Gaussian queues in communication Networks

Oleg V. Lukashenko

Karelian Research Centre of Russian Academy of Science, Russia

Convergence to FBM

$$\lim_{T\to\infty}\lim_{M\to\infty}\left\{\frac{\left(A(tT)-TM\dfrac{\mu_{on}}{\mu_{on}+\mu_{off}}t\right)}{T^{1/2}L^{1/2}(T)M^{1/2}},\ t\geq 0\right\} =_d \left\{cB_H(t),\ t\geq 0\right\}$$

which means that

$$A(tT)\approx TM\frac{\mu_{on}}{\mu_{on}+\mu_{off}}t+\sqrt{TL(T)M}\,cB_H(t)$$

$\left\{B_H(t),\ t\geq 0\right\}$ - Fractional Brownian motion

Queue with Gaussian input

Input traffic:

$$A(t) = mt + \sqrt{am}\, B_H(t)$$

Service rate:

Constant service rate c

Stationary overflow probability:

$$P(Q > b) = P\left(\sup_{t \in [0,\infty)} \left(A(t) - ct \right) \geq b \right)$$

$$\pi_n \equiv P\left(Q^n > b\right) = P\left(\sup_{t \in [0,\infty)} \left(\sum_{i=1}^{n} A_i(t) - nct \right) \geq nb \right)$$

For Brownian input:

$$P(Q > b) = \exp\left(-2 \frac{c-m}{m} b \right)$$

Asymptotics for Gaussian queues

- Large-buffer asymptotics
  - Logarithmic

$$\log P(Q>b) \sim g_1(b) \qquad \text{as} \quad b \to \infty$$

  - Exact

$$P(Q>b) \sim g_2(b) \qquad \text{as} \quad b \to \infty$$

- Many-sources asymptotics
  - Logarithmic

$$\log \pi_n \sim g_3(n) \qquad \text{as} \quad n \to \infty$$

  - Exact

$$\pi_n \sim g_4(n) \qquad \text{as} \quad n \to \infty$$

Large-buffer asymptotics

- [Duffield & O'Connell,1995]

Weybullian distribution of queue:

$$P(Q > b) \simeq e^{-\theta \cdot b^{2-2H}} \quad \text{as} \quad b \to \infty$$

$$\theta = \frac{1}{2}\left(\frac{c-m}{H}\right)^{2H} (1-H)^{-2(1-H)}$$

- [Hüsler & Piterbarg, 1999,2003]

- [Narayan, 1997]

Many-sources asymptotics

- [Botvich & Duffield,1995]

$$-\lim_{n\to\infty}\frac{1}{n}\log\pi_n = \inf_{t\in\mathbb{R}_+}\frac{\upsilon^2(t)}{2},\quad \upsilon(t)=\frac{b+ct}{t^H},\qquad t^* = \frac{H}{1-H}\frac{b}{c}$$

- [Likhanov & Mazumdar, 1999]

$$\pi_n \equiv P\left(\sup_{t\in\mathbb{N}}\left(\sum_{i=1}^{n} A_i(t) - nct\right)\geq nb\right) \sim \Psi\left(\upsilon\left(t^*\right)\sqrt{n}\right),\ n\to\infty$$

- [Debicki & Mandjes, 2002]

$$\pi_n \sim \frac{\beta_{2H}}{\pi}\frac{1}{\sqrt{H(1-H)}}\left(\frac{\upsilon\left(t^*\right)}{\sqrt{2}}\right)^{1/H-1} n^{1/2H-1/2}\Psi\left(\upsilon\left(t^*\right)\sqrt{n}\right),\ n\to\infty$$

$$\Psi(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-y^2/2}dy$$

System with finite buffer

Input traffic:

$$A(t) = m\,t + \sqrt{a\,m}\,B_H(t)$$

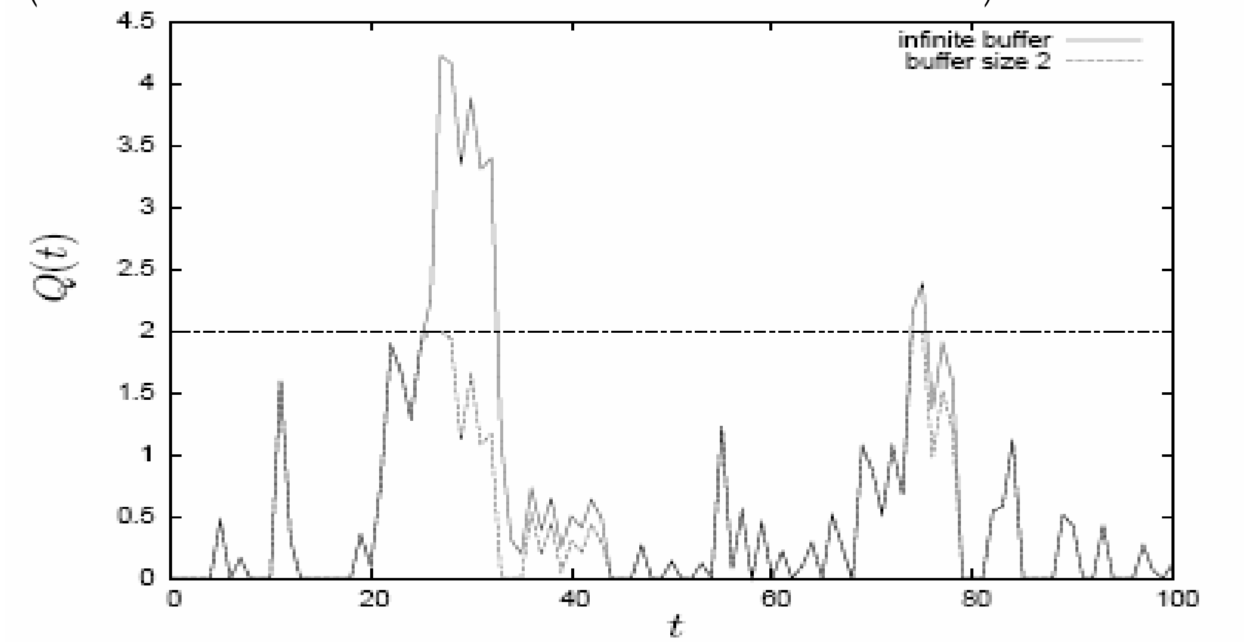
Service rate:

Constant service rate C

Queue length (in discrete scale):

$$Q(t) = \left(Q(t-1) - C + m + \sqrt{am}\left(B(t) - B(t-1)\right)\right)^+,\ \mathrm{t} = 0,1,\ldots$$

$$Q_b(t) = \min\left(\left(Q_b(t-1) - C + m + \sqrt{am}\left(B(t) - B(t-1)\right)\right)^+, b\right),\ \mathrm{t} = 0,1,\ldots$$



Overflow and loss probability

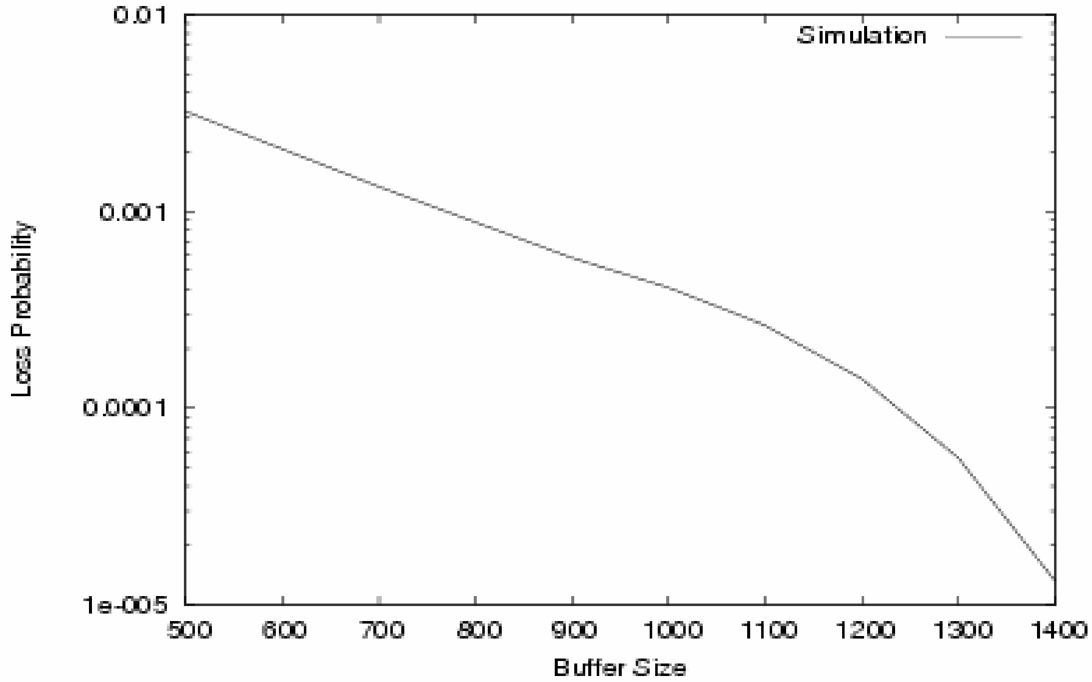
$$P_L(b,T) = \frac{\sum_{k=1}^{T} (Q_b(k-1) + m + \sqrt{am} B_H^*(k) - C - b)^+}{A(T)}$$

$$P_L(b) = \lim_{T \to \infty} P_L(b,T) = \frac{E(Q_b(n-1) + m + \sqrt{am} B_H^*(n) - C - b)^+}{m}$$

$$P(Q > b) = \lim_{N \to \infty} \frac{1}{N} \sum_{k=1}^{N} I(Q_k > b)$$

Estimation of loss probability

$$P_L(b) \approx \frac{EL}{mT}$$



$$EL = \frac{1}{N}\sum_{n=1}^{N} L_n$$

[Kim & Shroff, 2001]

$$P_L(b) \approx \frac{P_L(0)}{P(Q>0)} P(Q>b)$$

Relative Error

$$RE(\hat{p}_l) \triangleq \frac{\sqrt{Var(\hat{p}_l)}}{\mathrm{E}[\hat{p}_l]}$$

For MC-estimator

$$RE(\hat{p}_l) \sim \frac{1}{\sqrt{p_l N}} \qquad \textit{as} \qquad p_l \to 0$$

For $p_l \to 0,$ the number N of samples must be sufficiently large

Regenerative approach

$$\beta_{k+1} = \min\{t > \beta_k : Q(t-1) = 0, Q(t) > 0, k \geq 1\},\ \beta_0 = 0$$

- $Q(t)$ - workload of a system at time t
- EL - mean lost work per cycle
- EA - mean workload arrived per cycle
- $L_n(t)$ - lost work in [0; t], n – buffer size

$$\lim_{t \to \infty} \frac{L_n(t)}{A(t)} = \frac{EL}{EA} \equiv P_l$$

Confidence estimation

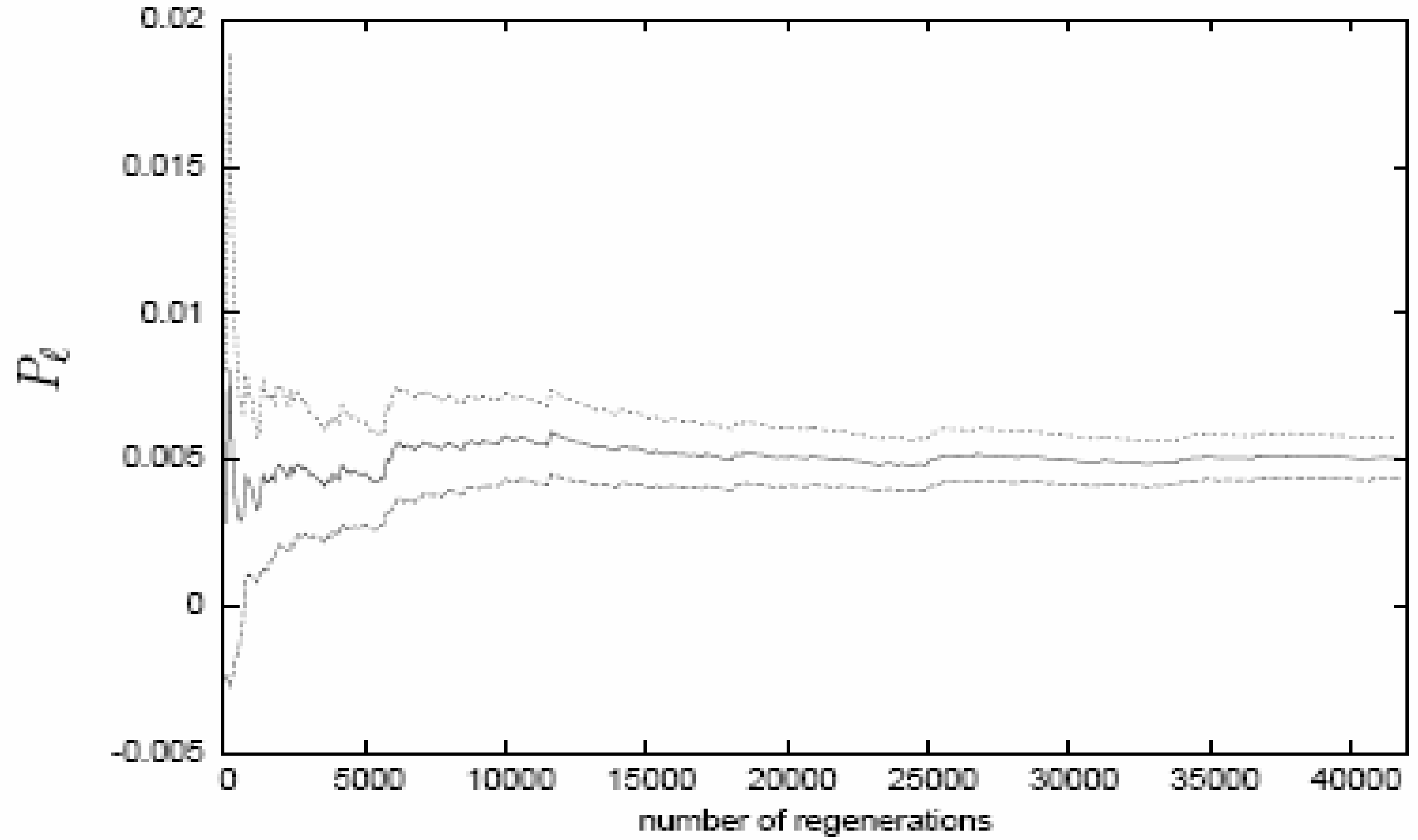
$$\hat{P}_L = \frac{\hat{L}}{\hat{A}} \qquad \hat{L} = \frac{1}{N}\sum_{n=1}^{N} L_n \qquad \hat{A} = \frac{1}{N}\sum_{n=1}^{N} A_n$$

an asymptotic 100(1 − δ)% confidence interval is given by

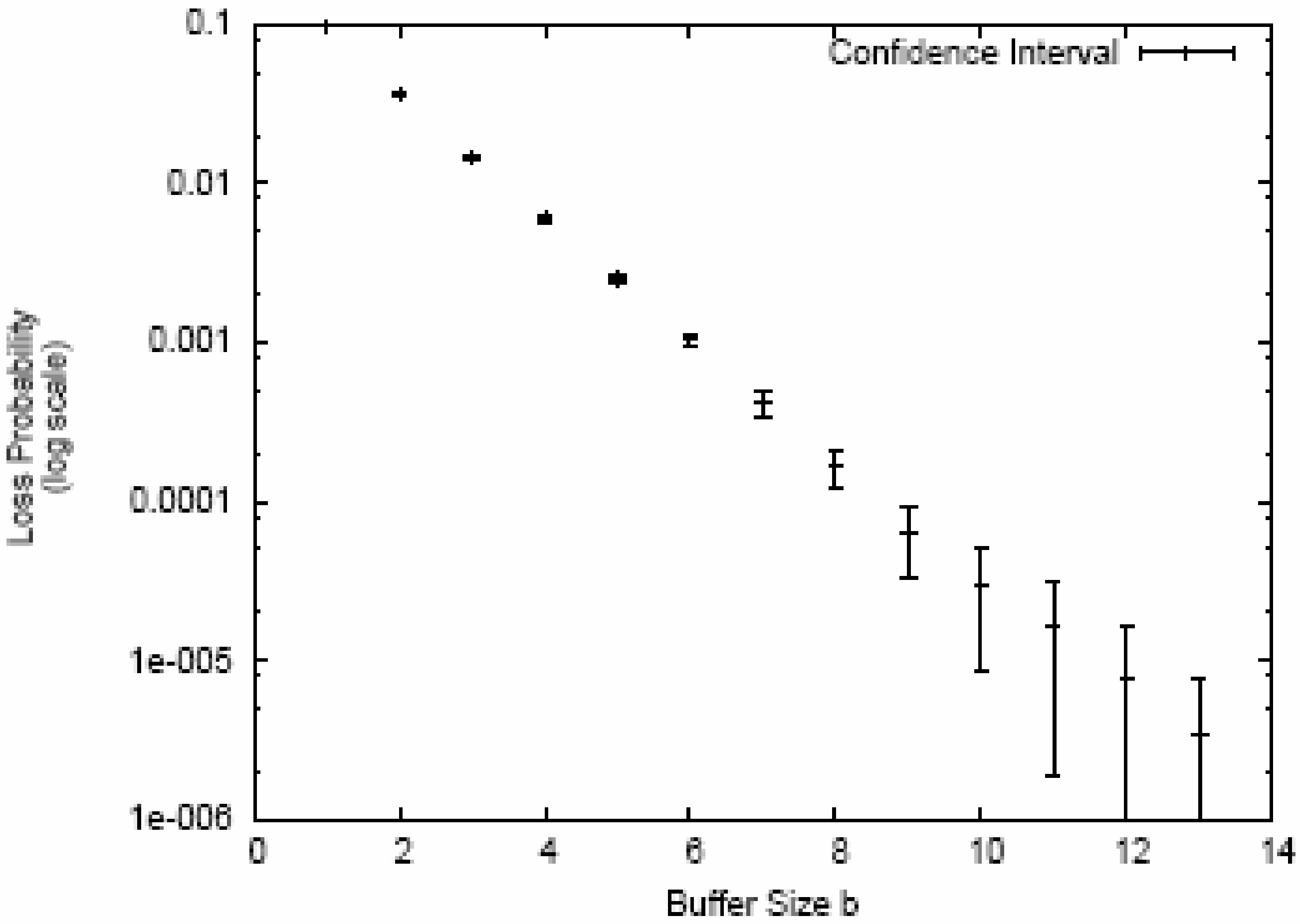
$$\hat{L} \pm z_{1-\delta/2}\hat{\eta} / \sqrt{N}$$

$$\hat{\eta}^2 = \frac{\frac{1}{N-1}\sum_{n=1}^{N}\left(L_n - \hat{P}_L A_n\right)^2}{\left(\frac{1}{N}\sum_{n=1}^{N} A_n\right)^2}$$

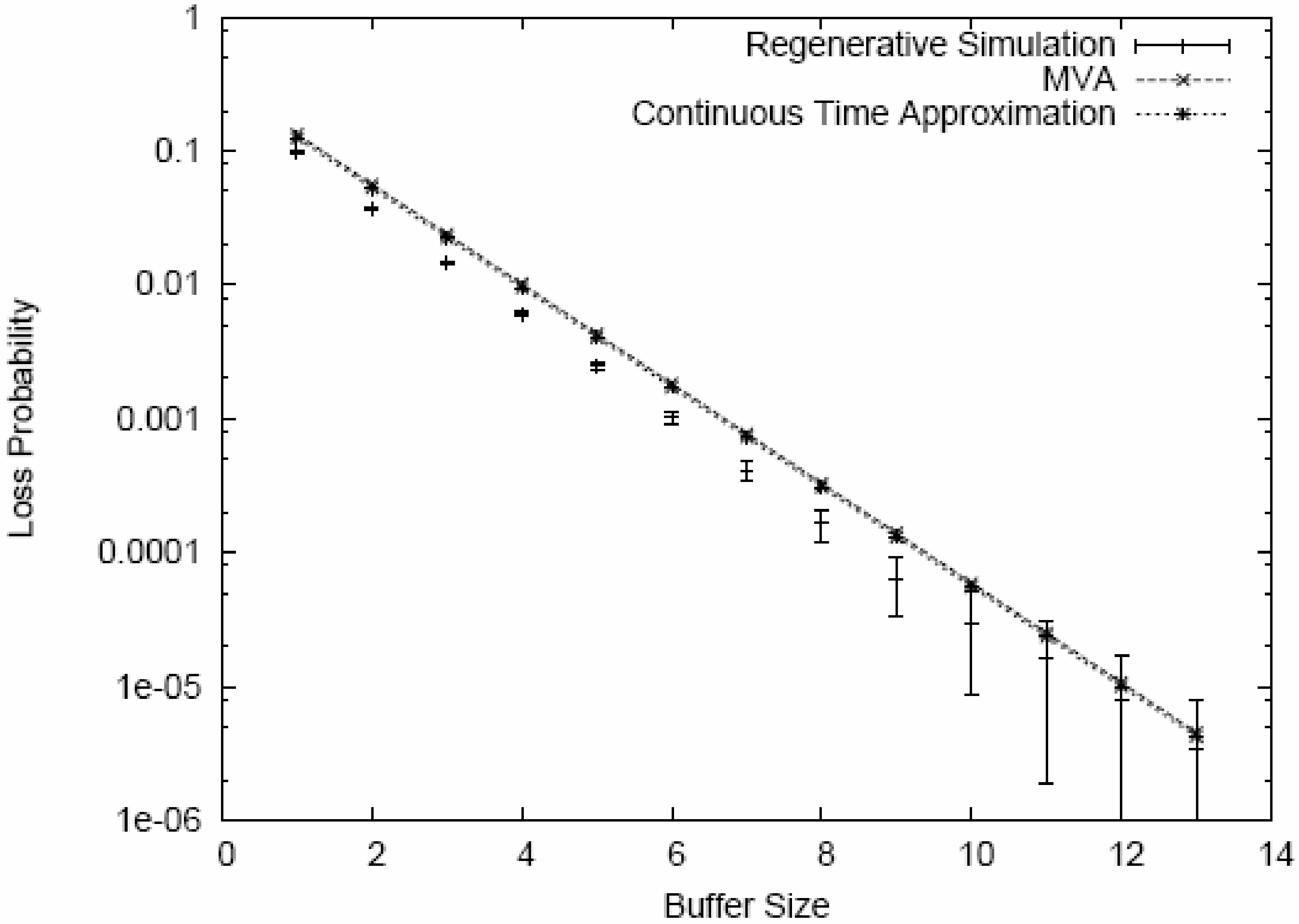
Confidence interval for P_l in BI/D/1/n



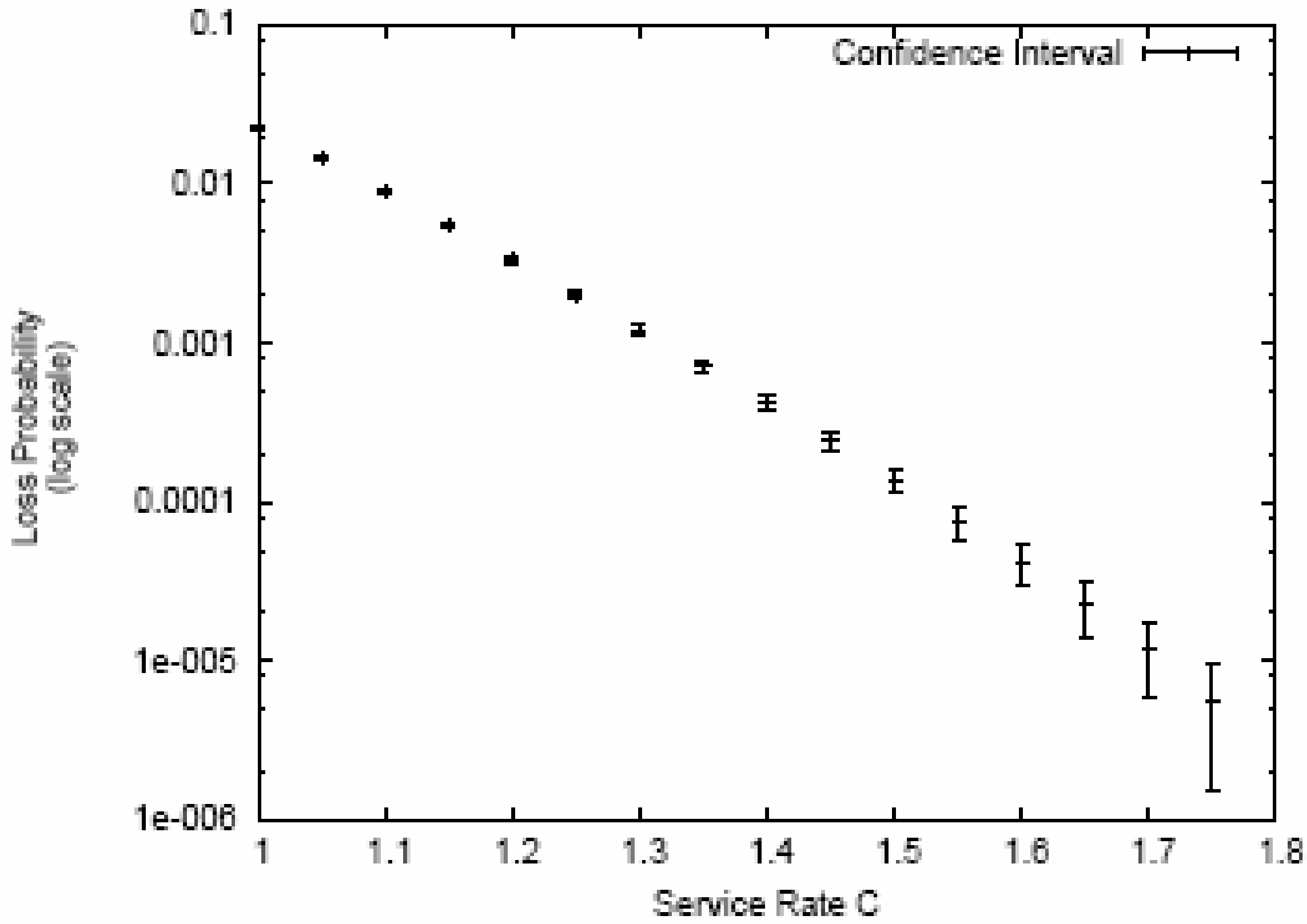
Dependence on buffer size



Estimate of P_l in BI/D/1/b



Dependence on service rate



References

- Botvich D., Duffield N. Large deviations the shape of the loss curve and economies of scale in large multiplexers. Queueing Systems, 1995, 20, 293-320.

- *Debicki K., Mandjes M.* Exact overflow asymtotics for queues with many Gaussian inputs. Report PNA-R0209 March 31, 2002.

- *Duffield N., O'Connell N. Large deviations and overflow probabilities for the general single server queue, with applications. Proceedings of the Cambridge Philosophical Society, 1995, 118, 363--374.*

- *Hüsler J., Piterbarg V. I. Extremes of a certain class of Gaussian processes. Stochastic Processes and their Applications, 1999, 83, 257—271.*

- *Kim H. S., Shroff N. B. Loss Probability Calculations and Asymptotic Analysis for Finite Buffer Multiplexers. IEEE/ACM Transactions on Networking, 2001, 9(6), 755--768.*

References

- Likhanov N., Mazumdar R. Cell loss asymptotics in buffers fed with a large number of independent stationary sources. Journal of Applied Probability, 1999, 36, 86-96.

- *Narayan O. Exact asymptotic queue length distribution for fractional Brownian traffic. Advances in Performance Analysis, 1998, 1, 39–63.*

- *Norros I.* A storage model with self-similar input, Queueing Systems, vol. 16, pp. 387-396, 1994.

- *Taqqu M. S., Willinger W., Sherman R. Proof of a fundamental result in self-similar traffic modeling. Computer communication review. 1997, 27: 5–23.*

- *Goricheva R. S., Lukashenko O. V., Morozov E. V., Pagano M.* Regenerative analysis of a finite buffer fluid queue, Proceedings of ICUMT 2010.

Thank you.