Long quantum transitions due to unstable semiclassical dynamics

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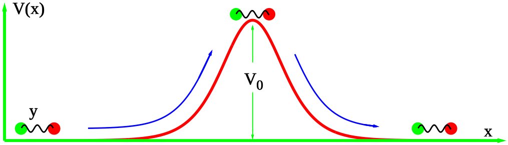
New tunneling mechanism

Creation of classically unstable "state"

Necessary conditions:

- $N \geq 2$
- Nonlinear interaction
- High energy $E > E_c > V_0$



New effect has been observed in:

- Regular systems ($N = 2$)

  Bezrukov, Levkov, ArXiv:quant-ph/0301022 (2003); J. Exp. Theor. Phys. **98**, 820 (2004).
- Systems with irregular dynamic ($N = 1.5,\ 2$)

  Takahashi, Ikeda, J. Phys. A **36**, 7953 (2003); Europhys. Lett. **71**, 193 (2005).

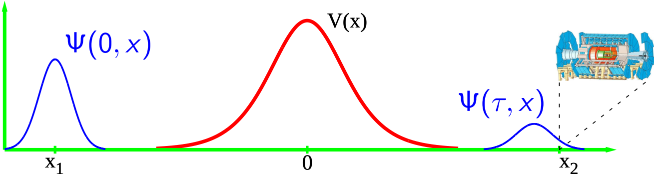
  Levkov, Panin, Sibiryakov, Phys. Rev. E **76**, 046209 (2007).
- Topological transitions in electroweak theory ($N = \infty$)

  Bezrukov, Levkov, Rebbi, Rubakov, Tinyakov, Phys. Rev. D **68**, 036005 (2003)
- Tunneling transitions in scalar field theory ($N = \infty$)

  Levkov, Sibiryakov, Phys. Rev. D **71**, 025001 (2005)

Traversal time

We define traversal time as follows



Traversal-time distribution

$$\rho(\tau) \propto \oint \mathbf{j}(\tau)\, d\mathbf{s}_{x_2}\ .$$

Due to probability conservation law

$$\rho(\tau) = \frac{1}{\mathcal{P}_\infty}\frac{d\mathcal{P}_\tau}{d\tau}\ , \quad \text{where} \quad \mathcal{P}_\tau = \int_{x>x_2} d\mathbf{x}\,|\Psi(\tau,\mathbf{x})|^2$$

Calculation $\rho(\tau)$

Below we adopt dimensionless units $m = \hbar = 1$.
Semiclassical approximation: $V_0 \sim l_0^2 \sim 1/g^2$, where $g^2 = \frac{\hbar}{l_0\sqrt{mV_0}} \ll 1$.
Wave function of the final state

$$\Psi(\tau, \mathbf{x}_f) = \int d\mathbf{x_i}\ \Psi(0, \mathbf{x}_i) \int [d\mathbf{x}(t)]\Big|_{\mathbf{x}_i}^{\mathbf{x}_f} \mathrm{e}^{iS[\mathbf{x}]} .$$

First, let us calculate $\mathcal{P}_\infty$:

- $S \propto 1/g^2 \Rightarrow$ saddle-point method.
- $\mathbf{x}(t)$ satisfies classical equations of motion.
- $\mathcal{P}_\infty = A \cdot \mathrm{e}^{-F/g^2}$, where A and F are functionals of the semiclassical trajectory $\mathbf{x}(t)$.

Straightforward method is inconvenient for finite values of τ.

Classical transitions: Lagrange multiplier method!

Time of motion in $x < x_2$:

$$T_{int}[\mathbf{x}] = \int_0^{+\infty} dt\, \theta(x_2 - x(t)) \Longrightarrow S[\mathbf{x}] \to S_{\tilde{\epsilon}}[\mathbf{x}] = S + \tilde{\epsilon}\,(T_{int}[\mathbf{x}] - \tau)$$

- $\tilde{\epsilon}$ — Lagrange multiplier.
- $\mathbf{x}_{\tilde{\epsilon}}(t)$ — saddle-point of modified action S_ϵ .
- The equation on $\tilde{\epsilon}$: $T_{int}[x_{\tilde{\epsilon}}] = \tau$.

Quantum transitions: Faddeev-Popov unity factor!

$$1 = \int_0^\tau d\tau'\, \delta(\tau' - T_{int}[\mathbf{x}(t)]) = \int_0^\tau d\tau' \int_{i\infty}^{-i\infty} \frac{id\epsilon}{2\pi g^2} \mathrm{e}^{\epsilon(\tau' - T_{int}[\mathbf{x}])/g^2}$$

$$\Psi(\tau, \mathbf{x}_f) = \int d\mathbf{x}_i\, \Psi(0, \mathbf{x}_i) \int [d\mathbf{x}(t)] \cdot \mathbf{1} \cdot \mathrm{e}^{iS}$$

All integrals are taken by saddle-point method.

- $x_\epsilon(t)$ is the saddle-point of $S_\epsilon[\mathbf{x}] = S[\mathbf{x}] + i\epsilon(T_{int}[\mathbf{x}] - \tau)/g^2$.
- Saddle-point value of ϵ is real!
- ϵ depends on $\tau \Rightarrow \mathrm{Re}\, T_{int}[\mathbf{x}_\epsilon] = \tau$.

Final result for the traversal-time distribution

$$\rho(\tau) = \mathcal{N}\sqrt{-\frac{d\epsilon}{d\tau}} \cdot A_\epsilon \cdot \mathrm{e}^{-F_\epsilon/g^2} ,$$

where A_ϵ and F_ϵ should be calculated on the modified trajectory $\mathbf{x}_\epsilon(t)$.

Stable trajectories:

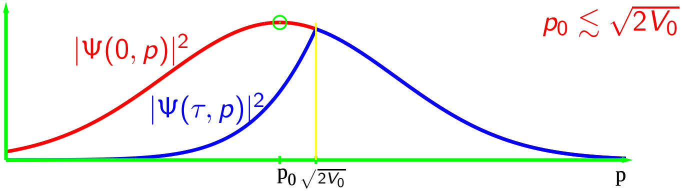
$$dF_\epsilon/d\tau = -2\epsilon \quad \Rightarrow \quad \epsilon = 0 \text{ — maximum } \rho(\tau) \quad \Rightarrow \quad S_\epsilon = S$$

One expands F_ϵ around $\tau = \mathrm{Re}\, T_{int}[\mathbf{x}_{\epsilon=0}] = \tau_0$:

$$\rho(\tau) = \frac{1}{\sqrt{2\pi}\sigma_\tau} \cdot \mathrm{e}^{-(\tau - \langle\tau\rangle)^2/2\sigma_\tau^2}$$

where $\langle\tau\rangle = \tau_0 \sim O(g^0)$ and $\sigma_\tau^2 \sim O(g^2)$.

One dimension



Total transition probability $\mathcal{P}_\infty = \int_0^\infty dp\, \mathcal{T}_p \cdot |\Psi_i(p)|^2$.
The energy of the semiclassical trajectory $E = \sqrt{2V_0}$!
In the vicinity of the barrier top

$$x_\epsilon(t) \to c_- \cdot \mathrm{e}^{-\omega_- t} + \epsilon c_+ \cdot \mathrm{e}^{+\omega_- t} \qquad \Rightarrow \qquad \tau \propto \log \epsilon\,.$$

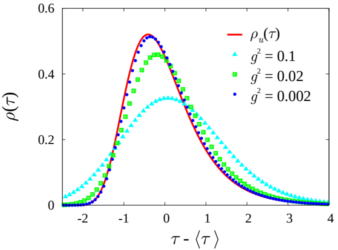
Substituting $x_\epsilon(t)$ into $\rho(\tau)$ one obtains

$$\rho_u(\tau) = \frac{2\epsilon_0}{g^2} \exp\left\{ -\omega_- \tau - \frac{2\epsilon_0}{g^2 \omega_-} \mathrm{e}^{-\omega_- \tau} \right\} , \qquad \omega_-^2 = -U''(0).$$

$$\rho_u(\tau) = \omega_- \exp\left\{-\omega_-(\tau - \langle\tau\rangle) - \gamma - \mathrm{e}^{-\omega_-(\tau-\langle\tau\rangle)-\gamma}\right\}$$

- Traversal-time distribution have unusual asymmetric form.
- Parameters of the distribution are large, $\langle\tau\rangle \sim \log(g^2)$, $\sigma_\tau^2 \sim O(g^0)$.
- The form of the distribution is universal.

Results for $V(x) = \frac{1}{g^2 \operatorname{ch}^2(gx)}$



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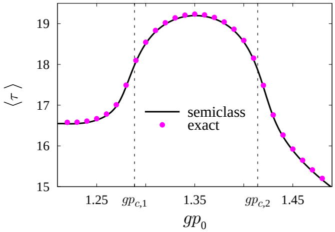


b

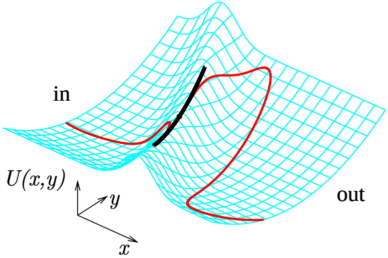
(a) Traversal–time distributions at $gp_0 = 1.35$. Points and solid line represent exact quantum mechanical results and analytical formula, respectively. (b) Average traversal time.

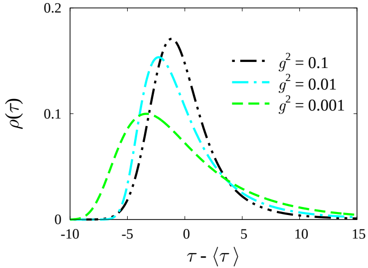
Two dimensions

- Momentum dispersion appears due to nonlinear interactions even if p_x is completely fixed in the in–state.
- Unstable orbits exist in some region $E > V_0$.
- If $E - V_0 \ll V_0$ the picture is similar to one-dimensional.

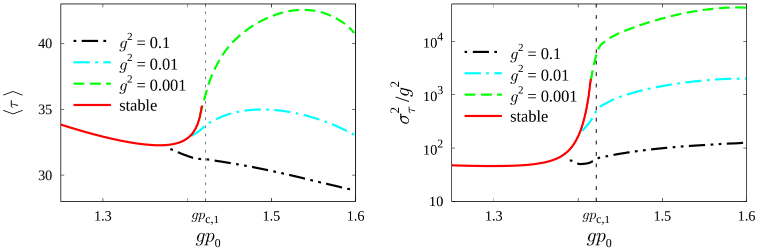
Consider the system

$$U(x, y) = \omega^2 y^2/2 + \frac{V_0}{g^2} \mathrm{e}^{-g^2(x+y)^2/2}$$





Distributions $\rho(\tau)$ in the case of sphaleron–driven tunneling for $(E - V_0)/V_0 \approx 0.2$.



Mean times of passing $\langle\tau\rangle$ and rescaled time dispersions σ_τ^2/g^2 as a function of p_0 for $g^2 E_y = 0.05$.

Summary

- Unusual form of $\rho(\tau)$ and related scalings of $\langle\tau\rangle$ and σ_τ^2 with g^2 can be used for experimental identification of unstable semiclassical dynamics.