Non-equilibrium Luttinger liquids:
Bosonization and tunneling spectroscopy

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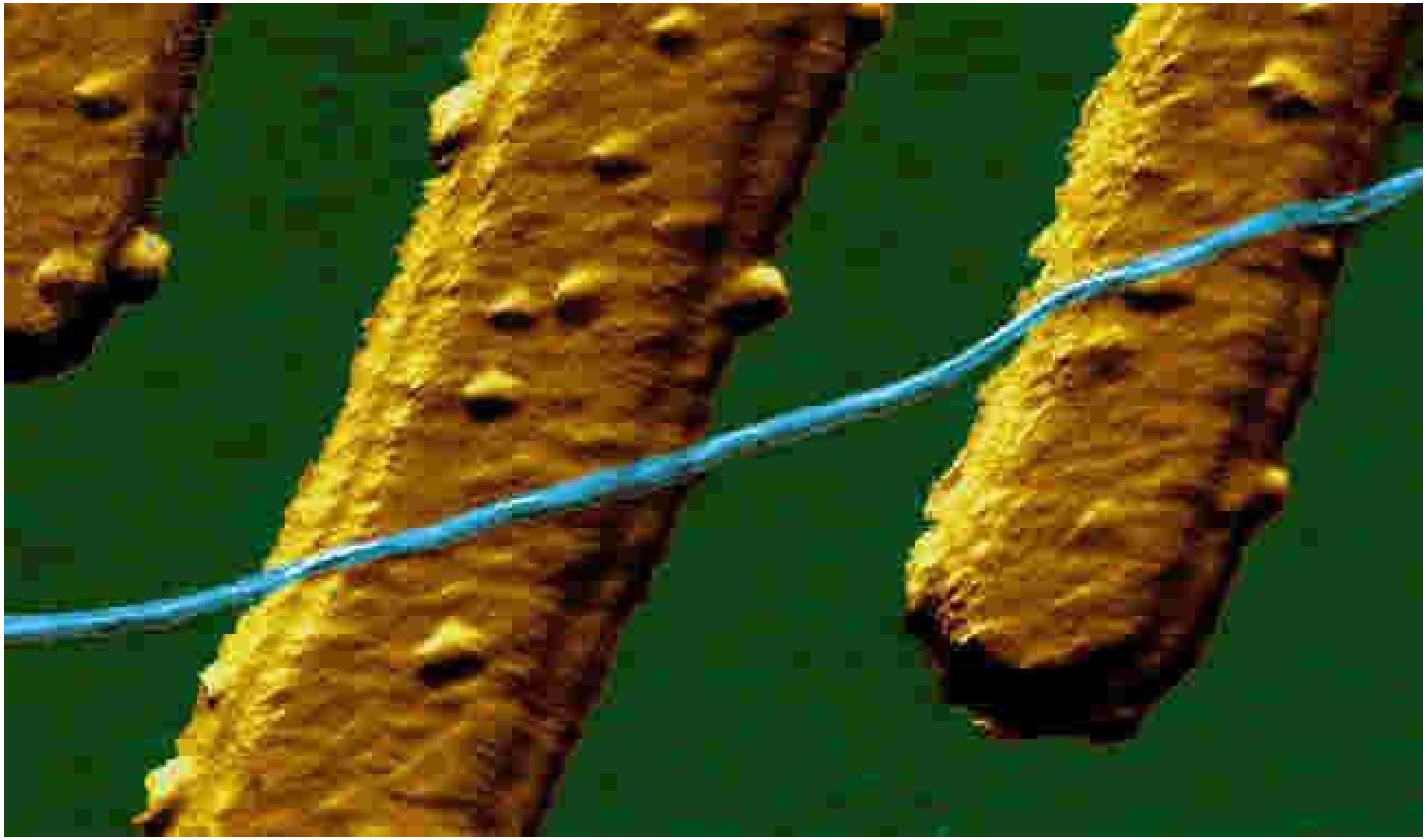
Plan

- **Non-equilibrium Luttinger liquid: Setups**
- **Partial non-equilibrium**
  - **Tunneling DOS, zero-bias anomaly (ZBA), dephasing**
  - **Energy relaxation**
- **Full non-equilibrium**
  - **Free electrons**
  - **Fermi-edge singularity**
  - **Interacting electrons (Luttinger liquid)**

Gutman, Gefen, ADM, Phys. Rev. Lett. 101, 126802 (2008)
Gutman, Gefen, ADM, arXiv:0903.3333, to appear in PRB
Gutman, Gefen, ADM, arXiv:0906.4076, submitted to PRL
Gutman, Gefen, ADM, in preparation

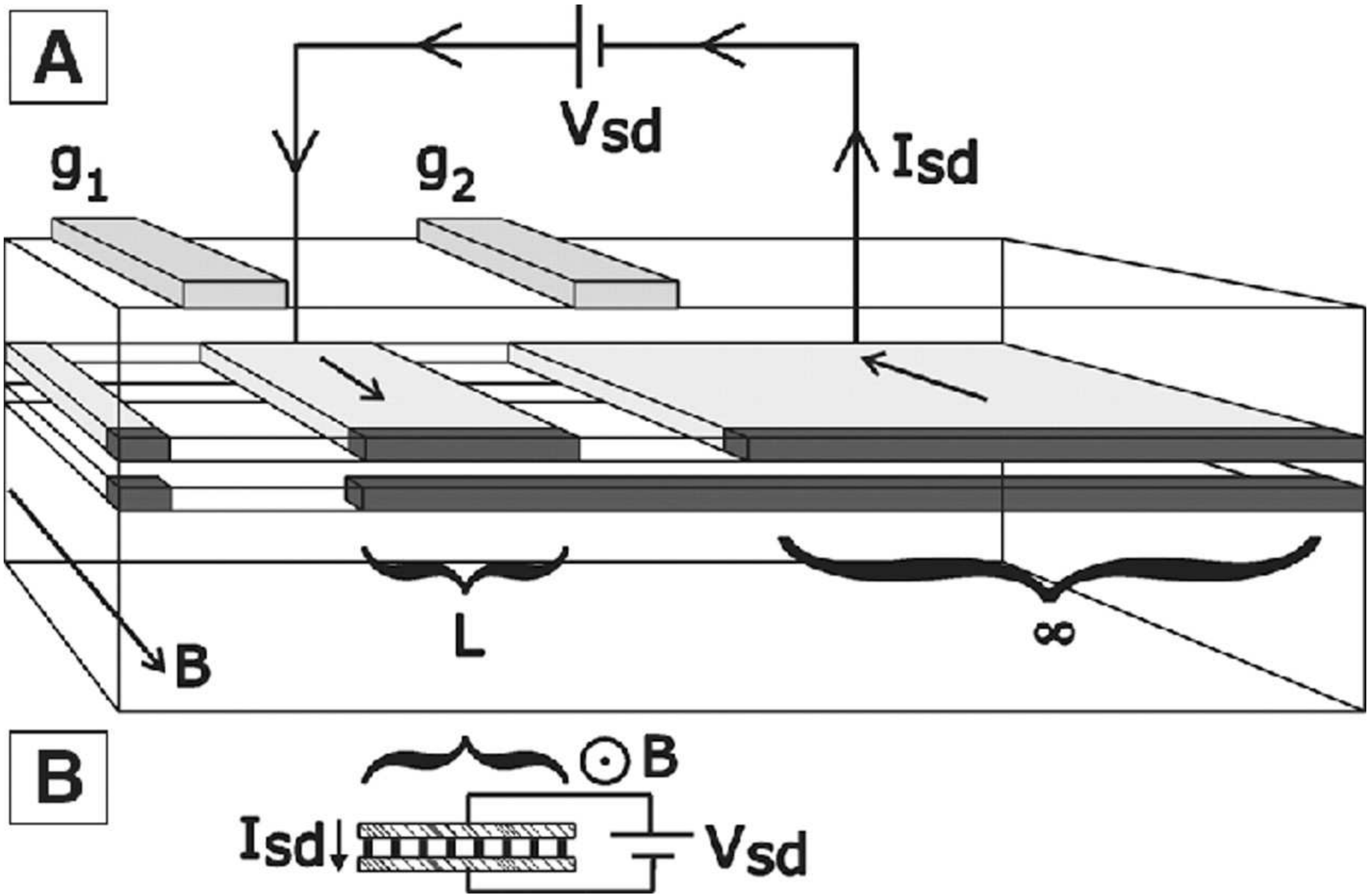
Quantum wires: Carbon nanotubes



Tans et al. (Dekker group), Nature '98

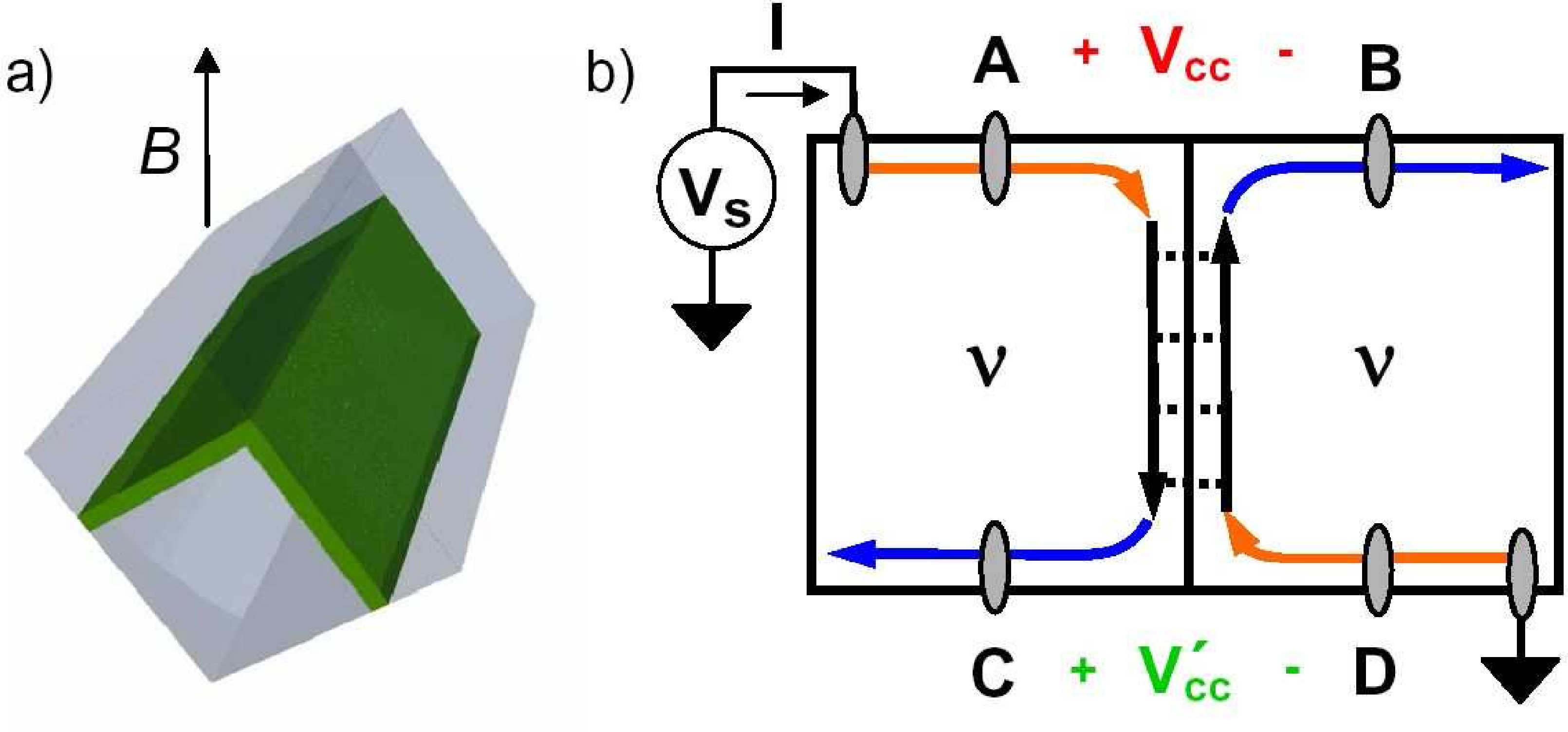
Semiconductor quantum wires:

Auslaender et al (Weizmann Inst./Bell Labs), Science 2002



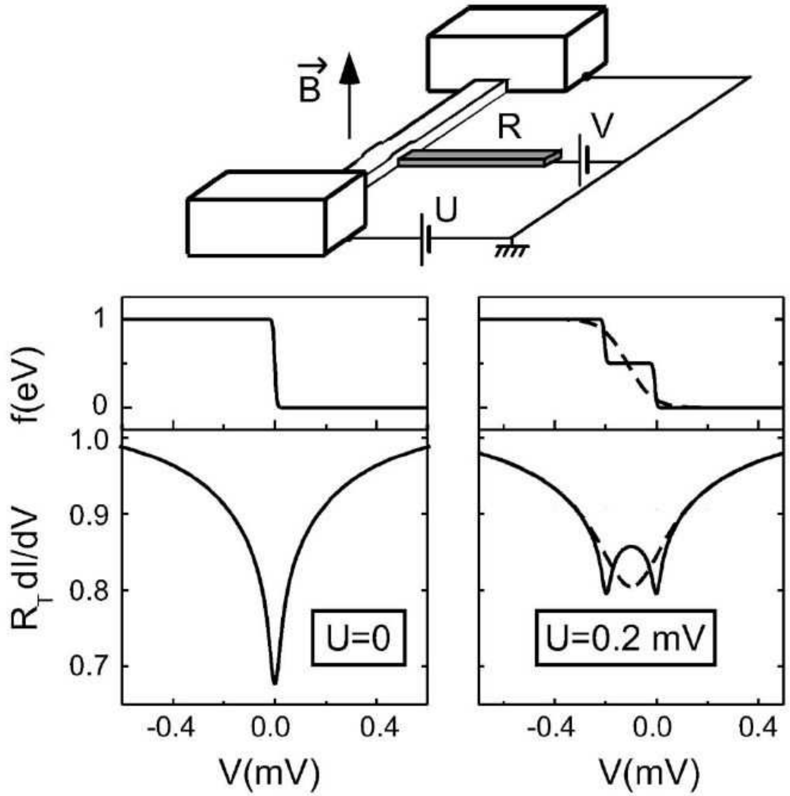
Quantum wires: Quantum Hall edges

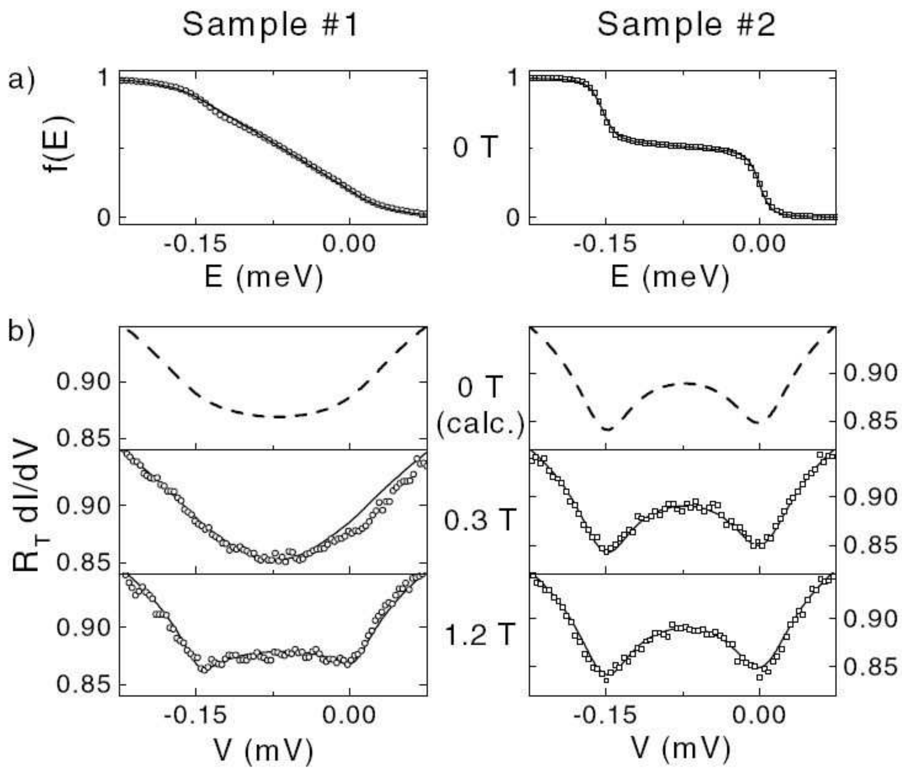
quantum wires with length of several mm



M. Grayson et al. APL'05, PRB'07

Tunneling spectroscopy of thick metallic wires





tunneling current $\longrightarrow$

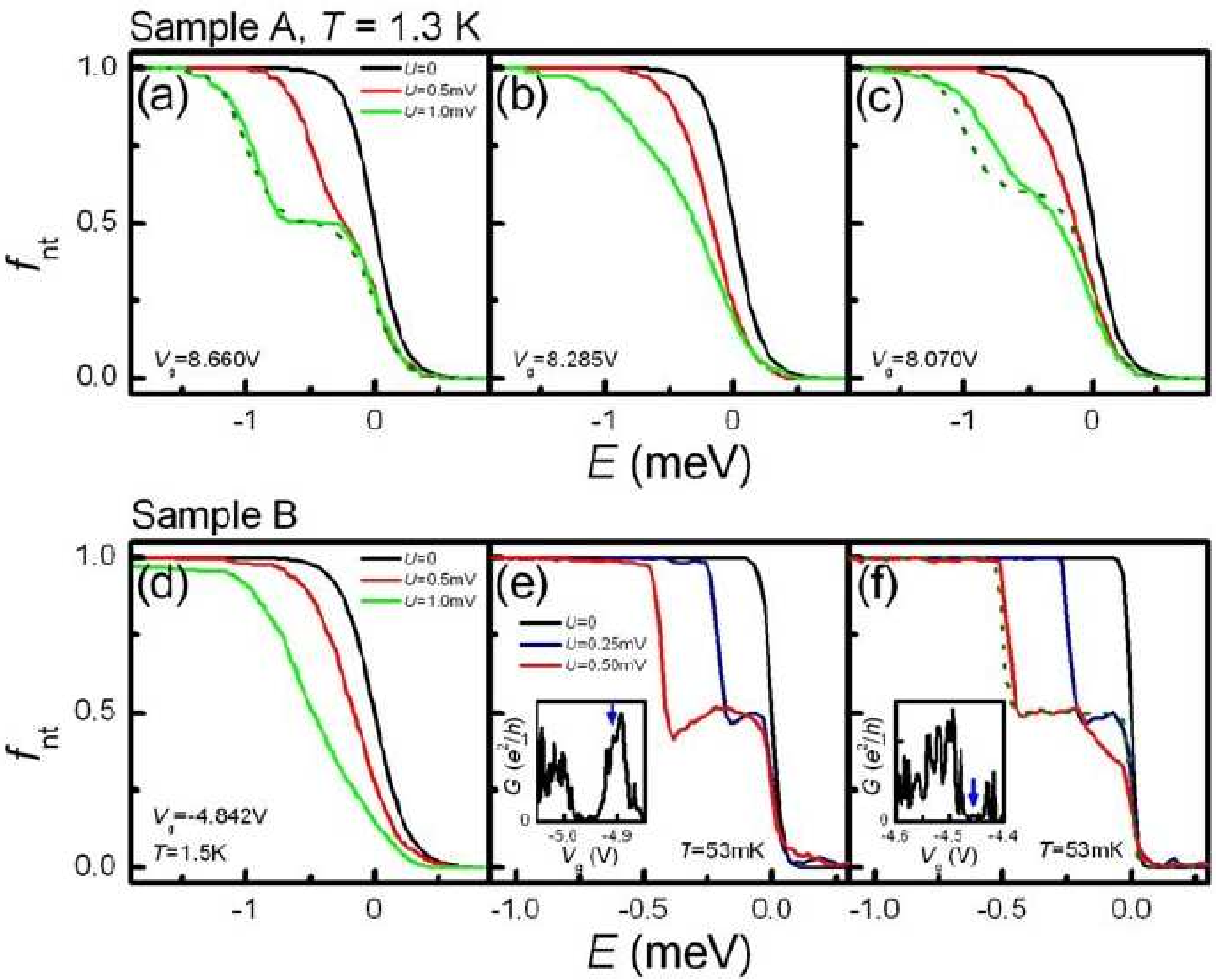
information about local Green functions $G^{\gtrless}(x, x; t)$ $\longrightarrow$

- tunneling DOS
- distribution function

Pothier, Gueron, Birge, Esteve, Devoret, PRL'97

Anthore, Pierre, Pothier, Esteve PRL'03

Tunneling spectroscopy of carbon nanotubes



Experiment: Y. Chen, T. Dirks, G. Al-Zoubi, N. Birge & N. Mason, PRL'08

1D: What is special?

- **Luttinger liquid:** strongly correlated system

  Strong (power-law) ZBA in a clean system

- **No energy relaxation in LL**

  (in the absence of inhomogeneities, neglecting non-linearity of spectrum and momentum dependence of interaction)

  Dephasing without energy relaxation ?!

- Equilibrium: exact solution via **bosonization**

  Non-equilibrium – ?

Luttinger Liquid in equilibrium: Crash course

$$H_0 = \sum_{k,\eta} v(\eta k - k_F)\psi_\eta^\dagger(k)\psi_\eta(k) \qquad \eta = R/L \longrightarrow \pm 1$$

$$H_{\text{e-e}} = \frac{g}{2}\int dx(\rho_R + \rho_L)^2 \qquad \rho_\eta = \psi_\eta^\dagger \psi_\eta$$

Bosonization: $H_0 = \pi v \int dx(\rho_R^2 + \rho_L^2)$

Canonical variables:

$$\partial_x \phi(x) = -\pi[\rho_R(x) + \rho_L(x)]; \qquad \partial_x \theta(x) = \pi[\rho_R(x) - \rho_L(x)]$$

$$\Pi(x) = \frac{1}{\pi}\partial_x \theta(x) \qquad [\phi(x), \Pi(x')] = i\delta(x - x')$$

$$H = \frac{1}{2\pi}\int dx[uK(\pi\Pi)^2 + \frac{u}{K}(\partial_x \phi)^2]$$

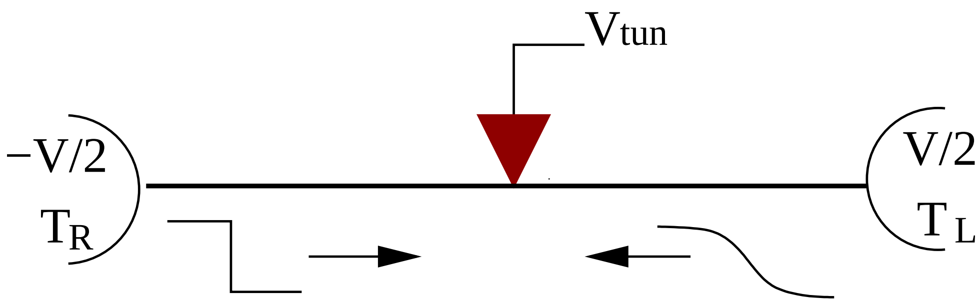
spectrum $\omega = uq \qquad u = v/K \qquad K = (1 + g/\pi v)^{-1/2}$

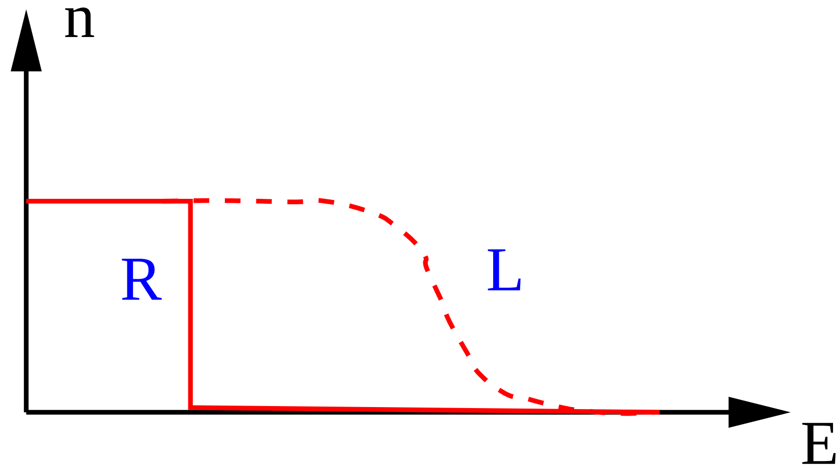
ferm. operators $\psi_{R,L}^\dagger(x) = \left(\frac{\Lambda}{2\pi v}\right)^{1/2} \hat{U}_{R,L}^\dagger e^{\pm i k_F x} e^{i[\theta(x) \mp \phi(x)]}$

Lagrangian formulation:

$$S[\theta, \phi] = \int d\tau dx \left\{ -\frac{i}{\pi}\partial_x \theta \partial_\tau \phi + \frac{1}{2\pi}[uK(\partial_x \theta)^2 + \frac{u}{K}(\partial_x \phi)^2] \right\}$$

Partially non-equilibrium LL

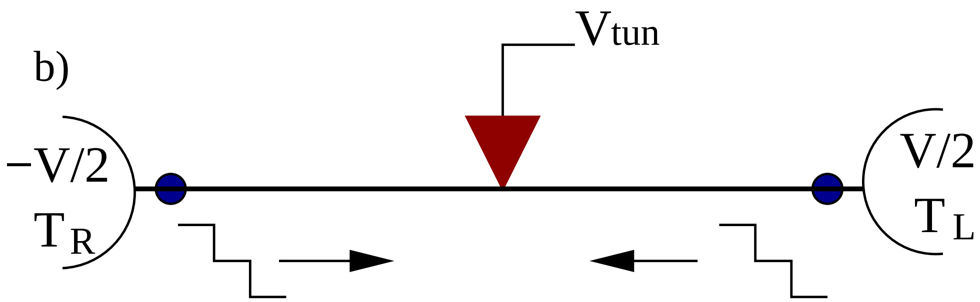


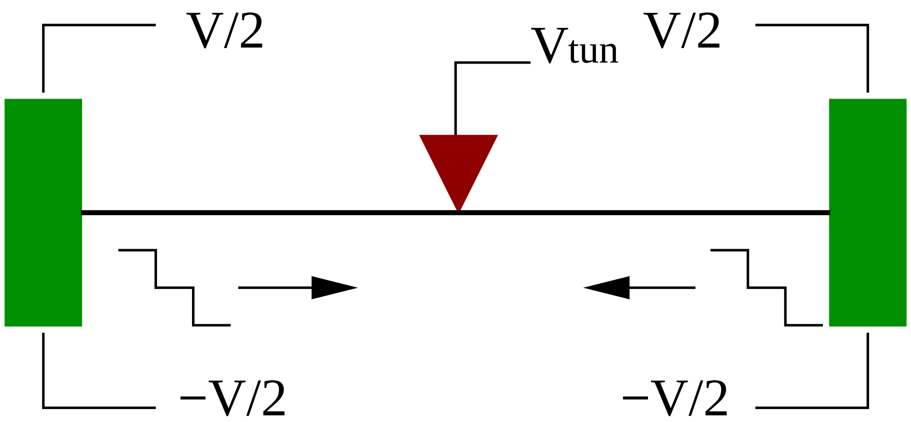


Left- and right-movers separately in equilibrium

but $V \neq 0\ ,\quad T_R \neq T_L$

Fully non-equilibrium LL: Schematic setups





Assume that LL part of the wire is clean;

interaction switches off $K(x) \to 1$ **towards the end of the wire**

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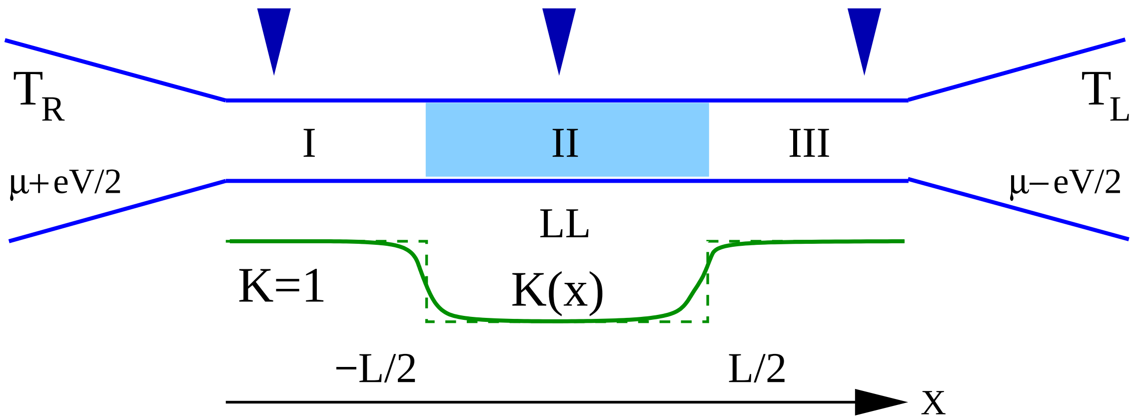
Luttinger Liquid in partial non-equilibrium



Hamiltonian $$H_0 = \sum_{k,\eta} v(\eta k - k_F)\psi^\dagger_\eta(k)\psi_\eta(k)$$

$$\eta = R/L \longrightarrow \pm 1$$

$$H_{\mathrm{e-e}} = \int dx \frac{g(x)}{2}(\rho_R + \rho_L)^2 \qquad K(x) = [1+g(x)/\pi v]^{-1/2}$$

Bosonization: **Fermionic operators** $$\psi_\eta \simeq \left(\frac{\Lambda}{2\pi v}\right)^{1/2} \exp(i\phi_\eta)$$

electron density $$\rho_\eta = \frac{\eta}{2\pi}\partial_x \phi_\eta$$

Tunneling spectroscopy: General results

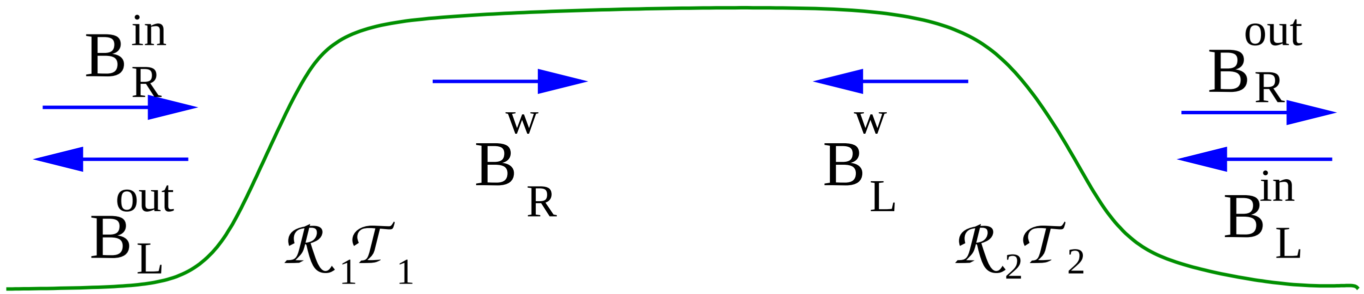
Keldysh Green functions $$G^{\gtrless}_{\eta}(x,t;x,0) = G^{\gtrless}_{\eta,0}(t) e^{-i\eta eVt/2} e^{-\mathcal{F}^{\gtrless}_{\eta}}$$

$$G^{\gtrless}_{\eta,0}(t) = -\frac{T}{2v}\frac{1}{\sinh \pi T(t \mp i/\Lambda)}$$ — free fermions

$$\mathcal{F}^{\gtrless}_{R} = \int_0^\infty \frac{d\omega}{\omega}\left[(B^{\mathrm{w}}_R - B^{(0)}_R)(1-\cos\omega t) + \gamma\Big((B^{\mathrm{w}}_R + B^{\mathrm{w}}_L)(1-\cos\omega t) \pm i\sin\omega t\Big)\right]$$

$\gamma = (K-1)^2/2K$ — interaction strength

$B_\eta(\omega)$ — plasmon distribution function



$$B^{\mathrm{w}}_R = \mathcal{T}_1 B^{\mathrm{in}}_R + \mathcal{R}_1 B^{\mathrm{w}}_L \qquad B^{\mathrm{out}}_L = \mathcal{R}_1 B^{\mathrm{in}}_R + \mathcal{T}_1 B^{\mathrm{w}}_L \qquad B^{\mathrm{in}}_\eta \equiv B^{(0)}_\eta$$

Non-interacting parts of the wire

- no effect on tunneling DOS
- fermionic distr. function: energy redistribution due to plasmon scattering

$$n_R(t) = n_{R,0}(t)e^{-\mathcal{F}_R(t)} = \frac{i}{2}e^{-ieVt/2}\left(\frac{T_R}{\sinh \pi T_R t + i0}\right)^{\mathcal{T}}\left(\frac{T_L}{\sinh \pi T_L t + i0}\right)^{\mathcal{R}}$$

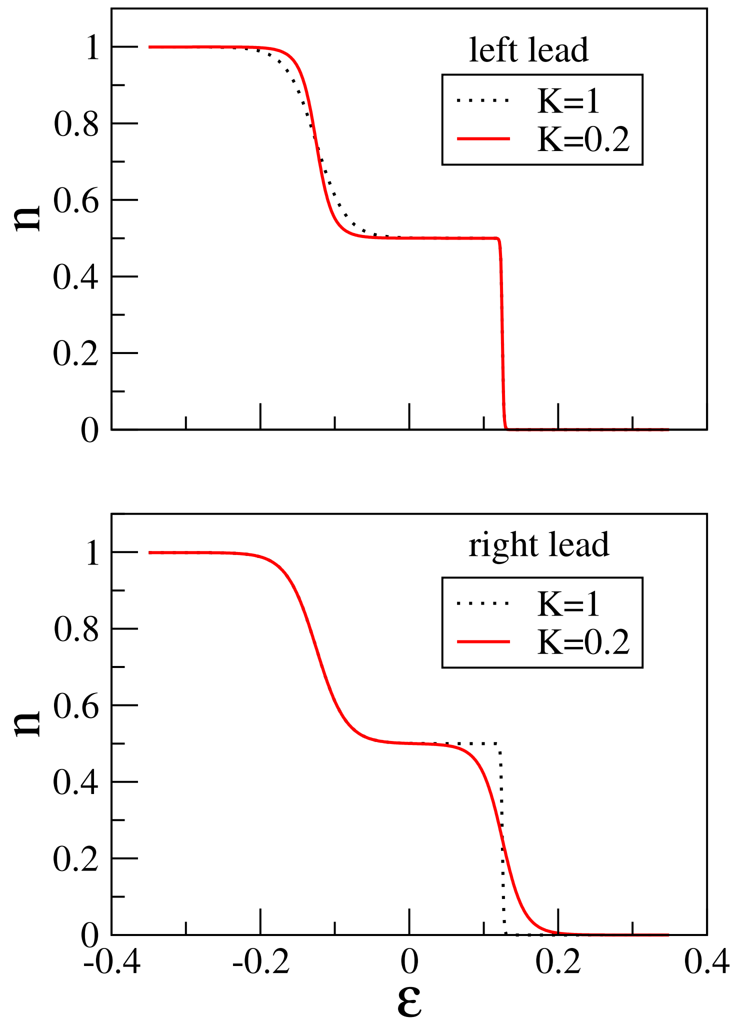


Figure:

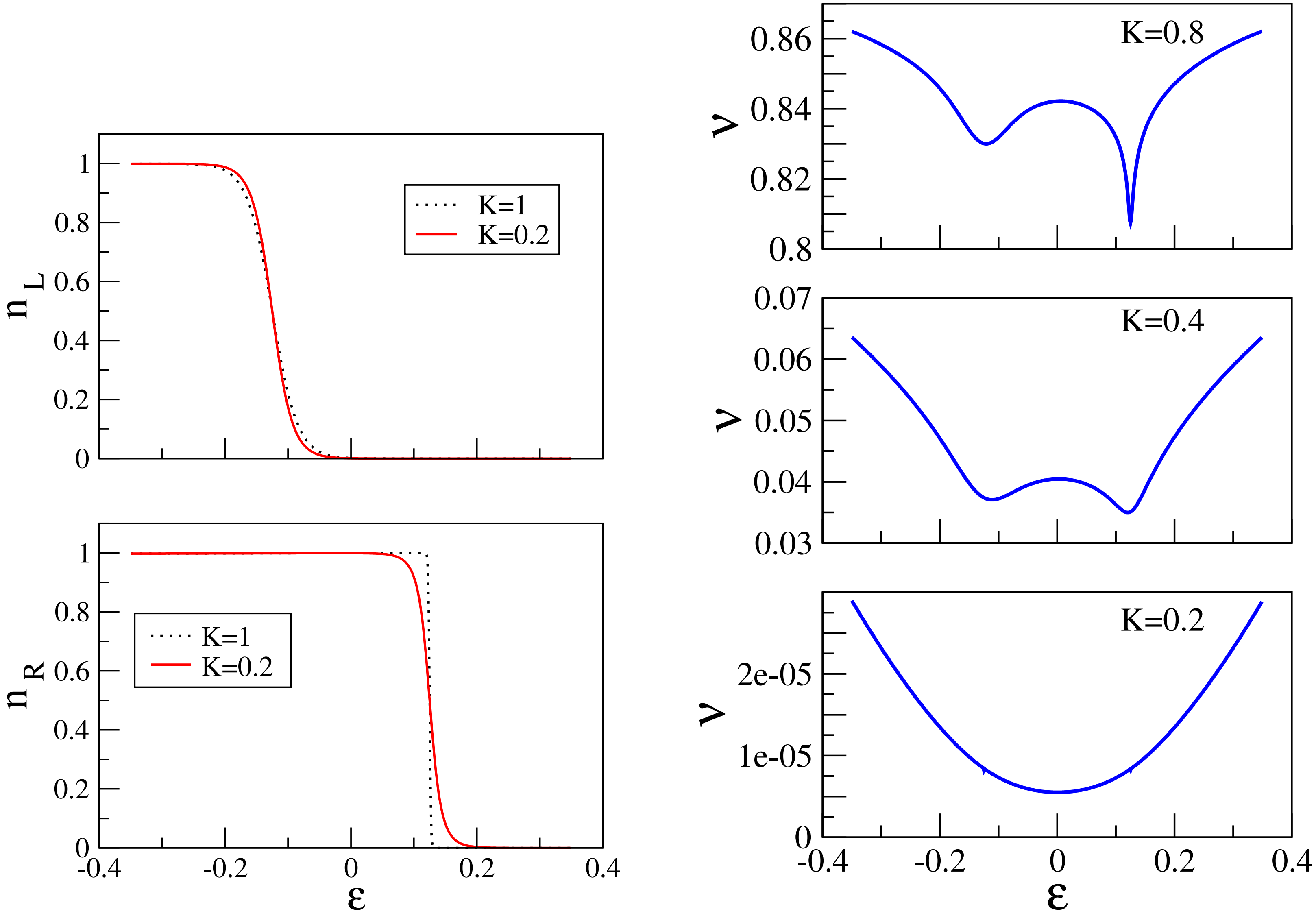
$n = n_R + n_L$

sharp boundaries

$T_R = 0.001, \quad T_L = 0.2, \quad eV = 0.25$

Interacting part of the wire

- distr. function $\quad n_\eta(t) = n_{\eta,0}(t)\exp\left\{-\int_0^\infty \frac{d\omega}{\omega}[B^{\rm w}_\eta(\omega) - B^{(0)}_\eta(\omega)](1-\cos\omega t)\right\}$
- broadening of $G^{\gtrless}(\epsilon)$, TDOS $\nu(\epsilon)$ by distribution function + dephasing



Relation to electric and thermal conductance

Electric current $$I = ev(N_R - N_L) = \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi}[n_R(\epsilon) - n_L(\epsilon)] = \frac{e^2}{h}V$$

Maslov, Stone '95; Ponomarenko '95; Safi, Schulz '95; Oreg, Finkelstein '96

Thermal current

$$I_E = v\partial_t \left[G_R^<(t,t') - G_L^<(t,t')\right]\Big|_{t=t'}$$ in non-interacting part

$$= \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi}\epsilon[n_R(\epsilon) - n_L(\epsilon)]$$

$$= \frac{1}{4\pi}\int_0^{\infty} d\omega\omega\mathcal{T}(\omega)[B_R^{(0)}(\omega) - B_L^{(0)}(\omega)]$$

$$= \frac{\pi}{12}\mathcal{T}(T_R^2 - T_L^2)$$ for ω-independent transmission

Fazio, Hekking, Khmelnitskii '98

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Bosonization of free fermions out of equilibrium

bosonized Keldysh action $\qquad S_0 = \sum_\eta (\rho_\eta \Pi_\eta^{a^{-1}} \bar\rho_\eta - i \ln Z_\eta[\bar\chi_\eta])$

$\rho, \bar\rho = (\rho_+ \pm \rho_-)/\sqrt{2}$ — classical and quantum components of density

$i \ln Z_\eta[\bar\chi_\eta] = \sum_n (-1)^{n+1} \bar\chi_\eta^n \mathcal{S}_{n,\eta}/n$

— partition function of free chiral fermions in the field $\bar\chi_\eta = \Pi_\eta^{a^{-1}} \bar\rho_\eta$

$\mathcal{S}_{n,\eta} = \langle \rho_{1,\eta} \rho_{2,\eta} \ldots \rho_{n,\eta} \rangle$ — density cumulants

Equilibrium: $\quad S_n = 0$ for all $n > 2$ $\longrightarrow$ gaussian theory

Out of equilibrium all $S_n \neq 0$ $\longrightarrow$ looks as interacting field theory

But crucial simplifications:

- Z_η depends only on $\bar\rho_\eta$ $\longrightarrow$ action linear in ρ_η

  $\longrightarrow$ integral over ρ_η can be performed, yields an equation fixing $\bar\rho_\eta$
- $Z_\eta[\bar\chi_\eta]$ is restricted to mass-shell $(\omega = \eta v q)$

Bosonization of free fermions out of equilibrium (cont'd)

Green function $G^{<}_{0,\eta}(0,\tau) = i\langle\psi^{\dagger}_{\eta}(0,0)\psi_{\eta}(0,\tau)\rangle$

fermionic operators $\hat{\psi}_{\eta}(x) \simeq (\Lambda/2\pi v)^{1/2} e^{\eta i k_F x} e^{i\hat{\phi}_{\eta}(x)}\,,\qquad \hat{\rho}_{\eta}(x) = \eta\partial_x\hat{\phi}_{\eta}/2\pi$

integration over ρ $\longrightarrow$ equation of motion $\partial_t\bar{\rho}_{\eta} + \eta v\partial_x\bar{\rho}_{\eta} = j(t,x)$

"source" $j(t,x) = \delta(x)[\delta(t) - \delta(t-\tau)]/\sqrt{2}$

solution $\longrightarrow$ $\bar{\rho}_{\eta}$ $\longrightarrow$ $\bar{\chi}_{\eta} = \Pi^{a^{-1}}_{\eta}\bar{\rho}_{\eta}$

$\longrightarrow$ world-line integral $\delta_{\eta}(t) = \sqrt{2}\int_{-\infty}^{\infty} d\tilde{t}\;\bar{\chi}_{\eta}(\tilde{t}-t, \eta v\tilde{t})$

Partition function Z_{η} depends on $\delta_{\eta}(t)$ only

Green function of Free Fermions (from bosonized theory)

$$G_{0,\eta}^{\gtrless}(\tau) = -\frac{1}{2\pi v}\frac{1}{\tau \mp i/\Lambda} Z_{\eta}^{\gtrless}[\bar{\chi}_{\eta}]$$

$$Z_{\eta}^{\gtrless}[\bar{\chi}_{\eta}] = \Delta_{\eta,\tau}(2\pi)$$

$$\Delta_{\eta}[\delta(t)] = \det\left[1 + \left(e^{-i\hat{\delta}} - 1\right)\hat{n}_{\eta}\right]$$ functional determinant

$\Delta_{\eta}[\delta(t)] = \Delta_{\eta,\tau}(\lambda)$ for rectangular pulse $\delta(t) = \lambda[\theta(t+\tau) - \theta(t)]$

Relation to the counting statistics problem !

Counting Statistics

generating function

$$\kappa(\lambda) = \sum_N e^{iN\lambda} p_N \,, \qquad \ln \kappa(\lambda) = \sum_{k=1}^{\infty} m_k \frac{(i\lambda)^k}{k!}, \qquad m_k = \langle\langle \delta N^k \rangle\rangle$$

N — number of particles passing in time τ

non-interacting fermions: $\kappa(\lambda) = \det[\hat{1} - \hat{n} + \hat{S}\exp(i\lambda)\hat{n}]$

$\hat{S}$ — scattering matrix

Levitov, Lesovik '93

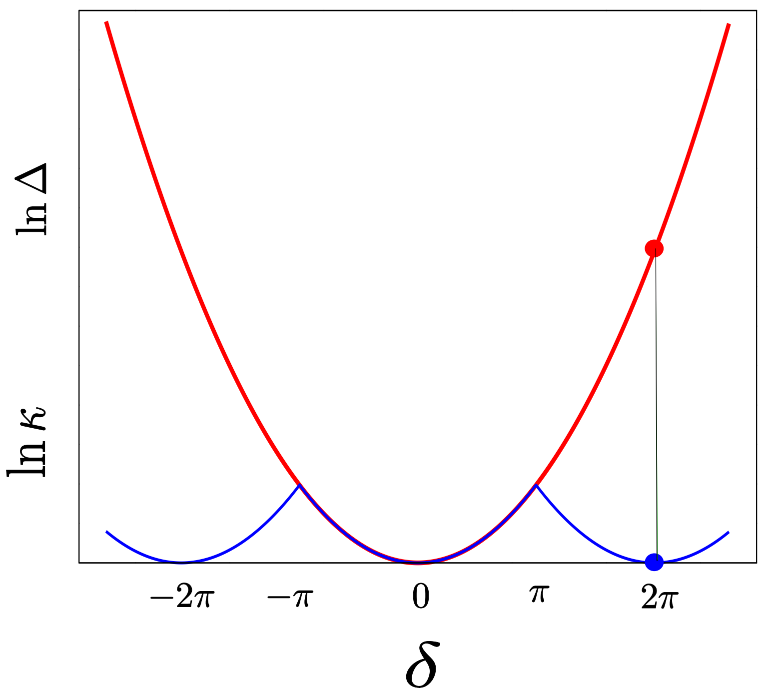
Analytic properties of $\Delta_{\eta,\tau}(\lambda)$ and $\kappa(\lambda)$

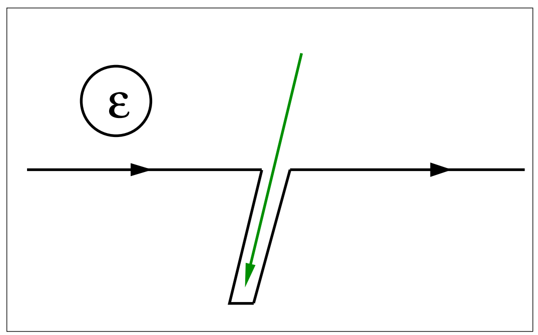
semiclassical limit

$$\ln \Delta_{\eta,\tau}(\lambda) = \frac{\tau}{2\pi\hbar}\int d\epsilon \ln[1 + (\exp i\lambda - 1)n_\eta(\epsilon)]$$

Equilibrium:

$$\ln \Delta_{\eta,\tau}(\lambda) = -\tau T \lambda^2/4\pi$$





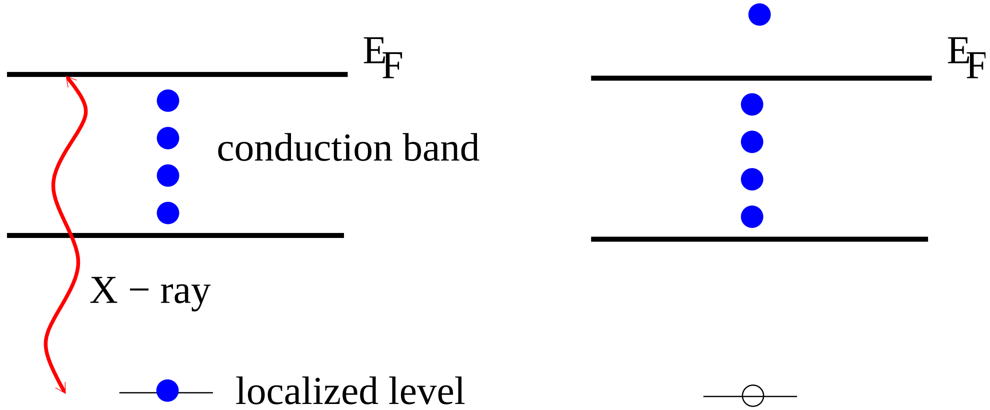
Fermi edge singularity in X-ray absorption

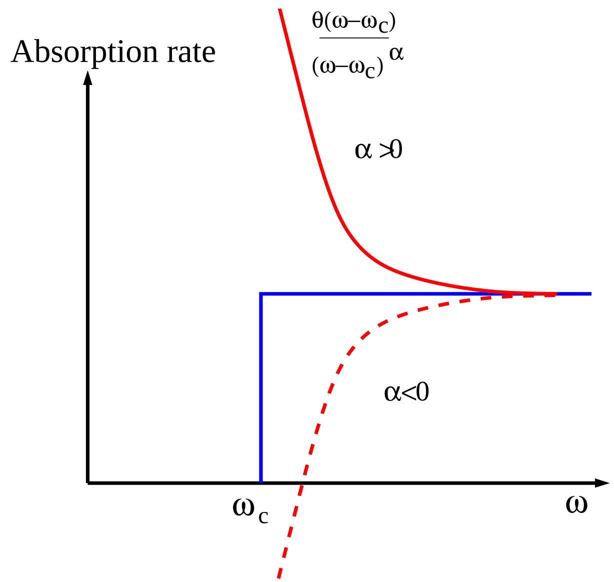
Hamiltonian

$$H = \sum_k \epsilon_k a_k^\dagger a_k + E_0 b^\dagger b + \sum_{k,k'} V_{k,k'} a_k^\dagger a_{k'} b b^\dagger$$

(band electrons a, localized level b)

X-ray coupling $\quad H_X = \sum W_k a_k^\dagger b e^{-i\omega t} + h.c.$





$$\alpha = \frac{2\delta}{\pi} - \left(\frac{\delta}{\pi}\right)^2$$

theory milestones:

Mahan '67 (exciton), Anderson '67 (orthogonality catastrophy)

Nozieres , De Dominicis '69 (diagrammatics)

Schotte, Schotte '69 (bosonization)

Fermi edge singularity out of equilibrium

Green function

$$G^{>}(\tau) = -i\,\mathrm{tr}\{\hat{\rho}_e e^{iH_0\tau}\psi(0)e^{-iH_1\tau}\psi^\dagger(0)\}$$

$$H_0 = -i\eta v\psi^\dagger\partial_x\psi \qquad\qquad H_1 = H_0 + \psi^\dagger(0)\psi(0)U$$

bosonic formalism:

$$G^{\gtrless}(\tau) = \mp\frac{i\Lambda}{2\pi v}\langle T_K e^{i\left(1-\frac{\delta}{\pi}\right)\hat{\phi}_{\mp}(0,\tau)} e^{-i\left(1-\frac{\delta}{\pi}\right)\hat{\phi}_{\pm}(0,0)}\rangle$$

$\delta = -U/2v$ — scattering phase

Analogous to free fermions but with source $\quad j \longrightarrow (1-\delta/\pi)j$

$\longrightarrow$ obtain the same result as for free fermions

but with the argument of the funct. det. $\;2\pi \longrightarrow 2(\pi-\delta)$

Fermi edge singularity out of equilibrium (cont'd)

$$G^{\gtrless}(\tau) \propto \Delta_\tau[2(\pi - \delta)]$$

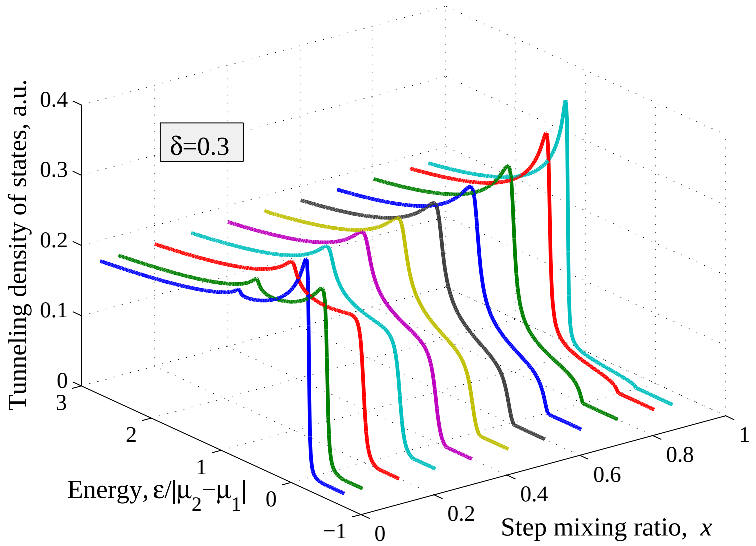
semiclassical limit

$$\ln \Delta_\tau(\delta) \simeq \frac{\tau}{2\pi\hbar} \int d\epsilon \log \left[1 + (e^{i\delta} - 1)n(\epsilon)\right] = -\tau/\tau_\phi$$

double step function $\quad n(\epsilon) = a n_0(\epsilon_-) + (1-a) n_0(\epsilon_+)$

$$\tau_\phi^{-1} = \frac{eV}{4\pi\hbar} \log \left[1 - 4a(1-a)\sin^2\delta\right]$$

ZBA dephasing broadening



Abanin, Levitov '04 :

fermionic approach, similar to functional bosonization

Interacting fermions: Luttinger liquid

Green function $G^{<}_{0,\eta}(\tau) = i\langle\psi^{\dagger}_{\eta}(0,0)\psi_{\eta}(0,\tau)\rangle$

Equation of motion

$$\partial_t\bar{\rho}_R + \partial_x\left((v+\frac{g}{2\pi})\bar{\rho}_R + \frac{g}{2\pi}\bar{\rho}_L\right) = j$$
$$\partial_t\bar{\rho}_L - \partial_x\left((v+\frac{g}{2\pi})\bar{\rho}_R + \frac{g}{2\pi}\bar{\rho}_L\right) = 0$$

scattering phase

$$\delta_\eta(t) = \sqrt{2}\int d\tilde{t}\,\bar{\chi}_\eta(\tilde{t}-t,\eta v\tilde{t}) \qquad \bar{\chi}_\eta = \Pi^{a^{-1}}_{\eta}\bar{\rho}_\eta$$

scattering phase expressed through asymptotic behaviour

$$\delta_\eta(t) = -2\pi\sqrt{2}\,\eta\lim_{\tilde{t}\to-\infty}\int_0^{\eta v(\tilde{t}+t)} d\tilde{x}\,\bar{\rho}_\eta(\tilde{x},\tilde{t})$$

Luttinger liquid: Results

Green function $$G_R^{\gtrless}(\tau) = -\frac{\Delta_L[\delta_L]\Delta_R[\delta_R]}{2\pi v(\pm i\Lambda)^{\gamma}(1 \mp i/\Lambda)^{1+\gamma}}$$

$$\delta_\eta(t) = \sum_{n=0}^{\infty} \delta_{\eta,n} w_\tau(t, t_n)$$ sequence of pulses

$$w_\tau(t, \tilde{t}) = \theta(t - \tilde{t} + \tau) - \theta(t - \tilde{t})$$ rectangular pulse of duration τ

$$t_n = (n + 1/2 - 1/2K)L/u$$ positions of pulses

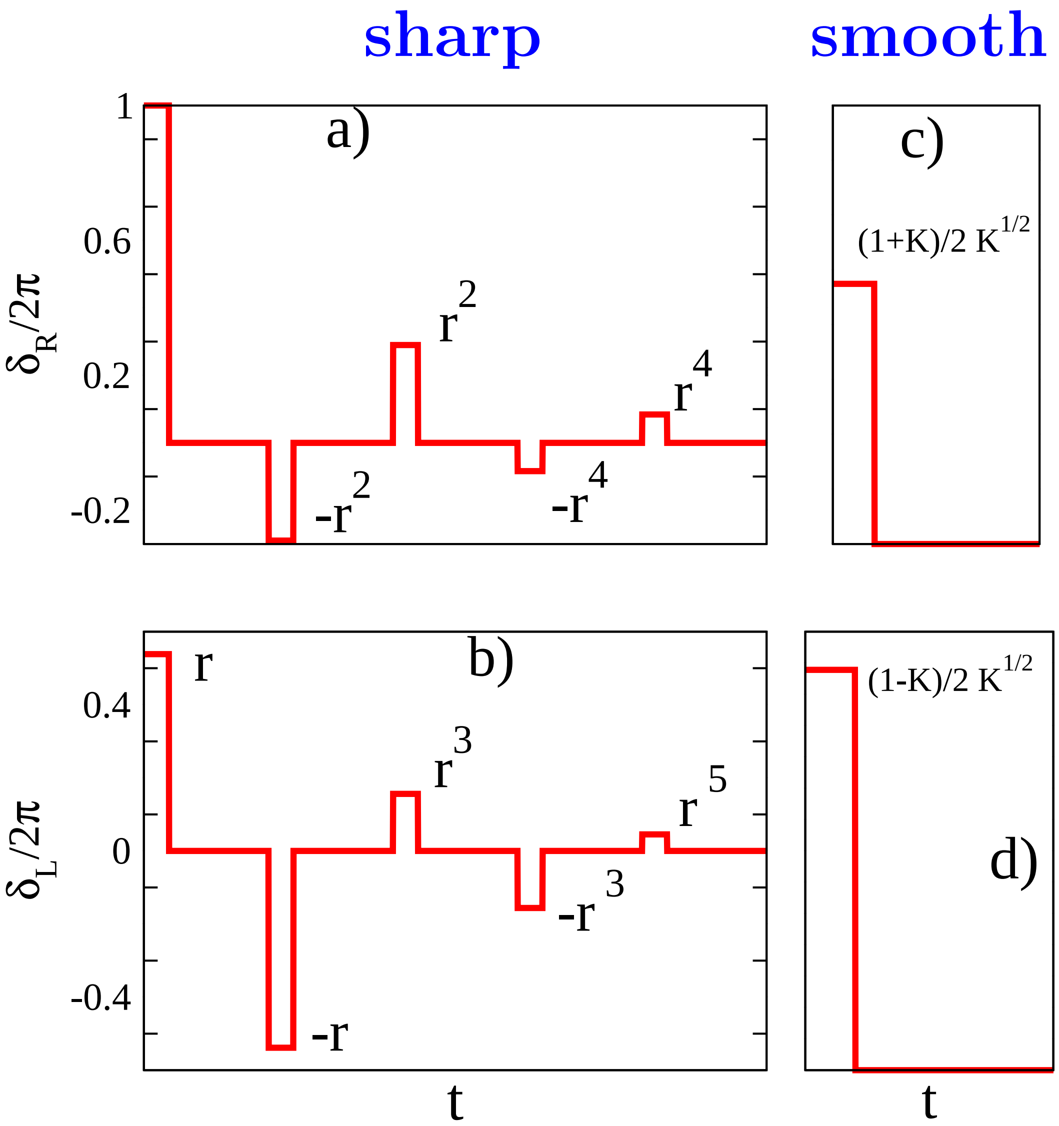
amplitudes of pulses:

$$\delta_{\eta,2m} = \pi t_\eta \frac{1 + \eta K}{\sqrt{K}} r_L^m r_R^m \qquad \delta_{\eta,2m+1} = -\pi t_\eta \frac{1 - \eta K}{\sqrt{K}} r_\eta^m r_{-\eta}^{m+1}$$

$r_{R/L}$, $t_{R/L}$ — plasmon reflection/transmission coefficients

Fractionalization of 2π pulse at the tunneling and at boundaries

Sharp vs smooth boundaries



$$r = \frac{1-K}{1+K}$$

for long wires $\tau \ll L/u$

$$\Delta_\eta[\bar{\theta}_\eta(t)] \simeq \prod_{n=0}^{\infty} \Delta_{\eta,\tau}[\delta_{\eta,n}]$$

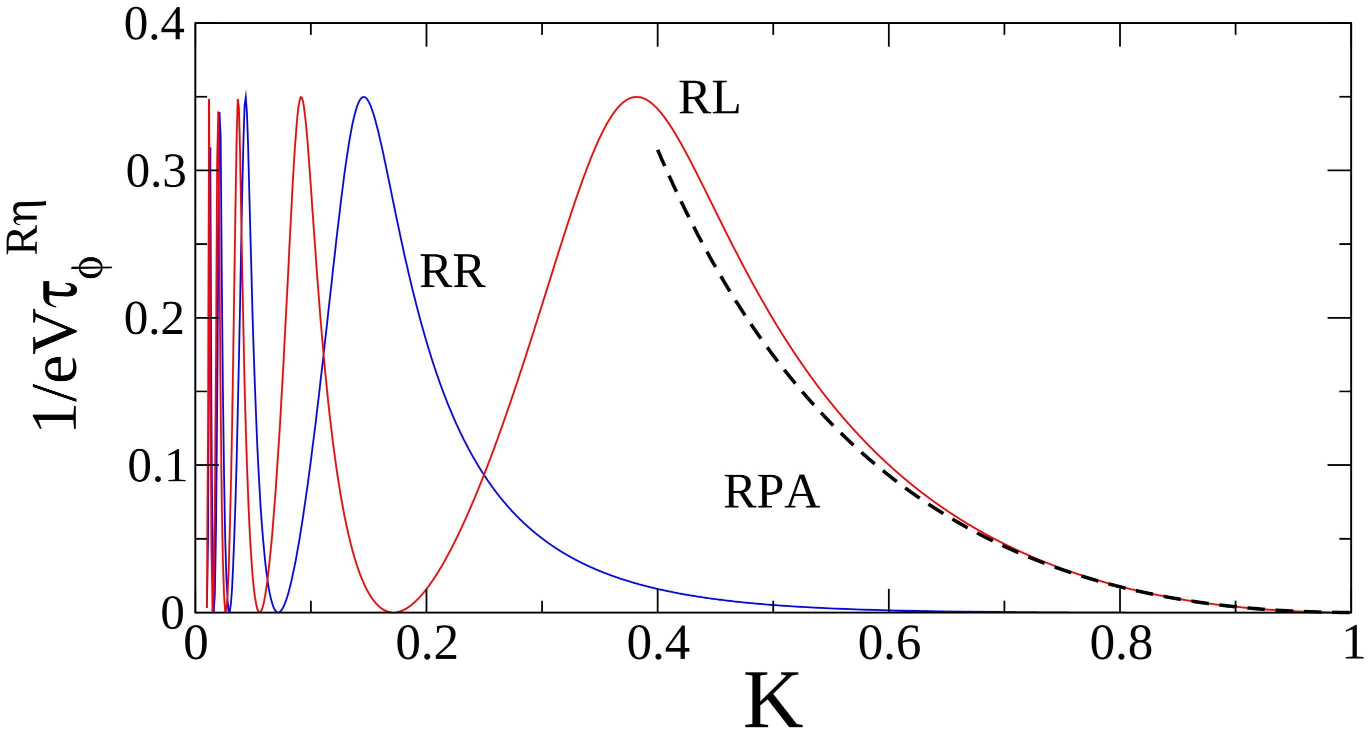
Fractionalization in LL: cf. Safi, Schulz; Le Hur, Halperin, Yacoby et al

Non-equilibrium dephasing (smooth boundaries)

$$1/\tau_\phi^R = 1/\tau_\phi^{RR} + 1/\tau_\phi^{RL}$$

dephasing broadening of ZBA

$$1/\tau_\phi^{R\eta} = \frac{eV}{2\pi} \ln\left[1 - 4a_\eta(1-a_\eta)\sin^2\frac{\pi(1+\eta K)}{2\sqrt{K}}\right]$$



- $1/\tau_\phi^{RR}$: RPA is violated even for weak interaction
- oscillations of dephasing

Summary

- Bosonization technique out of equilibrium
  - free fermions
  - fermi edge singularity
- Tunneling Spectroscopy of LL: exact solution via bosonization
  - $G^{\gtrless}$ in terms of $\Delta[\delta_\eta(t)]$
  - Energy distribution:

    plasmon scattering on the boundaries affects $n(\epsilon)$
  - Dephasing: broadening of ZBA

    dominant mechanism of broadening near sharp edges

    double-step distribution: oscillatory dephasing rate, breakdown of of RPA
  - Generalizations (no time to discuss): spin,

    Green function at different spatial points (e.g. Aharonov-Bohm setup), ...
- Relation to counting statistics