Regenerative approach for retrial queuing system

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Introduction

We present a new retrial systems which can be used to describe:

- ALOHA type multiple access protocols [3];
- short TCP transfers [2].

Estimation of blocking probability in M/G/1/1-type retrial queuing system .

Description of the model

M/G/1/1-type retrial system (Σ):

- Poisson input with rate λ ;
- general service time with rate μ $(ES = 1/\mu)$;
- arrivals who find the server busy **join the infinite capacity orbit**, and then return to the system after exponentially distributed retrial time with rate μ_0.

The total input stream to the server consists of two (generally, dependent) streams:

- Poisson λ-input of primary customers;
- the input of retrial customers with a rate $\tilde{\mu}_0 \le \mu_0$, $(\tilde{\mu}_0 = \mu_0$ when the orbit is not empty, and , $\tilde{\mu}_0 = 0$ otherwise).

Auxiliary system $(\hat{\Sigma})$

- The same Poisson λ-input;
- The same service times ;
- An independent Poisson input with rate μ_0;
- arriving customer who finds the server busy **leaves the system forever and do not affect the future state**.

The server in system Σ is less loaded than than server in $\hat{\Sigma}$ $(\tilde{\mu}_0 \leq \mu_0)$.

We expect: the stability of the orbit in auxiliary implies stability of the orbit in original system [1].

Stability condition

P_{loss}- stationary loss probability in system $\hat{\Sigma}$(always exists). It has been proved in [1] that the following condition:

$$(\lambda + \mu_0)P_{loss} < \mu_0. \qquad (1)$$

is sufficient (and often necessary) for stability of the orbit in system Σ.

Erlang models

Stability condition (1) for the c-server Erlang model (that is M/G/c/c loss system):

$$\frac{((\lambda+\mu_0)/\mu)^c}{c!}\left[\sum_{n=0}^{c}\frac{((\lambda+\mu_0)/\mu)^n}{n!}\right] < \frac{\mu_0}{\lambda+\mu_0}, \quad (2)$$

reducing for M/G/1/1 system (c=1):

$$\frac{\lambda+\mu_0}{\mu} < \frac{\mu_0}{\lambda+\mu_0}. \quad (3)$$

Regenerative analysis (notation)

- $\{t_n\}$ - the arrival instants of λ-customers in both systems;
- $N(t)$ - the orbit size at instant t;
- $\nu(t)$ - the state of server (0 or 1) at instant t;
- $X = X(t) = \{N(t) + \nu(t), t \geq 0\}$;
- $N(t_n^-) = N_n,\ \nu(t_n^-) = \nu_n,\ X(t_n^-) = X_n := N_n + \nu_n, n \geq 1;$
- $R(t)$ - the total number of rejected customers in interval [0,t];
- $A(t)$ - the total number of calls (λ and μ_0-customers) in interval [0,t];
- R - the number of orbit customers during regeneration cycle
- A - the total number of arrivals during regeneration cycle.

System regenerates at the instants when the λ-customers find the server empty in case of empty orbit. Regenerations of the discrete-time basic process $\{X_n\}$:

$$\beta_{n+1} = \inf_k (k > \beta_{n+1} : X_k = 0),\ n \geq 0,\ \beta_0 = 0. \qquad (4)$$

Stability case

$\{R(t), t \geq 0\}$ is positive recurrent [1] with regenerations $\{\beta_n\}$ so, w.p.1:

$$\lim_{t\to\infty} \frac{R(t)}{A(t)} = \frac{ER}{EA}. \qquad (5)$$

I_n - indicator of rejection of n-th custumer. If β is aperiodic, then:

$$I_n \Rightarrow I. \qquad (6)$$

By uniform integrability of indicators holds:

$$P(I_n = 1) \to P_{orb} := EI, \quad n \to \infty. \qquad (7)$$

Also by the regenerative theory:

$$\hat{P}_{orb}(n) := \frac{\sum_{k=1}^{n} I_k}{n} \to \frac{ER}{EA} = P_{orb}, \quad n \to \infty. \qquad (8)$$

Both limit ratios give the same expression for stationary probability P_{orb} .
Original system is less loaded, and one expect than in the stability region X is positive recurrent (that is $E\beta < \infty$) and thus:

$$\hat{P}_{orb}(n) \to P_{orb} \leq P_{loss}, \quad n \to \infty. \qquad (9)$$

Instability case

In the instability region the limit $\tilde{\mu}_0 \to \mu_0$ is expected, and we also expect that the estimator $\hat{P}_{orb}(n)$ must approach P_{loss} .

The process X is **not positive recurrent regenerative.**

Quasi-regenerations :

$$\alpha_{n+1} = \inf_k (k > \alpha_{n+1} : \nu_k = 0), \; n \geq 0, \; \alpha_0 = 0. \qquad (10)$$

One expect: more the system is unstable
the less difference $|P_{loss} - \hat{P}_{orb}(n)|$ must be as $n \to \infty$.

After a finite (w.p.1) time t_0 , the orbit will never be empty, and for $t \geq t_0$ systems Σ, $\hat{\Sigma}$ couple. Quasi-regenerations become **classical regenerations for the isolated process** $\{\nu_n\}$ **.**

As a result, the estimator $\hat{P}_{orb}(n)$ will converge to P_{loss} w.p.1 as $n \to \infty$.

Simulation results

Estimation of $\hat{P}_{orb}(n)$ both in stability and instability regions using regenerative/quasi-regenerative simulation for M/G/1/1 system with and G=M or G=Pareto.

Denote: $$\delta = P_{loss} - \hat{P}_{orb}(n) = \frac{\lambda + \mu_0}{\lambda + \mu_0 + \mu} - \hat{P}_{orb}(n).$$

Denote by Γ the difference between two sides of stability condition (3):

$$\Gamma = \frac{\mu_0}{\lambda + \mu_0} - \frac{\lambda + \mu_0}{\lambda + \mu_0 + \mu}.$$

- $\Gamma > 0$ in the stability region;
- $\Gamma < 0$ in the instability region;
- $\Gamma = 0$ at the boundary.

We expect:
In stability region:
- If $\Gamma \uparrow$, then the difference $\delta \uparrow$;
- If $\Gamma \downarrow 0$, then $\delta \downarrow 0$.

In instability region: $\delta \approx 0$.

Simulations of M/M/1/1 systems are presented in Table 1, where n is the number of regenerations (4) in the stability region , or the number of quasi-regenerations (9) in the instability region (λ =1).

Table 1: Simulation of M/M/1/1

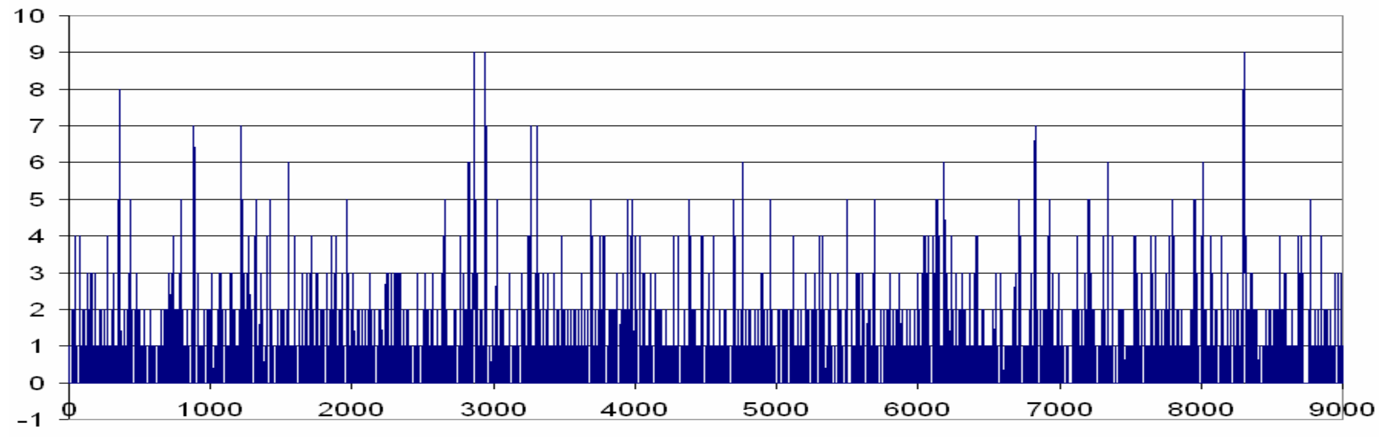
μ_0	μ	n	P_{loss}	$\hat{P}_{orb}(n)$	δ	Γ
2	4	11222	0,4286	0,2965	0,1321	0,2381
4	3	8051	0,625	0,4799	0,1451	0,175
0,6	3	1753	0,3478	0,3365	0,0113	0,0272
6	1,3	572	0,8434	0,8331	0,0103	0,0137
2	1,4	6417	0,6818	0,6791	0,0027	-0,0151
4	1,1	3529	0,8197	0,8235	-0,0038	-0,0197
0,5	2,3	12092	0,3947	0,3953	-0,0006	-0,0614
0,1	2	12947	0,3548	0,3526	0,0022	-0,2639
0,1	0,1	1679	0,9167	0,916	0,0007	-0,825

Similar results are obtained for M/Pareto/1/1 systems with service time $P(S \geq x) = (x / x_0)^{-2,5}, \; x \geq x_0$ in Table 2.

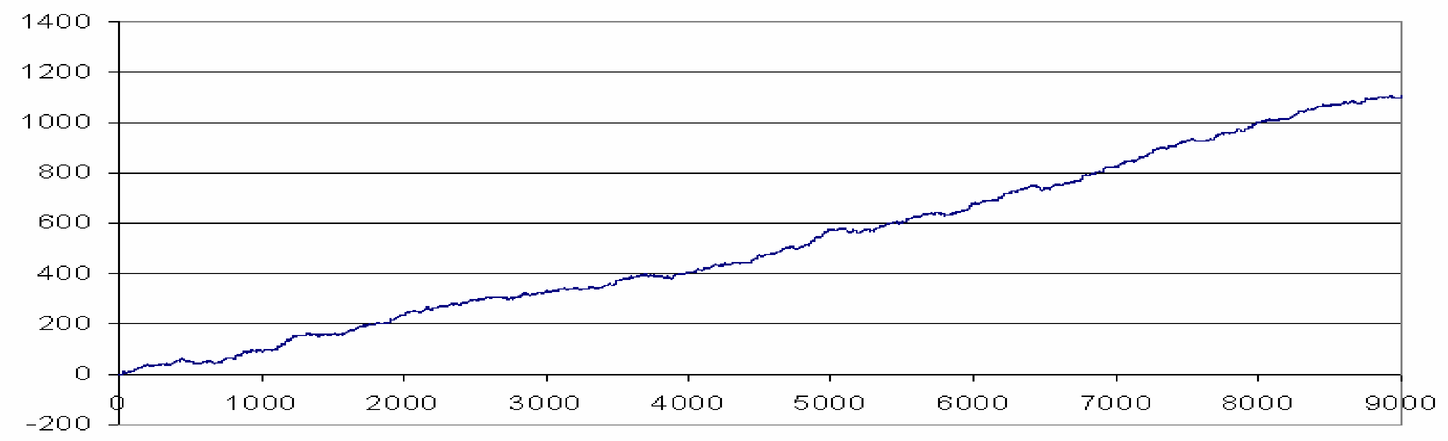
Table 2: Simulation of M/Pareto/1/1

μ_0	x_0	n	P_{loss}	$\hat{\mathsf{P}}_{orb}(n)$	δ	Γ
2,7	0,1	14335	0,9569	0,2057	0,7512	0,3483
1	0,1	12715	0,9231	0,1808	0,7423	0,2500
2	0,2	7809	0,9000	0,4033	0,4967	0,1667
1,65	0,37	339	0,8112	0,5986	0,2126	0,0023
2	0,5	5652	0,7826	0,7159	0,0667	-0,0476
0,1	0,4	11537	0,6226	0,4224	0,2002	-0,3322

Dynamics of orbit



Stable dynamics of orbit in M/M/1/1, $\mu_0 = 4,\ \mu = 3.$



Instable of orbit in M/M/1/1, $\mu_0 = 0{,}5,\ \mu = 2{,}3.$

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Thank you!