Quantum Interference in Disordered Luttinger Liquids

Andrey Yashenkin

Petersburg Nuclear Physics Institute, Gatchina, Russia

Igor Gornyi *(FZK & Ioffe)*

Alexander Mirlin *(FZK/UniKa & PNPI)*

Dmitry Polyakov *(FZK)*

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Alexander Mirlin *(FZK/UniKa & PNPI)*

Dmitry Polyakov *(FZK)*

Andrey Yashenkin *(PNPI)*

Dmitry Aristov *(PNPI)*

Dmitry Bagrets *(FZK)*

Alexander Dmitriev *(Ioffe)*

Yuval Gefen *(Weizmann)*

Dmitry Gutman *(UniKa)*

Valentin Kachorovskii *(Ioffe)*

Motivation: Why Disordered Luttinger Liquid?

- **Experiment:** Nanotechnology — new 1D systems

- **Theory (1D):** Interaction, no disorder: Luttinger liquid
  (Strongly Correlated Electron System)

  Disorder, no interaction:
  Localization $\xi \sim l$ no diffusion

Challenging Problem: To build up the theory of Disordered Strongly Correlated Electron Systems

Electrical conductivity $\sigma(T, B)$ – ?

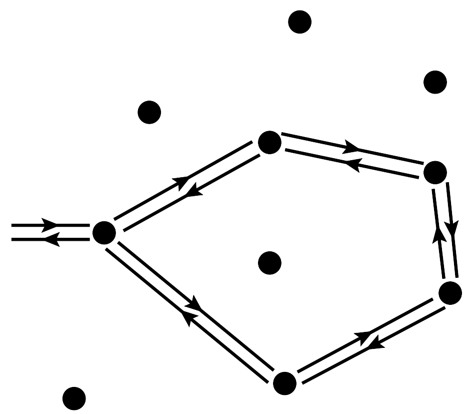
Key question: Are the notions of mesoscopics applicable to Luttinger liquid?

Outline

- Disordered Fermi Liquid in 2D
- Disordered Luttinger Liquid: Model and Formalism
- Single-Particle Properties and Singlet Channel of Weak Localization. Memory Effect
- Triplet Channel and Magnetoconductivity

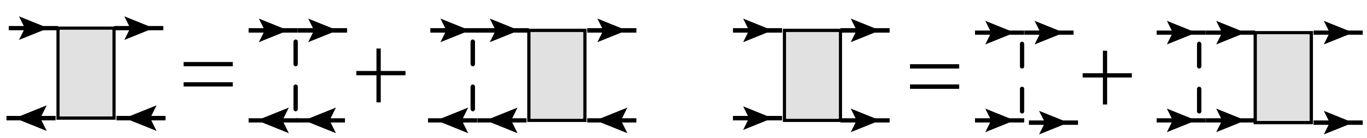
Impurity Scattering. Diffusion. Quantum Interference

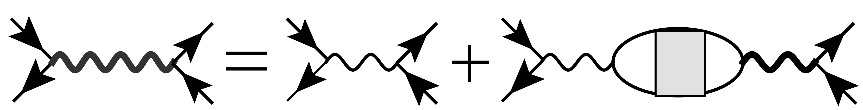
No interference: $\rightarrow$ **classical Drude formula:** $\qquad \sigma_D = \frac{n\,e^2\,\tau}{m}$



Closed Paths: $|A_1 + A_2|^2 = |A_1|^2 + |A_1|^2 + 2\,\mathrm{Re}\,(A_1\,A_2^*) \approx 4|A_1|^2$

Constituents: **Diffusons, Cooperons, RPA-screened interactions**





Weak Localisation and Memory Effect in 2D

Weak Localization **Memory Effect**

Corrections to σ_D due to Quantum Interference (WL)
and Non-Markovian Scattering (ME)

$$\frac{\Delta\sigma_{WL}}{\sigma_D} \propto \frac{\lambda_F}{l} \ln \frac{l}{l_\phi}$$

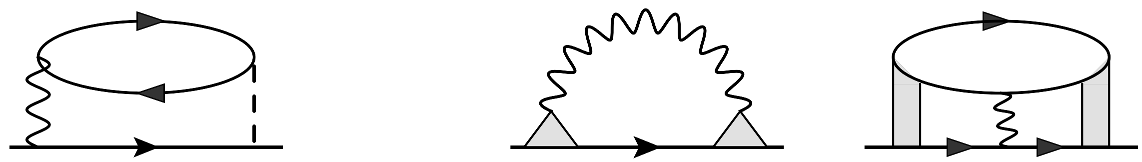
l_ϕ **– Dephasing Length** l **– Mean Free Path**

Diffusive regime: $l_\phi \gg l$ **WL is more important than ME**

Strong Localization occurs when Localization Length $\xi \sim l_\phi$

Magnetic field destroys interference ("Kills the Cooperons") →
Orbital Magnetoconductivity $F(\omega_H \tau_\phi)$ **where** $\omega_H \sim \frac{e\,D\,B}{c}$

Friedel Oscillations. Interaction Corrections in 2D



Gold-Dolgopolov **Aronov-Altshuler**

Corrections to σ_D due to Scattering off Friedel Oscillations near Single (GD) or Multiple (AA) Impurities

$$\frac{\Delta\sigma_{AA}}{\sigma_D} \propto \frac{\lambda_F}{l} \ln T\tau$$

At $T\tau \ll 1$ T-dependence of AA dominates over GD

Zeeman Magnetoconductivity $F(\omega_z/T)$ where $\omega_z \sim g\mu B$

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Beyond Perturbation Theory → NLσM for WL (Efetov, Wegner, ..) and for AA (Finkenstein, ..)

Disordered Luttinger Liquid: Hamiltonian and Parameters

- Single-channel infinite wire: right(left) movers ψ_μ, $\mu = \pm$
- Spinless (spin-polarized, $\sigma = +$) or spinful ($\sigma = \pm$) electrons
- Linear dispersion, $\epsilon_k = k v_F$
- Short-range weak e-e interaction, $\alpha \equiv V(0)/2\pi v_F \ll 1$
- No e-e backscattering; g-ology with g_2 and g_4
- White-noise weak ($\epsilon_F \tau_0 \gg 1$) disorder, $\langle U(x)U(x')\rangle = \delta(x-x')/2\pi\nu_0\tau_0$

$$H = \sum_{k,\mu,\sigma} v_F(\mu k - k_F)\psi^\dagger_{\mu\sigma}(k)\psi_{\mu\sigma}(k) + H_{\text{ee}} + H_{\text{dis}}$$

$$H_{\text{ee}} = \frac{1}{2}\sum_{\mu,\sigma,\sigma'}\int dx\left\{\psi^\dagger_{\mu,\sigma}\psi_{\mu,\sigma}\, g_2\, \psi^\dagger_{-\mu,\sigma'}\psi_{-\mu,\sigma'} + \psi^\dagger_{\mu,\sigma}\psi_{\mu,\sigma}\, g_4\, \psi^\dagger_{\mu,\sigma'}\psi_{\mu,\sigma'}\right\}$$

$$H_{\text{dis}} = \sum_{\sigma}\int dx\left\{\mathcal{U}\ \psi^\dagger_{+,\sigma}\psi_{-,\sigma} + \mathcal{U}^*\ \psi^\dagger_{-,\sigma}\psi_{+,\sigma}\right\}$$

Renorm Group Renormalization of Disorder

Integrate out $\mathrm{T} < \epsilon < \epsilon_{\mathrm{F}}$ $\longrightarrow$ T**-dependent static disorder**

(Giamarchi & Schulz '88, Mattis '74, Luther & Peschel '74)

$$\frac{\tau(T)}{\tau_0} = \left(\frac{T}{\epsilon_F}\right)^{2\alpha} \quad \longrightarrow \quad \sigma^{\mathrm{D}}(T) = \frac{e^2 n_e \tau(T)}{m} \propto T^{2\alpha}$$

Physically : Friedel oscillations (≈GD)

$T\tau > 1$ $(l > l_T)$ **: independent renormalization of weak impurities**

Localization : $l_\phi \sim \xi$ $\longrightarrow$ **1D :** $l_\phi \sim l$

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Quantum corrections: Ballistic Regime $l_\phi \ll l$ **!**

Functional Bosonization

Bosonization → boson (plasmon) modes → good in clean case

Quantum transport + disorder → both electron and plasmon modes

Functional bosonization: Fogelby '76, Lee & Chen '88, Yurkevich'01,

Yurkevich & Lerner '05

Interacting problem:

- Hubbard-Stratonovich decoupling of interaction:

$$\exp\left\{-S_{int}[\psi^\dagger,\psi]\right\} = \int \mathcal{D}\phi \exp\left\{-\frac{1}{2}\phi V_0^{-1}\phi + i\phi\psi^\dagger\psi\right\}$$

- coupling between ψ and ϕ can be gauged out:

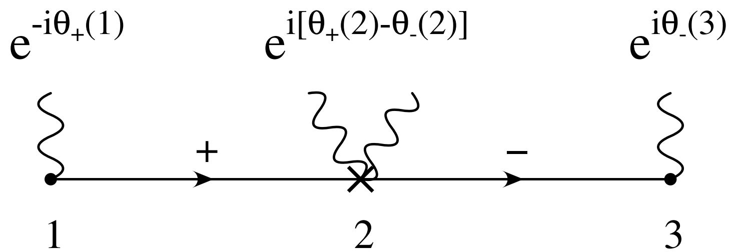
$$\psi_\mu(x,\tau) \to \psi_\mu(x,\tau)\, e^{i\theta_\mu(x,\tau)}\ , \qquad (\,\partial_\tau \mp i v_F \partial_x\,)\,\theta_\mu(x,\tau) = \phi(x,\tau)$$

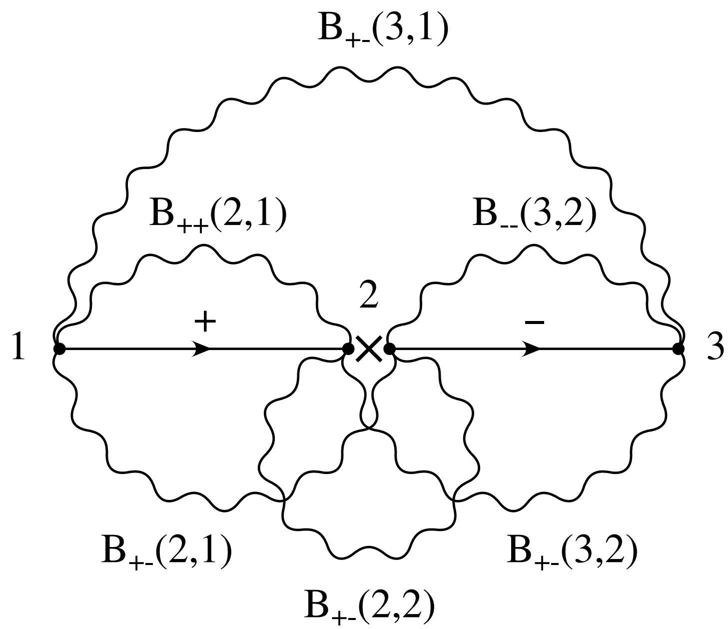
- In clean Luttinger liquid RPA is exact (Larkin-Dzyaloshinskii Theorem) :

$$\langle \varphi(x,\tau)\varphi(0,0)\rangle = V(x,\tau), \quad \langle \theta_\pm\theta_\pm\rangle = T\sum_n \int \frac{dq}{2\pi}\ \frac{V_{\pm\pm}(q,\Omega_n)e^{iqx-i\Omega_n\tau})}{(\pm vq - i\Omega_n)(\pm vq - i\Omega_n)}$$

Diagrammatics for Functional Bosonization

Backscattering points

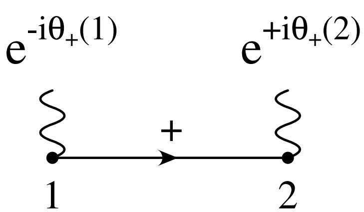


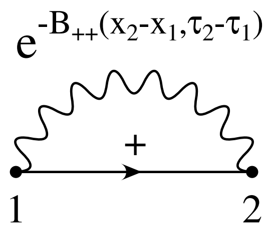


"Exponentiated" Interaction: all points where chirality alters must be interconnected!

Dirty RPA (GMP'05, GMP'07) justified if $l_T \ll l$ and $\alpha \ll 1$

Fermion Green's Function in (x,τ)-representation





$$G_+(x,\tau) = g_+(x,\tau)\,\exp\left[-B_{++}(x,\tau)\right]$$

$$g_+(x,\tau) = -\frac{iT}{2v}\,\frac{1}{\sinh[\pi T(x/v + i\tau)]}$$

$$G_+(x,\tau) = -\frac{i}{2\pi u}\,\frac{\pi T}{\sinh[\pi T(x/u + i\tau)]}$$

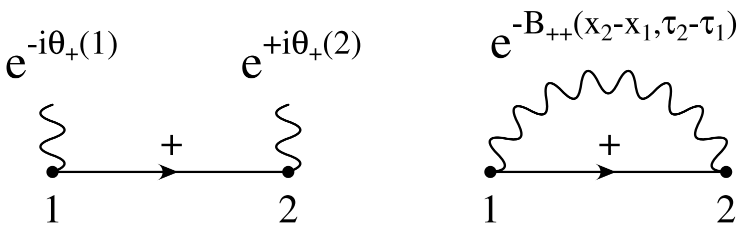
$$\times\left\{\frac{\pi T/\Lambda}{\sinh[\pi T(x/u + i\tau)]}\,\frac{\pi T/\Lambda}{\sinh[\pi T(x/u - i\tau)]}\right\}^{\alpha_b/2}$$

$$\alpha_b = \frac{(u-v)^2}{2uv} \sim \alpha^2 \qquad u^2 = v^2\,(1+2\alpha) \qquad \alpha = \frac{g}{2\pi v}$$

Spinless case, $\alpha \ll 1$: almost pole

Singularity: trajectory $x(t) = u\,t$. Path Integral Method is OK

Fermion Green's Function in (x, τ)-representation



$$G_+(x,\tau) = g_+(x,\tau)\ \exp\left[-B_{++}(x,\tau)\right]$$

$$g_+(x,\tau) = -\frac{iT}{2v}\frac{1}{\sinh[\pi T(x/v + i\tau)]}$$

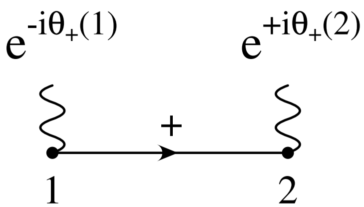
$$G_+(x,\tau) = -\frac{i}{2\pi\sqrt{uv}}\ \left\{\frac{\pi T}{\sinh[\pi T(x/v+i\tau)]}\frac{\pi T}{\sinh[\pi T(x/u+i\tau)]}\right\}^{1/2}$$

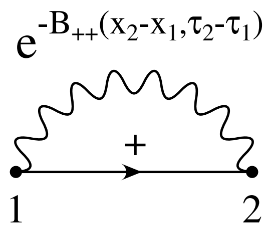
$$\times\left\{\frac{\pi T/\Lambda}{\sinh[\pi T(x/u+i\tau)]}\frac{\pi T/\Lambda}{\sinh[\pi T(x/u-i\tau)]}\right\}^{\alpha_b/4}$$

$$\alpha_b = \frac{(u-v)^2}{2uv} \sim \alpha^2 \qquad u^2 = v^2\,(1+4\alpha) \qquad \alpha = \frac{g}{2\pi v}$$

Spinful case, $\alpha \ll 1$: branch cuts. Spin Charge Separation. Velocities between v and u are accessible. Path Integral is more involved

Fermion Green's Function in (x,τ)-representation





$$G_+(x,\tau) = g_+(x,\tau)\ \exp\left[-B_{++}(x,\tau)\right]$$

$$g_+(x,\tau) = -\frac{iT}{2v}\frac{1}{\sinh[\pi T(x/v + i\tau)]}$$

$$G_+(x,\tau) \simeq -\frac{i}{2\pi\sqrt{uv}}$$

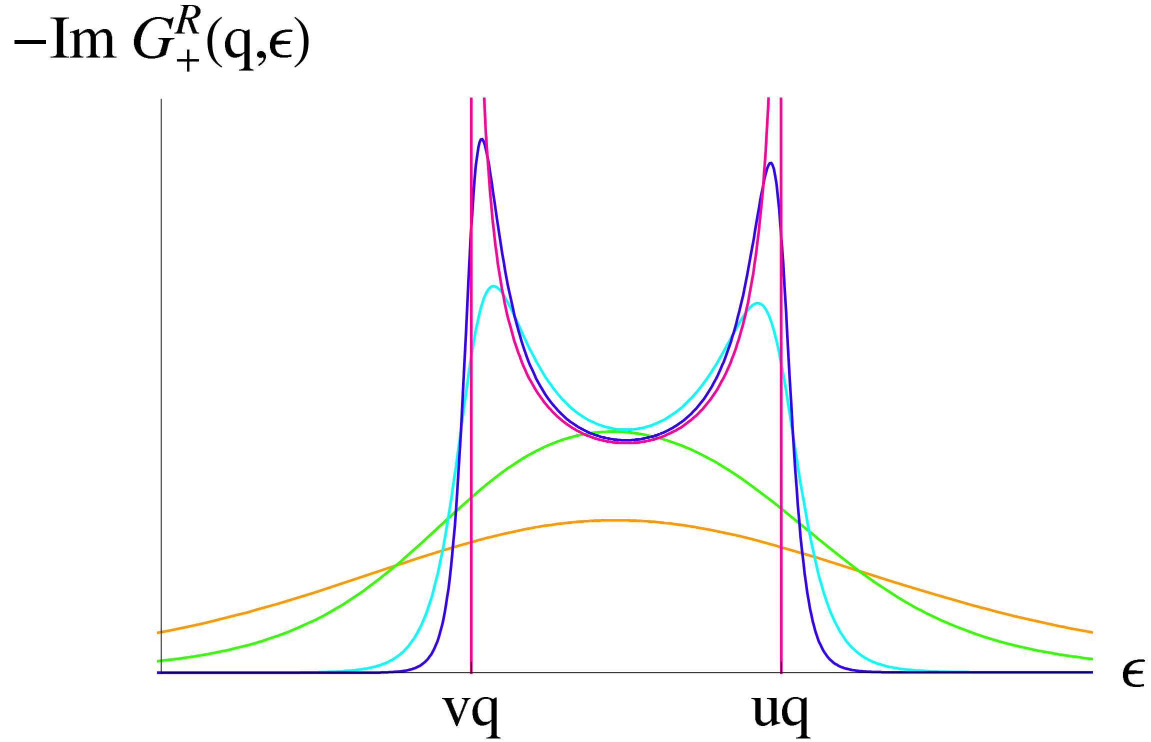
$$\times\left\{\frac{\pi T}{\sinh[\pi T(x/v+i\tau)]}\frac{\pi T}{\sinh[\pi T(x/u+i\tau)]}\right\}^{1/2}$$

$$\alpha_b = \frac{(u-v)^2}{2uv} \sim \alpha^2 \qquad u^2 = v^2\,(1+4\alpha) \qquad \alpha = \frac{g}{2\pi v}$$

Let $\alpha_b \to 0$ $(\sim g_2 \equiv 0)$

Spinful case, $\alpha \ll 1$, $g_2 \equiv 0$: Spin Charge Separation survive!

Fermion Green's Function in (q, ε)-representation

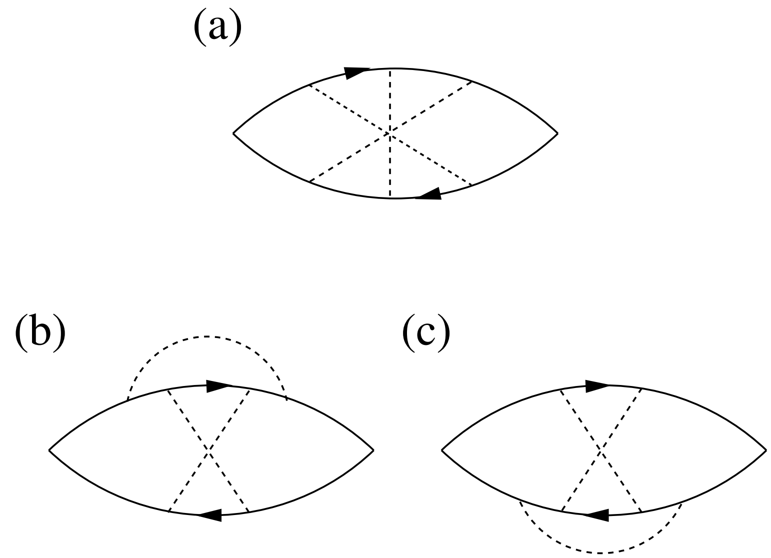


$$G_+^{R(A)}(q,\varepsilon) = \frac{2\, l_{ee}}{\sqrt{uv}}\,\mathcal{P}(\pm\kappa_u)\,\mathcal{P}(\pm\kappa_v)$$

$$\kappa_{\mathrm{u}} = (\varepsilon/\mathrm{u} - \mathrm{q})\, l_{ee}, \quad \kappa_{\mathrm{v}} = (\varepsilon/\mathrm{v} - \mathrm{q})\, l_{ee}, \quad {l_{ee}}^{-1} \approx \pi\alpha \mathrm{T}/\mathrm{v}.$$

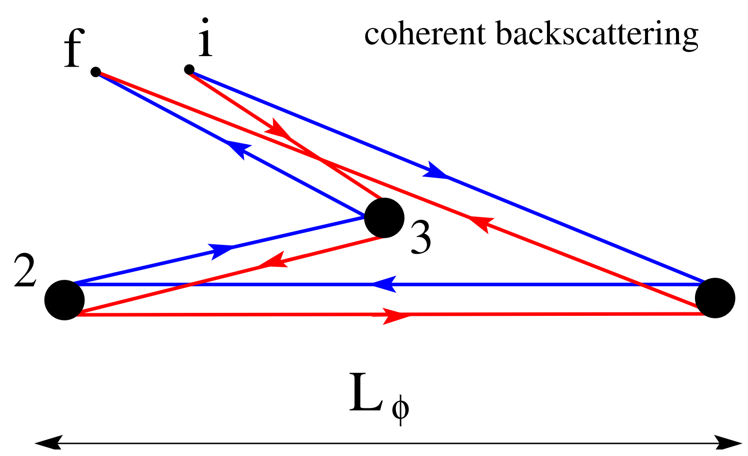
$$\mathcal{P}(z) = \frac{\Gamma\left[(1 - 2\, iz)/4\right]}{\Gamma\left[(3 - 2\, iz)/4\right]}$$

Weak Localization in 1D



fully dressed by e-e interactions

"Minimal Loop" : Three-impurity Cooperon $l_\phi \ll l$



Weak Localization and Memory Effect in 1D

Spinless case:

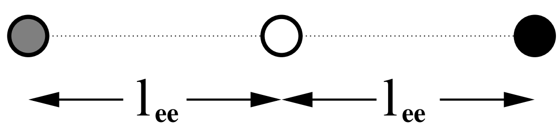
$$\frac{\Delta\sigma_{\mathrm{WL}}}{\sigma_{\mathrm{D}}} \sim -\int_0^l \frac{dx_a}{l}\int_0^l \frac{dx_b}{l}\exp\left(-\frac{x_a x_b}{l l_{ee}}\right) \sim -\frac{l_{ee}}{l}\ln\frac{l}{l_{ee}} \sim -\left(\frac{l_\phi}{l}\right)^2 \ln\frac{l}{l_\phi}$$

Spinful case:

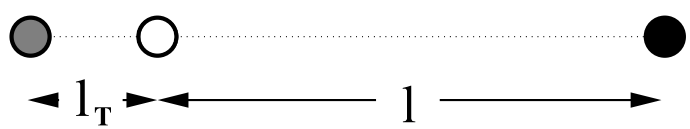
$$\frac{\Delta\sigma_{WL}}{\sigma_D} \sim -\int \frac{\exp(-x_a/l_{ee})\,dx_a}{l}\int \frac{\exp(-x_b/l_{ee})\,dx_b}{l} \sim -\left(\frac{l_{ee}}{l}\right)^2 \sim -\left(\frac{l_\phi}{l}\right)^2$$

$$\frac{\Delta\sigma_{ME}}{\sigma_D} \sim -\left(\int_0^\infty \frac{\exp(-x_c/l)\,dx_c}{l}\right)\left(\int_0^{\alpha x_c} \frac{\exp(-2x_a/l_T)\,dx_a}{l}\right) \sim -\frac{l_T}{l}$$

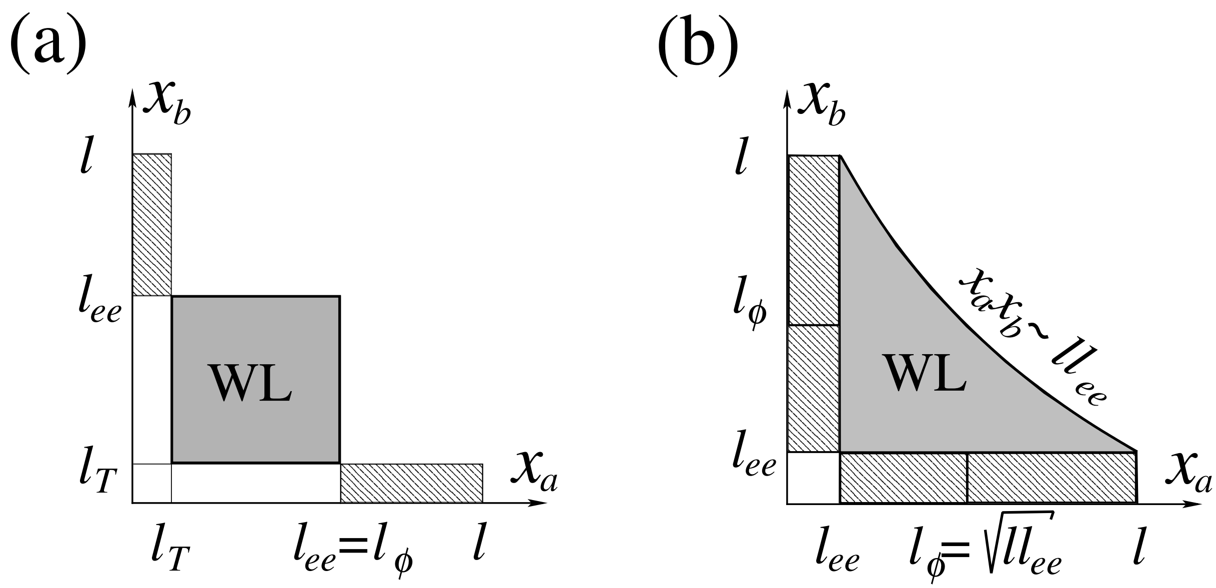
(a)



(b)



Spinful vs Spinless



Spinful **Spinless**

Spinless case: WL always dominates over ME

Spinful case: WL dominates when $\alpha < l_\phi / l < 1$

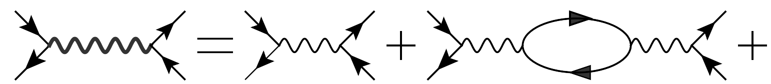
Summary of Singlet Weak Localization and Memory Effect

	e-e scattering length l_{ee}	dephasing length l_ϕ	WL correction $\Delta\sigma_{\rm WL}/\sigma_{\rm D}$	ME correction $\Delta\sigma_{\rm ME}/\sigma_{\rm D}$
spinless	$\frac{v}{\alpha^2 T}$	$\frac{1}{\alpha}\left(\frac{vl}{T}\right)^{1/2}$	$-\left(\frac{l_\phi}{l}\right)^2 \ln\frac{l}{l_\phi} \sim -\frac{v\ln(\alpha^2 Tl/v)}{\alpha^2 Tl}$	$-\frac{l_{ee}}{l} \sim -\frac{v}{\alpha^2 Tl}$
spinful	$\frac{v}{\alpha T}$	$\frac{v}{\alpha T}$	$-\left(\frac{l_\phi}{l}\right)^2 \sim -\left(\frac{v}{\alpha Tl}\right)^2$	$-\frac{l_T}{l} \sim -\frac{v}{Tl}$

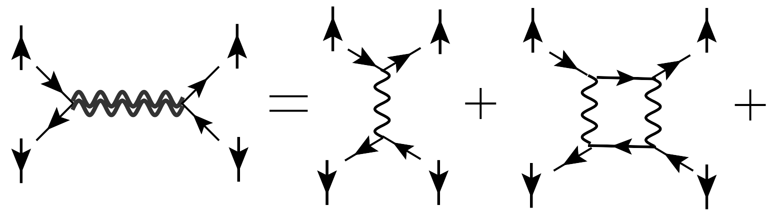
Quantum interference in spinless and spinful Luttinger liquid is parametrically different

Functional Bosonization in Triplet Channel

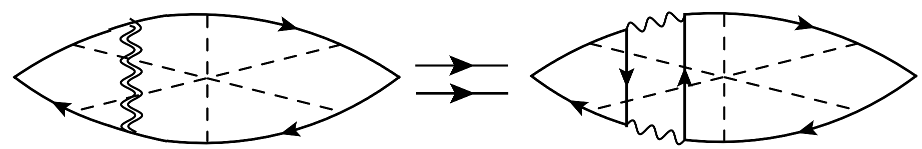
Singlet interaction:



Triplet interaction:



Triplet propagator $\sim \alpha^2$

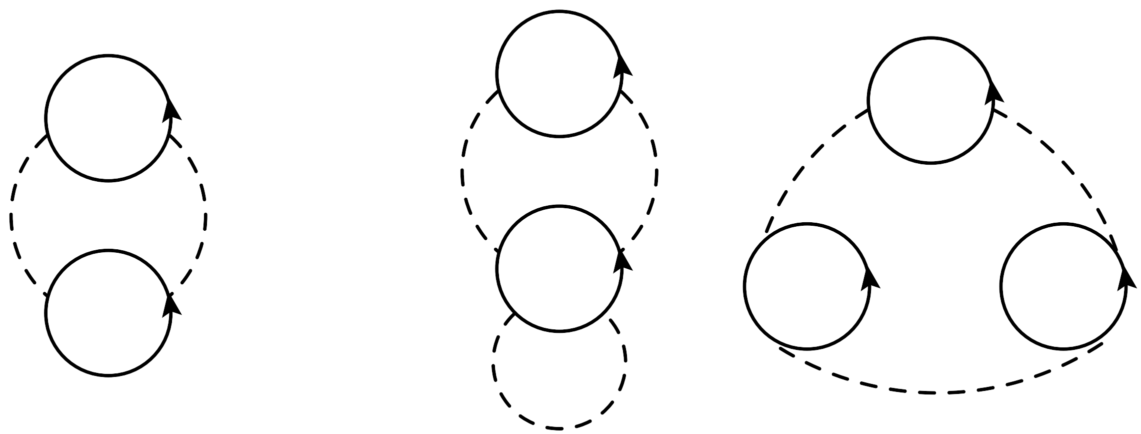


Spin → Matrix Functional Bosonization → T-ordering?

Dzyaloshinskii-Larkin Theorem → Closed Fermionic loops should be interconnected by Impurity lines

To given N-th order in Impurities → Diagrams with $<= N$ closed loops → Interconnect → Average using Singlet Propagators

Triplet WL and Aronov-Altshuler Correction



Aronov-Altshuler | **Triplet Weak Localization**

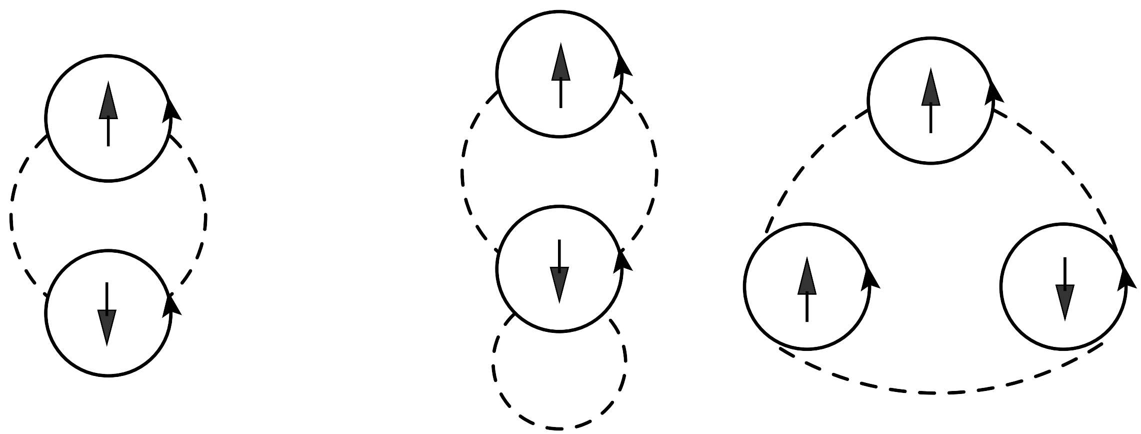
$$\frac{\Delta\sigma_{AA}}{\sigma_D} \propto -\alpha \frac{l_T}{l}$$

?

Smaller than Singlet WL. But: Magnetoconductivity!

Zeeman Magnetoconductivity of a Single Channel Wire

Single Channel → No Orbital Contribution → Zeeman:



Aronov-Altshuler **Triplet Weak Localization**

Diagrams with opposite Spins in different Fermion cycles contribute

$$\delta_{AA}(B) \propto \frac{\Delta\sigma_{AA}}{\sigma_D} F\left(\omega_z/T\right) \qquad ?$$

$F(x \to 0) \propto x^2$ **and** $F(x \to \infty) \to const$

Summary

- **The notions of classical Mesoscopics are applicable to Strongly Correlated Electron Systems (Non Fermi Liquids):**
  - **Dephasing Time**
  - **Weak Localization**
  - **Strong (Anderson) Localization**
  - **Mesoscopic Fluctuations,** *etc.*
- **Magnetoconductivity in a Single Channel Quantum Wire**
- **Formalism for Mesoscopics of Luttinger Liquid**
  - **Functional Bosonization**