quad = \quad -\frac{1}{N}\, (\partial_i \alpha)\, f(r)$$

The rotation matrix U provides the *orientation* of the string in the $SU(N)$ space

The boundary conditions for the profile functions are



$$
\begin{aligned}
\phi_1(0) &= 0, \qquad & \phi_2(0) &\neq 0 \\
f_{NA}(0) &= 1, \qquad & f(0) &= 1
\end{aligned}
$$

and

$$
\begin{aligned}
\phi_1(\infty) &= \sqrt{\xi}, \qquad & \phi_2(\infty) &= \sqrt{\xi} \\
f_{NA}(\infty) &= 0 \qquad & f(\infty) &= 0.
\end{aligned}
$$

The profiles satisfy the 1st order BPS equations

$$
\begin{aligned}
\partial_r \, \phi_1(r) &= \frac{1}{Nr} \left(f(r) \; + \; (N-1) f_{NA}(r) \right) \phi_1(r) \\
\partial_r \, \phi_2(r) &= \frac{1}{Nr} \left(f(r) \; - \; f_{NA}(r) \right) \phi_2(r) \\
\partial_r \, f(r) &= r \, \frac{N g_1^2}{4} \left((N-1) \phi_2(r)^2 \; + \; \phi_1(r)^2 \; - \; N\xi \right) \\
\partial_r \, f_{NA}(r) &= r \, \frac{g_2^2}{2} \left(\phi_1(r)^2 \; - \; \phi_2(r)^2 \right) .
\end{aligned}
$$

The tension of the string is

$$
T = 2\pi \, \xi .
$$

The string orientation U can be unambigiously parametrized by the modulus $n^l \in C$:

$$\frac{1}{N} U \begin{pmatrix} 1 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots & 1 & 0 \\ 0 & 0 & \ldots & -(N-1) \end{pmatrix} U^{-1} \;=\; -\, n^i \, \overline{n}_l \;+\; \frac{1}{N} \cdot \mathbf{1}^i{}_l$$

with a condition

$$\overline{n}_l \cdot n^l \;=\; 1.$$

Thus n^l are orientational collective coordinates

This defines $2(N-1)$ degrees of freedom, since CP(N-1) theory can be obtained from a gauge theory, and one phase can be removed

To find the effective worldsheet model, one needs to substitute the bosonic string solution q^{kA}, $A_i^{\mathrm{U}(1)}$, *etc*, depending on the orientational coordinates n^l into the original action, to calculate the so-called overlap of the zero-modes

In a supersymmetric model, one also needs to substitute the fermionic zero-modes which depend both on orientational and on fermionic orientational coordinates

The fermionic zero-modes can be obtained by applying $\mathcal{N}=2$ supersymmetry transformations to the bosonic solution q^{kA}. For example, the superorientational modes are

$$
\begin{aligned}
\bar{\psi}_{\dot{2}Ak} &= \frac{\phi_1^2 - \phi_2^2}{\phi_2} \cdot n\bar{\xi}_L \\
\bar{\tilde{\psi}}^{kA}_{\dot{1}} &= -\frac{\phi_1^2 - \phi_2^2}{\phi_2} \cdot \xi_R \bar{n} \\
\lambda^{11\ SU(N)} &= i\sqrt{2}\,\frac{x^1 - i\,x^2}{r^2}\frac{\phi_1}{\phi_2} f_N \cdot n\bar{\xi}_L \\
\lambda^{22\ SU(N)} &= -\,i\sqrt{2}\,\frac{x^1 + i\,x^2}{r^2}\frac{\phi_1}{\phi_2} f_N \cdot \xi_R \bar{n}
\end{aligned}
$$

where ξ_R, ξ_L are the superorientational collective coordinates

In the $\mathcal{N}=1$ theory, half of supersymmetry is lost, and with this approach one can only recover half of the zero-modes – the left-handed modes.

In order to obtain the right-handed modes on has to solve the Dirac equations

$$4\,\frac{i}{g_1^2}\left(\overline{\not{\partial}}\lambda^f_{\mathrm{U(1)}}\right)+i\sqrt{2}\mathrm{Tr}\,\left(\overline{\psi}q^f\ +\ \overline{q}^f\overline{\widetilde{\psi}}\right)-4{\delta_2}^f\sqrt{\frac{N}{2}}\mu\,\overline{\lambda}_2^{\mathrm{U(1)}}\ =\ 0$$

$$\frac{i}{g_2^2}\left(\overline{\not{\mathcal{D}}}\lambda^f_{\mathrm{SU(N)}}\right)\ +\ \frac{i}{\sqrt{2}}\left(\overline{\psi}T^aq^f\ +\ \overline{q}^fT^a\overline{\widetilde{\psi}}\right)T^a\ -$$
$$-\ {\delta_2}^f\,\mu\,\overline{\lambda}_2^{\mathrm{SU(N)}}\ =\ 0$$

$$\cdots$$

$$\cdots$$

The equations are not solveable exactly, and one can resolve them in a limit of small or large μ.

It turns out that the right-handed zero-modes have $1/r$-tails

$$\overline{\widetilde{\psi}}_{\rm i} \quad \simeq \quad \frac{\phi_2}{r}\, n\overline{n}\, \zeta_R \quad \sim \quad \frac{\sqrt{\xi}}{r}\, n\overline{n}\, \zeta_R$$

These long-range tails reflect the presence of the Higgs branch.

One expects to be able to substitute these zero-modes into the microscopic action in order to obtain the effective sigma model

It turns out that CP(N-1) does not admit $\mathcal{N} = (0,2)$ generalization.

However, the model we are deriving is not exactly CP(N-1)

$$
\begin{aligned}
\mathcal{L}_{CP(N-1)} \;=\;& |\partial_k n|^2 \;+\; (\overline{n}\partial_k n)^2 \;+\; \overline{\xi}_L\, i\partial_R\, \xi_L \;+\; \overline{\xi}_R\, i\partial_L\, \xi_R \\
&-\; i\,(\overline{n}\partial_R n)\,\overline{\xi}_L\xi_L \;-\; i\,(\overline{n}\partial_L n)\,\overline{\xi}_R\xi_R \\
&+\; \overline{\xi}_L\xi_R\overline{\xi}_R\xi_L \;-\; \overline{\xi}_R\xi_R\overline{\xi}_L\xi_L
\end{aligned}
$$

$$
\overline{n}_l n^l \;=\; 1, \qquad \overline{n}_l \xi^l \;=\; \overline{\xi}_l n^l \;=\; 0,
$$

$$
\partial_R \;=\; \partial_0 \;+\; i\partial_1, \qquad\qquad \partial_L \;=\; \partial_0 \;-\; i\partial_1
$$

Our theory possesses supertranslational modes ζ_R which mix with the orientational modes

Therefore, our model is extended with ζ_R, and it was argued by Edalati and Tong that it has to be $CP(N-1) \times C$ sigma model

Based on $\mathcal{N} = (0,2)$ superfield formalism, they were able to build the $\mathcal{N} = (0,2)$ $CP(N-1) \times C$ sigma model

The major ingredient in the (modification of the) CP(N-1) model is the quadratic superpotential

$$\hat{\mathcal{W}}_{1+1} \;\; = \;\; \frac{1}{2}\,\delta\,\Sigma^2$$

which reflects the presence of $\mu\mathcal{A}^2$ in the microscopic theory. Also the right-handed constraints are now modified

$$\bar{\xi}_R \cdot n \;\; \neq \;\; 0, \qquad\qquad \bar{n} \cdot \xi_R \;\; \neq \;\; 0$$

The latter constraints can be restored via a shift

$$
\begin{aligned}
\bar{\xi}_R &\to \bar{\xi}_R - \frac{m_W}{\sqrt{2}}\,\delta\,\zeta_R \bar{n} \\
\xi_R &\to \xi_R - \frac{m_W}{\sqrt{2}}\,\bar{\delta}\,\bar{\zeta}_R n
\end{aligned}
$$

This, together with the quadratic superpotential

$$
\hat{\mathcal{W}}_{1+1} = \frac{1}{2}\,\delta\,\Sigma^2
$$

leads to mixing of supertranslational and superorientational modes ζ_R and ξ_R.

In terms of components, upon the elimination of all the auxiliary fields the $\mathcal{N} = (0, 2)$ takes the form

$$
\begin{aligned}
\mathcal{L}^{(0,2)}_{1+1} \;=\; & \bar{\zeta}_R \, i\partial_L \, \zeta_R \;+\; \ldots \;+ \\
& + \; |\partial n|^2 \;+\; (\bar{n}\partial_k n)^2 \;+\; \bar{\xi}_R \, i\partial_L \, \xi_R \;+\; \bar{\xi}_L \, i\partial_R \, \xi_L \\
& - \; i \, (\bar{n}\partial_L n) \, \bar{\xi}_R \xi_R \;-\; i \, (\bar{n}\partial_R n) \, \bar{\xi}_L \xi_L \\
& - \; \gamma \, (i\partial_L \bar{n}) \xi_R \zeta_R \;-\; \bar{\gamma} \, \bar{\xi}_R (i\partial_L n) \bar{\zeta}_R \;+\; |\gamma|^2 \, \bar{\xi}_L \xi_L \bar{\zeta}_R \zeta_R \\
& + \; \left(1 \,-\, |\gamma|^2\right) \, \bar{\xi}_L \xi_R \bar{\xi}_R \xi_L \;-\; \bar{\xi}_L \xi_L \bar{\xi}_R \xi_R
\end{aligned}
$$

where

$$
\gamma \;=\; \frac{\sqrt{2}\,\delta}{\sqrt{1 + 2|\delta|^2}} \, .
$$

Our goal is to find γ (or δ) in terms of the microscopic parameter μ

We calculate the bifermionic mixing term

$$- \gamma \, (i\partial_L \overline{n})\xi_R \zeta_R$$

from the microscopic theory

The result is

$$\delta \quad = \quad \text{const} \cdot \sqrt{\ln \frac{g_2^2 \mu}{m_W}} \,, \qquad\qquad \text{as } \mu \ \to \ \infty \ .$$

and

$$\delta \quad = \quad \text{const} \cdot \frac{g^2 \mu}{m_W} \qquad\qquad \text{small } \mu.$$

We do a similar calculation in the M-model

$$\mathcal{L}_{\rm quark} \;=\; \int d^2\theta\, d^2\overline{\theta}\; \frac{2}{h}\,{\rm Tr}\,\overline{M}M \;+\; \int d^2\theta\; QM\widetilde{Q}$$

Once there is finite h, the Higgs branch is lifted and μ can be taken to infinity

In this case one needs the zero-modes of the fermionic M and quark fields

By solving Dirac equations (in the small h limit), and calculating the overlap

$$\delta \;=\; \text{const} \cdot \sqrt{\ln\, g_2^2/h}$$

Here h needs to be small, but finite, no need to take it to zero

Conclusion

- Supersymmetric QCD theories with $\mathcal{N} > 1$ can be brought into the confinement regime for the monopoles
- $SU(N) \times U(1)$ with N flavours
- Confinement is essentially non-abelian (non-abelian strings)
- Monopoles can be studied as kinks of the 2-dimensional worldsheet model: in particular the masses match

- Attempts to break supersymmetry to $\mathcal{N}=1$ lead to massless modes on the Higgs branch
- One can introduce a meson field which gives quark a dynamical mass and lift the Higgs branch
- The worldsheet dynamics of the non-abelian strings is described by $C \times CP(N-1)$

- Considering it as a modification of the $CP(N-1)$ model, the modification parameter of the 2-d superpotential is given as

$$\delta \quad = \quad \mathrm{const} \cdot \sqrt{\ln \frac{g_2^2 \mu}{m_W}} \; , \qquad\qquad \text{as } \mu \;\to\; \infty$$

in softly broken $\mathcal{N}=2 \;\to\; \mathcal{N}=1$ SQCD, and

$$\delta \quad = \quad \text{const} \cdot \sqrt{\ln g_2^2/h} \; , \qquad\qquad \text{as } h \;\to\; 0$$

in the M-model

- The next step is to introduce masses to the quark fields (Δm_{AB})
- The massive heterotic worldsheet model is already being considered
- Another direction of extension is to consider $N_F > N_c$, where the strings become semi-local, and size moduli appear on the string worldsheet (simultaneously an extension of the Non-Abelan duality to the $\mathcal{N} = 1$ case)

Work done in collaboration with Alexei Yung and Mikhail Shifman, FTPI, Minnesota