# Soliton–antisoliton production in high–energy particle collisions

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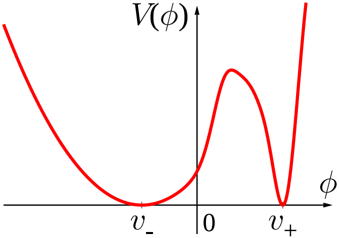
July 4, 2008

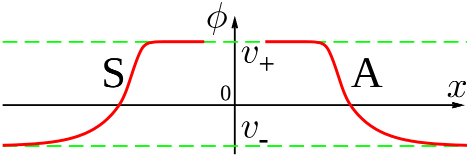
# Topological solitons in weakly coupled theories

$\boxed{\phi}$ $(1+1)$ $\hbar = c = 1$

$$S = \frac{1}{g^2}\int dxdt \left[(\partial_\mu \phi)^2/2 - V(\phi)\right]$$

- semiclassical parameter
- coupling constant ( $\phi = g\phi'$)

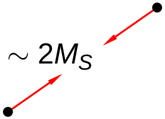


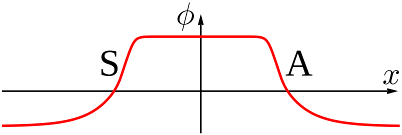




Properties: $l_S \sim m^{-1}$
$M_S \sim m/g^2$

# From particles to solitons

$E \sim 2M_S$






$\lambda \sim 1/E \sim g^2/m$ $\Downarrow$

$l_S \sim 1/m$

Exponential suppression!

$$\boxed{\mathcal{P}(E) \approx A \cdot \mathrm{e}^{-F(E)/g^2}}$$

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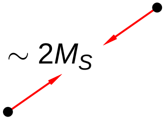

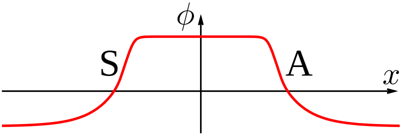


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- Coherent–state “estimate”

  $\bar{n}_S \sim M_S/m \sim 1/g^2$

  $\langle 2|S\rangle \sim \mathrm{e}^{-\bar{n}_S/2} \sim \mathrm{e}^{-\mathrm{c}/g^2}$

*Drukier, Nussinov (1982)*

*Banks at al. (1990)*

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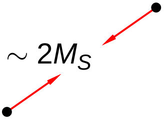

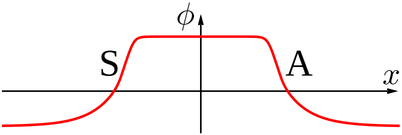


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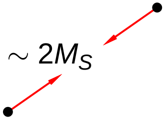

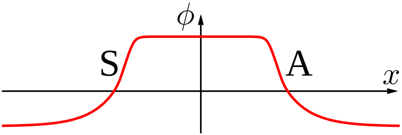


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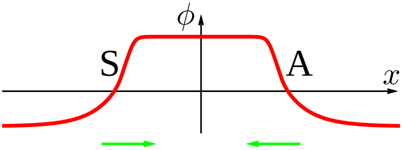
No reliable estimate of $\mathcal{P}(E)$ so far!

Aim: calculate semiclassically $F(E)$.

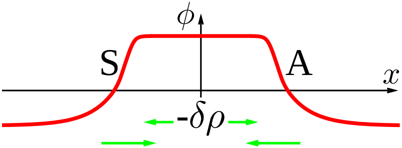
# Introducing potential barrier

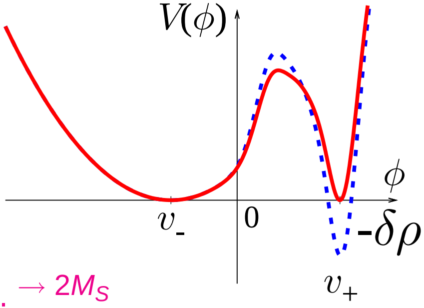



$\Longrightarrow$ Many particles

Attraction!

Not a tunneling process?




Critical bubble $\Leftrightarrow$ Barrier top

$\delta\rho \longrightarrow 0$ : cr. bubble $\longrightarrow$ SA; $E_{\text{cr.b.}} \to 2M_S$

# In–state

*Rubakov, Son, Tinyakov, 1992*

Not semiclassical!

$$E \sim m/g^2$$

RST conjecture: $F(E)$ universal

Does not depend on details of the in–state

$$\mathcal{P}(E,N) = \sum_{i,f} \left| \langle i | \underbrace{\hat{P}_E \hat{P}_N}_{\text{Projectors}} \hat{S} | f \rangle \right|^2$$

- $N \ll 1/g^2 \Rightarrow F(E,N) \approx F(E)$
- $N \gg 1 \Rightarrow$ semiclassical in–states

Checks of universality:

- Field theory

  *Tinyakov, 1991*

  *Mueller, 1992*

- Toy QM models

  *Bonini et al, 1999*

  *Levkov et al, 2009*

$$\boxed{F(E) = \lim_{g^2 N \to 0} F(E,N)}$$

# Semiclassical method

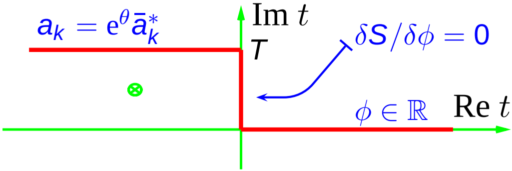
*Rubakov, Son, Tinyakov, 1992*

$$\mathcal{P}(E,N) = \sum_{i,f} \left|\langle i|\hat{P}_E\hat{P}_N\hat{S}|f\rangle\right|^2 = \int d\phi_i d\phi_f \left|\int d\phi_i'[d\phi]\, \mathrm{e}^{i(S+B)}\right|^2$$

$$\langle\phi_i|\hat{P}_E\hat{P}_N|\phi_i'\rangle = \mathrm{e}^{iB(\phi_i,\phi_i')} \qquad \langle\phi_i'|\hat{S}|\phi_f\rangle = \int[d\phi]\,\mathrm{e}^{iS[\phi]}$$

$S \propto 1/g^2 \Rightarrow$ Saddle–point method!

$$\boxed{\phi(x,t) \in \mathbb{C}}$$

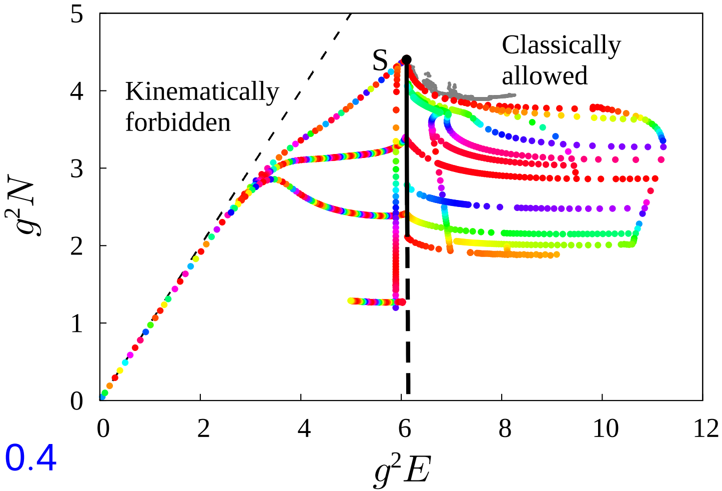



$$\mathcal{P} = A\cdot\mathrm{e}^{-F/g^2}$$

$$F(E,N) = 2g^2\,\mathrm{Im}(S+B)$$

# Numerical solutions

$$V(\phi) = \frac{1}{2}(\phi+1)^2 \left[1 - v \cdot f\left(\frac{\phi-1}{a}\right)\right], \qquad f(x) = \mathrm{e}^{-x^2}\left(1 + x^3 + x^5\right)$$




$\delta\rho = 0.4$

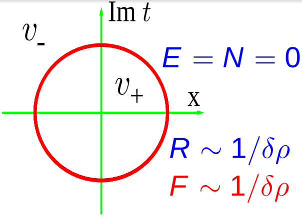
# $\delta\rho \to 0$: thin–wall limit!

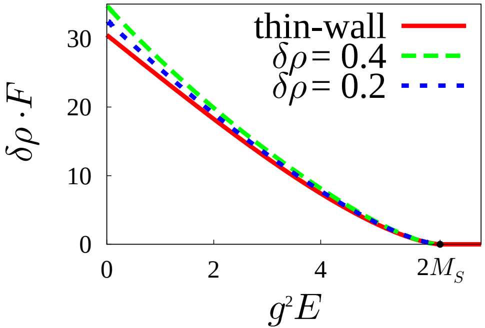
$$F(\delta\rho) = F_{-1}/\delta\rho + F_0 + O(\delta\rho)$$

$$F_{-1}(E, N) = E_S^2 \left( \pi - 2\arcsin\frac{E}{2E_S} - \frac{E}{E_S}\sqrt{1 - \frac{E^2}{4E_S^2}} \right)$$

*Voloshin, Selivanov, 1986*

*Rubakov et al, 1991*



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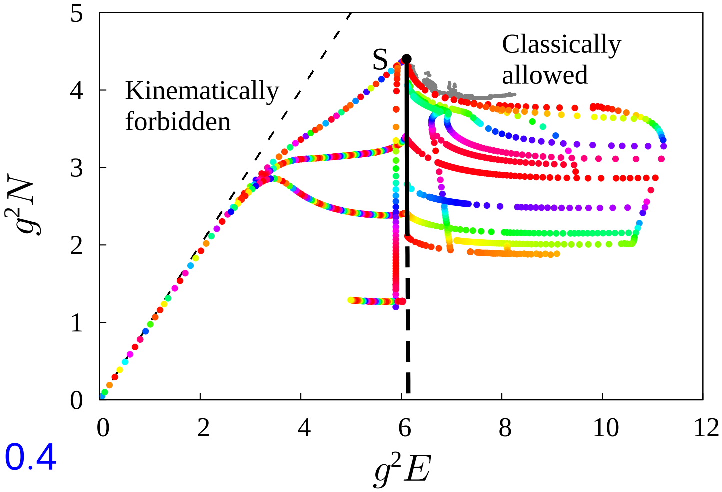
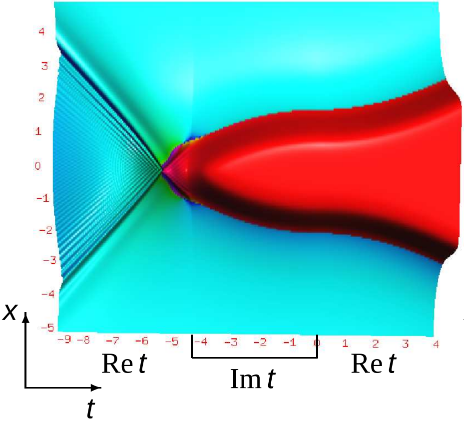


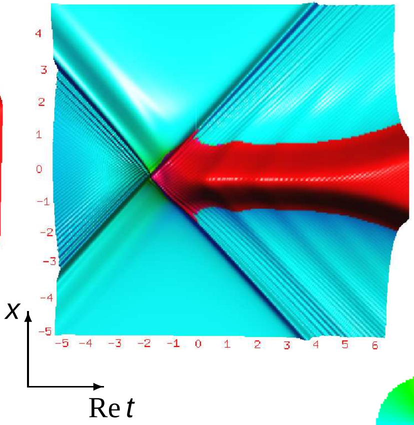

$\delta\rho = 0.4$

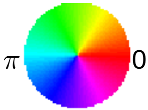
# Going to $E > 2M_S$

$E \approx 5.48$ $(2M_S \approx 6.23)$ $E \approx 9.06$

$N \approx 2.39$, $\delta\rho = 0.4$ $N \approx 2.47$, $\delta\rho = 0.4$



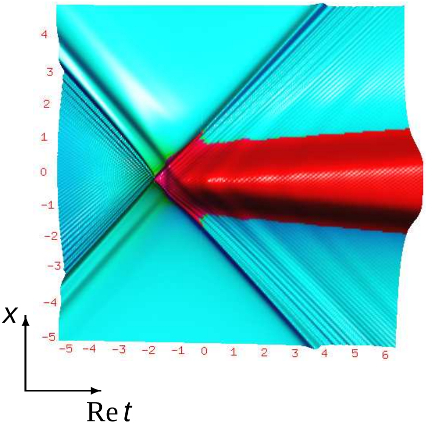



# Limit $\delta\rho \to 0$

$E \approx 8.95$

$N \approx 2.42$, $\delta\rho = 0.02$

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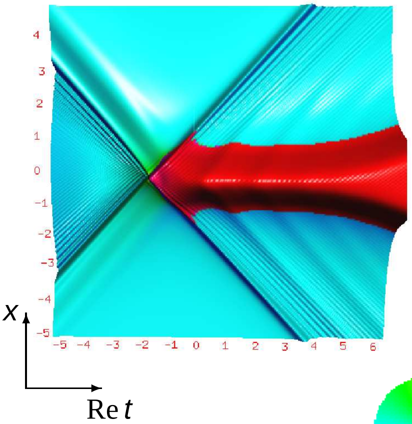
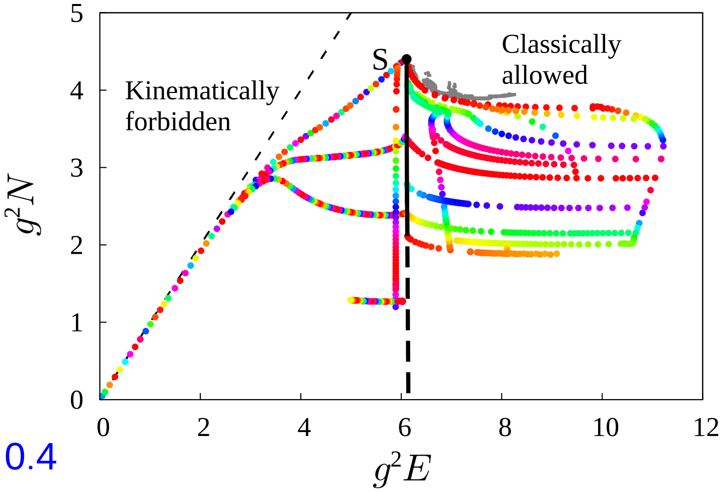
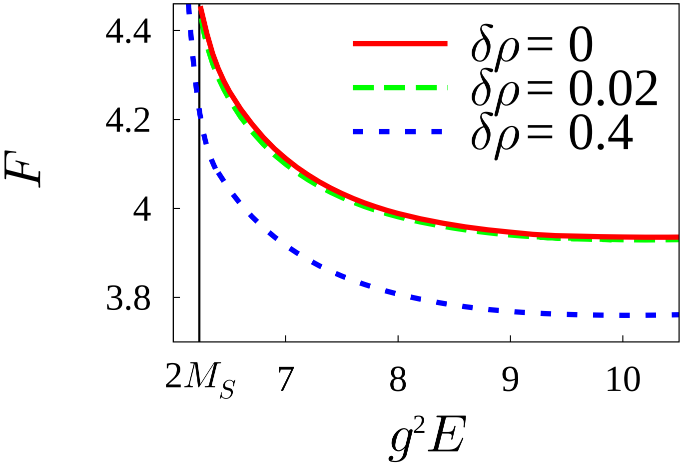



# Extrapolating to $N \to 0$




$\delta\rho = 0.4$

# Result



# Conclusions

- The probability of SA creation in high–energy collisions is

$$\mathcal{P}(E) \approx \mathrm{e}^{-F(E)/g^2}$$

- Method is applicable in $2D$ scalar field models.

Generalizations to other models?