Correlations of the local density of states in quasi-one-dimensional wires

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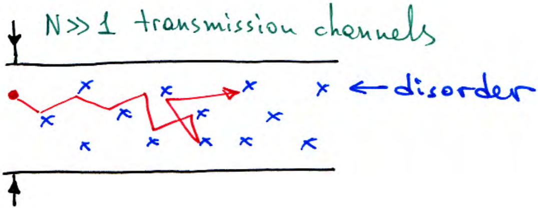
Object of study

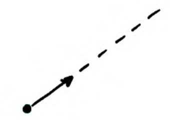
Localization in quasi-one-dimensional wires



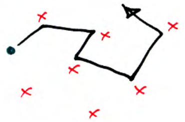
(single-particle quantum mechanics in a disordered potential)

Anderson localization

1. Free particle

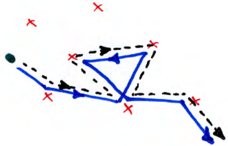


2. Classical diffusion:
$L^2 \propto t$



3. Quantum interference:
$$|A_1 + A_2|^2 = |A_1|^2 + |A_2|^2 + 2\mathrm{Re}\, A_1^* A_2$$

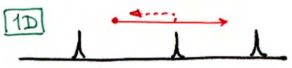
4. Localization corrections:



In 1 and 2 dimensions, interference effects suppress the diffusion completely at arbitrary strength of disorder: the particle stays in a finite region of space (localization)

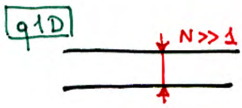
Localization in one dimension: 1D vs q1D

Particle on a line (1D):



$$\xi \sim l$$

Thick wire (q1D):



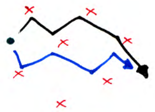
$$\xi \sim Nl$$

ξ — localization length
l — mean free path

Rescaled to the localization length ξ, localization looks similar in the two models.
Which properties are universal?

Quantitative description of localization

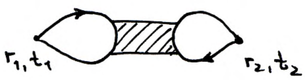
Localization is not visible in the average of a single Green's function:



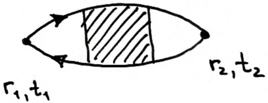
$\langle G(r) \rangle$ decays at the length scale of the mean free path

Averaging two Green's functions (TWO types of averages):

1. $\langle G(1,1)G(2,2) \rangle$
(correlations of DOS)

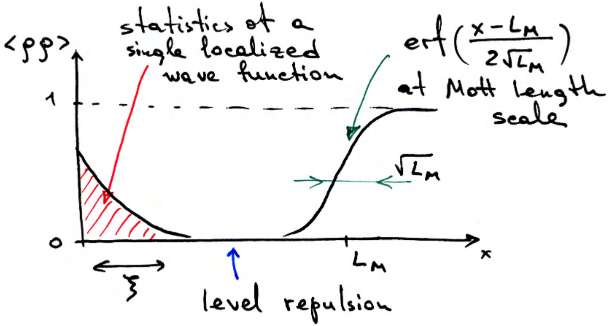


2. $\langle G(1,2)G(2,1) \rangle$
(response function)



Exact results in one dimension

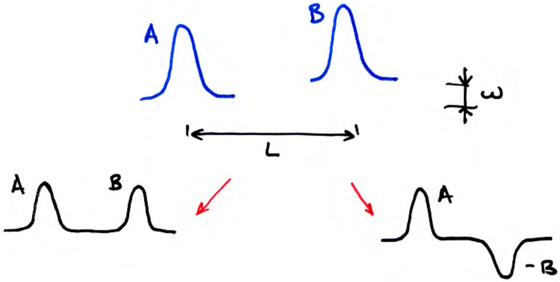
$\langle \rho_E(r_1)\rho_{E+\omega}(r_2)\rangle$ with $\omega \ll \Delta_\xi$
(Δ_ξ: level spacing over the localization length ξ)
[Gor'kov, Dorokhov, Prigara, '83] $\Leftarrow$ **strictly 1D model**



compare with q1D

$L_M \sim \log(\Delta_\xi/\omega)$ — Mott length scale

Mott argument (wave function hybridization)



1. For short distances ($x \lesssim \xi$), the two eigenfunctions have the same profile (single localized wave function)

2. Hybridization is important as long as the splitting
$\Delta_\xi \exp(-L/2\xi) > \omega \quad \Leftrightarrow \quad L < L_M$

$\Rightarrow$ Mott length $L_M = 2\xi \ln(\Delta_\xi/\omega)$

Exact approach in q1D (sigma model)

Averaging over disorder $\Rightarrow$ Nonlinear supersymmetric sigma model [Efetov, '83]

For simplicity, we consider the unitary symmetry class (time-reversal symmetry completely broken: e.g., by a magnetic field).

$$Z = \int [DQ]\, e^{-S}\,, \qquad S = -\frac{1}{4}\,\mathrm{STr}\int dx \left[\frac{1}{2}\left(\frac{dQ}{dx}\right)^2 + i\omega\Lambda Q\right]$$

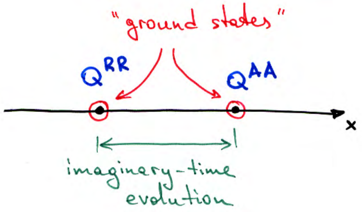
x and ω in the units of ξ and Δ_ξ, respectively
Q is a 4×4 supermatrix with constraint $Q^2 = 1$ (from $N \gg 1$),
fermion-boson (FB) and retarded-advanced (RA) sectors

$$\Lambda = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}_{RA}$$

Transfer-matrix formalism

Relation to the correlations of the density of states:

$$R(\omega, x) \equiv \nu^{-2} \langle \rho_E(0) \rho_{E+\omega}(x) \rangle = \frac{1}{2} \left[1 - \mathrm{Re} \langle Q^{RR}_{BB}(0) Q^{AA}_{BB}(x) \rangle \right]$$



$$R(\omega, x) = 1 + \frac{1}{2} \mathrm{Re} \langle \Psi_0 | e^{-Hx} | \Psi_0 \rangle$$

$\Psi_0(\lambda_B, \lambda_F)$ – known ground state (in terms of Bessel functions) [Skvortsov, Ostrovsky '06, D.I., Skvortsov, '08]

Separation of variables

Luckily, the variables in the Hamiltonian separate

$$\lambda_B \in [1, +\infty) \,, \qquad \lambda_F \in [-1, 1]$$

$$H = H_B + H_F$$

$$H_B = -\partial_{\lambda_B}(\lambda_B^2 - 1)\partial_{\lambda_B} + \Omega\lambda_B$$
$$H_F = -\partial_{\lambda_F}(1 - \lambda_F^2)\partial_{\lambda_F} - \Omega\lambda_F$$

where $\Omega = -i\omega/2$.

Fermionic part: compact, can be solved perturbatively in ω.

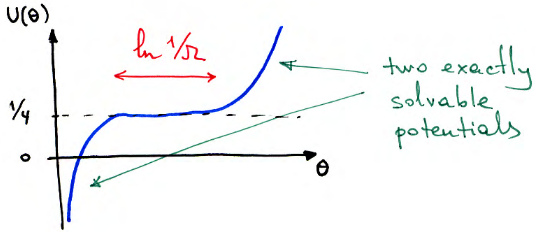
Bosonic part: non-compact, expansion contains both powers and logarithms of ω.

For calculation, we assume Ω real positive, then analytically continue.

Bosonic sector: matching Legendre and Bessel asymptotics

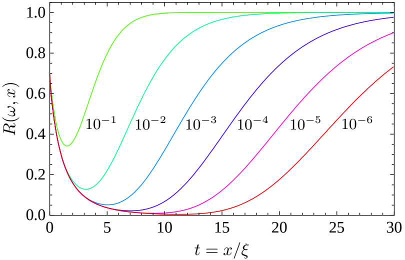
If one "unfolds" the λ_B axis ($\lambda_B = \cosh\theta$)

$$H_\theta = -\frac{d^2}{d\theta^2} + U(\theta)\,, \qquad U(\theta) = \frac{1}{4} - \frac{1}{4\sinh^2\theta} + \Omega\cosh\theta$$



Eigenstates may be constructed order by order in Ω by matching the asymptotics of Legendre (at small θ) and modified Bessel (at large θ) functions (technical part)

Results



The leading asymptotics is the same as in 1D: single-wave-function correlations at small x and $\mathrm{erf}\left(\frac{x-L_M}{2\sqrt{L_M}}\right)$ at large x

Subleading terms in ω are different. At $x \lesssim \xi$

$$R(x,\omega) = R_0 + O(\omega^2 \ln^2 \omega) \qquad \text{in q1D}$$

$$R(x,\omega) = R_0 + O(\omega^2 \ln \omega) \qquad \text{in 1D}$$

Summary and perspectives

- We have obtained a perturbative expansion in ω (including log's) of the correlations of DOS in q1D (in the unitary symmetry class)
- We have confirmed the universal properties of 1D/q1D localization
    - for the single-wave function statistics (known result)
    - at the Mott length scale (new result)

  and studied non-universal corrections in ω

- The method and results will be useful for further studies of localization in 1D/q1D:
    - dynamical response function $\langle G_E^R(0,x) G_{E+\omega}^A(x,0)\rangle$
    - improving the hybridization argument (especially at the Mott length scale)
    - finite number of channels (crossover from $N=1$ to $N=\infty$)
    - other symmetry classes?